Title: Tensor network description of 3D Quantum Gravity and Diffeomorphism Symmetry

Speakers: Bianca Dittrich

Collection: Tensor Networks: from Simulations to Holography III

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Abstract: In contrast to the 4D case, there are well understood theories of quantum gravity for the 3D case. Indeed, 3D general relativity constitutes a topological field theory (of BF or equivalently Chern-Simons type) and can be quantized as such. The resulting quantum theory of gravity offers many interesting lessons for the 4D case.

In this talk I will discuss the quantum theory which results from quantizing 3D gravity as a topological field theory. This will also allow a derivation of a holographic boundary theory, together with a geometric interpretation of the boundary observables.

The resulting structures can be interpreted in terms of tensor networks, which provide states of the boundary theory.
I will explain how a choice of network structure and bond dimensions constitutes a complete gauge fixing of the diffeomorphism symmetry in the gravitational bulk system. The theory provides a consistent set of rules for changing the gauge fixing and with it the tensor network structure. This provides an example of how diffeomorphism symmetry can be realized in a tensor network based framework.

I will close with some remarks on the 4D case and the challenges we face there.


From quantum gravity to
tensor networks
... and back again
Bianca Dittrich
(Perimeter Institute)
Tensor networks: from simulation to holography III
Perimeter Institute, Nov 2020

# Overview

A. Tensor networks from (2+1)D quantum gravity.

Quantum geometry.
Diffeomorphism symmetry.
Boundary theories.

[BD, Donnelly, Riello, …. to appear]

Papers on quasi-local (2+1)D holography
with short introduction to (2+1)D q-gravity:

[BD, Goeller,Livine, Riello 1710.04202 [hep-th] ]

[BD, Goeller,Livine, Riello 1710.04237 [hep-th] ]

[Goeller,Livine, Riello 1912.01968 [hep-th] ]

Short summary paper:

[BD, Goeller,Livine, Riello 1803.02759 [hep-th] ]

B. Tensor networks for quantum gravity.

(Decorated) Tensor network algorithms.
(3+1) Quantum gravity and tensor networks.

Decorated TNW algorithm for 3D gauge theories:

[BD, Mizera, Steinhaus 1409.2407 [gr-qc]]

[Cunningham, BD, Steinhaus 2002.10472 [hep-th]]

4D (numerically) effective spin foam models:

[Asante, BD, Haggard 2004.07013 [gr-qc]]
+ to appear very soon

References: cannot mention all relevant references here, ask me if you are interested in specific topics.

# 3D gravity as a topological phase

3D general relativity is a topological field theory.
No local degrees of freedom.
Solutions are locally flat (for $\Lambda = 0$) or homogeneously curved (for $\Lambda \neq 0$).

Can construct reasonable theories of quantum gravity.
Which you may already know in some form.

- $\Lambda = 0$, Riemannian signature

| •SU(2) - BF - theory (Palatini action)<br>•Integral over flat SU(2) connections<br>•(lattice) gauge theory at zero coupling | $\simeq$ | •Ponzano-Regge Model 1968<br>•made more precise within loop quantum gravity | $\simeq$ | •Chern-Simons for $SU(2) \times \mathbb{R}^3$<br>•'Doubled' or 'non-chiral' Chern-Simons |
|---|---|---|---|---|

- Why is $\Lambda = 0$ interesting for holography?

| (asymptotic) AdS boundary<br>$\downarrow$<br>conformal symmetry group | (asymptotically) flat boundary<br>$\downarrow$<br>(inf. dim.) BMS-symmetry group | But also interesting holographic duals at finite boundary! |
|---|---|---|
| [Brown-Hennaux 1986] | [Bondi-(van der Burg)-Metzner-Sachs 1962]<br>[3D: Barnich et al] | |

# 3D gravity as a topological phase

- $\Lambda > 0$ (deSitter), Riemannian signature: All gauge symmetries are compact.

| | | | | |
|---|---|---|---|---|
| •BF $+\Lambda$BB action<br>•introduces a quantum deformation<br>•(lattice) gauge theory with $SU(2)_k$ | $\simeq$ | •$SU(2)_k$ Turaev-Viro Model<br>•$SU(2)_k$ string net model<br>[Levin-Wen] | $\simeq$ | •Chern-Simons for $SU(2)_k \times \overline{SU(2)_k}$<br>• Reshetikhin-Turaev model |

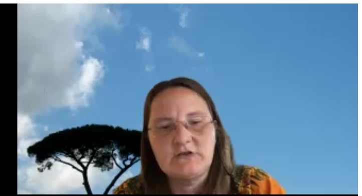
- $\Lambda < 0$ (AdS), Riemannian signature

| | | |
|---|---|---|
| •BF $+\Lambda$BB action<br>•introduces a quantum deformation<br>•(lattice) gauge theory with $SU(2)_q$ | •$SU(2)_q$ Turaev-Viro Model with $q$ real<br>[Dupuis-Girelli, et al] | •Chern-Simons combinatorical quantization<br>[Alekseev-Grosse-Schomerus]<br>[Schroers, Meusburger, …] |

- similarly for semi-Riemannian signature (non-compact rotation gauge groups)

- work here with (best understood) flat case
- quantum group deformations introduce a (mild) non-locality

- for flat case: gauge (shift) symmetry is non-compact (equivalent to diffeomorphisms)
  gauge fixing very well understood [Louapre-Freidel, Bonzom-Smerlak]



# Tensor networks, quantum geometry and diffeomorphisms

3D (and 4D): Quantum gravity states can be interpreted as tensor network states and come with a precise quantum geometric interpretation.

3D: The choice of tensor network connectivity, bond dimensions and (SU(2)-invariant) tensors can be understood as a complete gauge fixing of bulk diffeomorphisms.

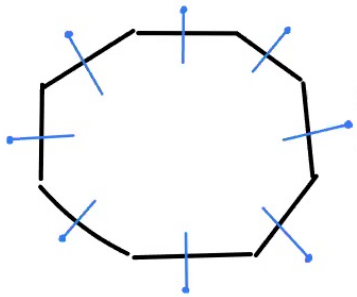
# Tensor network states from quantum gravity

1-dim circular boundary, discretized with edges of equal lengths

$\downarrow$

Boundary Hilbert space: spin chain Hilbert space with spin rep $j_\partial$

$$\mathscr{H}_\partial = \otimes_{\text{edges}} V_{j_\partial}$$



$m = -j_\partial, -j_\partial + 1, \ldots, j_\partial$

bond dim $= 2j_\partial + 1$

# Tensor network states from quantum gravity

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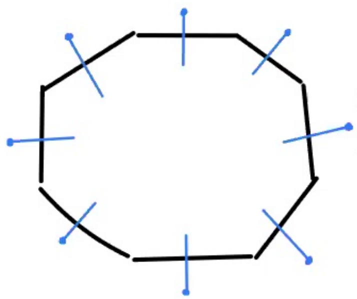
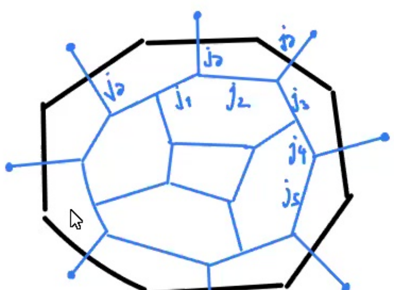


$m = -j_\partial, -j_\partial + 1, \ldots, j_\partial$

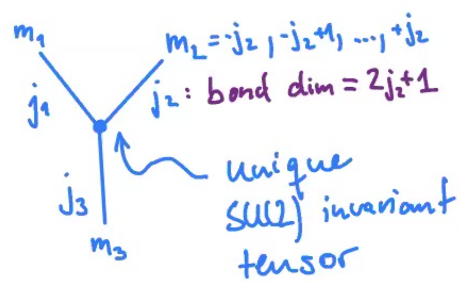
bond dim $= 2j_\partial + 1$

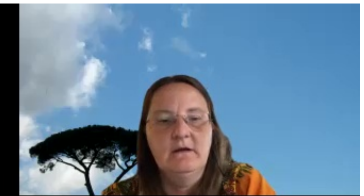
quantum gravity (bulk) state $\longrightarrow$ tensor network state



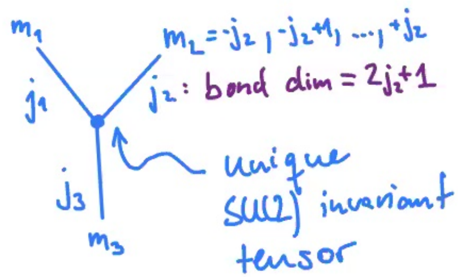

(spin network basis state)

spin network evaluation
(on flat connection)

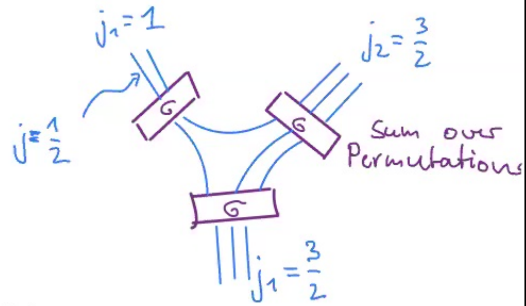


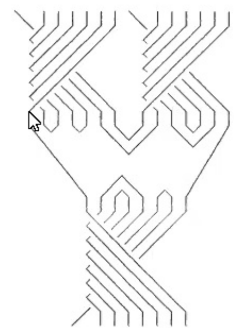



# Tensor network states from quantum gravity




spin network evaluation can be defined by



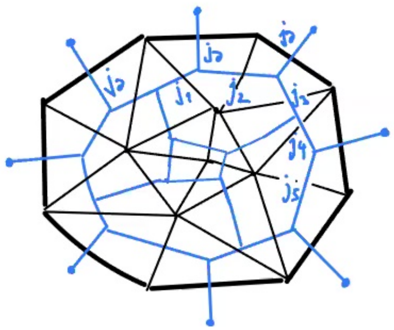



Penrose: spin networks 1971
(diagrammatic calculus)

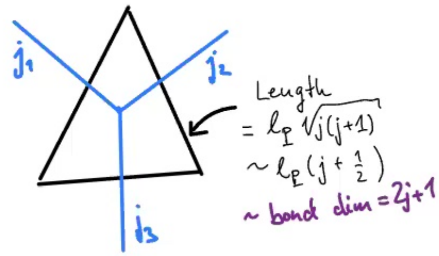
[J. Eisert: spaghetti picture]

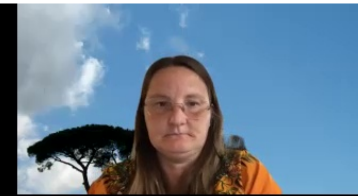
# Quantum geometry for tensor networks

Spin network states come with a precise (rigorous) quantum geometric interpretation.




Spin network defines quantum geometry
for 2D triangulation dual to the underlying network.
Bond dimension of network link ~ length of edge.



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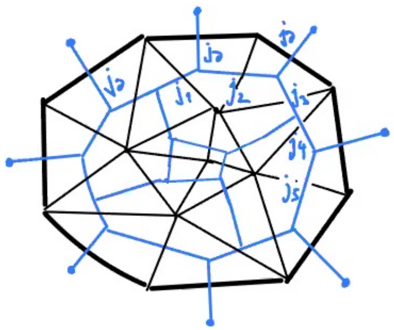


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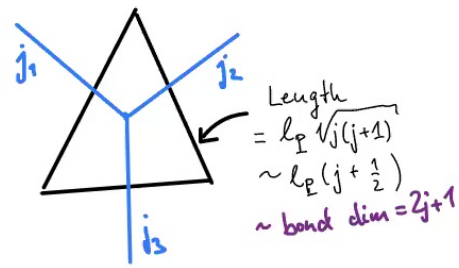
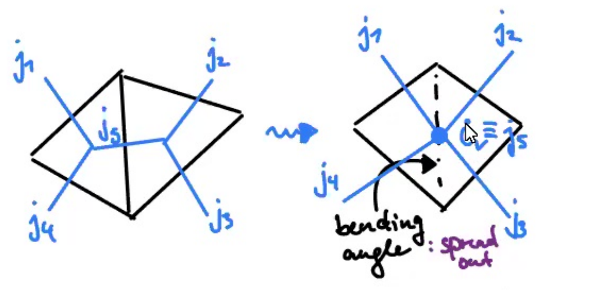
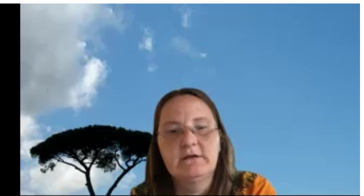



Can contract three-valent vertices
to higher-valent ones:




four-valent invariant tensor
describing quadrangle



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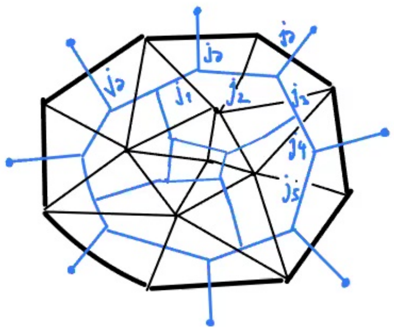


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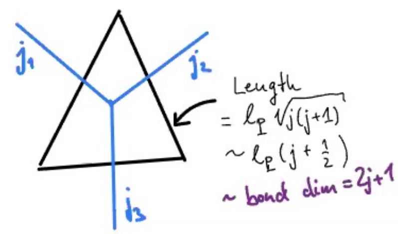


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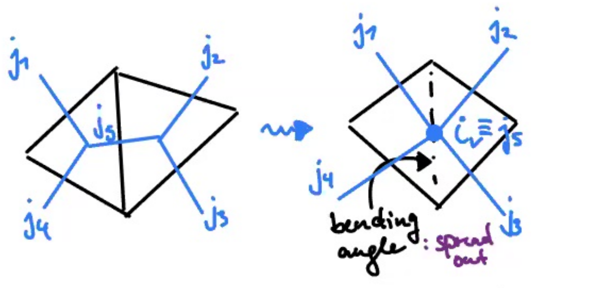
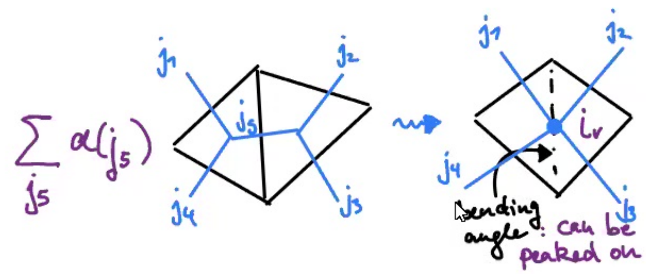
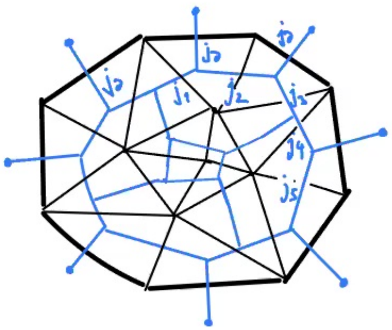



four-valent invariant tensor
describing quadrangle

More general invariant tensors through superposition
-define 2D geometry embedded into 3D space

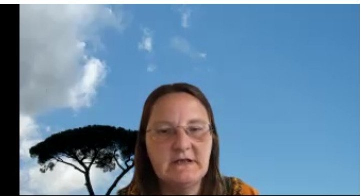


# Quantum geometry for tensor networks



Bond dimension of network link ~ length of edge
Entanglement between triangles ~ length of edge

→ Ryu-Takayanagi conjecture



Holographic tensor network conjecture (inspired by the AdS case):

Tensor network states which come with the interpretation as a regular (flat) quantum geometry are good candidates for low energy states for the boundary theory.

[BD, Donnelly, Riello, to appear]

Questions:

1. This fixes a particular discretization. What about diffeomorphism symmetry?

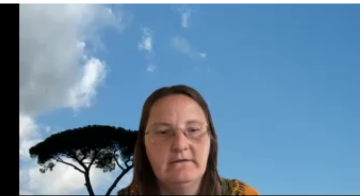
   Choice of bulk tensor network = complete gauge fixing of diffeos.

2. What are the boundary theories?

   Finite boundary: Embarrassment of riches.
   Asymptotic boundary: signs of universality and BMS.
   [BD, Goeller, Livine, Riello, 2017-19]

# Diffeomorphisms in the discrete

[Reviews: BD 2008, BD 2012]



1. A change in triangulation.

2. Change of edge lengths through change in relative position of vertices. (Shift symmetry in BF)

We will see that both notions are connected.
Conjecture: triangulation invariance equivalent to diffeomorphism invariance.

[Bahr, BD, Steinhaus 2011 ]

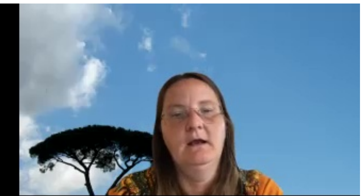
Equivalence relation for bulk states ⟵ Change in 2D triangulation ⟶ Interpretation as time evolution = diffeomorphism

- same as in string nets
- resulting from identities for the invariant tensors
- do not change boundary state

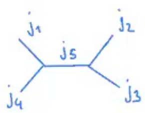
- Pachner moves generate all possible changes

- gluing of tetrahedra to 2D hypersurface leads to an evolving hypersurface

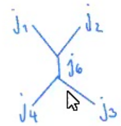
# Diffeomorphisms in the discrete



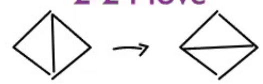
| Equivalence relation for bulk states | ← | Change in 2D triangulation | → | Interpretation as time evolution = diffeomorphism |
|---|---|---|---|---|

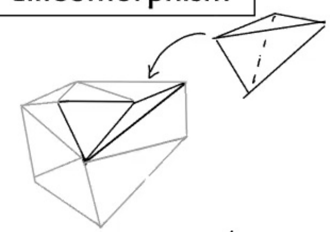



$$= \sum_{j_6} F^{145}_{326}$$

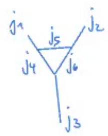



2-2 Move

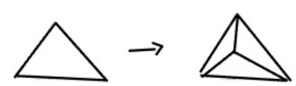


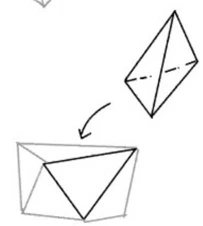


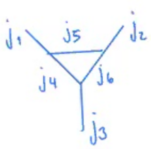

$$= \sum_{j_4,j_5,j_6} F^{123}_{645}$$



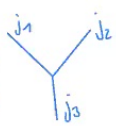

1-3 Move



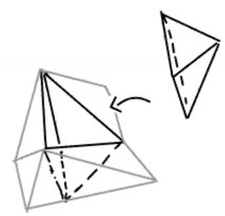





$$= \frac{v_4 v_6}{v_3} F^{123}_{645}$$



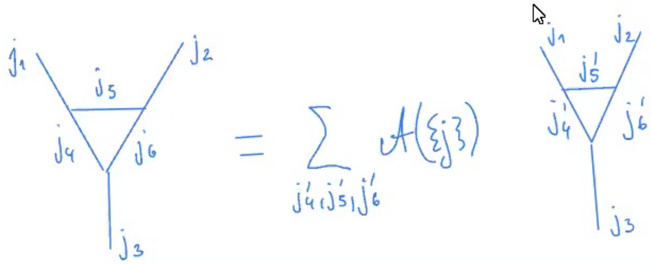

3-1 Move

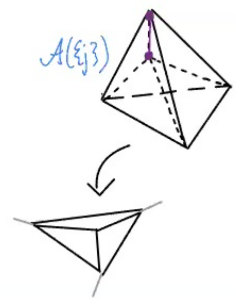


Do not change the boundary state.

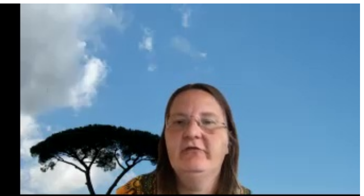
# Diffeomorphisms in the discrete

There is acertain combinations of these moves, which preserve the triangulation, e.g. (1-3) + (2-2) + (3-1).




Inner vertex position is a gauge degree of freedom = vertex translation.



With sum over gauge orbit (inner vertex position): Plaquette stabilizer for string nets.
Here a non-proper projector.

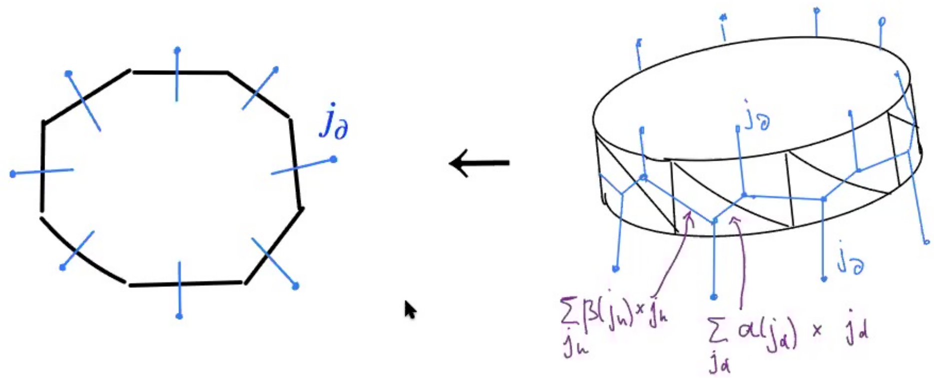
Gives one version of the Wheeler-DeWitt equation = Hamiltonian for quantum gravity = generator of diffeos.

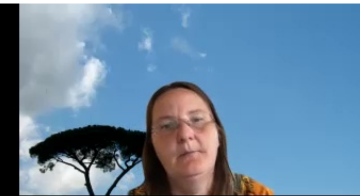
Can represent one and the same tensor network state via different (spin labelled) networks.
But there might be a "simplest" representation: computationally preferred gauge.

# Boundary theory: time evolution

Time evolution of 1D boundary $\longrightarrow$ 2D quantum geometry

We already know how to describe a 2D quantum geometry: with spin networks.



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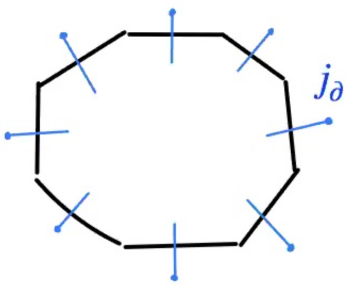
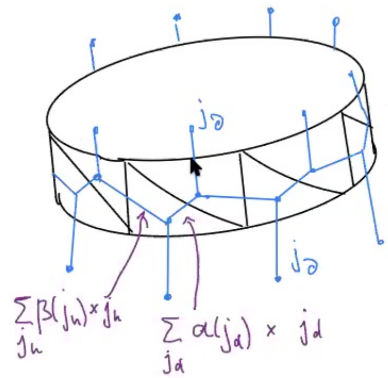
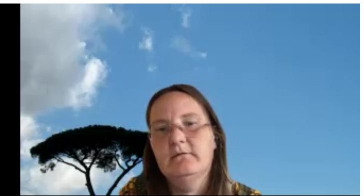



$\longleftarrow$




Quantum geometry of annulus defines transfer matrix for boundary theory.

Different boundary theories for different boundary quantum geometries.



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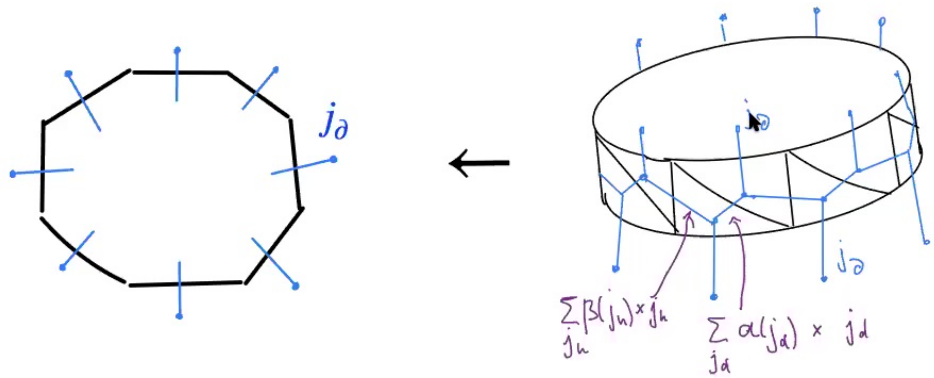



Quantum geometry of annulus defines transfer matrix for boundary theory.

Different boundary theories for different boundary quantum geometries.

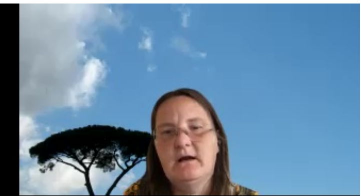
Example: For $j_\partial = 1/2$ we can map boundary theories to (subspace of couplings) for 6-vertex model.

[BD, Goeller, Livine, Riello 2017]

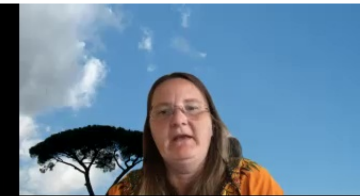
Which quantum geometries lead to integrable theories?

Rich subject: Interpretation of spin network evaluation as statistical partition functions.

[L. Kauffman: Knots and Physics; BD, Hnybida 2013; Bonzom, Livine 2013]



# Dual theory for asymptotic boundary: BMS symmetry



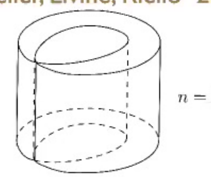
Can define quantum geometry describing a large (through large boundary spin), semiclassical, geometry.

(Semi-classical) Evaluation of torus partition function reproduces BMS character + trans-Planckian corrections.

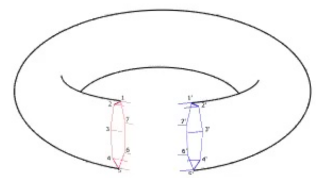
[BD, Goeller, Livine, Riello 2017]

Consistent with perturbative (1-loop) Evaluation in continuum (metric) theory.

[Barnich et al 2015]



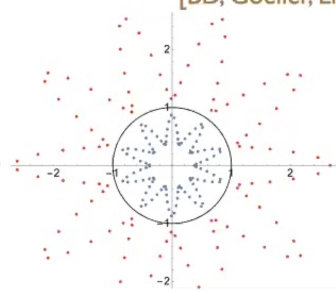

The BMS character has a certain singularity structure in the torus moduli parameter.
It actually almost completely defined by this singularity structure.



Emergence of universality:
These singularities are reproduced also for simpler boundary geometries (fixed, small boundary spin),
in the limit of infinitely many plaquettes in the angular direction: another way to reach infinitely large boundary.

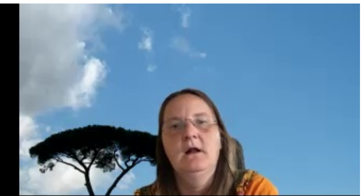
[BD, Goeller, Livine, Riello 2017]

Singularities can be regularized by using superpositions in edge lengths.




[Goeller, Livine, Riello 2019]

# Geometric interpretation of boundary field



A form of bulk reconstruction.

Can express the same boundary theory with different (field) variables:

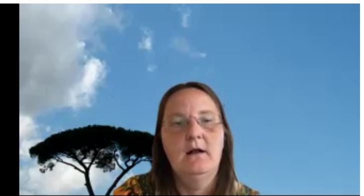
| | Boundary theory | Boundary field | |
|---|---|---|---|
| deep quantum boundary geometries (small j) → | Spin chains or Vertex models | Magnetic Indices | [BD, Goeller, Livine, Riello 17] |
| Semi-classical boundary geometries (large j) → | SU(2) Sigma model | SU(2) group Variables | [BD, Goeller, Livine, Riello 17] |
| General boundary triangulation with fixed spin. → | RSOS like models | SU(2) rep label: geodesic lengths | [Riello 18] |

Describe embedding of surface into flat 3D space

Geodesic lengths variables lead to Liouville like boundary theory in the continuum (limit). Generalizes to dS, AdS and 4D flat sector.

[Bonzom, BD 15]
[Asante, BD, Hopfmueller 19]
[Asante, BD, Haggard 18]

# Tensor network algorithms as a tool for quantum gravity



(3+1)D gravity: not a topological theory.

Various lattice models.

Spin foams originate from a topological quantum field theory (BF).
Introduce non-trivial dynamics via (lots of) defect excitations.

Open question: Phase diagram.

# Tensor networks as a tool for quantum gravity

The 3D models for quantum gravity are "quantum mechanical" partition functions: with an $i$.

Long history of failed attempts to construct quantum gravity via a Wick rotation (as classical partition function).
One reason: the action is unbounded from below.

Similarly (spin foam) partition functions are with an $i$.
Derive from a topological field theory, but are not topological.
A very open question: continuum limit or phase diagram.

[BD 2014 (review), Steinhaus 2020 (review)]

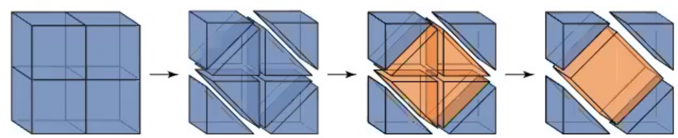
Cannot use Monte-Carlo method for simulations.

Use tensor networks. But is very challenging for (3+1)D and the rather complicated quantum gravity models.

Started with lower dimensional 'analogue models'.

[BD, Eckert, Martin-Benito 2011, …,
BD, Schnetter, Seth, Steinhaus 2016]

For (2+1)D gauge systems: decorated TNW algorithms.
Accounts for the somewhat non-local observable structure in gauge theories.

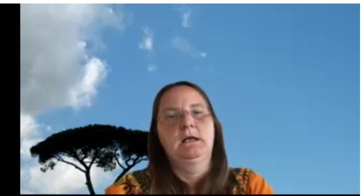


[BD, Mizera, Steinhaus 2014,
Delcamp, BD 2016]

Use spin network basis.

# 3D Decorated TNW algorithm in fusion basis

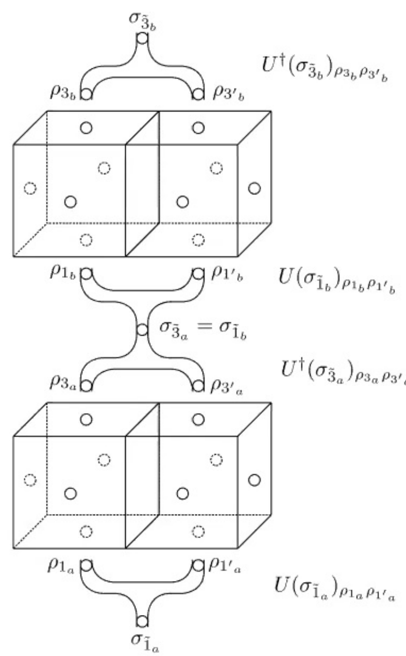
[Cunningham, BD , Steinhaus 2020]



- For lattice gauge theory (with Abelian, non-Abelian or quantum deformed gauge groups)
- For anyon systems
- To study phase transitions between different discrete values of the cosmological constant. [BD 2018]

- Fusion basis has coarse graining already in-built.

- Different from spin network basis:
  structure preserved under coarse graining.



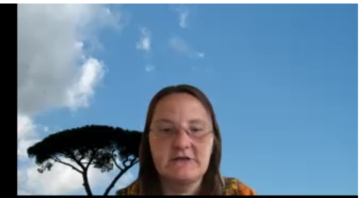
# Quantum gravity and quantum geometry in 4D

Tensor networks: entanglement as a measure for geometry.

Suggest areas, instead of lengths, as fundamental variables.
But these need to be constrained in order to define a suitable quantum geometry.

Areas do emerge as fundamental variables in (3+1)D loop quantum gravity and spin foams.
These models can be derived from a constrained SO(4) BF action. [Perez: Living Reviews 2012]
Leads again to spin network states, where spins give now discrete area eigenvalues.
The 3D spin network states can be interpreted as tensor network states (for 2D surfaces).

# Quantum gravity and quantum geometry in 4D



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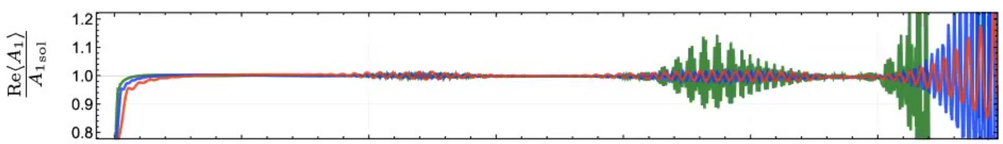
Constraints change dynamics from topological to non-topological.
But there is an interesting anomaly in the constraint algebra, related to discrete area eigenvalues.
Does this endanger the semi-classical dynamics? Debated for over 10 years.

Introduced a numerically accessible 'effective' spin foam model:
First explicit numerical calculation on (small) lattice: Can recover semi-classical regime.

[Asante, BD, Haggard
to appear in PRL 2020,
+ to appear very soon]




Plan: Use tensor networks to investigate refinement/ continuum limit.
Hope: The continuum limit restores diffeomorphism symmetry. [BD 2014 (review)]

# Summary

- Precise quantum geometry associated to spin network states (in (2+1)D and (3+1)D)
  — which can be taken to define tensor network states (in 1D and 2D).
- Matches well with the framework of reconstructing geometry from entanglement.
- Quantum gravity framework defines dynamics of boundary theory, depends on boundary geometry.
  For asymptotic (large radius) boundary: regain boundary symmetry (BMS) group. [BD, Goeller, Livine, Riello, 2017-19]
- Many issues to be understood: e.g. relation between boundary geometry and integrability.

For (3+1)D quantum gravity:

- Tensor network algorithms as a tool (so far the only one) to investigate the phase diagram of truly quantum models. [BD, Mizera, Steinhaus 2014, Delcamp, BD 2016]
- Need algorithms for (3+1)D for (quite involved) gauge theories. [Cunningham, BD , Steinhaus 2020]