Title: A measurement-based variational quantum eigensolver

Speakers: Luca Dellantonio

Collection: Tensor Networks: from Simulations to Holography III

Date: November 19, 2020 - 1:00 PM

URL: http://pirsa.org/20110031

Abstract: In this talk I will speak about the meeting point of two models that have raised interest in the community in the last years. From one side, we looked at measurement-based quantum computing (MBQC), which is an alternative to circuit-based quantum computing. Instead of modifying a state via gates, MBQC achieves the same result by measuring auxiliary qubits in a graph. From the other side, we considered variational quantum eigensolvers (VQEs), that are one of the most successful tools for exploiting quantum computers in the NISQ era. In our work, we present two measurement-based VQE schemes. The first introduces a new approach for constructing variational families. The second provides a translation of circuit-based to measurement-based schemes. Both schemes offer problem-specific advantages in terms of the required resources and coherence times. We apply them, respectively, to the Schwinger model and the two-dimensional Z(2) lattice gauge theory.

# A measurement-based variational quantum eigensolver

11/19/2020
Tensor Networks: from Simulations to Holography III

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Presented by: Luca Dellantonio

In collaboration with: **Ryan R. Ferguson**, Abdulrahim Al Balushi, Karl Jansen, Wolfgang Dür and Christine Muschik

arXiv:2010.13940 [quant-ph]

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# Outline:

- Introduction
- MB-VQE
- The Schwinger model
- The perturbed toric code
- Summary & conclusions

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# A **measurement-based** variational quantum eigensolver

Robert Raussendorf, Daniel E. Browne, and Hans J. Briegel

Phys. Rev. A 68, 022312 (2003)
(arXiv:quant-ph/0301052 for a more recent version)

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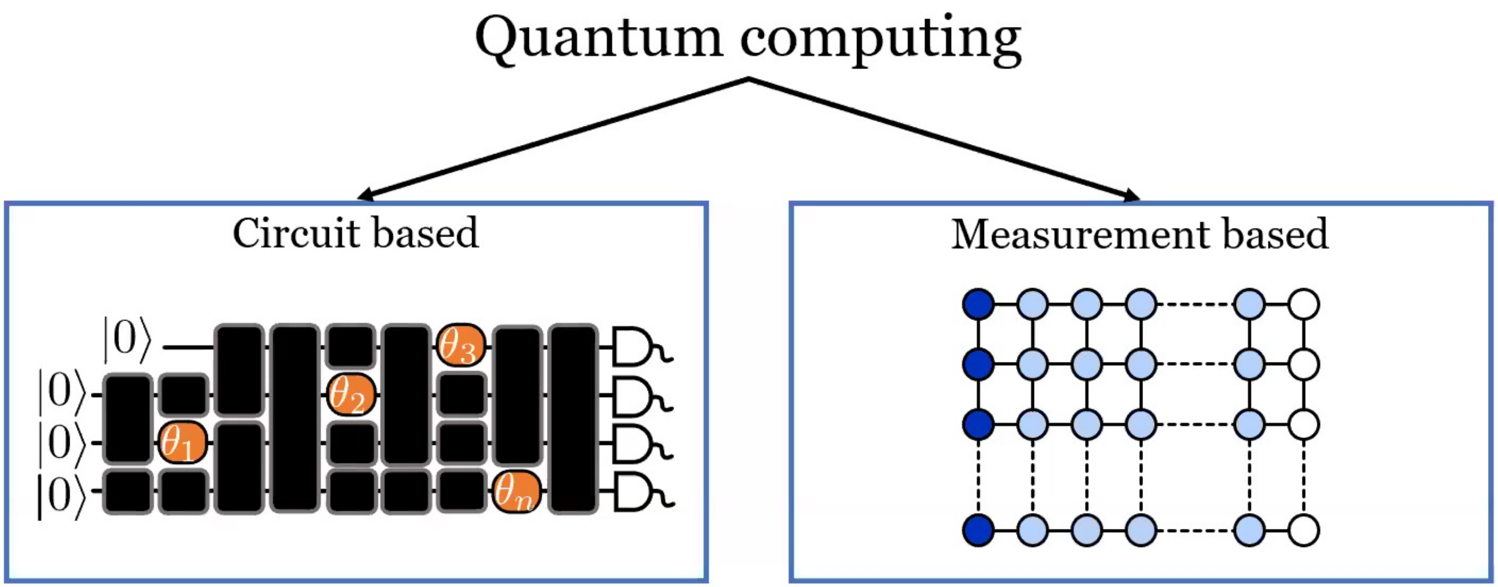
# A measurement-based variational quantum eigensolver

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# A measurement-based variational quantum eigensolver

$$|+\rangle \; —\boxed{\phi}— \; |0\rangle + e^{i\phi}|1\rangle$$

$$\boxed{\phi} = e^{i\frac{\phi}{2}\hat{Z}}$$

$$|\pm\rangle \propto |0\rangle \pm |1\rangle$$

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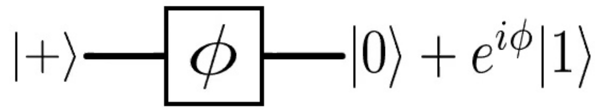
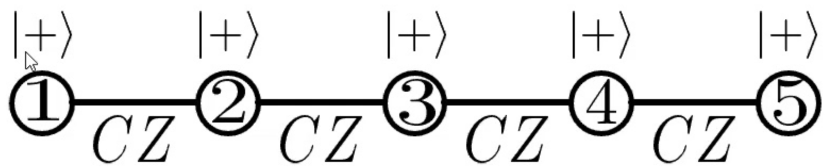
# A **measurement-based** variational quantum eigensolver

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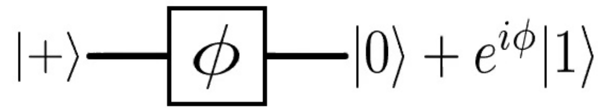


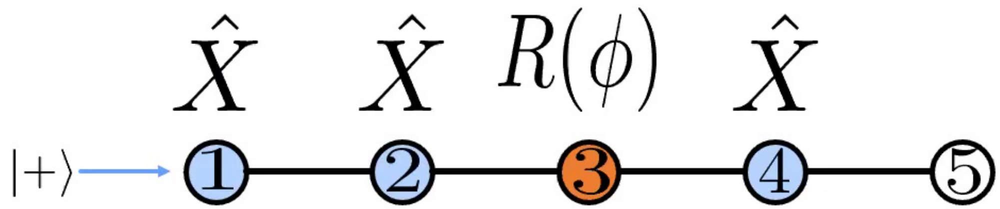


$$R(\phi) = \left\{ \frac{|0\rangle + e^{i\phi}|1\rangle}{2}, \frac{|0\rangle - e^{i\phi}|1\rangle}{2} \right\}$$

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# A measurement-based variational quantum eigensolver

$$|+\rangle \text{—} \boxed{\phi} \text{—} |0\rangle + e^{i\phi}|1\rangle$$

$$|+\rangle \longrightarrow ⑤ = |0\rangle + e^{i\phi}|1\rangle$$

$$R(\phi) = \left\{ \frac{|0\rangle + e^{i\phi}|1\rangle}{2}, \frac{|0\rangle - e^{i\phi}|1\rangle}{2} \right\}$$

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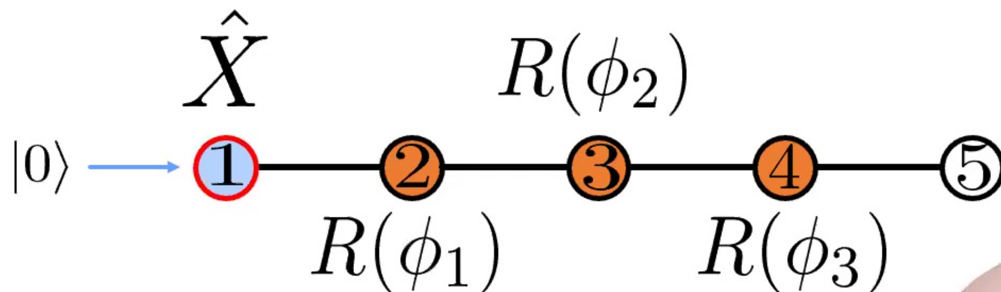
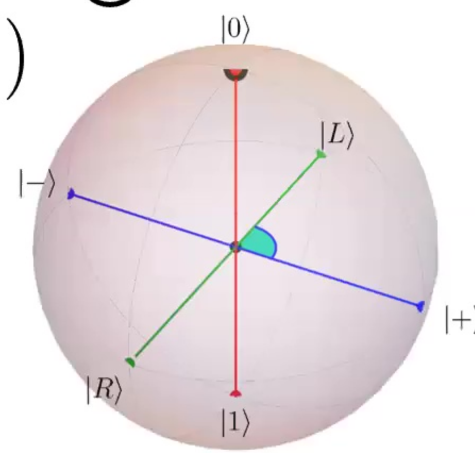



'Current' state: $|0\rangle$

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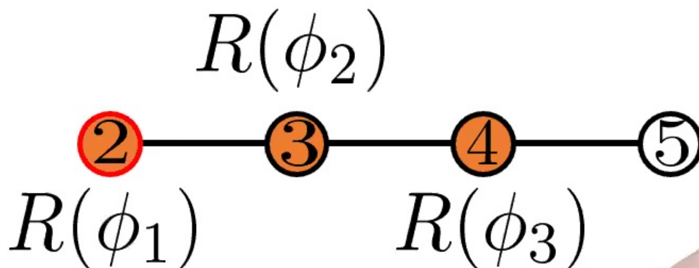
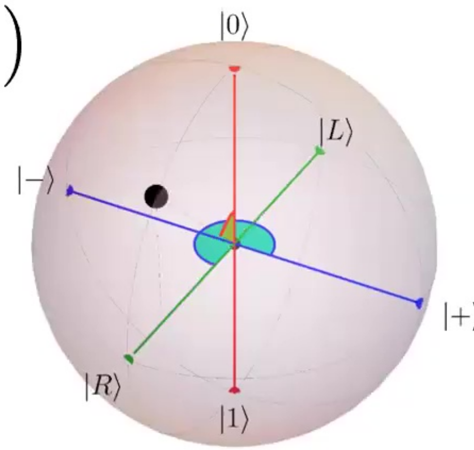



'Current' state: $e^{i\frac{\phi_1}{2}\hat{X}}|0\rangle$

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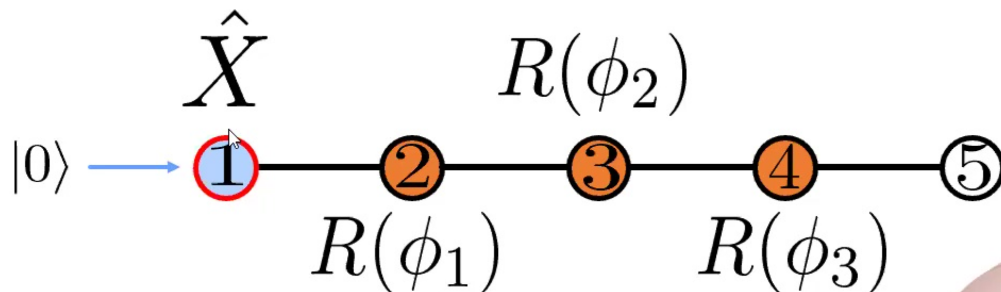


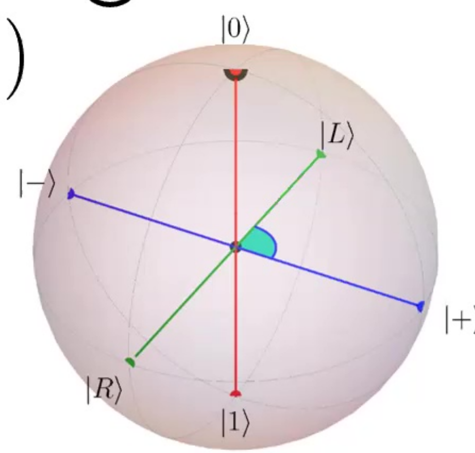


'Current' state: $|0\rangle$

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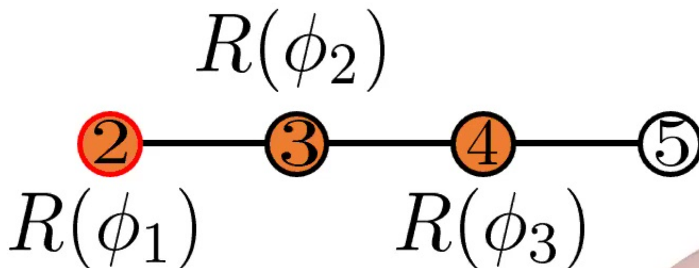
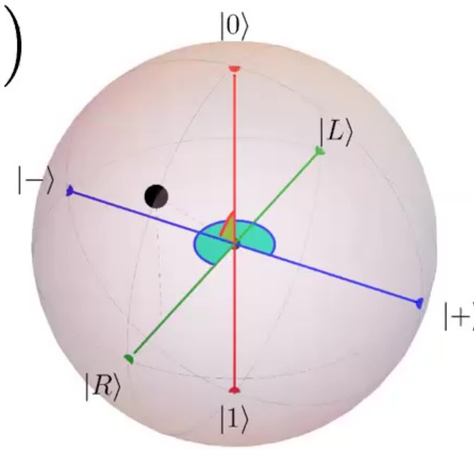



'Current' state: $e^{i\frac{\phi_1}{2}\hat{X}}|0\rangle$

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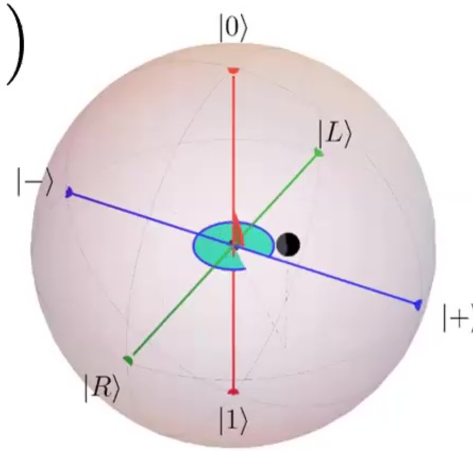
# A measurement-based variational quantum eigensolver

$R(\phi_2)$


$R(\phi_3)$

'Current' state: $e^{i\frac{\phi_2}{2}\hat{Z}}e^{i\frac{\phi_1}{2}\hat{X}}|0\rangle$

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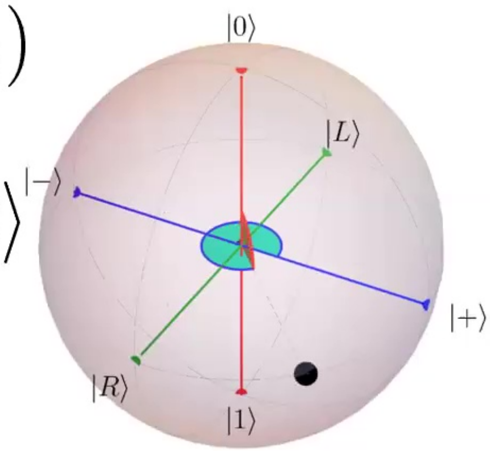
# A measurement-based variational quantum eigensolver


$R(\phi_3)$

'Current' state: $e^{i\frac{\phi_3}{2}\hat{X}}e^{i\frac{\phi_2}{2}\hat{Z}}e^{i\frac{\phi_1}{2}\hat{X}}|0\rangle$

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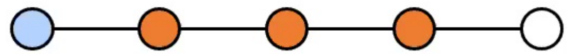
# A measurement-based variational quantum eigensolver

## Universal set of gates

- Arbitrary rotations [1]
- CNOT [1]

...

- And several others [2,3,4]




[1]: R. Raussendorf et al. Phys. Rev. Lett.86,5188 (2001).

[2]: P. Walther et al. Nature434, 169 (2005).

[3]: M. Zwerger et al. Sci Rep4, 5364(2014).

[4]: D. E. Browne et al. arXiv quant-ph/0603226 (2006).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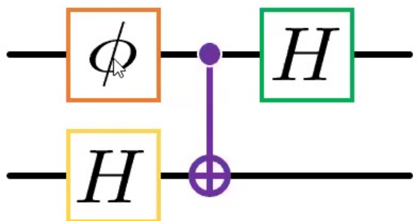
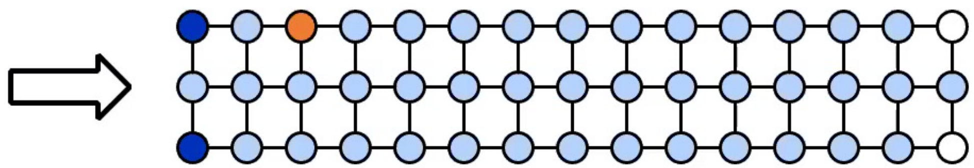
# A measurement-based variational quantum eigensolver



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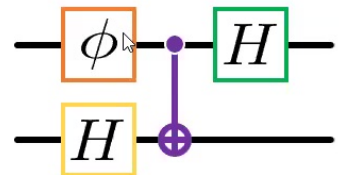
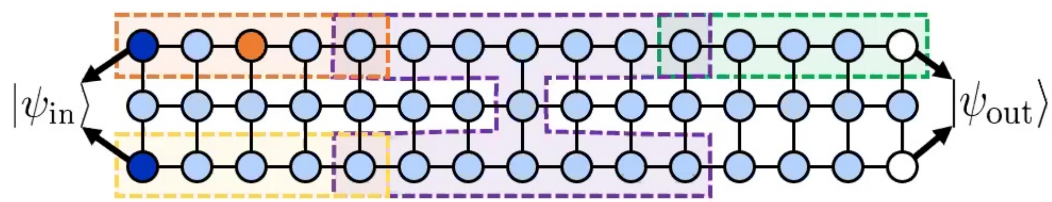
# A measurement-based variational quantum eigensolver

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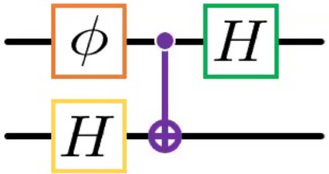
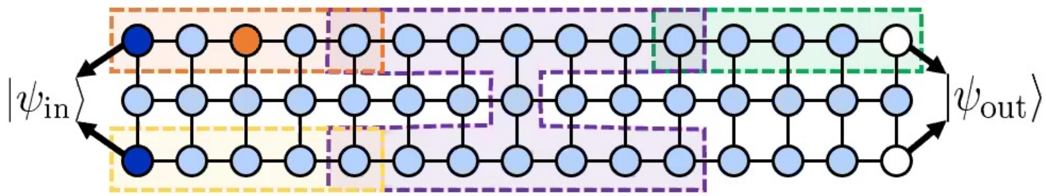



Gottesmann Knill theorem

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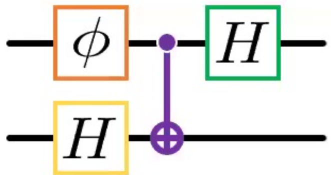
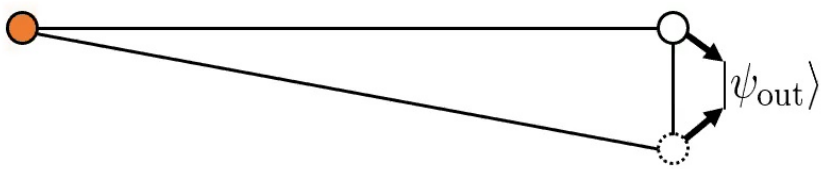



Gottesmann Knill theorem

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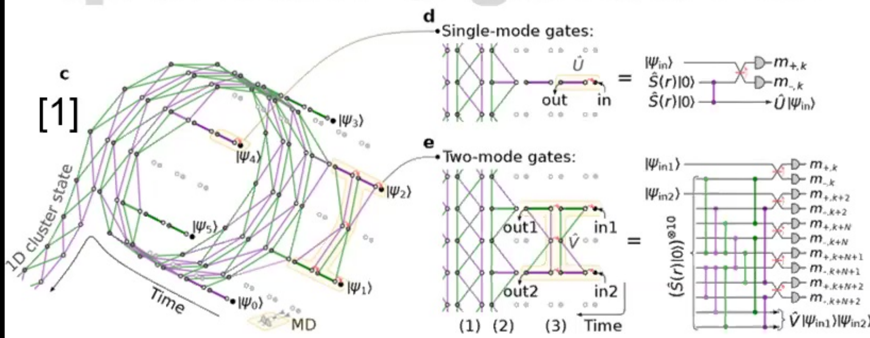
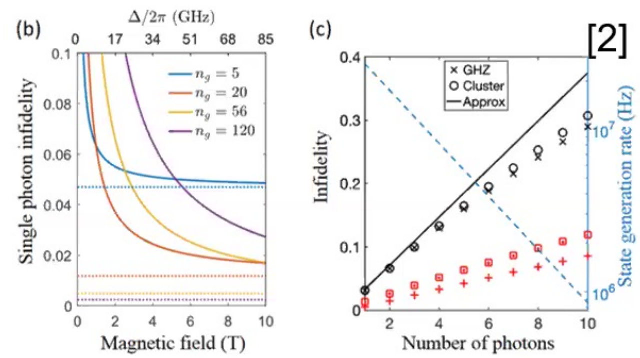
# A measurement-based variational quantum eigensolver

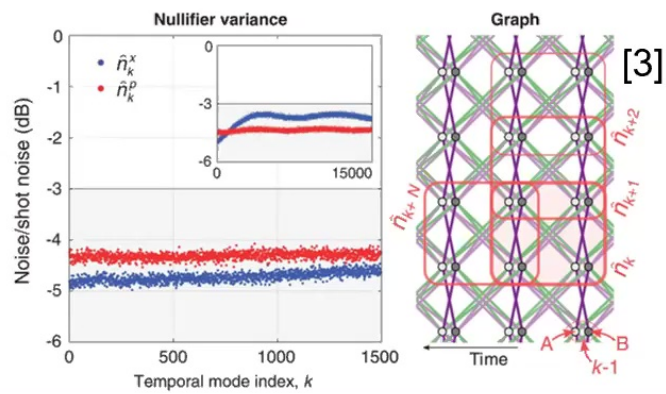
[1]




[2]




[3]




[1]: M. V. Larsen et al., arXiv:2010.14422 [quant-ph], (2020).
[2]: K. Tiurev et al., arXiv:2007.09295 (2020).
[3]: W. Asavanant et al., Science366, 373 (2019).
[4]: P. Walther et al., Nature434, 169 (2005).

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# A measurement-based variational quantum eigensolver

J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik,
New J. Phys.18, 023023 (2016).

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# A measurement-based variational quantum eigensolver

$$\hat{H} = \text{target Hamiltonian}$$

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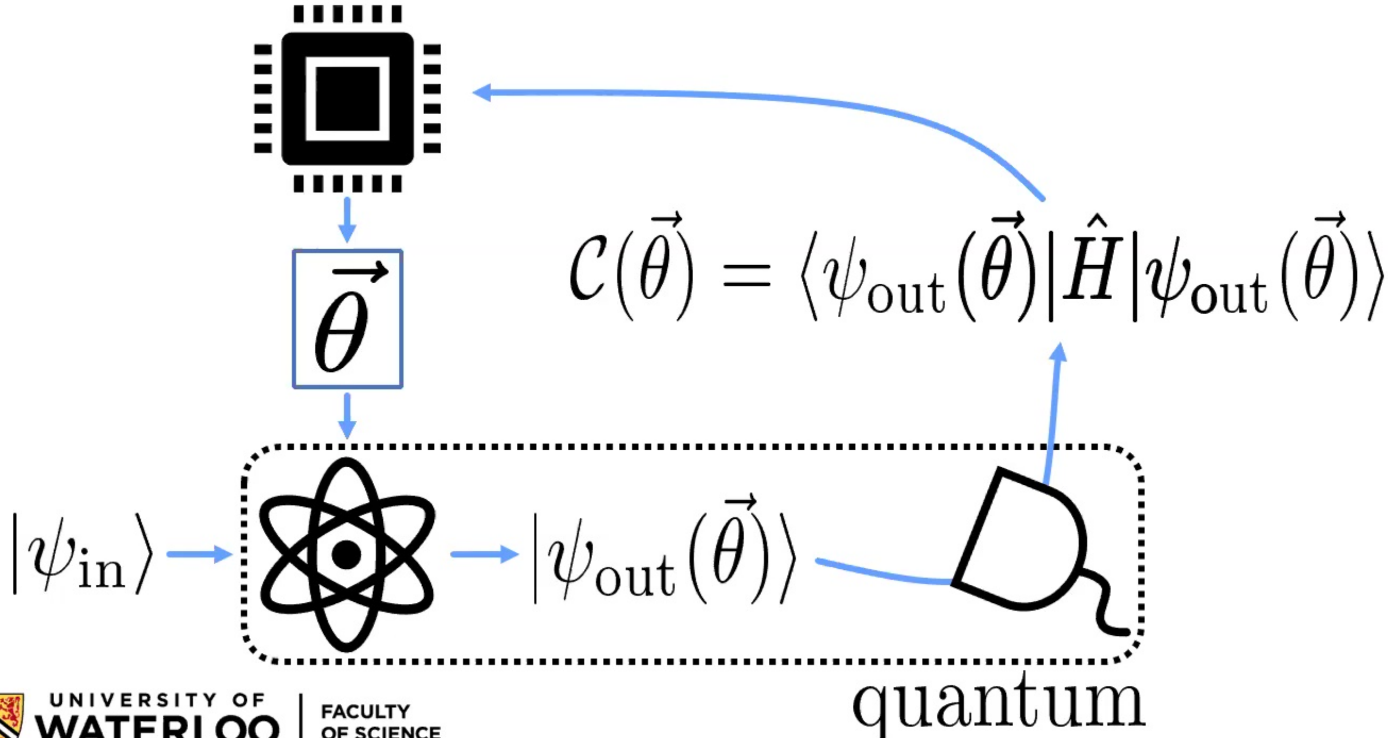
# A measurement-based variational quantum eigensolver

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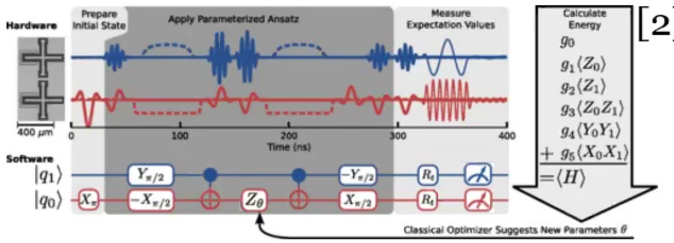
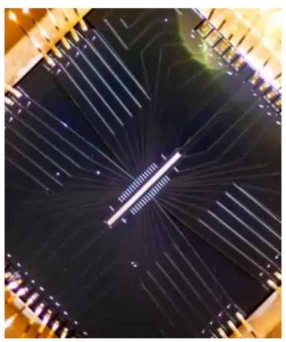
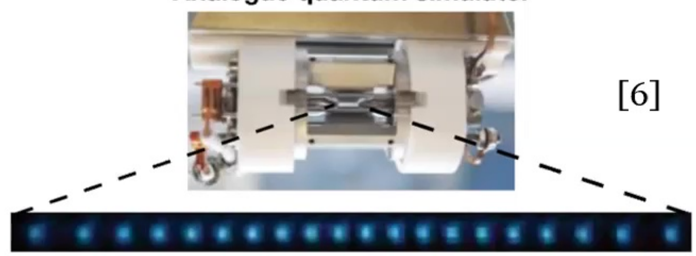



[2]



[3]




[6]


[1]: Hsuan-Hao Lu et al
Phys. Rev. A 100, 012320

[3]: O. Shehab et al.
Phys. Rev. A 100, 062319

[2]: P. J. J. O'Malley *et al.*
Phys. Rev. X 6, 031007

[4]: Sam McArdle et al.
Rev. Mod. Phys. 92, 015003

[5]: Peruzzo *et al.*
*Nat Commun* 5, 4213 (2014).

[6]: *Kokail et al.*
*Nature* volume 569, pages 355–360 (2019)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# Outline:

- Introduction
- **MB-VQE**
- The Schwinger model
- The perturbed toric code
- Summary & conclusions

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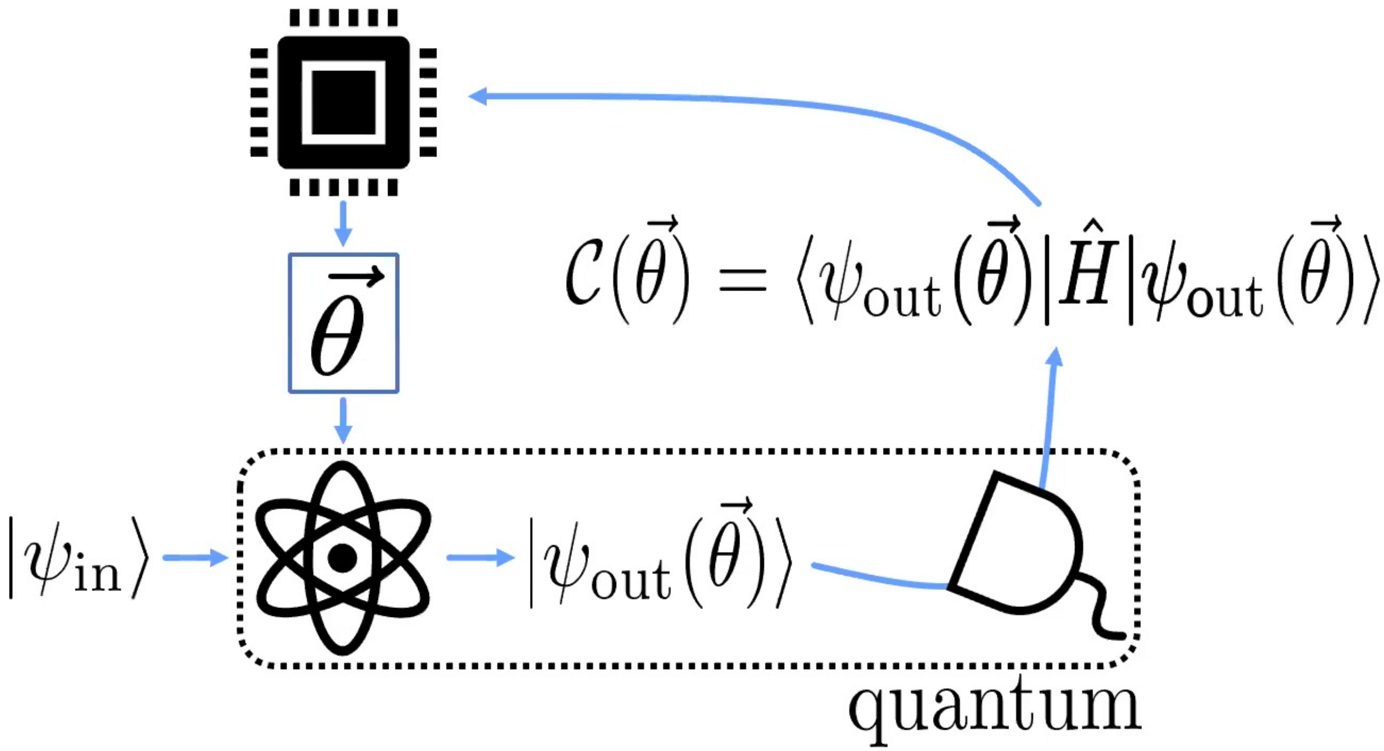
# General framework:

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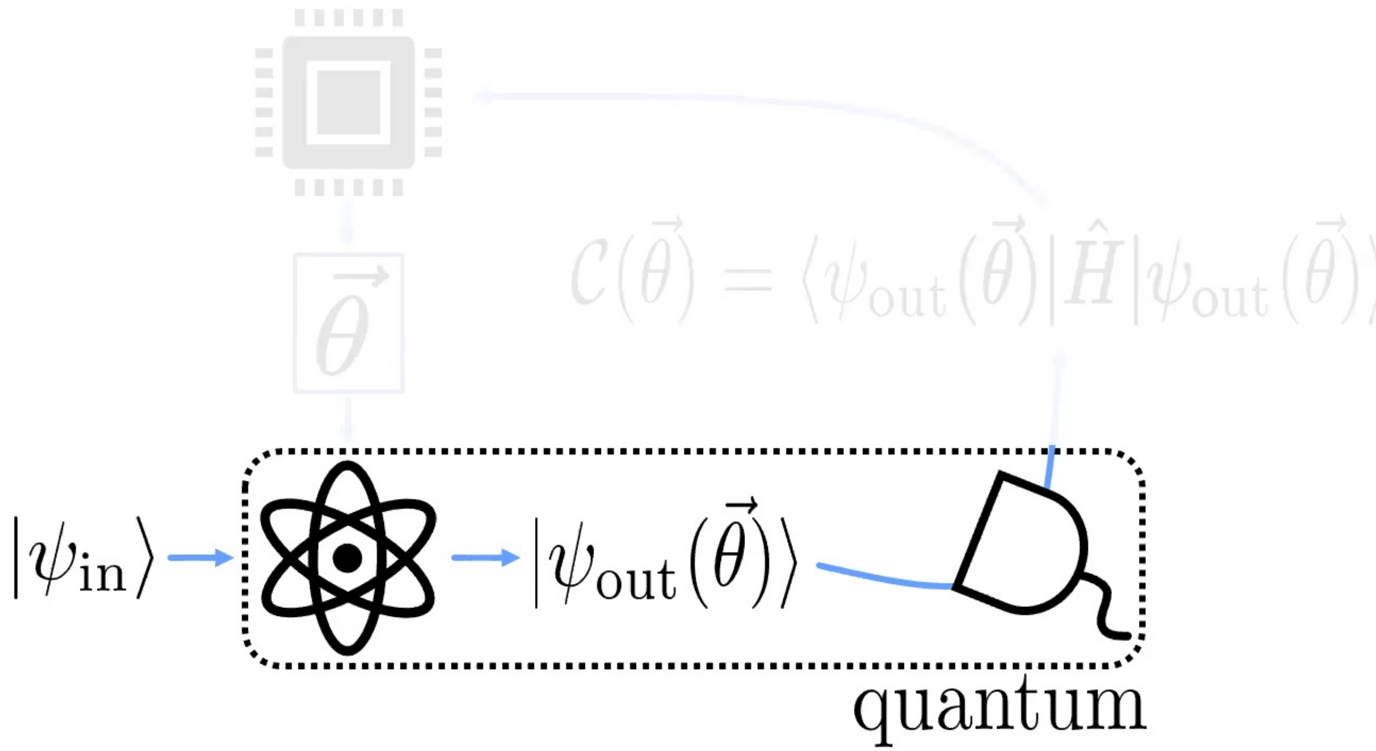
# General framework:

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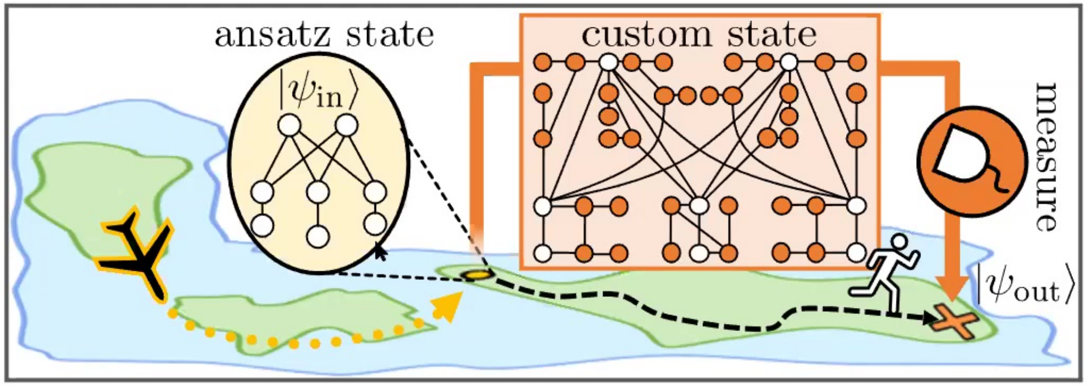
# General framework:




- Find a suitable ansatz state $|\psi_{\text{in}}\rangle$ to start in the right corner of the Hilbert space
- From $|\psi_{\text{in}}\rangle$, construct the custom state by adding auxiliary qubits
- Measure the auxiliary qubits in rotated bases $R(\theta)$
- Estimate the cost function $\mathcal{C}(\vec{\theta})$ with the resulting state $|\psi_{\text{out}}\rangle$

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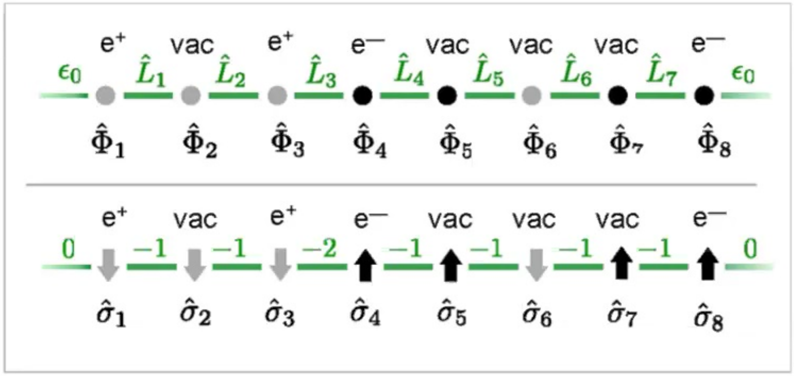
# The Schwinger model:




[1]

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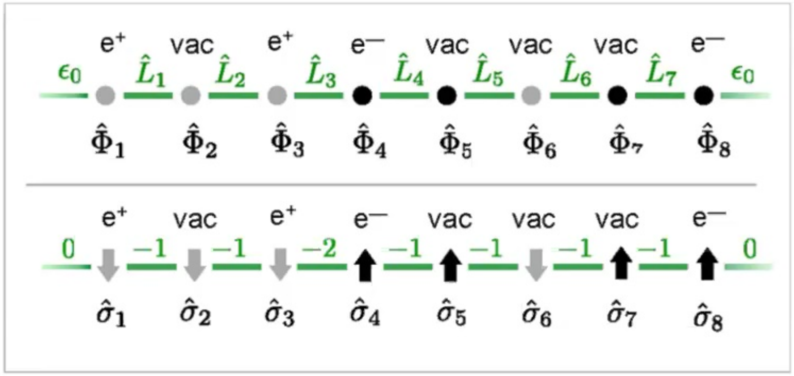
[1]: Muschik, Christine, et al. *New Journal of Physics* 19.10 (2017): 103020.

# The Schwinger model:




$$\hat{H} = \frac{J}{2}\sum_{n=1}^{S-2}\sum_{k=n+1}^{S-1}(S-k)\hat{Z}_n\hat{Z}_k - \frac{J}{2}\sum_{n=1}^{S-1} n\text{mod}2\sum_{k=1}^{n}\hat{Z}_k$$
$$+w\sum_{n=1}^{S-1}\left(\hat{\sigma}_n^{+}\hat{\sigma}_{n+1}^{-} + \text{H.C.}\right) + \frac{\mu}{2}\sum_{n=1}^{S}(-1)^n\hat{Z}_n$$

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[1]: Muschik, Christine, et al. *New Journal of Physics* 19.10 (2017): 103020.

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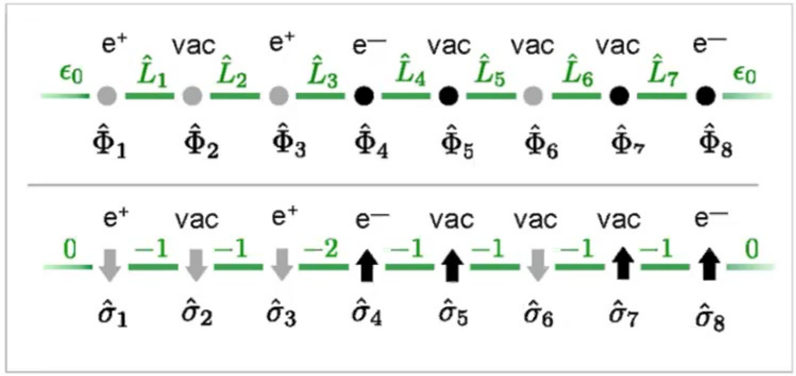


Use a (MB-)VQE!

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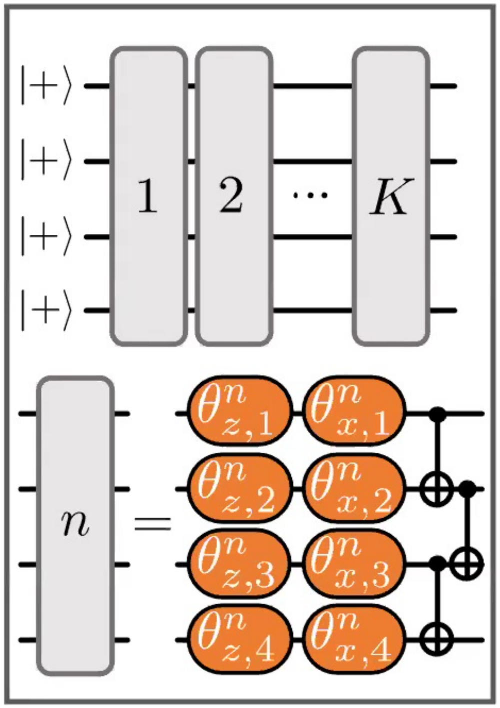
[1]: Muschik, Christine, et al. *New Journal of Physics* 19.10 (2017): 103020.

# The Schwinger model:

Circuit

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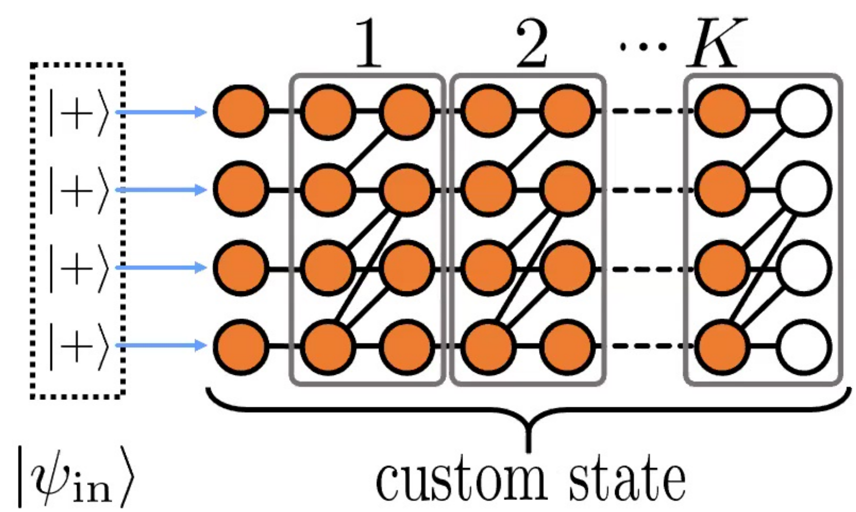
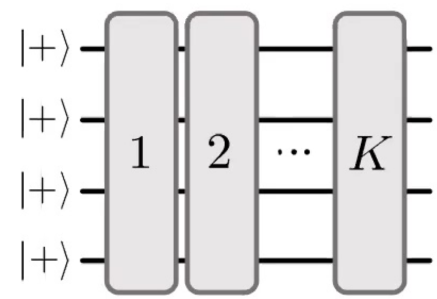
# The Schwinger model:

Equivalent graph state

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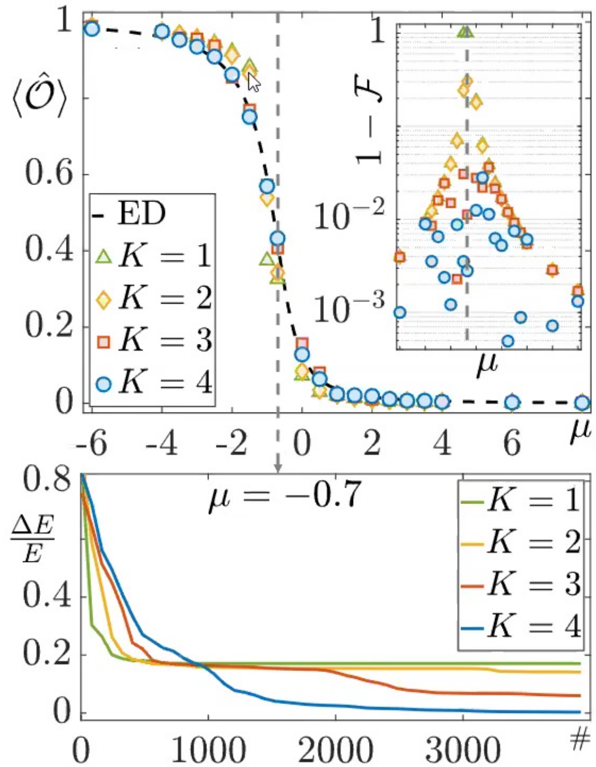


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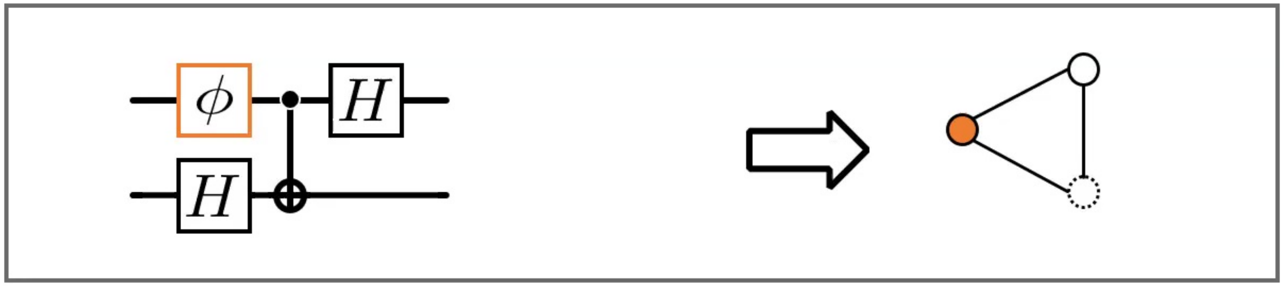


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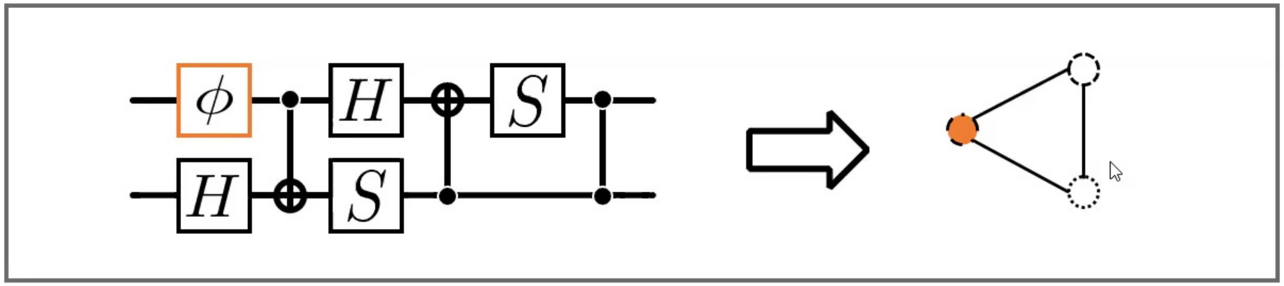


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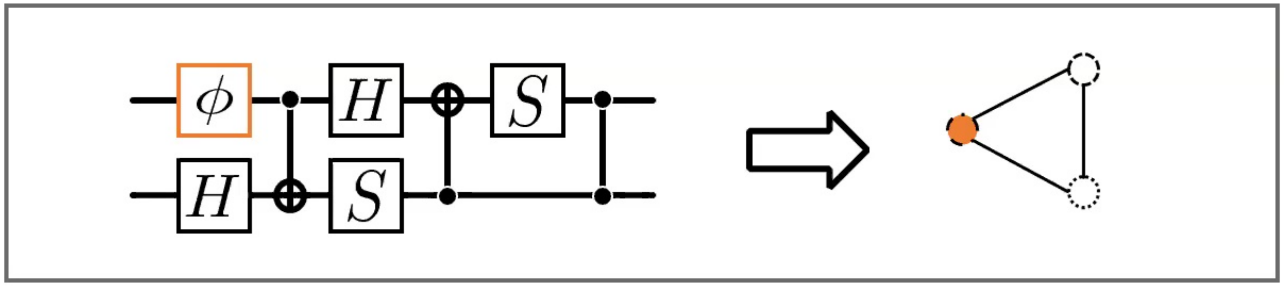


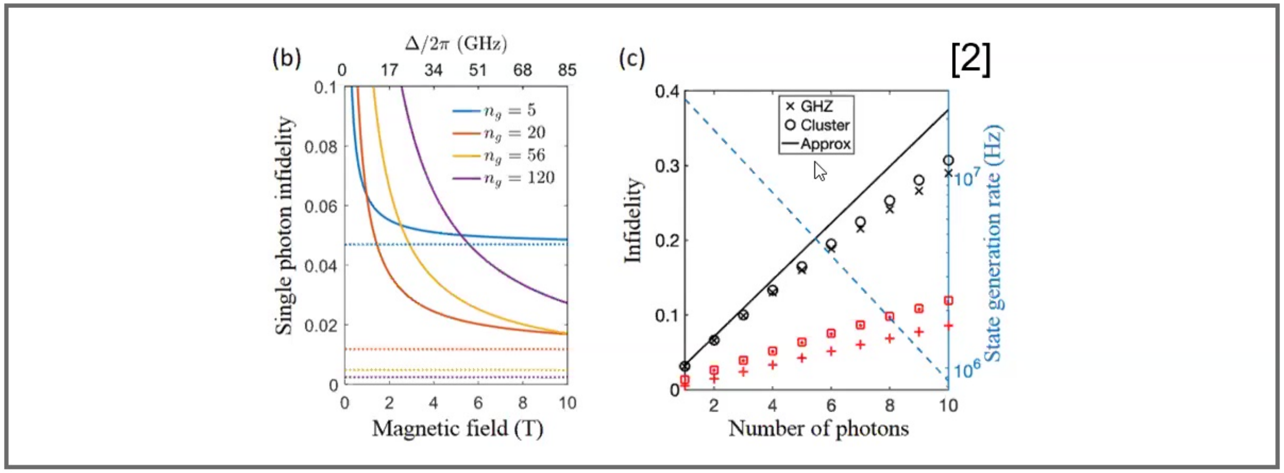


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[2]: K. Tiurev et al., arXiv:2007.09295 (2020).

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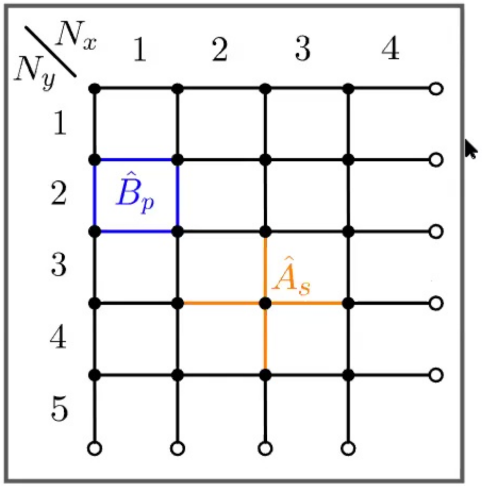
# The perturbed toric code:




- $2N_xN_y$ qubits on the links
- $2N_xN_y - 2$ independent stabilizers $\hat{A}_s$ and $\hat{B}_p$

$$\hat{H}_{\mathrm{tc}} = -\sum_p \hat{B}_p - \sum_s \hat{A}_s$$

4 'code words' of the toric code: the logical states $|r,s\rangle_L$, $(r,s) = 0, 1$.

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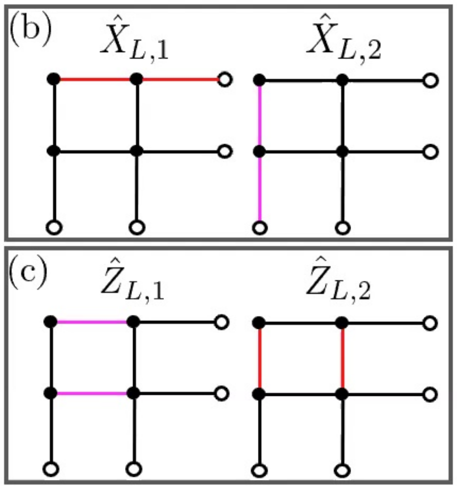
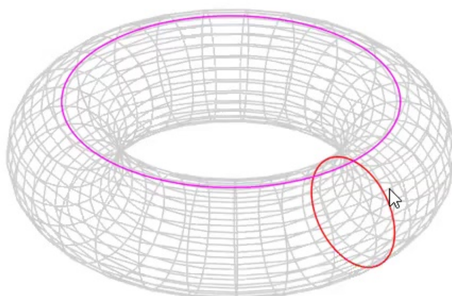
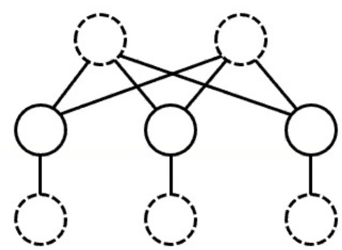
# The perturbed toric code:



$$|r, s\rangle_L$$

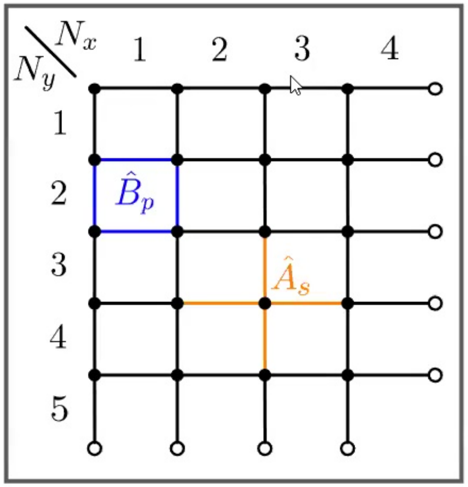
$$\hat{H}_{\mathrm{tc}} = -\sum_p \hat{B}_p - \sum_s \hat{A}_s - (-1)^r \hat{Z}_{L,1} - (-1)^s \hat{Z}_{L,2}$$



$$|0, 0\rangle_L$$

# The perturbed toric code:

Perturbation




$$\hat{H}_{\rm p} = \sum_n \lambda_n \hat{Z}_n$$

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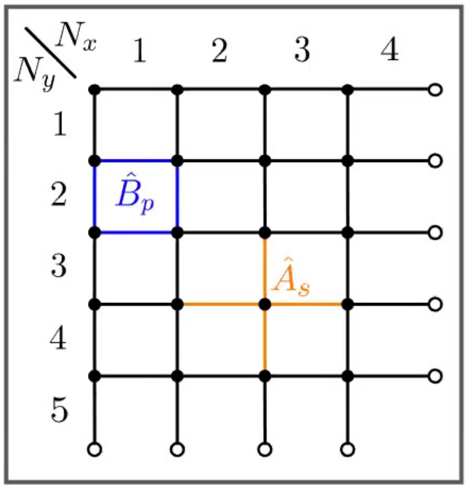
# The perturbed toric code:

Equivalence with $Z(2)$ lattice gauge theory in 2D




$$\hat{H}_{\mathrm{p}} = \sum_n \lambda_n \hat{Z}_n$$ → Electric energy

$$\hat{H}_{\mathrm{tc}} = -\sum_p \hat{B}_p - \sum_s \hat{A}_s$$ → Gauss law

Magnetic energy

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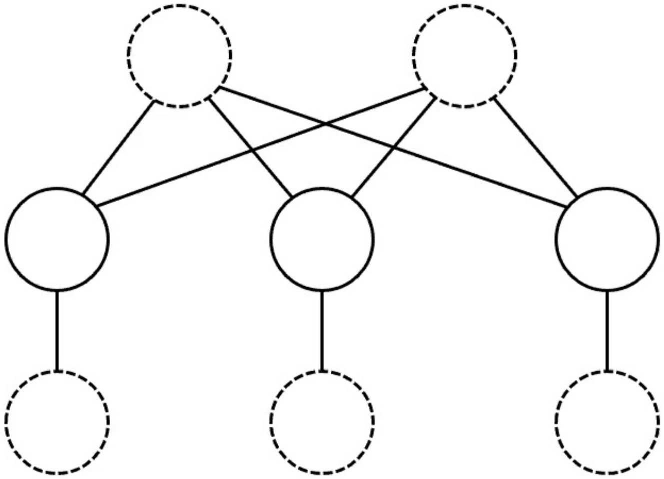
# The perturbed toric code:

Ansatz state



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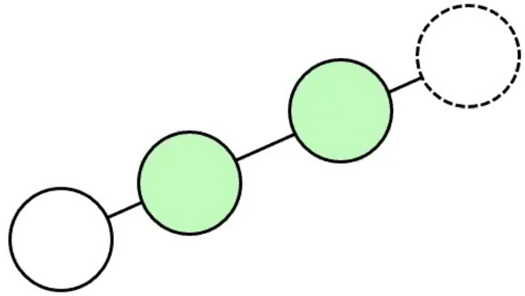
# The perturbed toric code:

Qubit decoration



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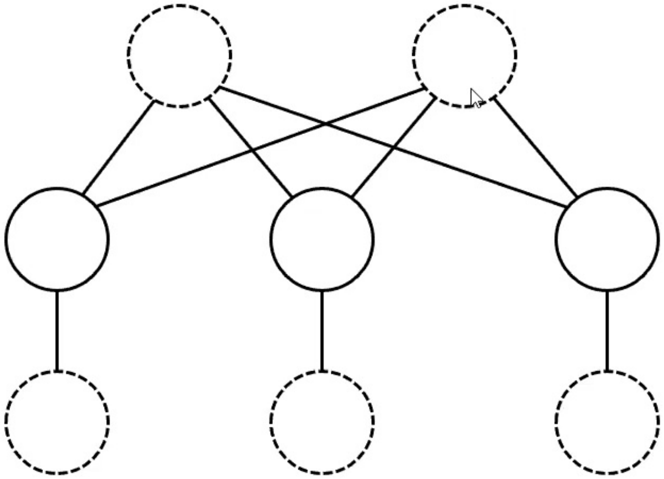
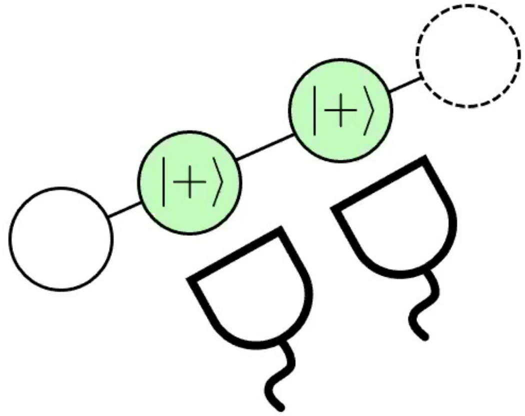
# The perturbed toric code:

Ansatz state



# The perturbed toric code:

Qubit decoration

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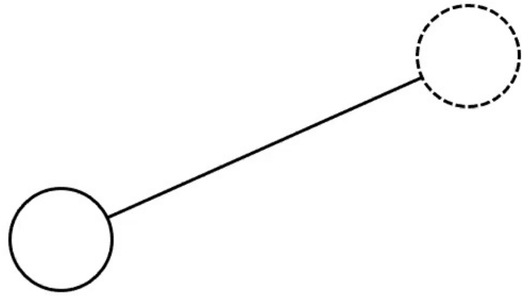
Qubit decoration



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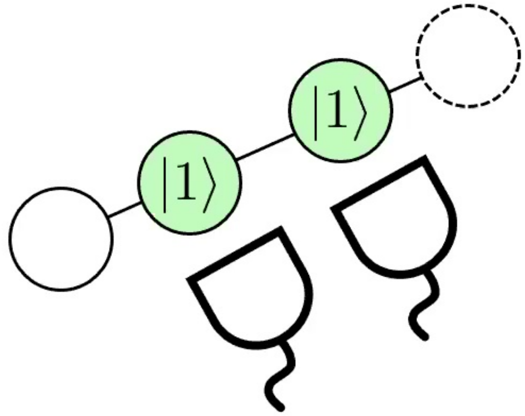
Qubit decoration

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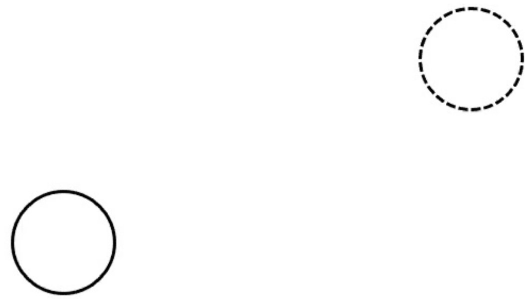
# The perturbed toric code:

Qubit decoration



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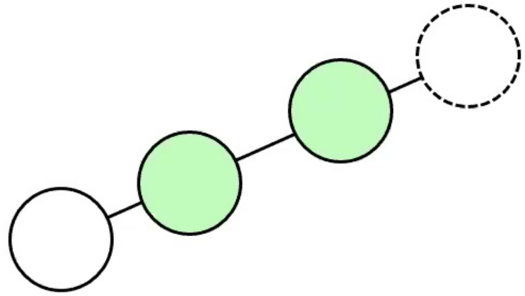
Qubit decoration



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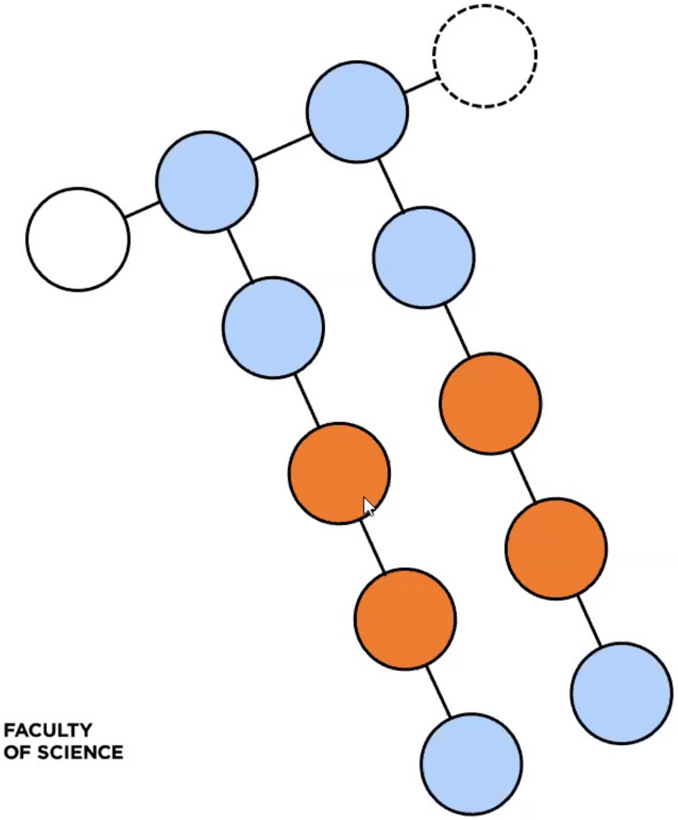
Qubit decoration



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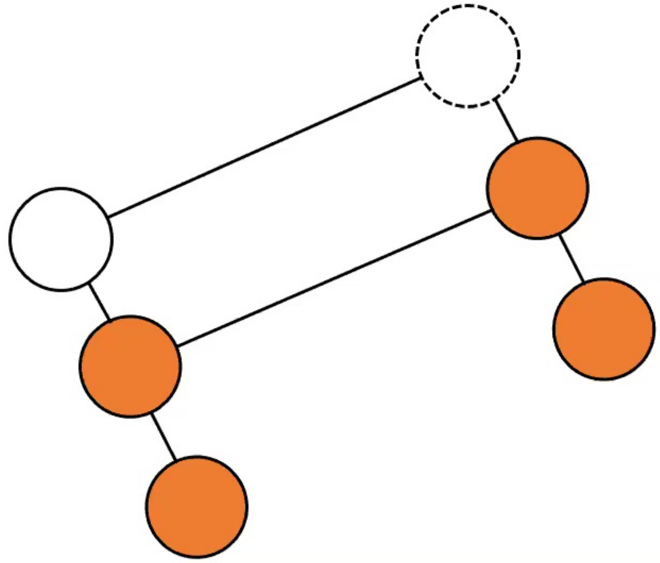
# The perturbed toric code:

Qubit decoration



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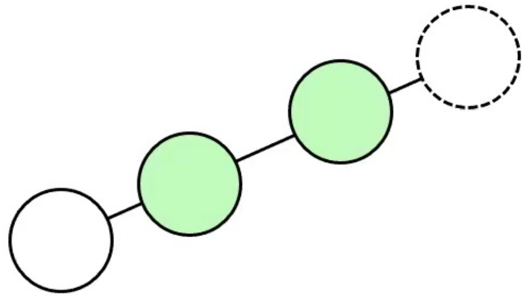
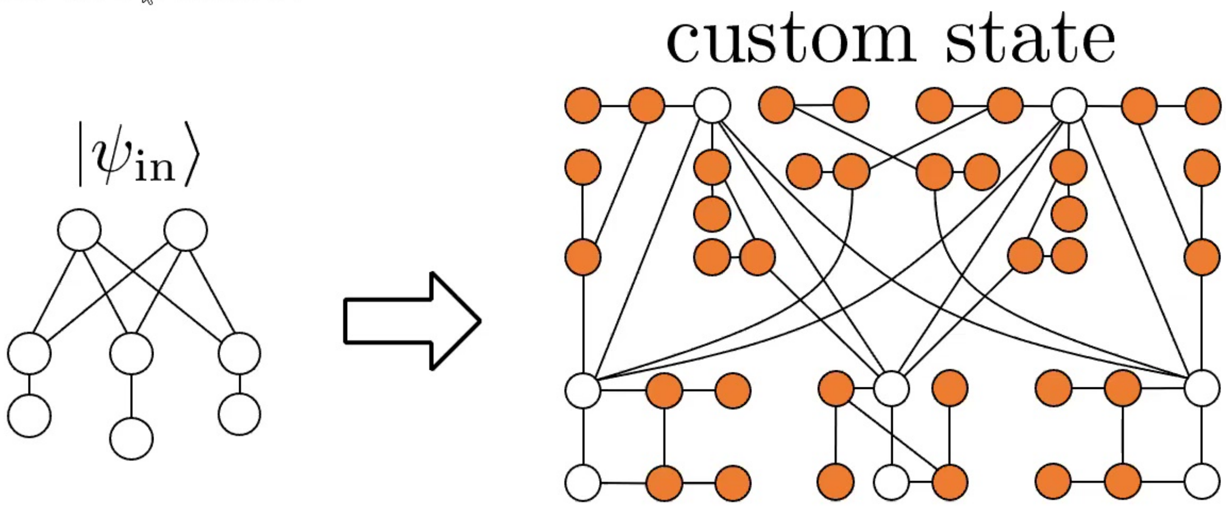
Qubit decoration

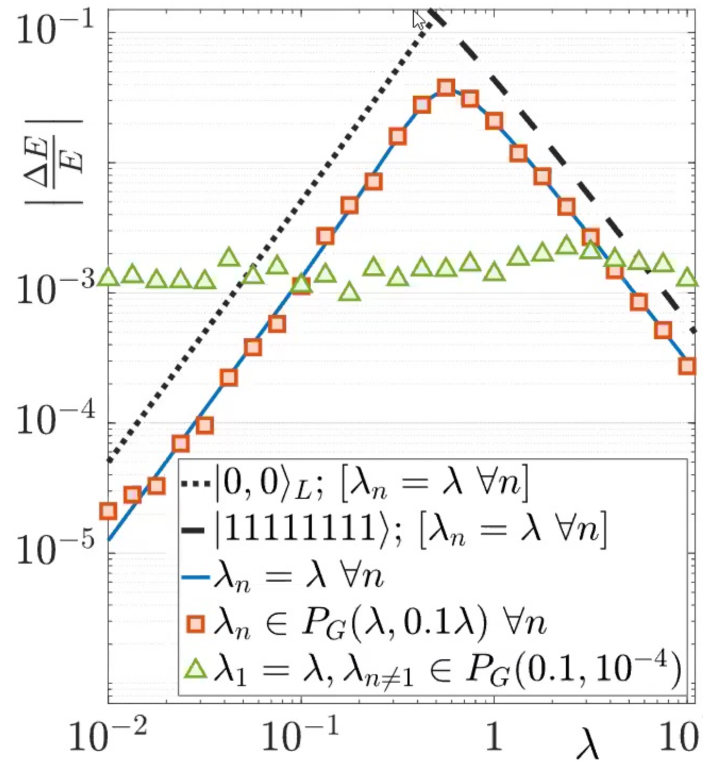


# The perturbed toric code:

Qubit decoration



# The perturbed toric code:




$$\hat{H}_\text{p} = \sum_n \lambda_n \hat{Z}_n$$

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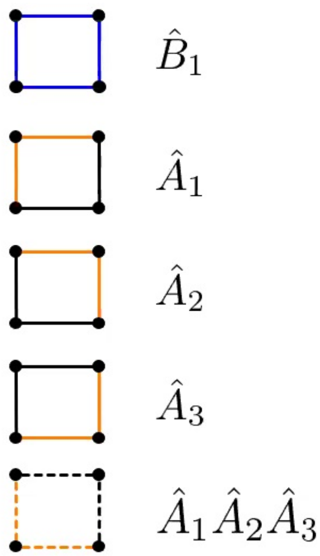
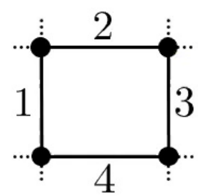
# The perturbed toric code:

Toy example (open boundary conditions)




$$\hat{H}_0 = -\hat{B}_1 - \hat{A}_1 - \hat{A}_2 - \hat{A}_3$$

$$\hat{H}_{\mathrm{p}} = \lambda_1 \hat{Z}_1$$

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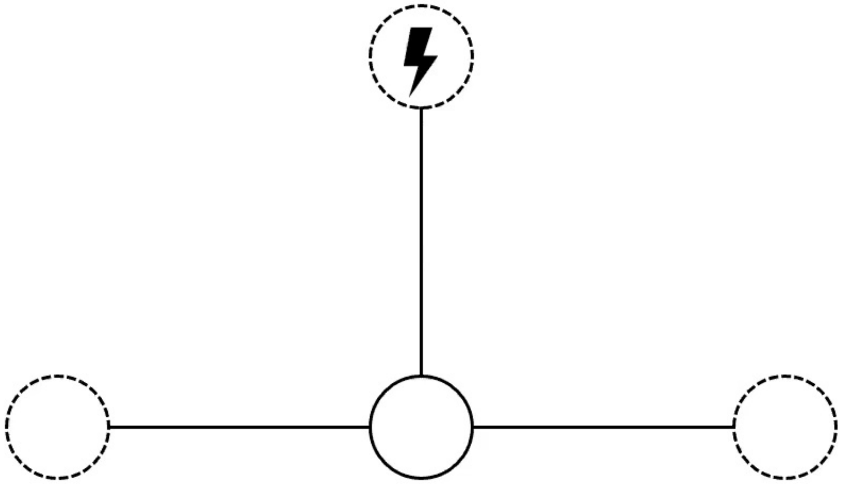
# The perturbed toric code:

Toy example (open boundary conditions)



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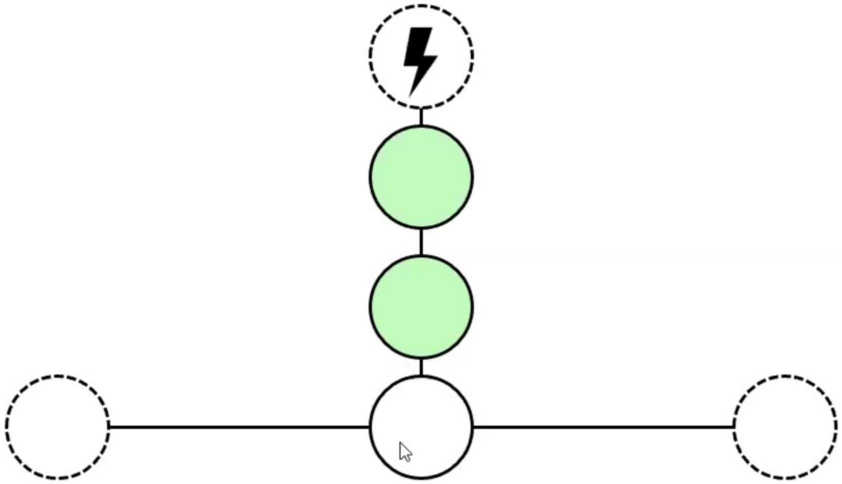
Toy example (open boundary conditions)



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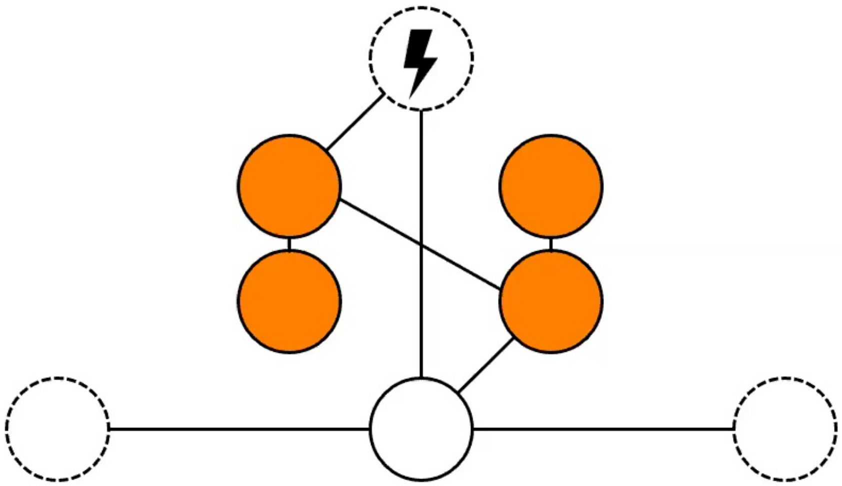
Toy example (open boundary conditions)



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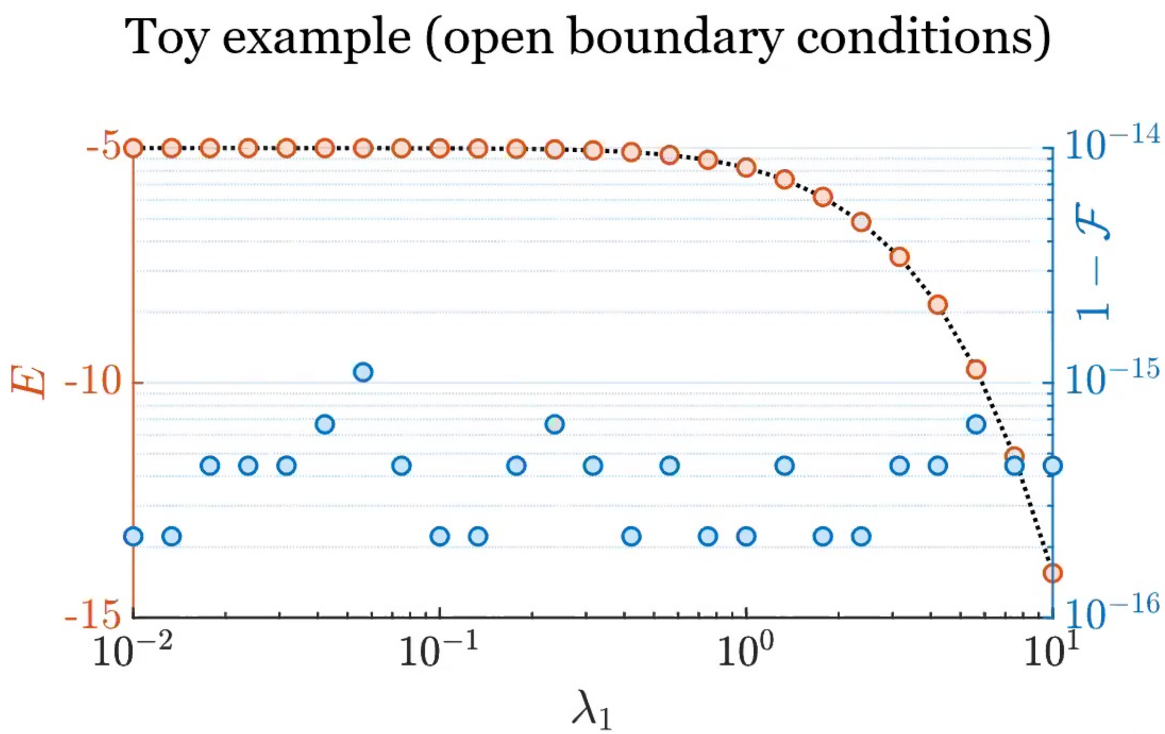
# The perturbed toric code:

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# Summary & conclusions

## MB-VQE

- GS is in a graph's neighbourhood (perturbed coric code)
- Large Clifford component (Schwinger model)
  - New (MB-)VQE designs?

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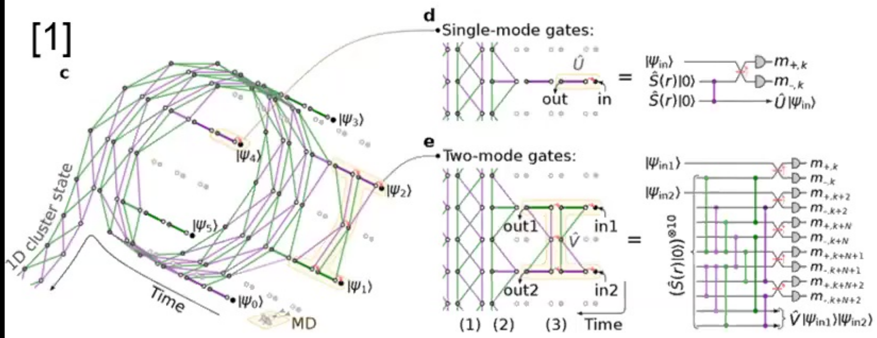
# Summary & conclusions

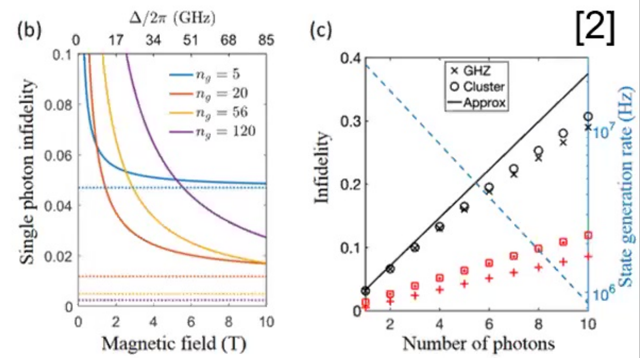
## MB-VQE

- New platforms for variational algorithms! [1,2,3,4]

[1]




[2]





[1]: M. V. Larsen et al, arXiv:2010.14422, (2020).
[2]: K. Tiurev et al, arXiv:2007.09295 (2020).
[3]: W. Asavanant et al, Science366, 373 (2019).
[4]: P. Walther et al, Nature434, 169 (2005).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# Summary & conclusions

## MB-VQE

- Noise analysis
  - ➢ Better resilience against noise sources? [1,2,3,4]

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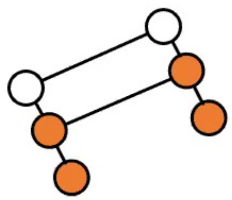
[1] R. Raussendorf et al, Phys.Rev. A 68, 022312 (2003).
[2] M. Zwerger et al, App. Phys. B 122,50 (2015).
[3] M. Zwerger et al, Sci Rep4, 5364(2014).
[4] M. Zwerger et al, Phys. Rev. Lett.110, 260503 (2013).

# Summary & conclusions

## MB-VQE



- More tailored decorations
  - Investigating the effect of different decorations
  - Less auxiliary qubits required
  - Less variational parameters

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# Collaborators

**Ryan R. Ferguson**

A. Al Balushi

Karl Jansen

Wolfgang Dür

arXiv:2010.13940 [quant-ph]

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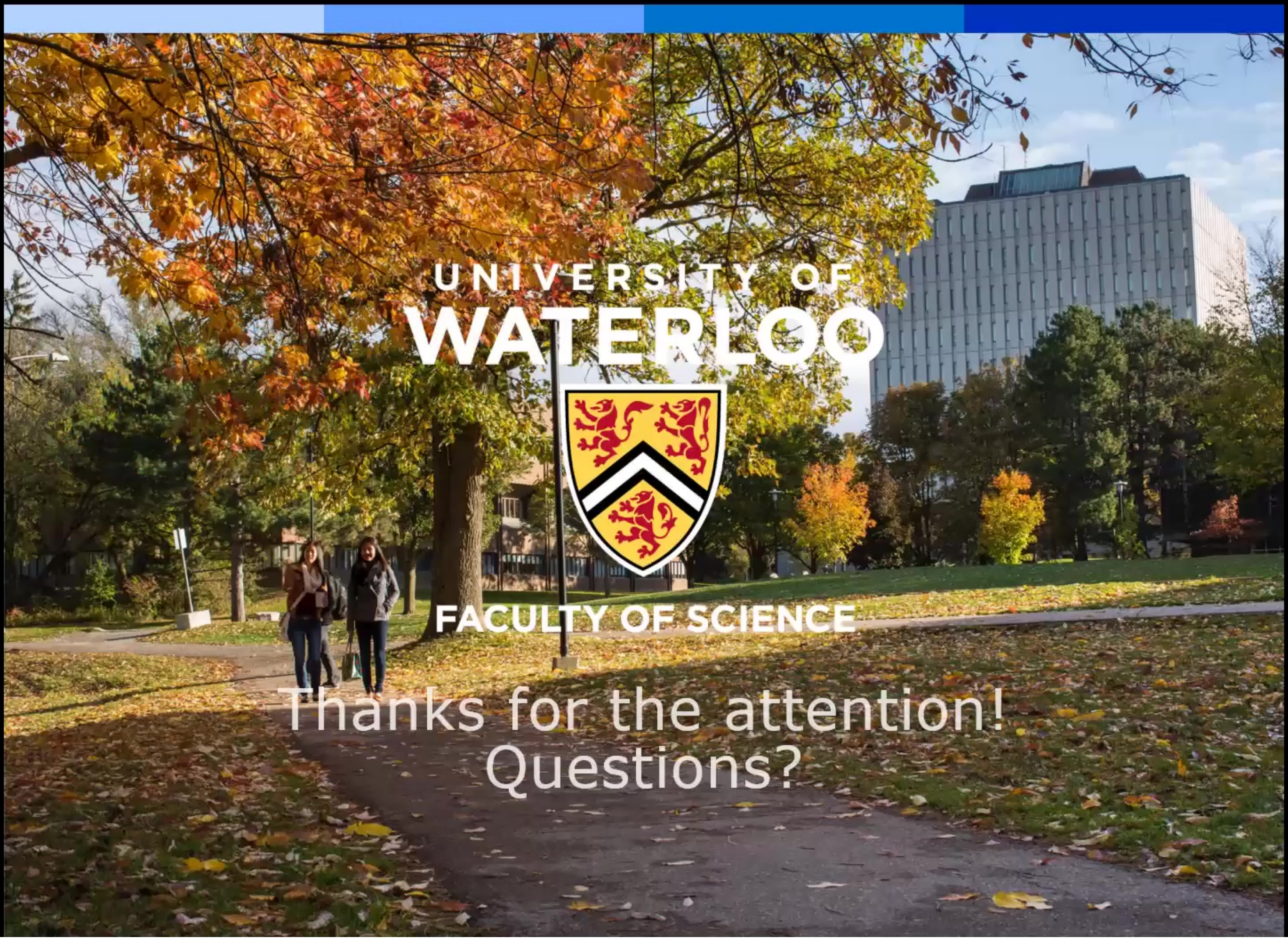
Christine Muschik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Thanks for the attention!
Questions?