Title: Towards a realistic holographic tensor network: From p-adic CFT to (minimal) CFT2

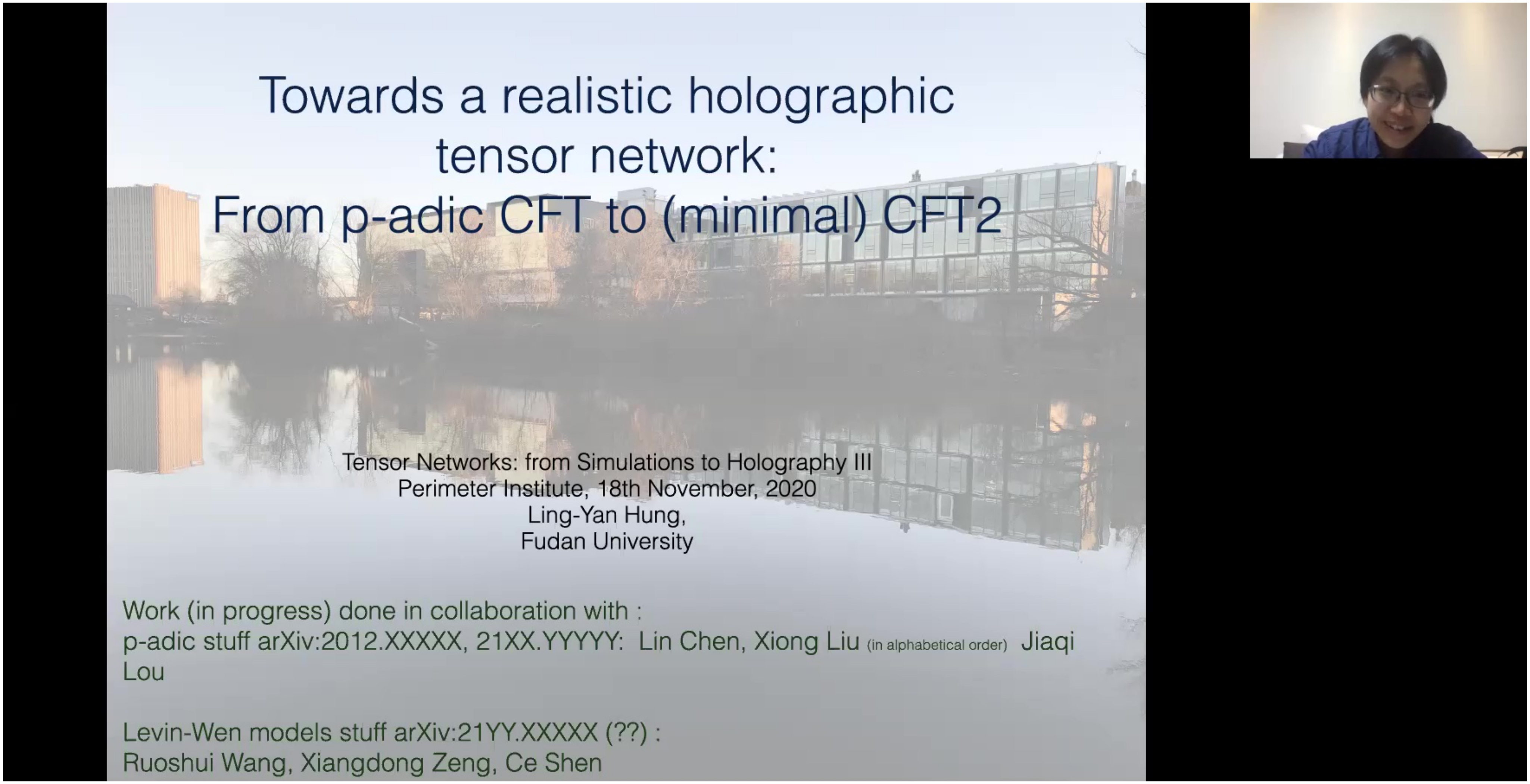
Speakers: Ling-Yan Hung

Collection: Tensor Networks: from Simulations to Holography III

Date: November 18, 2020 - 8:00 AM

URL: http://pirsa.org/20110026

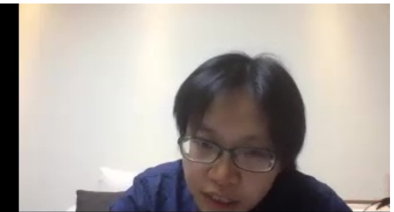
Abstract: The success of the Ryu-Takayanagi formula suggests a profound connection between the AdS/CFT correspondence and tensor networks. There are since many works on constructing examples, although it is very difficult to make them explicit and quantitative. We will discuss some new progress in the toy example of p-adic CFT where its tensor network dual was previously constructed explicitly [arXiv:1703.05445 , arXiv:1812.06059, arXiv:1902.01411], and how some analogue of Einstein equation on the graph emerges as we consider RG flow of these CFTs. These progresses inspire us to find a way to construct explicit holographic tensor networks of more realistic CFTs based on their connection with topological models with one higher dimension, at least in CFT2. We will present some preliminary results and works in progress.

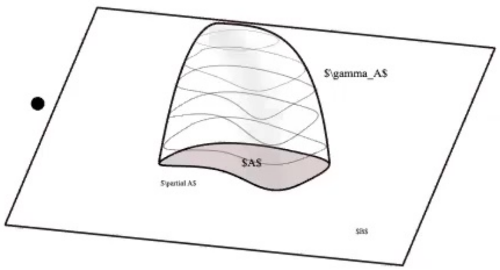

Towards a realistic holographic tensor network:
From p-adic CFT to (minimal) CFT2
Tensor Networks: from Simulations to Holography III
Perimeter Institute, 18th November, 2020
Ling-Yan Hung,
Fudan University
Work (in progress) done in collaboration with :
p-adic stuff arXiv:2012.XXXXX, 21XX.YYYYY: Lin Chen, Xiong Liu (in alphabetical order) Jiaqi Lou
Levin-Wen models stuff arXiv:21YY.XXXXX (??) :
Ruoshui Wang, Xiangdong Zeng, Ce Shen

Overview

- Tensor network and AdS/CFT
  - Part I p-adic tensor network
    - RG flow and Einstein equation ?
    - Wilson lines and p-adic black holes
  - Part II more realistic CFTs ???
  - Outlook

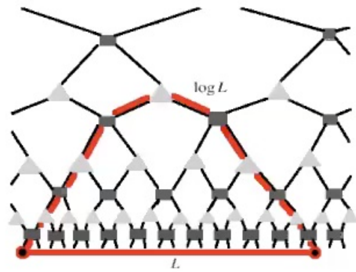
Holographic entanglement and the tensor network





$S_{EE} = \frac{A}{4G}$

v.s.



$S_{EE} \leq \mathcal{N} \log L$

Picture courtesy Orus

- For MERA type networks, it recovers a Ryu-Takayanagi type entanglement entropy swingle

P-adic CFT

- Consider putting a CFT on the p-adic field $x \in Q_p$
- a (1d) p-adic CFT lives on the (projective) Q_p

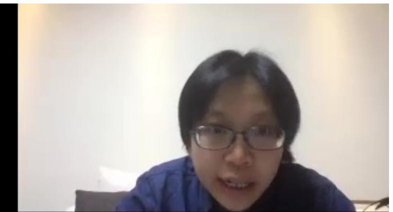
$$x = p^v(\sum_{m=0}^{\infty} a_m p^m) \quad |x|_p = p^{-v} \qquad (x, y)_p = |x - y|_p$$

- conformal symmetries = Mobius transformation

$$x \to \frac{ax+b}{cx+d} \qquad a, b, c, d \in Q_p, ad - bc \neq 0$$

$$PGL(2, Q_p)$$

-



Data of a p-adic CFT

E. Melzer, Int. J. Mod. Phys. A 4 (1989) 4877

1. List of primaries $\mathcal{O}_i(x)$ $\quad \mathcal{O}_i(x) \to |\frac{ad-bc}{(cx+d)^2}|_p^{-\Delta_i}\mathcal{O}_i(x)$

2. OPE

$$\mathcal{O}_i(x)\mathcal{O}_j(y) = \sum_k C_{ij}^k |x-y|_p^{\Delta_k - \Delta_i - \Delta_j}\mathcal{O}_k(y)$$

note: "no descendent": continuous in Qp — locally constant

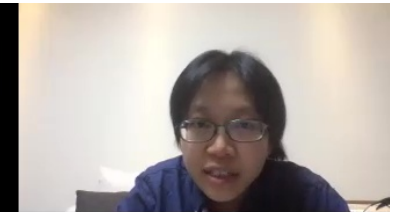
3. Correlation functions satisfying $PGL(2, Q_p)$ covariance

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = C_i \delta_{ij}|x-y|_p^{-2\Delta_i}$$

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}(z)_k\rangle = \frac{C_i C_{jk}^i}{|x-y|_p^{\Delta_{ij}}|x-z|_p^{\Delta_{ik}}|y-z|^{\Delta_{jk}}}$$

note: 3 point function actually depends only on 2 of the coordinates given any particular radial ordering of x,y,z

P-adic CFT



- Consider putting a CFT on the p-adic field $x \in Q_p$
- a (1d) p-adic CFT lives on the (projective) Q_p

$$x = p^v \left(\sum_{m=0}^{\infty} a_m p^m\right) \quad |x|_p = p^{-v} \qquad (x,y)_p = |x-y|_p$$

- conformal symmetries = Mobius transformation

$$x \to \frac{ax+b}{cx+d} \qquad a,b,c,d \in Q_p, ad-bc \neq 0$$

$$PGL(2, Q_p)$$

-

Data of a p-adic CFT

E. Melzer, Int. J. Mod. Phys. A 4 (1989) 4877

1. List of primaries $\mathcal{O}_i(x)$ $\quad \mathcal{O}_i(x) \to |\frac{ad-bc}{(cx+d)^2}|_p^{-\Delta_i} \mathcal{O}_i(x)$

2. OPE

$$\mathcal{O}_i(x)\mathcal{O}_j(y) = \sum_k C_{ij}^k |x-y|_p^{\Delta_k - \Delta_i - \Delta_j} \mathcal{O}_k(y)$$

note: "no descendent": continuous in Qp — locally constant

3. Correlation functions satisfying $PGL(2, Q_p)$ covariance

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = C_i \delta_{ij} |x-y|_p^{-2\Delta_i}$$

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}(z)_k\rangle = \frac{C_i C_{jk}^i}{|x-y|_p^{\Delta_{ij}} |x-z|_p^{\Delta_{ik}} |y-z|^{\Delta_{jk}}}$$

note: 3 point function actually depends only on 2 of the coordinates given any particular radial ordering of x,y,z

Consistency conditions:

("no descendents":
conformal blocks are trivial)

4pt function :

$$\langle \mathcal{O}_{i_1}(\infty)\mathcal{O}_{i_2}(1)\mathcal{O}_{i_3}(\eta)\mathcal{O}_{i_4}(0)\rangle = \begin{cases} \sum_\ell C^\ell{}_{i_1 i_2} C_{\ell i_3 i_4} |\eta|^{\Delta_\ell - \Delta_{i_3} - \Delta_{i_4}} & |\eta| \le |1-\eta| = 1 \\ \sum_\ell C^\ell{}_{i_1 i_4} C_{\ell i_2 i_3} |1-\eta|^{\Delta_\ell - \Delta_{i_2} - \Delta_{i_3}} & |1-\eta| \le |\eta| = 1 \\ \sum_\ell C^\ell{}_{i_1 i_3} C_{\ell i_2 i_4} |\eta|^{\Delta_{i_1} - \Delta_{i_2} - \Delta_{i_3}} & 1 \le |\eta| = |1-\eta| \end{cases} .$$

crossing symmetry now implies associativity

$$\sum_m C^m{}_{ij} C_{k\ell m} = \sum_m C^m{}_{ik} C_{j\ell m} \,,$$

p-adic AdS/CFT: Proposed bulk- Bruhat-Tits Tree

Gubser et al. Commun.Math.Phys. 352 (2017) no.3, 1019-1059 ;
Heydeman, Matthew *et al.* Adv.Theor.Math.Phys. 22 (2018) 93-176

	upper half plane $\mathbb{H}$	Bruhat-Tits tree $\mathbb{H}_p$
Isometry group G	$\mathrm{SL}(2,\mathbb{R})$	$\mathrm{PGL}(2,\mathbb{Q}_p)$
Isotopy group K	$\mathrm{SO}(2,\mathbb{R})$	$\mathrm{PGL}(2,\mathbb{Z}_p)$
Boundary	$\mathbb{R}$	$\mathbb{Q}_p$

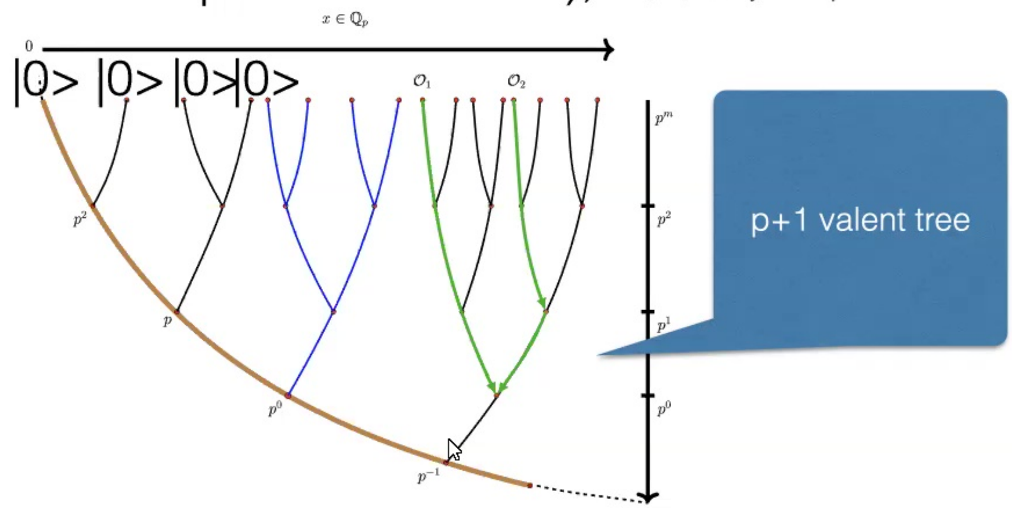
$\mathbb{H} \equiv \mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2,\mathbb{R})$ vs $\mathbb{H}_p \equiv \frac{\mathrm{PGL}(2,\mathbb{Q}_p)}{\mathrm{PGL}(2,\mathbb{Z}_p)}$

The bulk is discrete.

$PGL(2, Q_p)$ acts transitively.

Putting together the tensor network and the Bruhat-Tits tree: a proprosal for partition function and correlation functions

(tensor network of the partition function), HLY, Li, Melby-Thompson 2019



partition function

(C_{ij1} can be diagonalised)

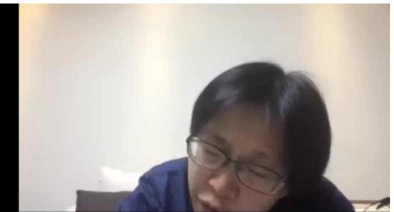
Graph Laplacian

$$\Box\phi(v) = \sum_{u\sim v}(\phi(u) - \phi(v))$$

- consider $G(v_1, v_2) = p^{-\Delta d(v_1,v_2)}$
- p+1 = valancy of graph
- We have $(\Box_{v_1} + m^2)G(v_1, v_2) = \mathcal{N}\delta_{v_1,v_2}$

$$m^2 = -\frac{1}{\zeta_p(\Delta-1)\zeta(\Delta)}, \qquad \zeta_p(s) \equiv \frac{1}{1-p^{-s}}$$

Putting together the tensor network and the BT tree

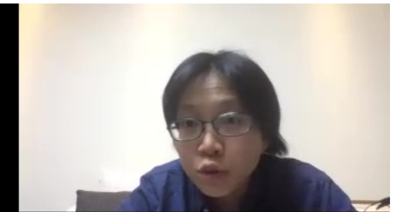


the labels of the tensors are primaries of the CFT

p=2 $$G_{I_1 I_2 I_3} = p^{-\Delta_{I_1} - \Delta_{I_2} - \Delta_{I_3}} C_{I_1 I_2 I_3}$$

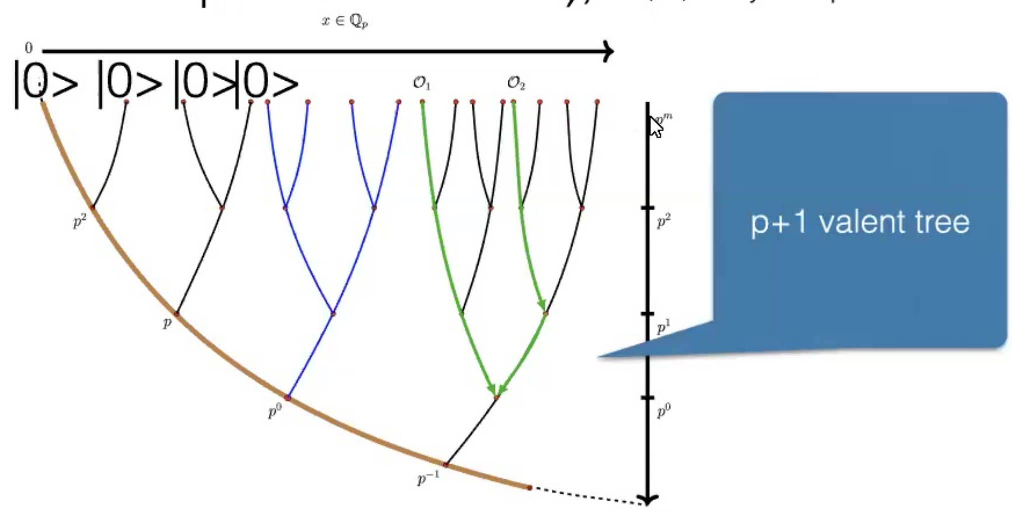
$$G_{I_1 I_2 I_3 \cdots I_{p+1}} = p^{-\Delta_{I_1} - \Delta_{I_2} - \Delta_{I_3} - \cdots \Delta_{I_{p+1}}} C_{I_1 I_2 I_3 \cdots I_{p+1}}$$

$$C_{I_1 \cdots I_n} = C^{J_1}_{I_1 I_2} C^{J_2}_{J_1 I_3} \cdots C_{J_{n-2} I_{n-1} I_n}$$



Putting together the tensor network and the Bruhat-Tits tree: a proprosal for partition function and correlation functions

(tensor network of the partition function), HLY, Li, Melby-Thompson 2019

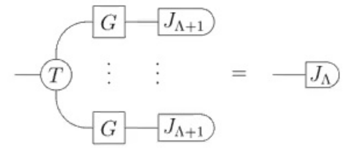


partition function

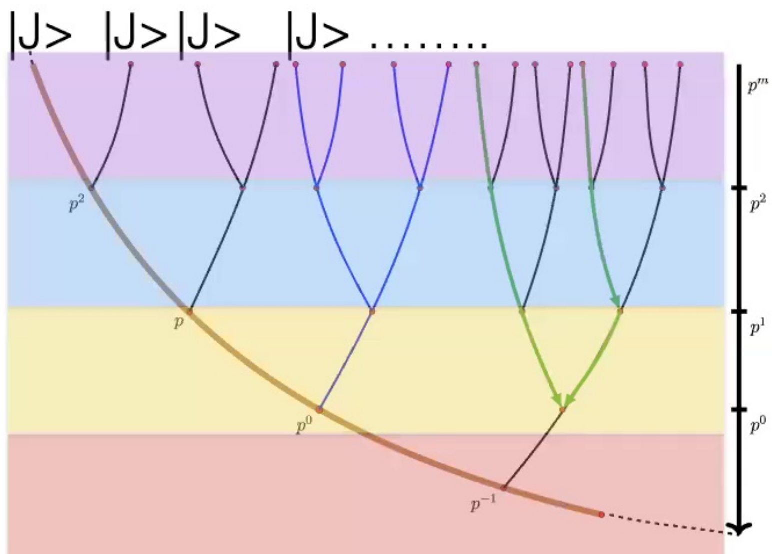
(C_{ij1} can be diagonalised)

P-adic RG flow

HLY, Li, Melby-Thompson 2019



Define RG flow $|J_n\rangle = |0\rangle + J_n^a|a\rangle$



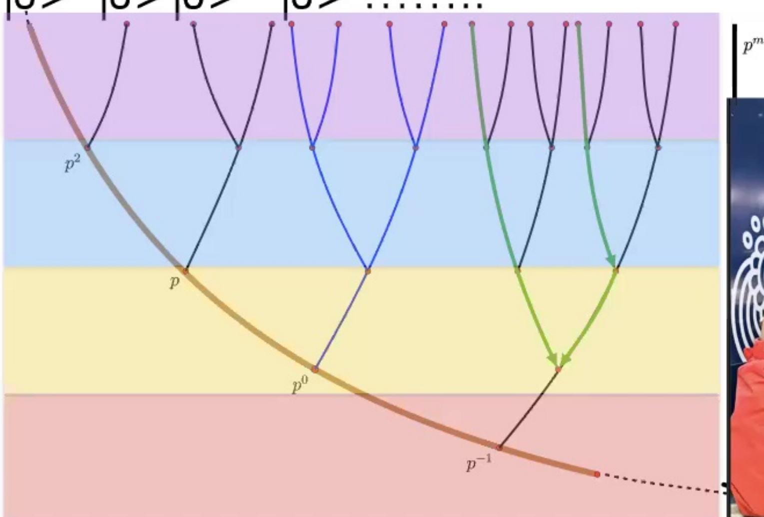
Is there some analogue of "Einstein Equation" that can describe this flow?

Deforming from pure pAdS space

Define RG flow

$$|J_n\rangle = |0\rangle + J_n^a|a\rangle$$

|J> |J> |J> |J> ....

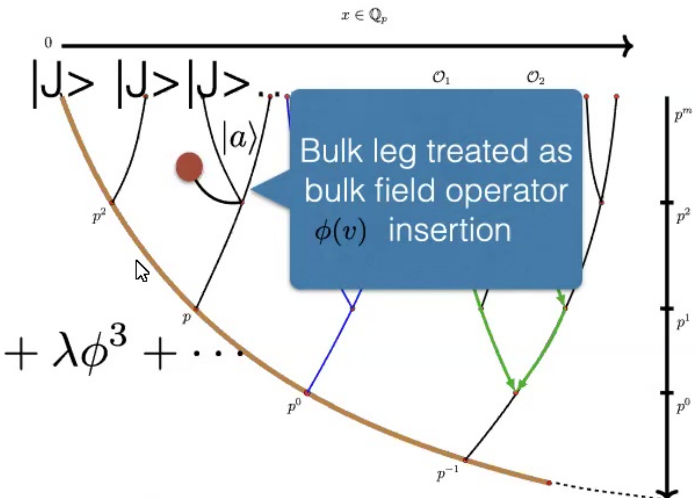


Lin Chen
(Fudan)

Xirong Liu
(Fudan)

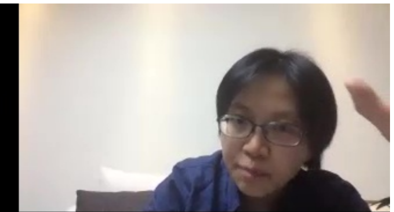
Recipe of Einstein equation (Some Guesses……)

1. Bulk expectation value of some scalar field. Compute stress tensor assuming the simplest possible kinetic term + possible interaction terms…



$$\partial\phi\partial\phi + m^2\phi^2 + \lambda\phi^3 + \cdots$$

$$\langle\phi_a(v)\rangle = \sum_{x_b,a} G^{\Delta_a}(x_b,v)J_a + \frac{1}{2}\sum_{a,b,c}\sum_{x_b,y_b,z} J_bJ_cC_{abc}G^{\Delta_b}(x_b,z)G^{\Delta_c}(y_b,z)G^{\Delta_a}(z,v) + \mathcal{O}(J^3)$$

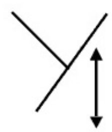


Recipe of Einstein equation (Some Guesses……)

2. A notion of distance on the links.

$$j_n \equiv 1 - \frac{|\langle J_{\Lambda-n}|J_{\Lambda-n-1}\rangle|^2}{\langle J_{\Lambda-n}|J_{\Lambda-n}\rangle\langle J_{\Lambda-n-1}|J_{\Lambda-n-1}\rangle}. \qquad ?$$

The overlap between the "renormalised boundary condition" with the next one is some kind of distance between two vertices? or some kind of angles between them = local curvature?



Recipe of Einstein equation (Some Guesses……)

2. A notion of distance on the links.

The overlap between the "renormalised boundary condition" with the next one is some kind of distance between two vertices? or some kind of angles between them = local curvature?

$$j_n = (\partial\phi)^2 + \lambda(\phi_{v_1} + \phi_{v_2})(\partial\phi)^2 \quad + O(J^4)$$

$$\lambda = -C^{\epsilon}_{\epsilon\epsilon}\frac{(p-1)(2p^{2\Delta} + p^{\Delta+1} + p)}{(p^{\Delta}+1)(p^{3\Delta} - p^2)}$$

$$\Box j_n \;=\; p(j_n - j_{n-1}) + (j_n - j_{n+1}).$$

$$\Box j_n = (-p^{2\Delta-1} - p^{2-2\Delta} + p + 1)(\partial\phi)^2 + C^{\epsilon}_{\epsilon\epsilon} C_0 \phi^3_{v_1} \quad + O(J^4)$$

$$C_0 \;=\; (p-1)p^{-6\Delta-2}\left(p - p^{\Delta}\right)^2\left(p^{\Delta}+p\right)\frac{\left(p^{8\Delta} + p^{9\Delta} + (p+1)p^{\Delta+5} + p^{2\Delta+5} - p^{3\Delta+3} - (2p+1)p^{4\Delta+3} - 2(p+1)p^{5\Delta+2} + 2p^{7\Delta+1} + p^6\right)}{\left(p^{\Delta}+1\right)\left(p^{3\Delta} - p^2\right)}$$

Recipe of Einstein equation (Some Guesses……)

2. A notion of distance on the links.

The overlap between the "renormalised boundary condition" with the next one is some kind of distance between two vertices? or some kind of angles between them = local curvature?

$$j_n = (\partial\phi)^2 + \lambda(\phi_{v_1} + \phi_{v_2})(\partial\phi)^2$$

$$\Box j_n = (-p^{2\Delta-1} - p^{2-2\Delta} + p + 1)(\partial\phi)^2 + C^{\epsilon}_{\epsilon\epsilon} C_0 \phi^3_{v_1}$$

+ O(J^4)

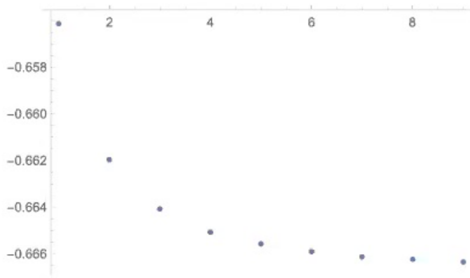
$$S = \sum_{\langle xy \rangle} (\kappa_{xy} - 2\Lambda) + \sum_{\langle xy \rangle} \frac{J_{xy}}{2} (\phi_x - \phi_y)^2 + \sum_x \frac{m^2}{2} \phi_x^2 ,$$

Gubser, Heydeman, Jepsen, Marcolli, Parikh, Saberi, Stoica, Trundy

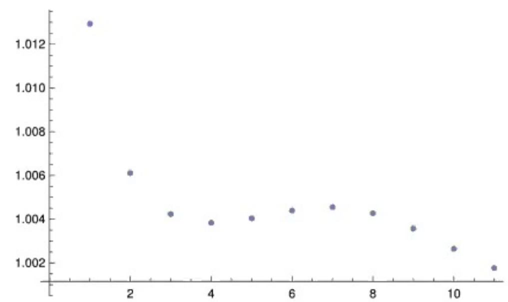
$$\kappa_{xy} + 2\frac{q-1}{q+1} = -\frac{q-3}{2(q+1)^2} \Box j_{xy} + O(j^2) .$$

$$\frac{1}{2(q+1)} \Box j_{xy} + O(j^2) ,$$

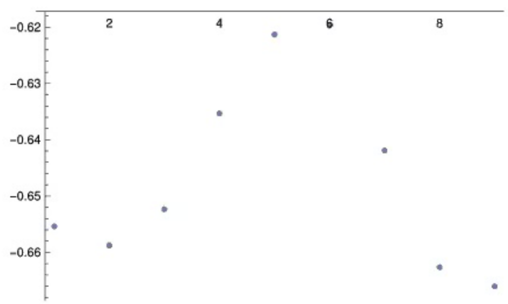
A remark: Monotonicity of “j” ?
testing it with the Ising fusion data,
tuning the conformal dimension (in progress)



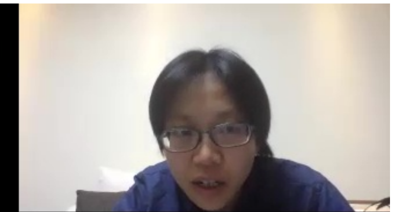
$\Delta_\sigma = \sqrt{2}$



$\Delta_\sigma = 0.2$



$\Delta_\sigma = 0.9$



We destroy monotonicity as the conformal dimension of sigma drops below 1 apparently….

Another way of deforming away from "pure AdS" — A view from *"p-adic Chern-Simons theory"*

The p-adic tensor network is a Wilson line network with gauge group PGL(2,Qp).

LYH, Li, Melby-Thompson 2018

Construct global gauge potential :

Take reference point $f_0^t = (1,0), \qquad g_0^t = (0,1)$

$$g(x) = \Gamma_p^{i_1}\Gamma_p^{i_2}\cdots\Gamma_p^{i_{d(P,x)}}$$

$$\langle\langle \vec{f},\vec{g}\rangle\rangle_v = \langle\langle \begin{pmatrix} p^n \\ 0 \end{pmatrix}, \begin{pmatrix} x^{(n)} \\ 1 \end{pmatrix} \rangle\rangle \qquad \longleftrightarrow \qquad \mathfrak{g}(v) = \begin{pmatrix} p^n & x^{(n)} \\ 0 & 1 \end{pmatrix}$$

$$W(x \to y) = g(x)g(y)^{-1}$$

$$\left[\prod_{i=1}^{3}\langle\Delta_i|\right]\hat{\mathfrak{W}}_{\Delta_i}(v_a \to v_i)|\mathcal{S}\rangle$$

We deform the p-adic connection getting help from the pure AdS case

$$A = \begin{pmatrix} -\frac{1}{2}d\rho & e^{\rho}dz \\ \frac{4G}{l}L(z)e^{-\rho}dz & \frac{1}{2}d\rho \end{pmatrix}.$$

- In Qp, the Lie algebra of the matrix group is not very well defined —
  because infinitesimal stuff (in terms of p-adic norm) exponentiated could lead to a divergent p-adic norm.

- Also the BT tree is discrete, and so the shortest Wilson lines should at least connect two nearest neighbours on the tree.
- Let us formally exponentiate the AdS result

$$\begin{aligned} \mathfrak{W}(v_1 \to v_2) &= P\exp\left(\int_{v_1}^{v_2} A_\mu(\xi)d\xi^\mu\right) \\ &= P\exp\left(\int_{z_1}^{z_2} A_z(\rho_1, z)dz\right) \cdot P\exp\left(\int_{\rho_1}^{\rho_2} A_\rho(\rho, z_2)d\rho\right). \end{aligned}$$

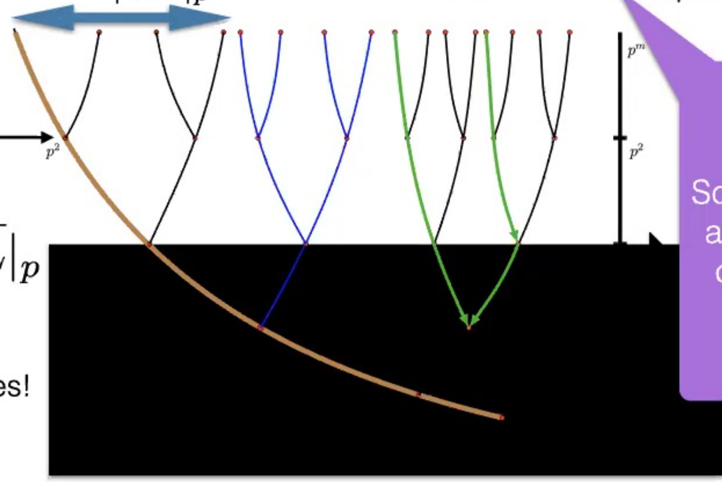
$$\mathfrak{W}(v_1 \to v_2) = \begin{pmatrix} e^{-\mathrm{d}\rho}\cosh\left(\frac{\mathrm{dz}\sqrt{L}}{\sqrt{k}}\right) & \frac{e^{\rho_1}\sqrt{k}\sinh\left(\frac{\mathrm{dz}\sqrt{L}}{\sqrt{k}}\right)}{\sqrt{L}} \\ \frac{e^{-\rho_2}\sqrt{L}\sinh\left(\frac{\mathrm{dz}\sqrt{L}}{\sqrt{k}}\right)}{\sqrt{k}} & \cosh\left(\frac{\mathrm{dz}\sqrt{L}}{\sqrt{k}}\right) \end{pmatrix} e^{\mathrm{d}\rho/2},$$

We deform the p-adic connection getting help from the pure AdS case

To ensure that the exponential has finite p-adic norm, we have $|dx|_p < \frac{1}{|\sqrt{L}|_p}$

Periodicity emerges: $\beta = \frac{1}{|\sqrt{L}|_p}$? $\rightarrow$ $S \sim |\sqrt{L}|_p$

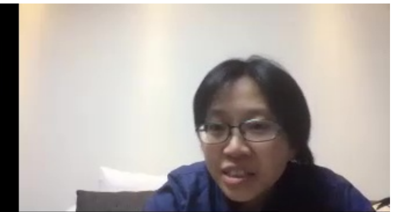
Locally the tenors take exactly the same value as before.

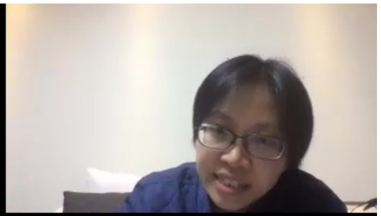


$r \leq |\sqrt{L}|_p$

When the bound is violated, the Wilson line expectation value vanishes! There is effectively a horizon (?!)

Scales in the same way as number of vertices cut by the "*horizon*".



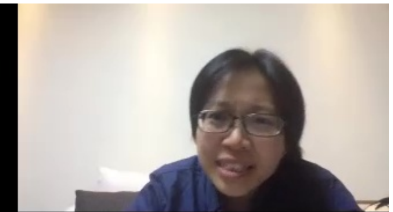


Part II -Towards building a holographic network describing realistic CFTs??

Ruoshui Wang
(Cornell)

Xiangdong Zeng
(Fudan)

Ce Shen
(Fudan)



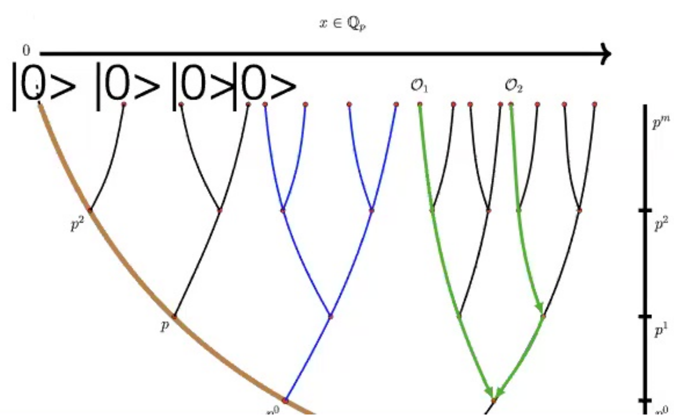
Towards building a holographic network describing realistic CFTs??

Some lessons learned…

- The tensor network describing the partition function rather than the wavefunction preserve more isometries allowing more quantitative manipulations.

- Our partition function takes the form of a "strange correlator"

You, Bi, Rasmussen, Slagle, Xu 2013



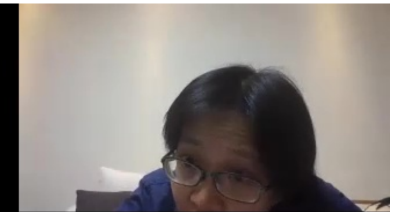
Minimal models and Levin Wen models

1. Tensor Categories can be used to construct Hamiltonians of CFT minimal models
Feiguin, Trebst, Ludwig, Troyes, Kitaev, Wang, Freedman PRL 2007;

2. The partition functions of minimal models can be thought of as imposing boundary conditions on a corresponding topological model defined using these tensor categorical data
Aaesen, Fendley, Mong J. Phys. A; Math. Theor. 2016; 2020 ;

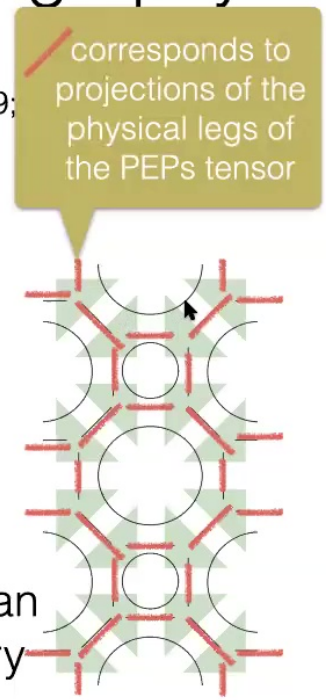
3. There is a strange correlator representation of these CFT partition functions — the overlap between a direct product state and a Levin - Wen wavefunction
Bal, Williamson, Vanhove, Bultinck, Haegeman, Verstraete PRL 2018;
Lootens, Vanhove, Verstraete PRL 2019

There are beautiful tensor network (PEPs) construction



Minimal models and Levin Wen models — and holography

- PEPS representation of Levin-Wen models
  Gu, Levin, Swingle, Wen PRB 2009; Buershaper, Aguado, Vidal PRB 2009; (More recently — the form we follow closely , is presented in Bultinck, Marien, williamson, Sahinoglu, Haegeman, Verstraete Annals of physics 2017; Williamson, Bultinck, Verstraete 2017)



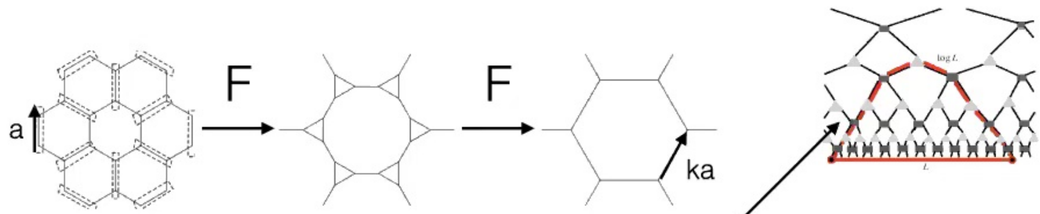
$$\langle \Omega_N | \Psi_a^{LW} \rangle$$

- The idea of Levin-Wen PEPs tensor network may recover some form of holography was discussed.
  Luo, Lake, Wu PRB 2017

- The Levin Wen wavefunction being topological, can be transformed using Alexandre moves to arbitrary triangulations. — the strange correlator can be coarse grained by repeated use of these moves of the Levin- Wen wavefunction

Minimal models and Levin Wen models — and holography

$|\Omega_N\rangle = \prod_i$ 

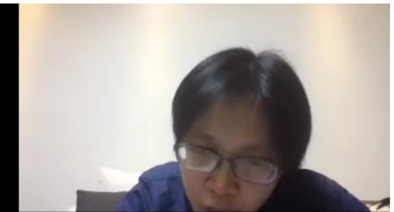


$$\langle \Omega_N | \Psi_a^{LW} \rangle \qquad \langle \Omega_N | F F | \Psi_{ka}^{LW} \rangle = \langle \Omega_{N-1} | \Psi_{ka}^{LW} \rangle$$

In previous works, this is used to assist usual procedure of tensor network renormalisation.

Evenbly, Vidal PRL 2014; Bal, Williamson, Vanhove, Bultinck, Haegeman, Verstraete PRL 2018; Lootens, Vanhove, Verstraete PRL 2019

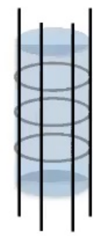
Here, we make the (perhaps obvious ;P) observation that $\langle \Omega_N | F F F \cdots$ looks like Euclidean AdS3. Isn't it in fact an analytic holographic tensor network !



Minimal models and Levin Wen models — and holography

Operator Insertion:

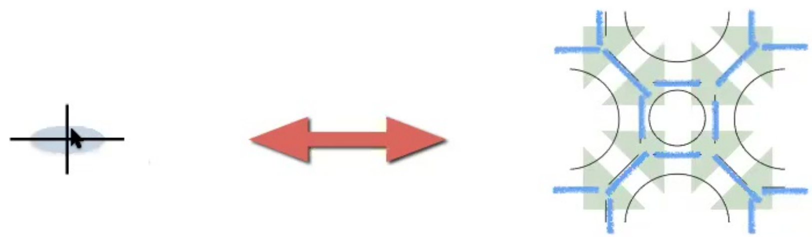
They are eigenstates of a cylinder



Boundary conditions vs operator insertion.

A different boundary projection

We check in the case of the Ising the primaries can be obtained from changing the boundary conditions of the Levin-Wen PEPs tensor network

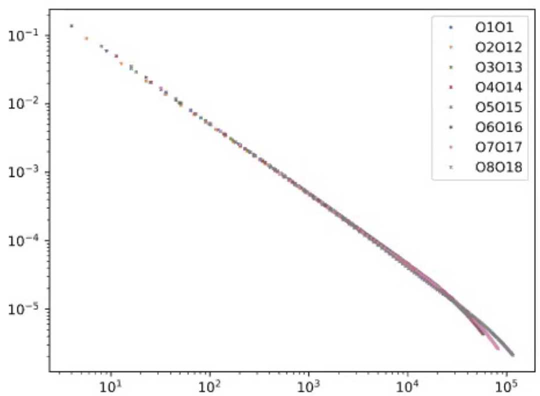


Looks like a direct analogue of the p-adic tensor network?

Minimal models and Levin Wen models — and holography

Preliminary result for a bulk boundary propagator:

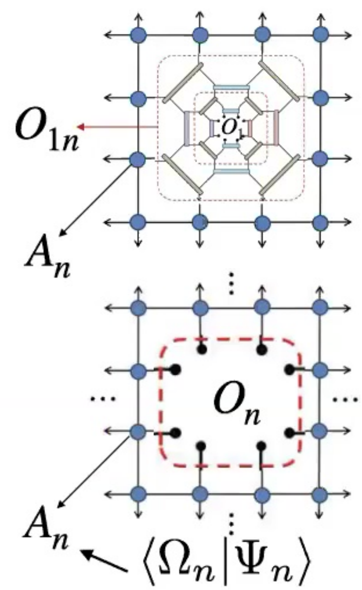
$< O_1 O_{1n} > \quad vs. \quad z_n x_n^2$



$$< O_n O_{1n} > \sim \left(\frac{z_n}{x_1^2 + z_n^2}\right)^{\Delta} = \left(\frac{1}{z_n(x_n^2+1)}\right)^{\Delta} \qquad x_1 = z_n x_n, \quad z_n = \left(\sqrt{2}\right)^{n-1}$$

er.. looks like the bulk insertion didn't recover the right descendants, but only the primary! — this should be an issue of correctly dealing with sub-AdS locality in the network .

Picture courtesy Vidal et al 2014



Bar, Can, Carroll, Chatwin-Davies, Hunter-Jones, Pollack, Remmen, 2015

Outlook

- Our result is some discretised realisation of Sung-Sik's new paper. arXiv:2009.11880
- It is suggested that the path-integral of a d-dimensional field theory can be thought of as the overlap of two wave functions in d+1 dimensions.

$$Z = \langle \mathbb{I} | S \rangle$$

- The identity being a state invariant under RG, so that one could evolve the bra with RG flow operator H, but then group it with S that leads to flow of the couplings — this is
  very close in spirit to the strange correlator holographic network that we studied here.

Outlook

- Quantitative control of descendants which could allow control of sub-AdS locality and gravitational excitations (?)
  You, Milsted, Vidal 2018, 2020; You, Vidal 2020

- Generalization to higher dimensions, and Categorical symmetry
  Verstraete et al ; Gaiotto, Kulp 20;
  Kong, Zheng 2017; Ji, Wen 2019; Kong, Lan, Wen, Zhang, Zheng 2020 …..