Title: A tensor-network approach to fixed-point models of topological phases

Speakers: Andreas Bauer

Collection: Tensor Networks: from Simulations to Holography III

Date: November 17, 2020 - 1:00 PM

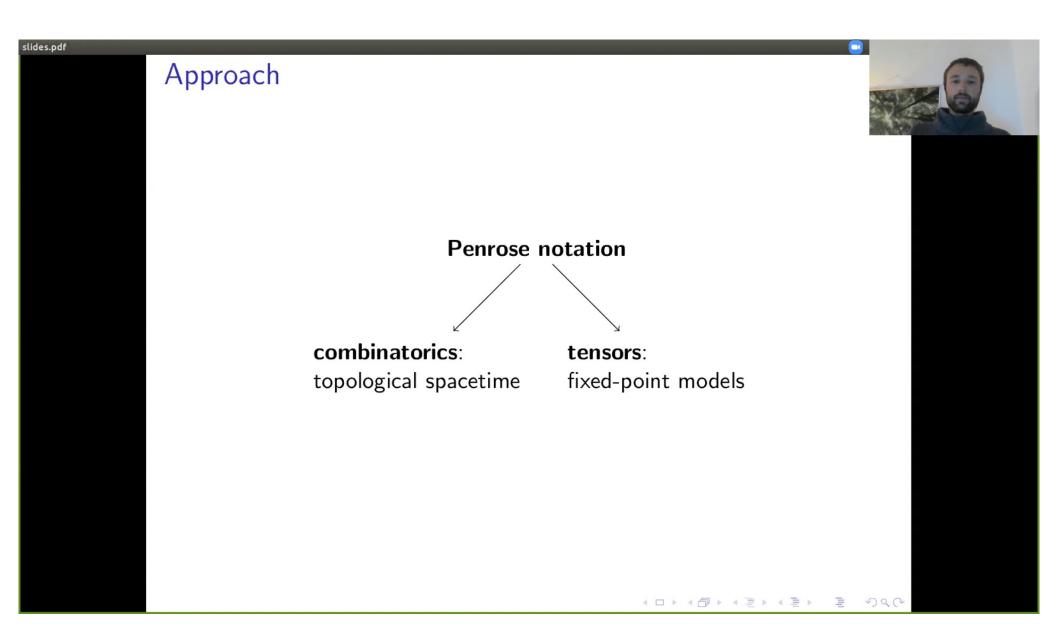
URL: http://pirsa.org/20110024

Abstract: "I will introduce a tensor-network based language for classifying topological phases via fixed-point models. The "models" will be tensor networks formalizing a discrete Euclidean path integral living in a topological space-time, and can be obtained from Hamiltonian models by Trotterizing the imaginary time evolution. Topological fixed-point models are invariant under topology-preserving space-time deformations. Space-time manifolds and homeomorphisms can be combinatorially represented by graph-like "networks", which together with "moves" form a "liquid". The networks can be interpreted as tensor networks, and the moves as equations which determine the fixed-point models. Different combinatorial representations of the same space-times yield new kinds of fixed-point models. Given the limited time, I will stick to very simple examples in 1+1 dimensions for this talk."

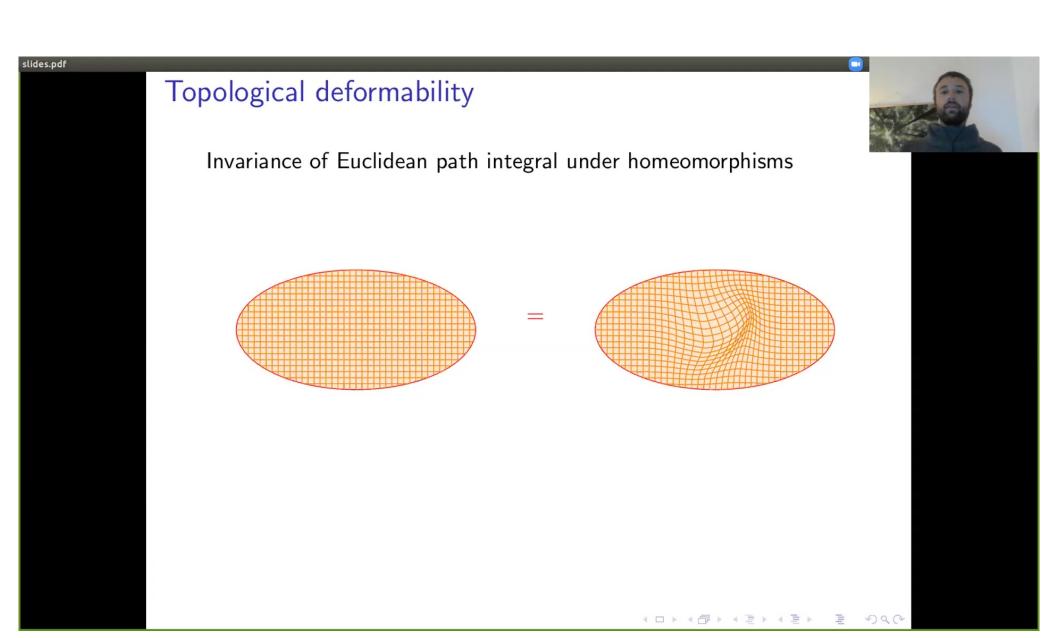
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slides.pdf A unified diagrammatic approach to topological fixed point models Andreas Bauer 17.11.2020

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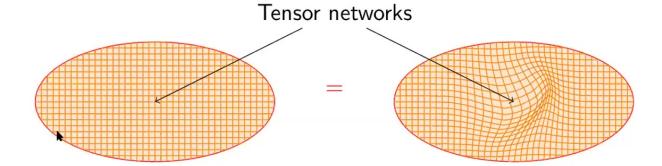
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Invariance of Euclidean path integral under homeomorphisms

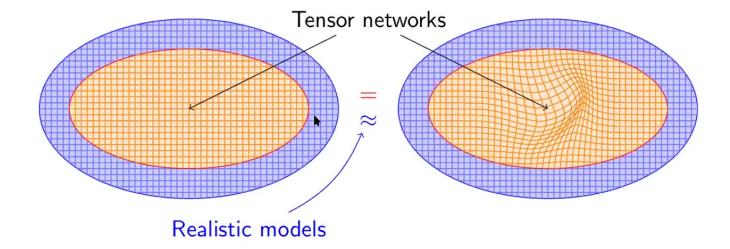




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Topological deformability

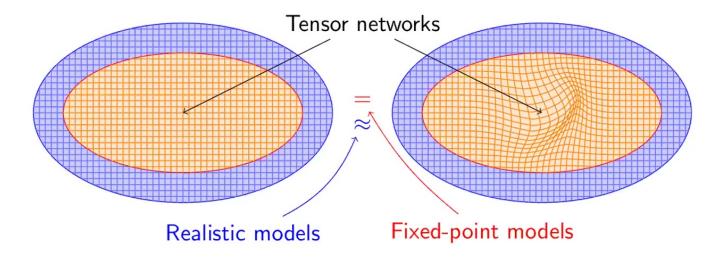
Invariance of Euclidean path integral under homeomorphisms



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Topological deformability

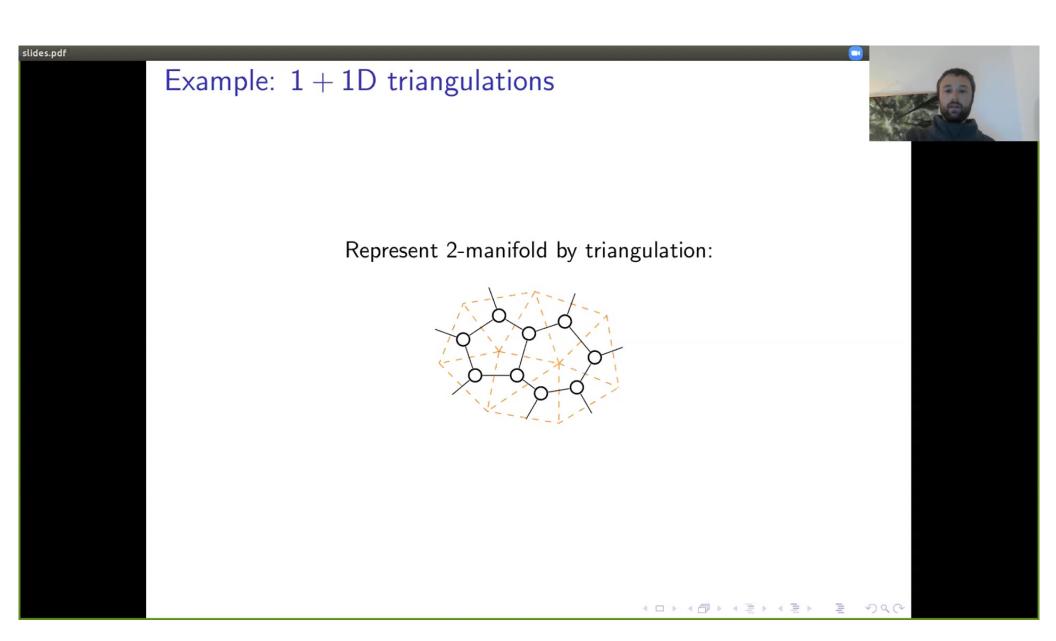
Invariance of Euclidean path integral under homeomorphisms



- ✓ Conventional topological order
- ✓ Ŝymmetry breaking order
- X Fracton order



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Represent homeomorphisms by Pachner moves:







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Example: 1+1D triangulations

#### Represent homeomorphisms by Pachner moves:

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Example: 1+1D triangulations



Represent homeomorphisms by Pachner moves:



- combinatorial structure = "liquid"
- ➤ 3-index tensor fulfiling the equations = "model of liquid"

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Example: 1+1D triangulations



Represent homeomorphisms by Pachner moves:

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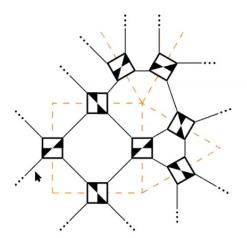
- combinatorial structure = "liquid"
- ➤ 3-index tensor fulfiling the equations = "model of liquid"
- equations can be interpreted in different "tensor types"



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 $1+1\mathsf{D}$  edge liquid

- ▶ There are other ways to represent 2-manifolds
- ► Toy example: One tensor at every edge



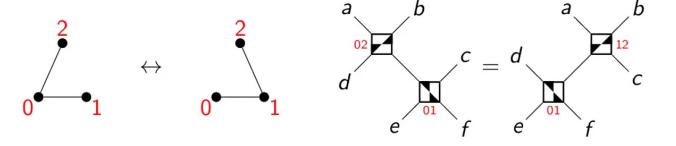


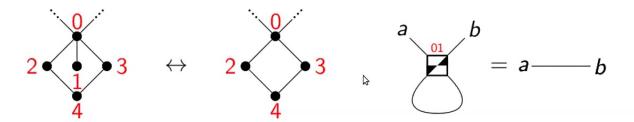
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# $1+1\mathsf{D}$ edge liquid



Moves:



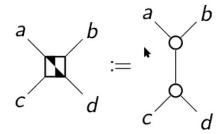


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# Equivalence between the two liquids

► Mapping edge-liquid → triangle-liquid:



► Mapping triangle-liquid → edge-liquid:

$$aa'$$
 $bb'$ 
 $= a$ 
 $b'$ 
 $b'$ 





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### Equivalence between the two liquids

► Mapping edge-liquid → triangle-liquid:

$$\begin{array}{c}
a \\
c
\end{array}
\qquad b \\
c
\qquad d$$

► Mapping triangle-liquid → edge-liquid:

$$aa'$$
 $bb'$ 
 $=$ 
 $a'$ 
 $b'$ 
 $b'$ 

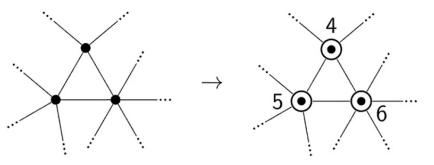
► Models for liquids describe the same phases



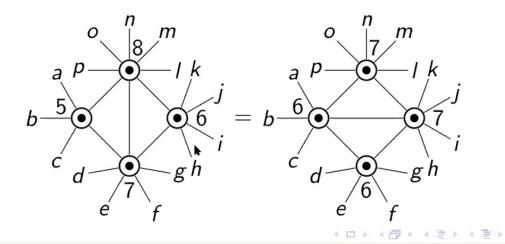
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# Vertex liquid

► Represent every vertex by a tensor

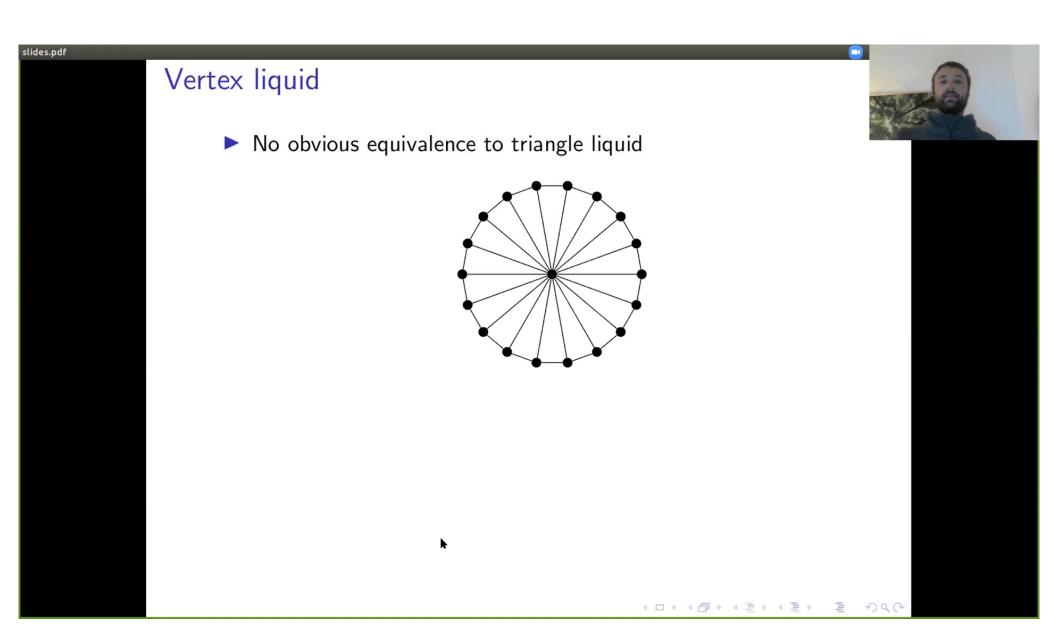


- ▶ 5-valent, 6-valent, . . . vertices represented by different tensors.
- Moves:



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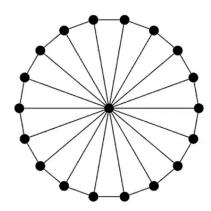


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### Vertex liquid



No obvious equivalence to triangle liquid



- ► ⇒ Could potentially contain models for more general phases
- ► Apparently not the case in 1+1D
- ► No standard boundary construction
- ▶ No standard commuting-projector construction
- ► Chiral phases in 2+1D?



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#### Other liquids



- ► Fixed-point models in any dimension, for boundaries/anyons/domain walls/defects, with/without orientation/spin structure/...
- ightharpoonup E.g., tensors as tetrahedra of 3D triangulation ightarrow string-net models
- ightharpoonup E.g., tensors at edges and faces of 3D cell complex ightharpoonup Kitaev quantum doubles for weak Hopf algebras
- More general liquids with possibly more general phases (chiral phases?)
- ► Conformal fixed-point models?



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