Title: Tensor networks for LGT: beyond 1D

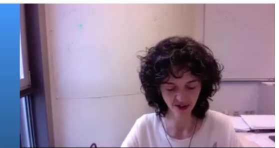
Speakers: Mari-Carmen Banuls

Collection: Tensor Networks: from Simulations to Holography III

Date: November 16, 2020 - 8:00 AM

URL: http://pirsa.org/20110020

Abstract: The suitability of tensor network ansatzes for the description of physically relevant states in one dimensional lattice gauge theories (LGT) has been demonstrated in the last years by a large amount of systematic studies, including abelian and non-abelian LGTs, and including scenarios where traditional Monte Carlo approaches fail due to a sign problem. While this establishes a solid motivation to extend the program to higher dimensions, a similar systematic study in two dimensions using PEPS requires dealing with specific considerations. Besides a larger computational costs associated to the higher spatial dimension, the presence of plaquette terms in LGTs hinders the efficiency of the most up-to-date PEPS algorithms. With a newly developed update strategy, nevertheless, such terms can be treated by the most efficient techniques. We have used this method to perform the first ab initio iPEPS study of a LGT in 2+1 dimensions: a Z3 invariant model, for which we have determined the phase diagram.



TNS for 2+1D Lattice Gauge Theories

Mari-Carmen Bañuls

with D. Robaina, P. Emonts, J.I. Cirac (MPQ), E. Zohar (HUJI)

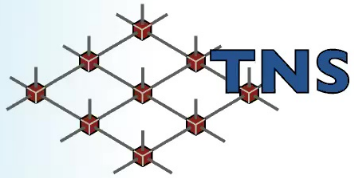
Max Planck Institut
of Quantum Optics
(Garching)

MCQST

November 16th, 2020

Using TNS for LGT

A NATURAL CONNECTION

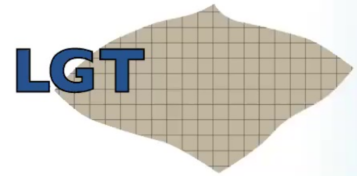


Non-perturbative for Hamiltonian systems

Extremely successful for 1D systems (MPS)

Promising developments for higher dimensions

ground states
low-lying excitations
thermal states
time evolution

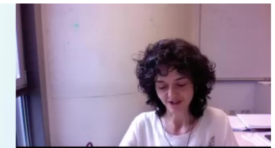


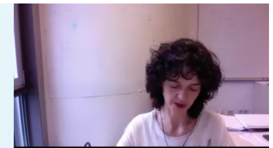
Non-perturbative way of solving QFT (QCD)

Mostly path-integral formalism & MC

4D lattice

spectrum
finite T
big 3+1 dimensional
chemical potential
time evolution





USING TNS FOR LGT

formal approach

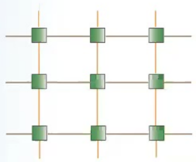
gauging the symmetry

explicitly invariant states

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

general prescriptions, U(1), SU(2)

numerical simulations **no sign problem**



TN describe partition functions (observables)

TRG approaches to classical and quantum models

Liu et al PRD 2013; Shimizu, Kuramashi, PRD 2014; Kawauchi, Takeda 2015;
review Meurice et al. 2010.06539

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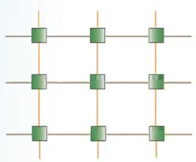

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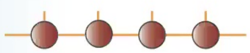


TN describe states

GENERAL STRATEGY

Hamiltonian formulation

acting on a Hilbert space

⟶ choose proper basis

Finite dimensional degrees of freedom

fermions

⟶ ✓ no sign problem

gauge bosons require attention

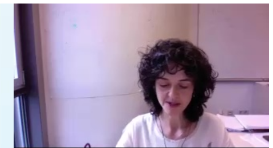
⟶ truncating, integrating out (also QLinks)

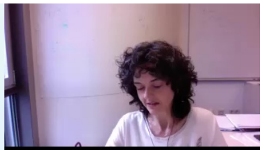
★ Common ingredients for quantum simulation

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

There are related topics…





There are related topics…

K. Cichy's talk on Thursday

TNS for other field theories

Thirring model, PRD100, 094504 (2019)
Thermal QFT dynamics, PRR2, 033301 (2020)

continuous TNS for QFT

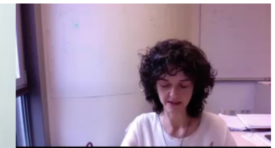
Verstraete, Cirac PRL104, 190405 (2010)
Tilloy, Cirac PRX9, 021040 (2019)

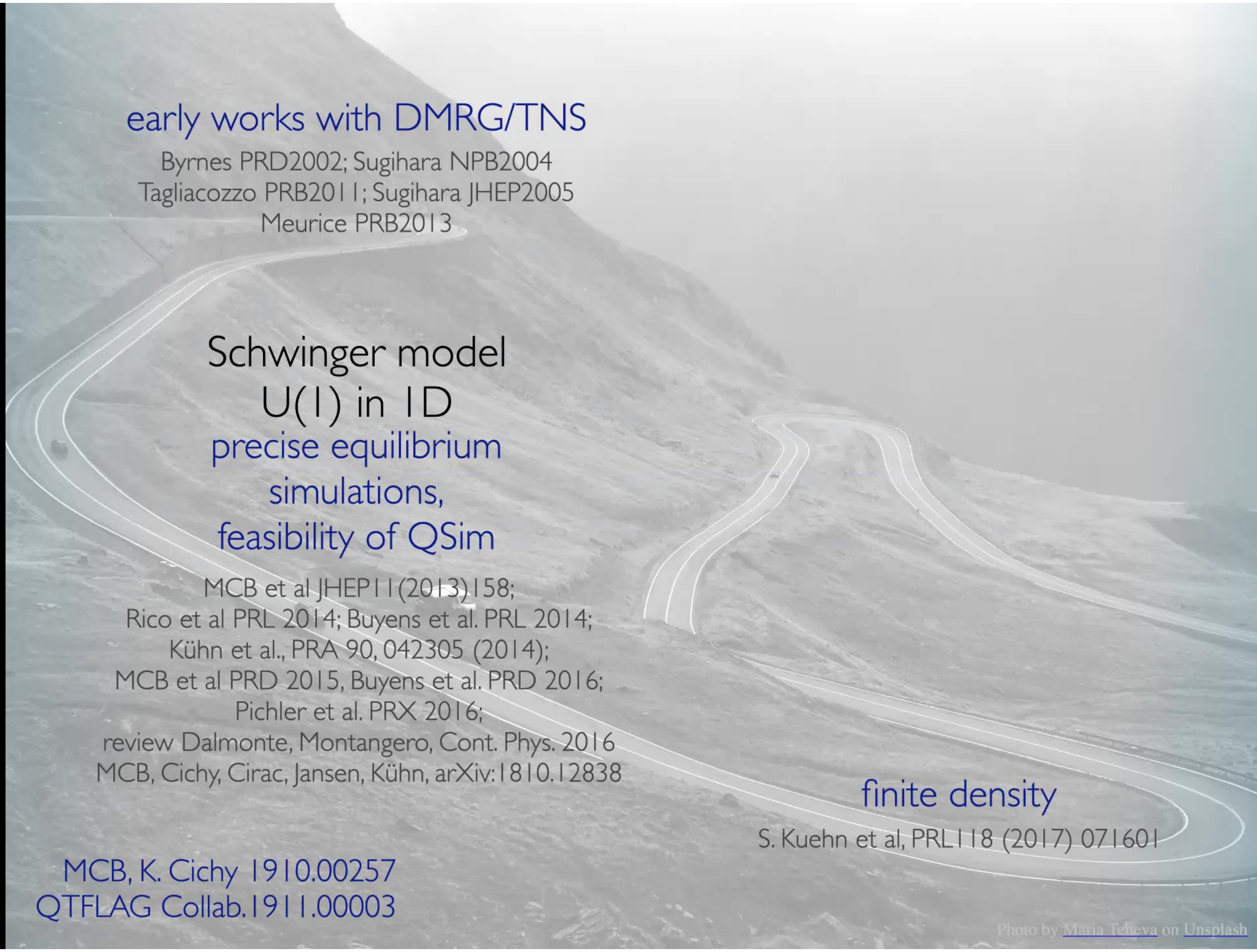

there is long way to go until LQCD
journey begins with 1+1D steps
Photo by Maria Teneva on Unsplash

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
Tagliacozzo PRB2011; Sugihara JHEP2005
Meurice PRB2013

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

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Schwinger model
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precise equilibrium
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MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016
MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838
finite density
S. Kuehn et al, PRL118 (2017) 071601
MCB, K. Cichy 1910.00257
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Photo by Maria Teneva on Unsplash


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MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

2+1 dimensions

Falser et al. arXiv:1911.09693
Robaina et al. arXiv:2007.11630
Emonts et al. PRD102, 074501 (2020)

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

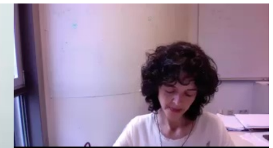
SU(3)QLM

Silvi et al, PRD 2019

finite density

S. Kuehn et al, PRL118 (2017) 071601

Photo by Maria Teneva on Unsplash



So far…

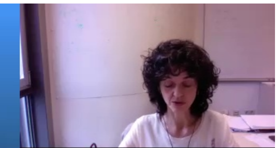
TNS feasibility for LQFT thoroughly tested in 1+1D

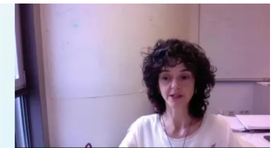
- high numerical precision attainable (controlled errors)
- spectrum, thermal equilibrium, finite density, (some) dynamics
- Abelian and non-Abelian models

some problems where std techniques don't work

…next step: simulations in 2+1 D

BEYOND ID





TNS FOR 2+2 D LGT

few results with other TNS

Tagliacozzo, Vidal PRB 2011
Felser et al. arXiv:1911.09693

explicitly gauge invariant PEPS

- restricted ansatz calculations

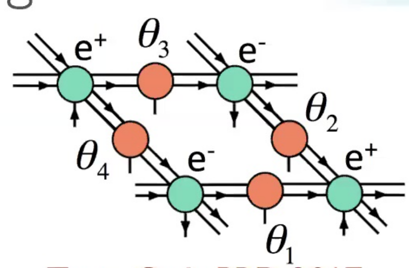
Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

- also fully Gaussian PEPS

Zohar, Cirac PRD 2018
Emonts et. al., PRD 102, 074501 (2020)

standard PEPS toolbox contains all ingredients for full variational computation

- computational cost, required D
- plaquette terms



Zapp, Orús PRD 2017

PEPS FOR LGT

computational cost, required D

great recent progress

Corboz PRB94, 035133 (2016)
Vanderstraeten et al. PRB 94, 155123 (2016)

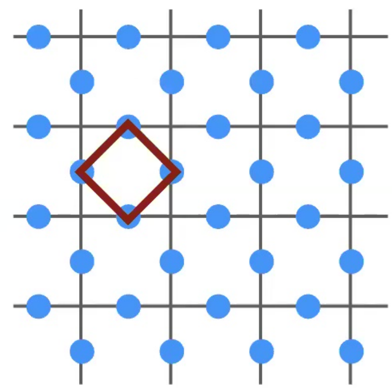
outperform other methods

Corboz PRB93, 045116 (2016)
PRL 113, 046402 (2014)

plaquette terms

optimizing four tensors at once ⇒ much larger cost

Dusuel et al. PRL 2011
Schulz et al. NJP 2012



A NEW STRATEGY

generalizes a strategy proposed for digital quantum simulation

Zohar et al. PRL 118, 070501 (2017); PRA 95, 023604 (2017); JPA 50, 085301(2017)

plaquette terms reduced to nearest neighbour at the cost of introducing ancillary dof

can also be used to impose Gauss' law

incorporate in iPEPS imaginary time evolution

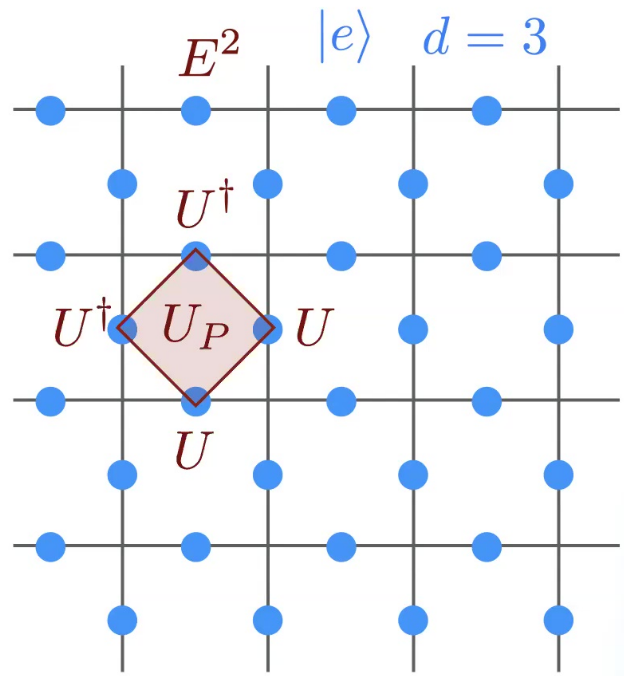
use it to study the phase diagram of Z_3 pure LGT

D. Robaina, MCB, J. I. Cirac, arXiv:2007.11630

Z_3 LGT

$$E|e\rangle = e|e\rangle$$

$$e \in \{-1, 0, 1\}$$



$$H = H_{\mathrm{E}} + H_{\square}$$

$$H_{\mathrm{E}} = \frac{g^2}{2} \sum_{\ell} E_{\ell}^2$$

$$H_{\square} = -\frac{1}{2g^2} \sum_{\square} U_P + U_P^\dagger$$

Z_3 LGT

$$E|e\rangle = e|e\rangle$$

$$e \in \{-1, 0, 1\}$$

$$U|e\rangle = |e-1\rangle$$

$$H = H_{\mathrm{E}} + H_{\square}$$

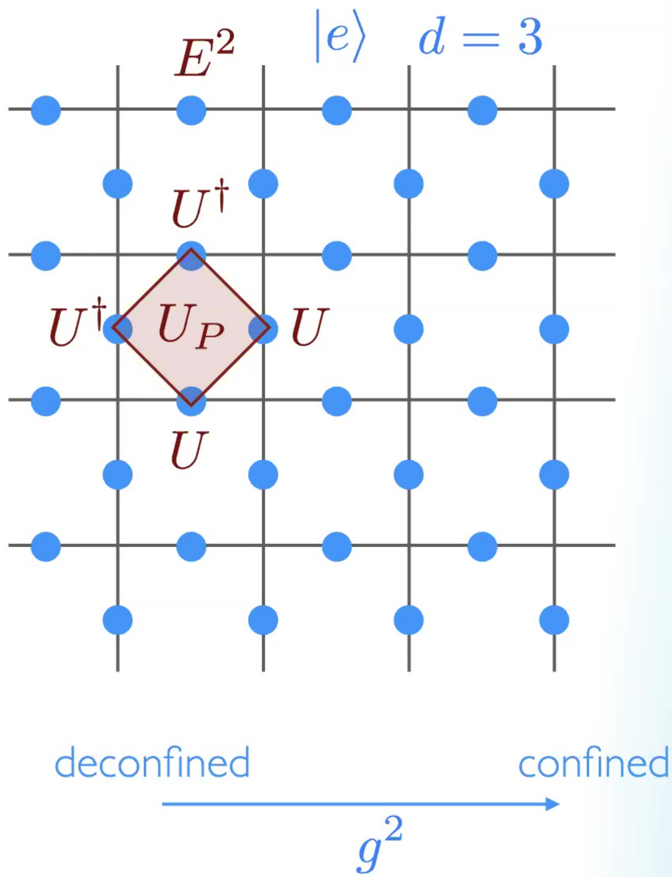
electric

$$H_{\mathrm{E}} = \frac{g^2}{2}\sum_{\ell} E_{\ell}^2$$

magnetic

$$H_{\square} = -\frac{1}{2g^2}\sum_{\square} U_P + U_P^\dagger$$

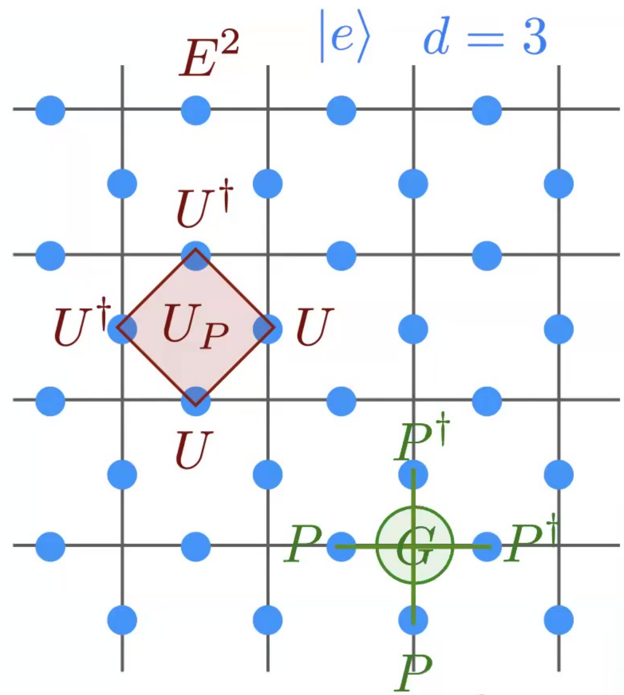
$Z_N \rightarrow U(1)$ as $N \rightarrow \infty$



Z_3 LGT

$$H = H_{\mathrm{E}} + H_{\square}$$

$$[H, G(x)] = 0$$



$$P = e^{i\frac{2\pi}{3}E}$$

$$e^{i\frac{2\pi}{3}(E_\ell + E_d - E_r - E_u)}$$

$\mathbb{Z}_3$ LGT

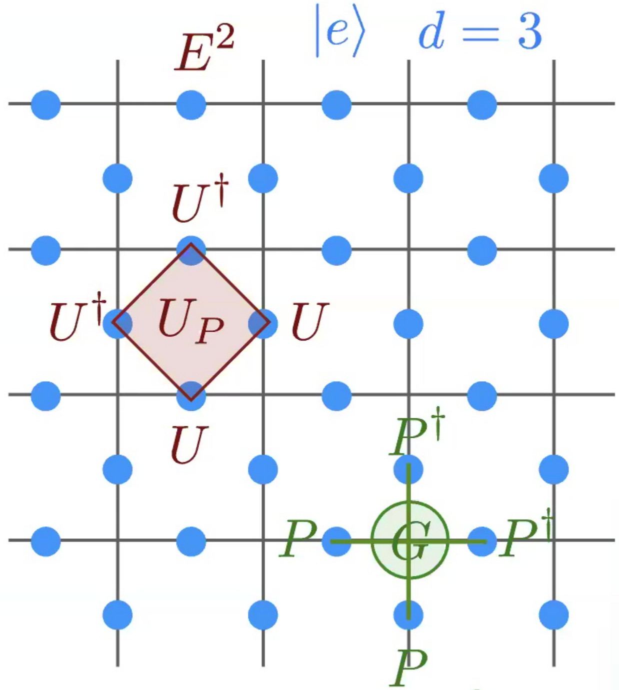
$$H = H_{\mathrm{E}} + H_{\square}$$

$$[H, G(x)] = 0$$

physical states: Gauss' law

$$G(\mathbf{x})|\psi\rangle = e^{\frac{2\pi i}{3} q(\mathbf{x})}|\psi\rangle$$

$q \in \{-1, 0, 1\}$



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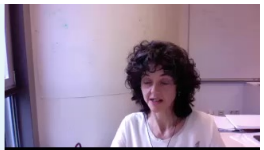
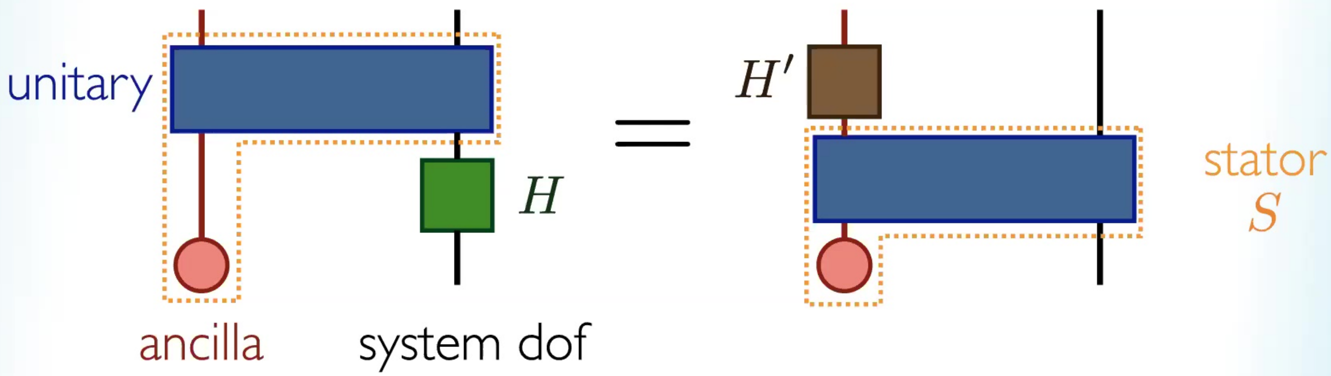


EFFECTIVE PLAQUETTE TERMS

strategy proposed for digital quantum simulation



$$SH = H'S$$

Zohar et al. PRL 118, 070501 (2017)
Zohar et al. PRA 95, 023604 (2017)
Zohar, JPA 50, 085301(2017)

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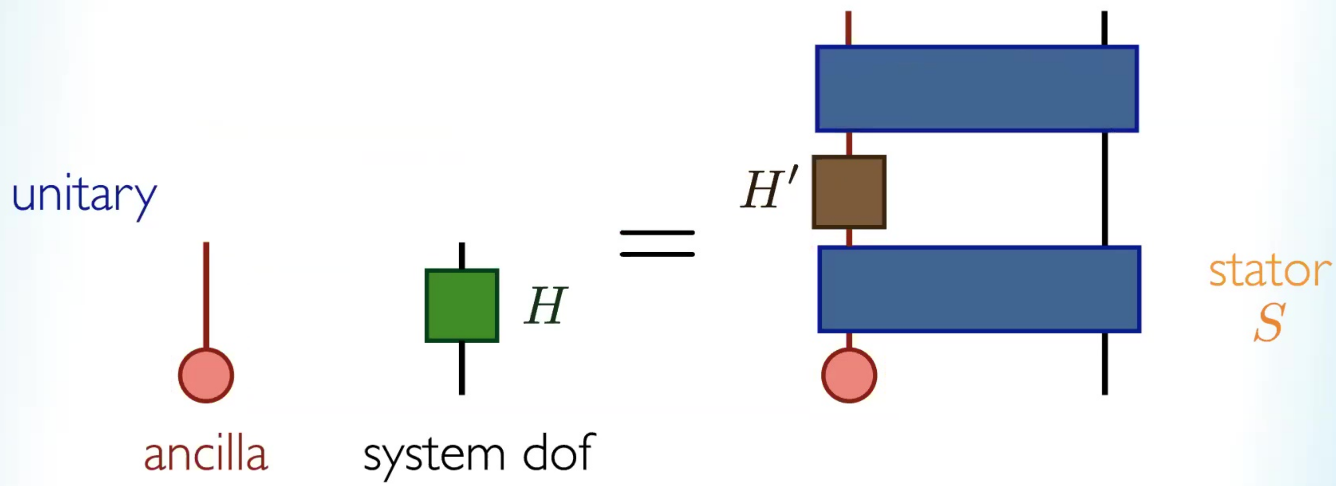

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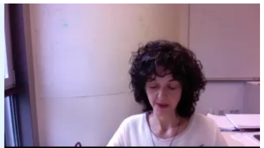

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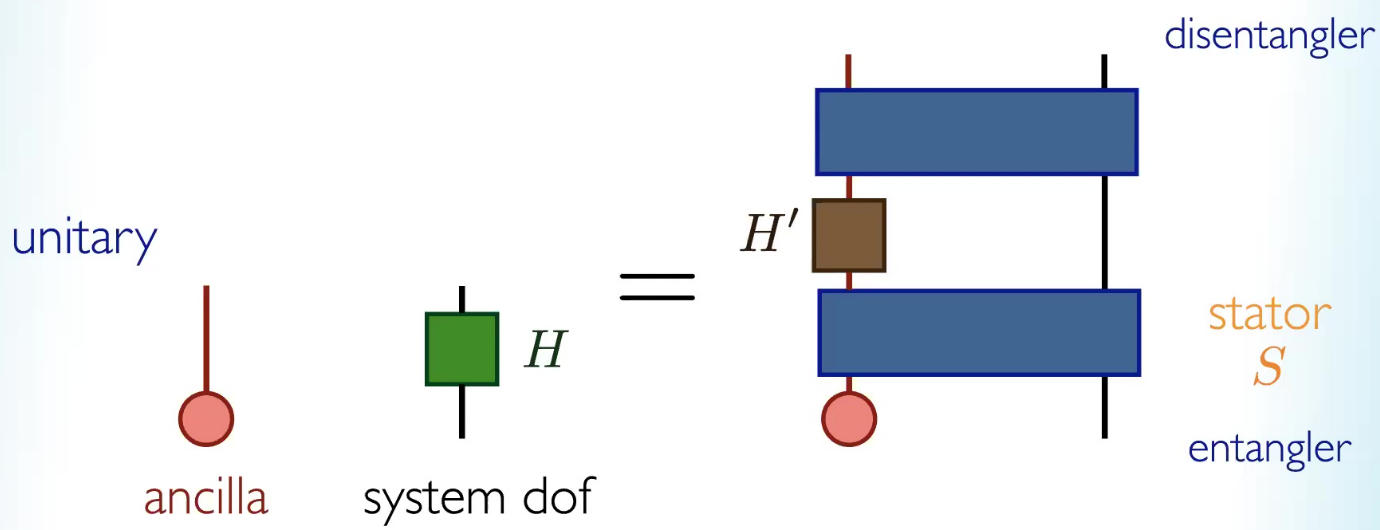

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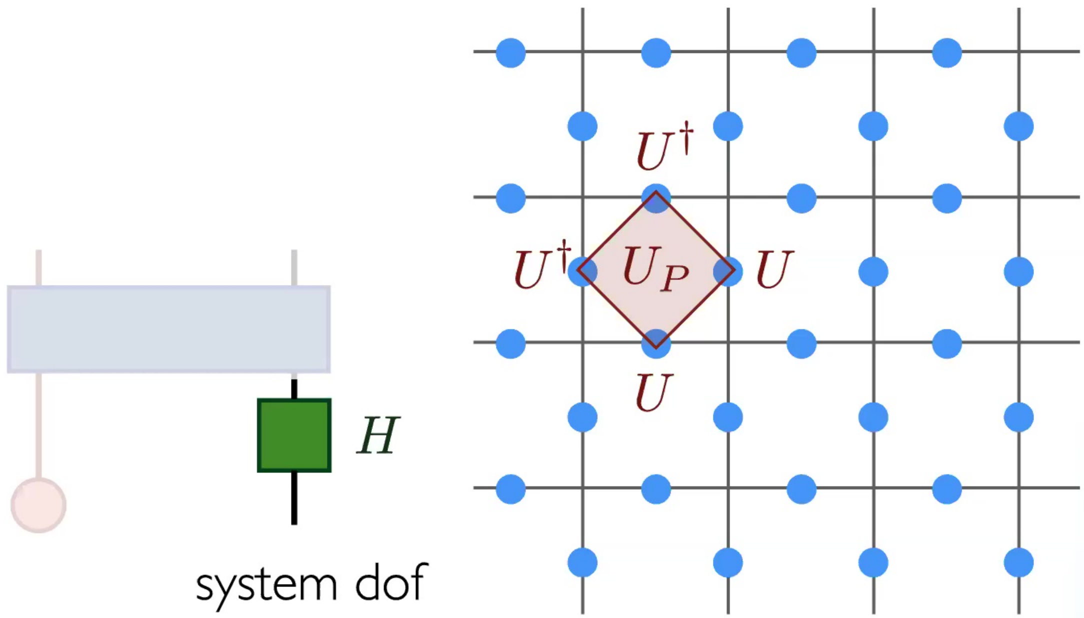

Zohar et al. PRL 118, 070501 (2017)
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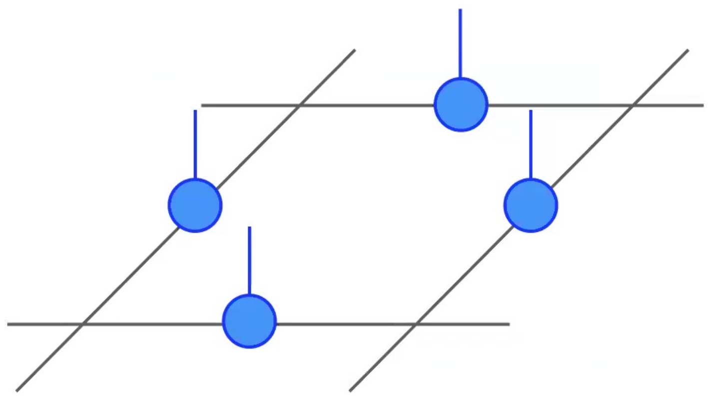
Zohar, JPA 50, 085301(2017)

EFFECTIVE PLAQUETTE TERMS

$S = W_{\square}|\text{in}\rangle$



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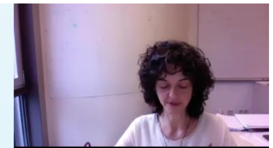
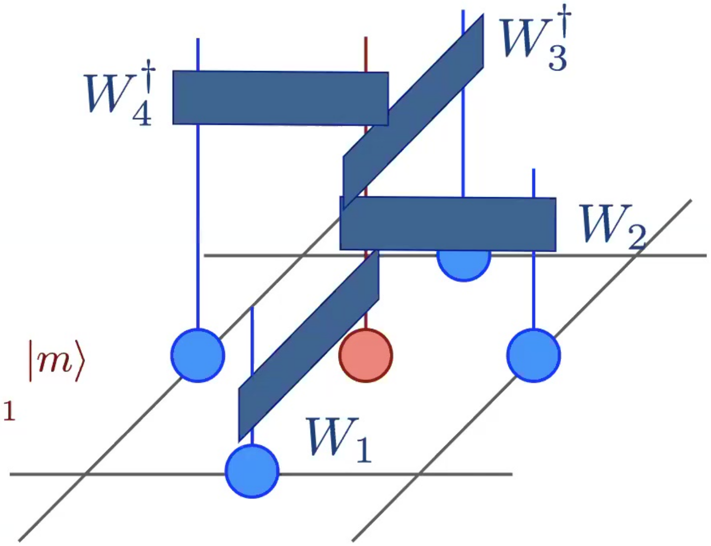


EFFECTIVE PLAQUETTE TERMS

$$S = W_{\square}|\text{in}\rangle$$

$$|\text{in}\rangle = \frac{1}{\sqrt{3}} \sum_{m=-1,0,1} |m\rangle$$



$$W = U \otimes P_1^{\text{anc}} + \mathbb{I} \otimes P_0^{\text{anc}} + U^\dagger \otimes P_{-1}^{\text{anc}}$$

Zohar et al. PRL 118, 070501 (2017)
Zohar et al. PRA 95, 023604 (2017)
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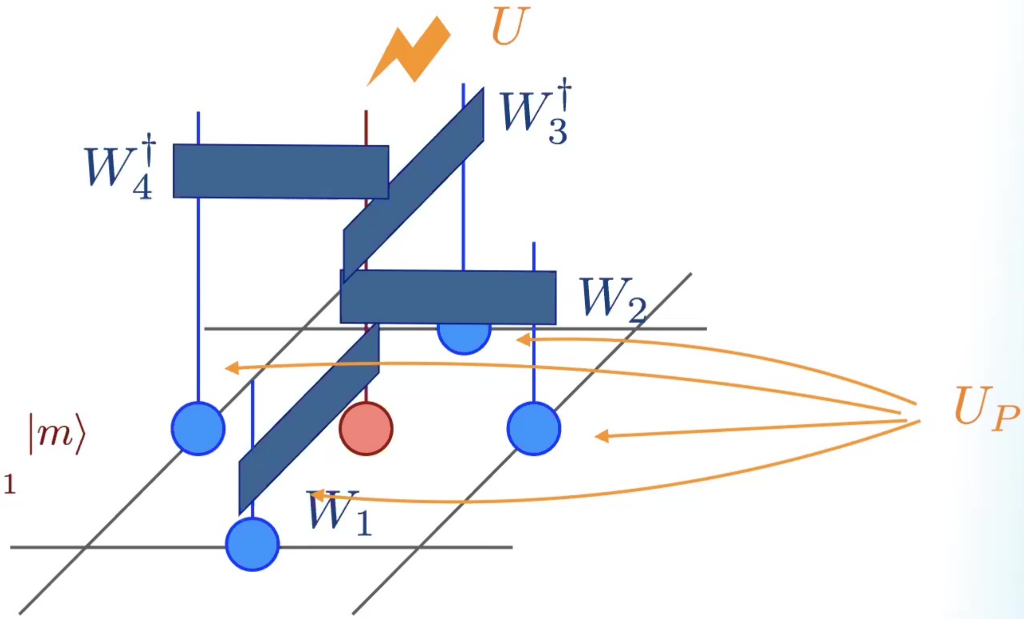

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Zohar, JPA 50, 085301(2017)

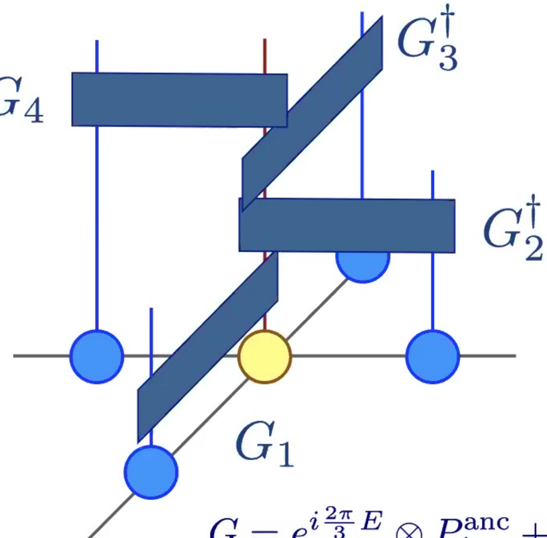
GAUSS' LAW

projector for external charge on a vertex

$$P_q = \frac{1}{3} \sum_{m=-1,0,1} e^{im\frac{2\pi}{3}(E_1 - E_2 - E_3 + E_4 - q(\mathbf{x}))}$$

$$S_G = G_+ |\text{in}\rangle$$

$$|\text{in}\rangle = \frac{1}{\sqrt{3}} \sum_{m=-1,0,1} |m\rangle$$

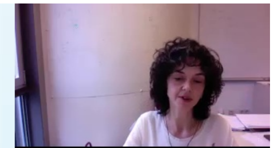


$$G = e^{i\frac{2\pi}{3}E} \otimes P_1^{\text{anc}} + \mathbb{I} \otimes P_0^{\text{anc}} + e^{-i\frac{2\pi}{3}E} \otimes P_{-1}^{\text{anc}}$$

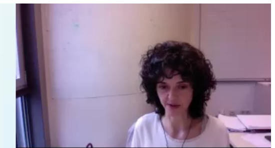
$$S_G P_q = \tilde{P}_q S_G$$

$$\tilde{P}_q = \mathbb{I} + e^{-i\frac{2\pi q}{3}} U + e^{i\frac{2\pi q}{3}} U^\dagger$$

Robaina, MCB, Cirac, arXiv:2007.11630



iPEPS SIMULATION



imaginary time evolution

$$\frac{e^{-\tau H}|\Phi\rangle}{\| e^{-\tau H}|\Phi\rangle \|} \rightarrow |E_0\rangle$$

local

using ancilla construction

$$e^{-\tau H} = \lim_{n\to\infty} \left(e^{-\frac{\delta_\tau}{2} H_E} e^{-\delta_\tau H_\square} e^{-\frac{\delta_\tau}{2} H_E} \right)^n$$

simple update used to evolve the tensors

environment contraction with CTM

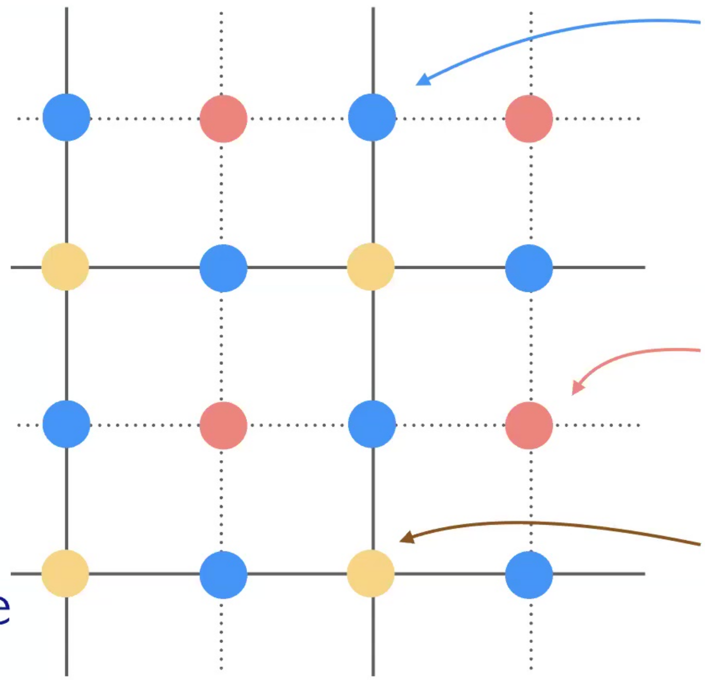
SyTen toolkit by C. Hubig https://syten.eu/

Robaina, MCB, Cirac, arXiv:2007.11630

iPEPS SIMULATION

unit cell

4×4



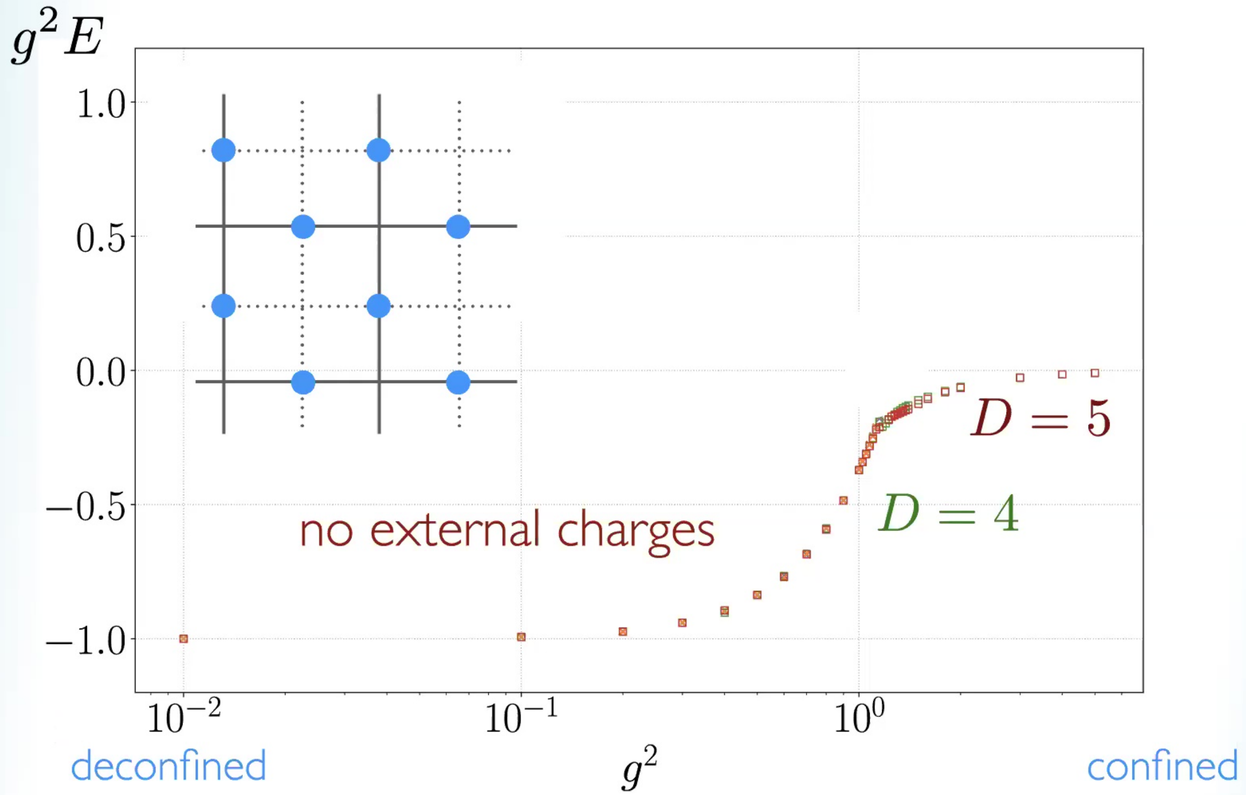
charge sector imposed at the beginning with vertex projector

Robaina, MCB, Cirac, arXiv:2007.11630

PHASE DIAGRAM

first order phase transition

Blöte, Swendsen, PRL 1979
Bhanot, Creutz, PRD 1980



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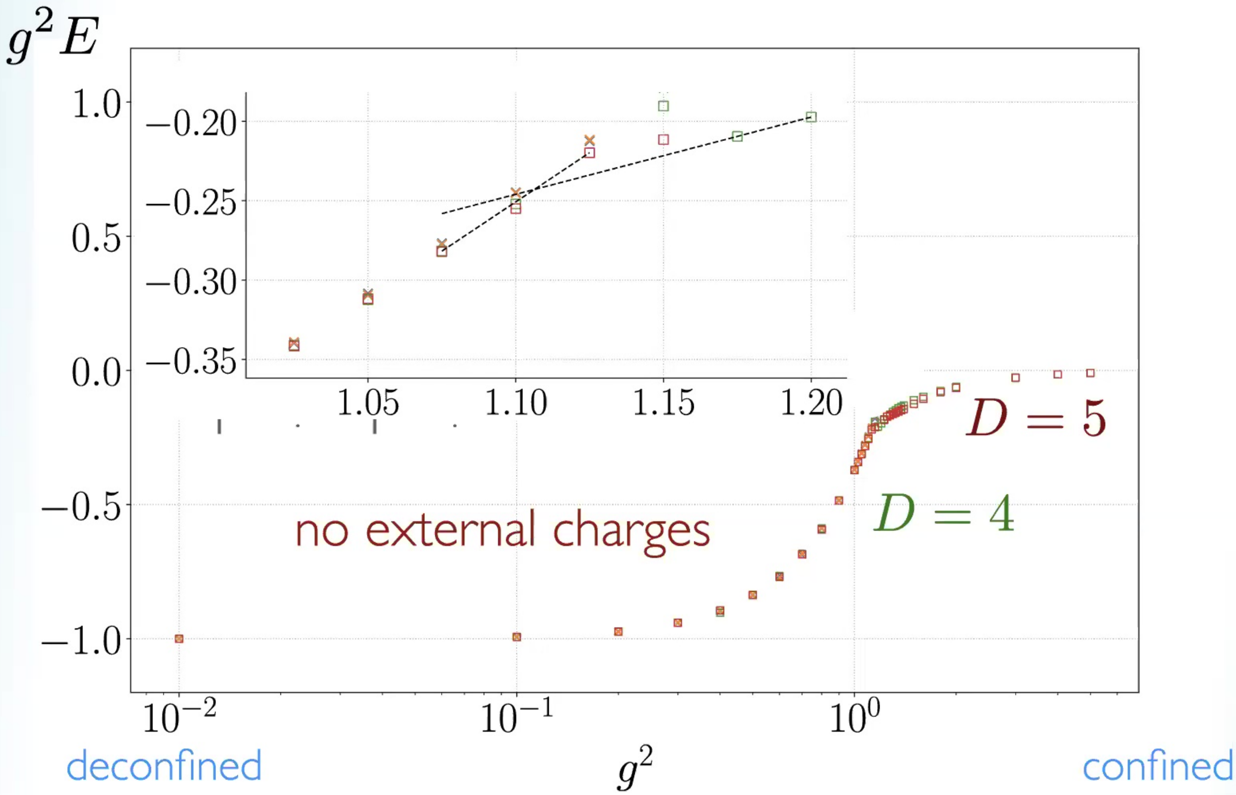

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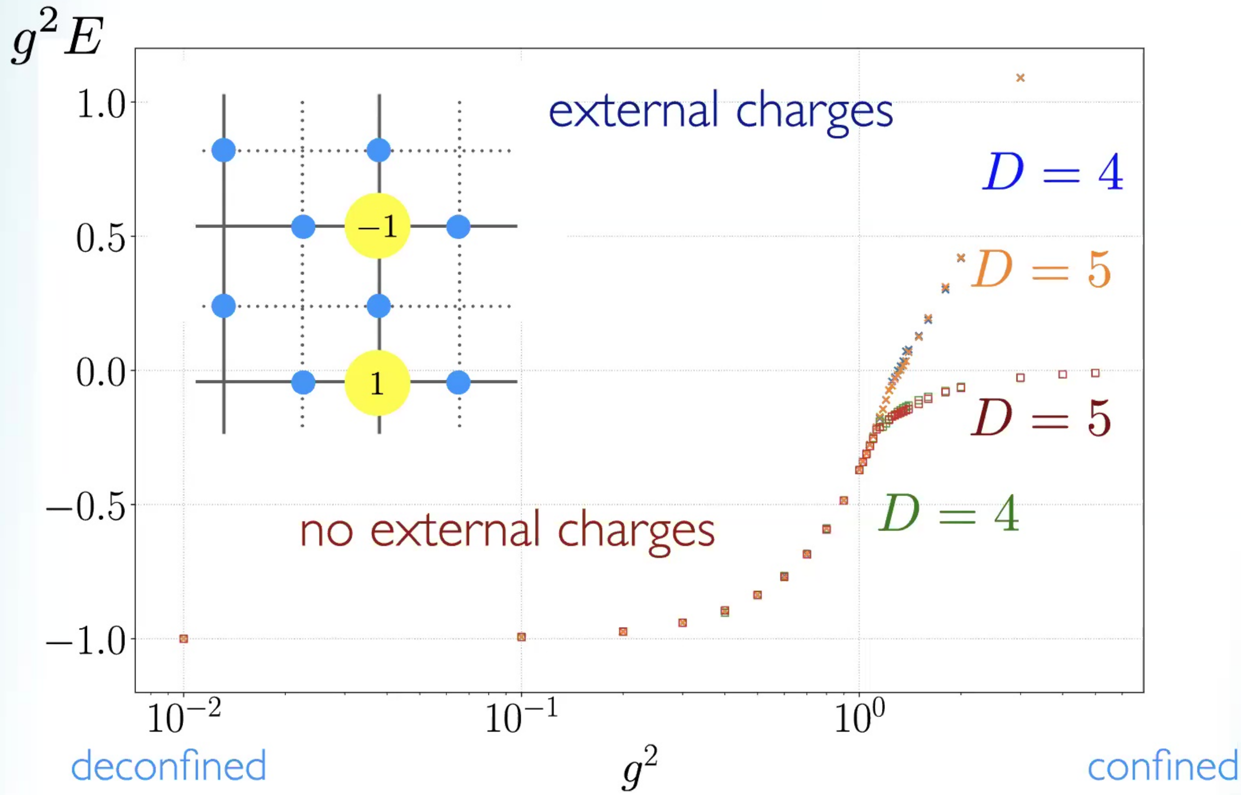
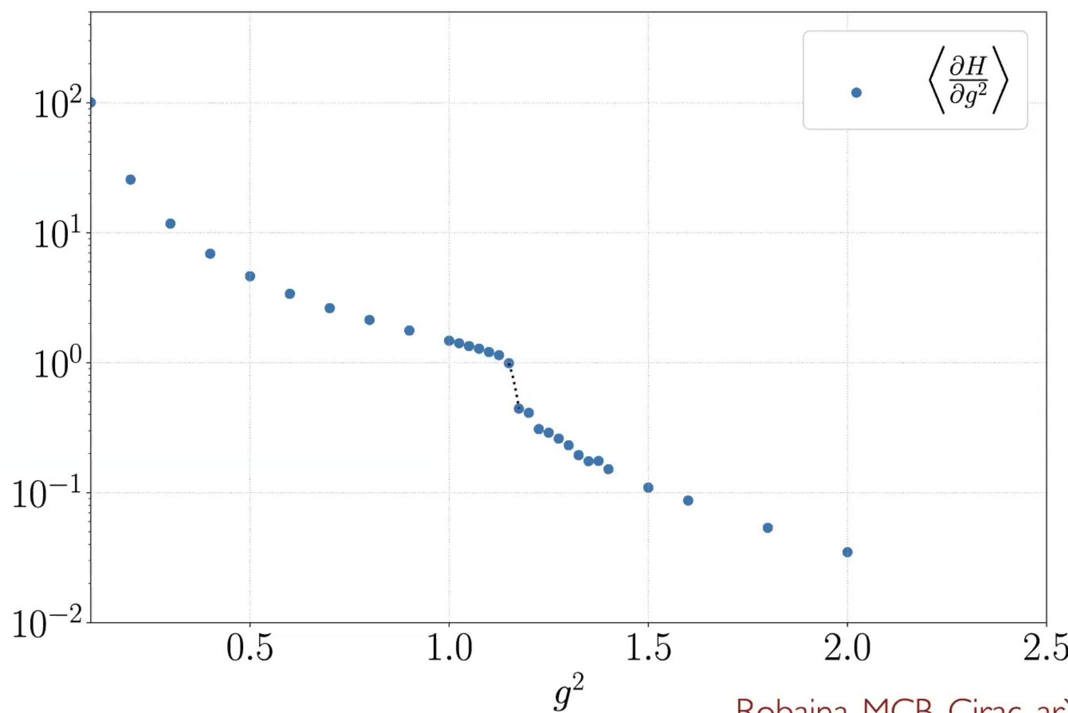


PHASE DIAGRAM

first order phase transition

discontinuity in derivative of energy

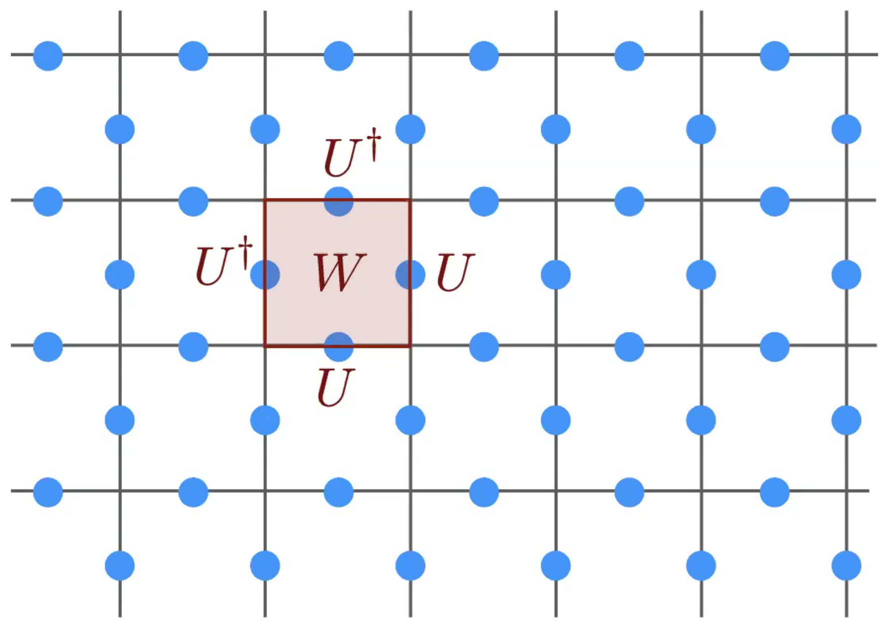


Robaina, MCB, Cirac, arXiv:2007.11630

WILSON LOOP

to distinguish between deconfined and confined phases

decay with area



Robaina, MCB, Cirac, arXiv:2007.11630

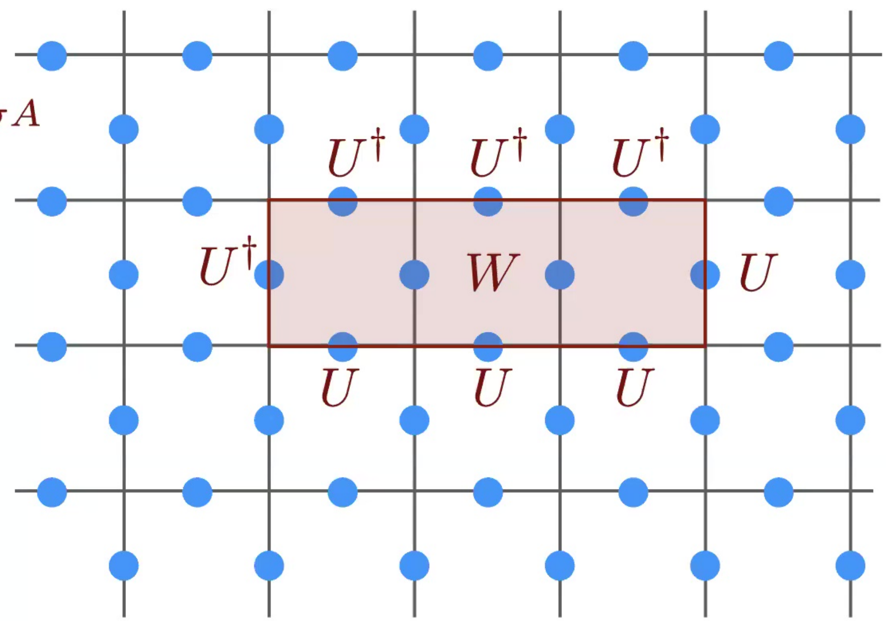
WILSON LOOP

to distinguish between deconfined and confined phases

decay with area

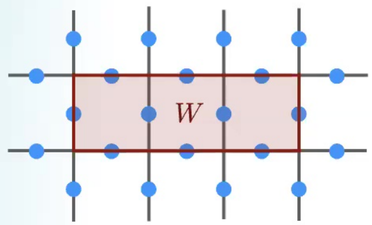
$$\langle W_A \rangle \approx e^{-\sigma A}$$

string tension



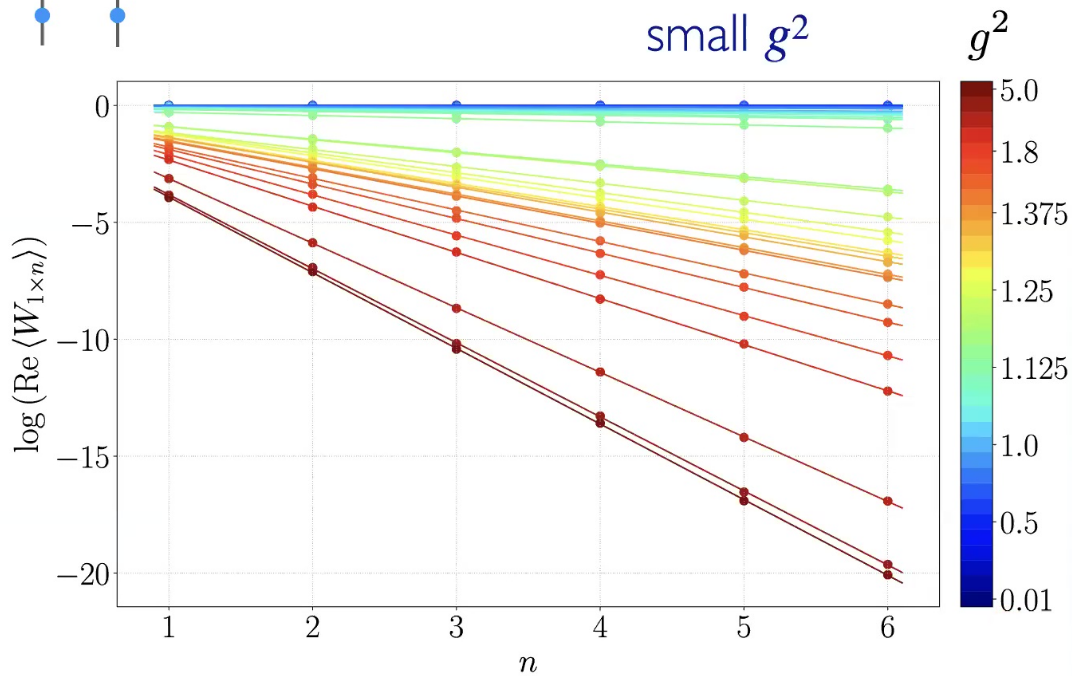
Robaina, MCB, Cirac, arXiv:2007.11630

WILSON LOOP



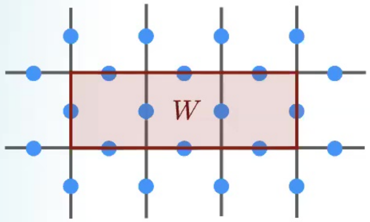
$W_{1\times n}$

small g^2

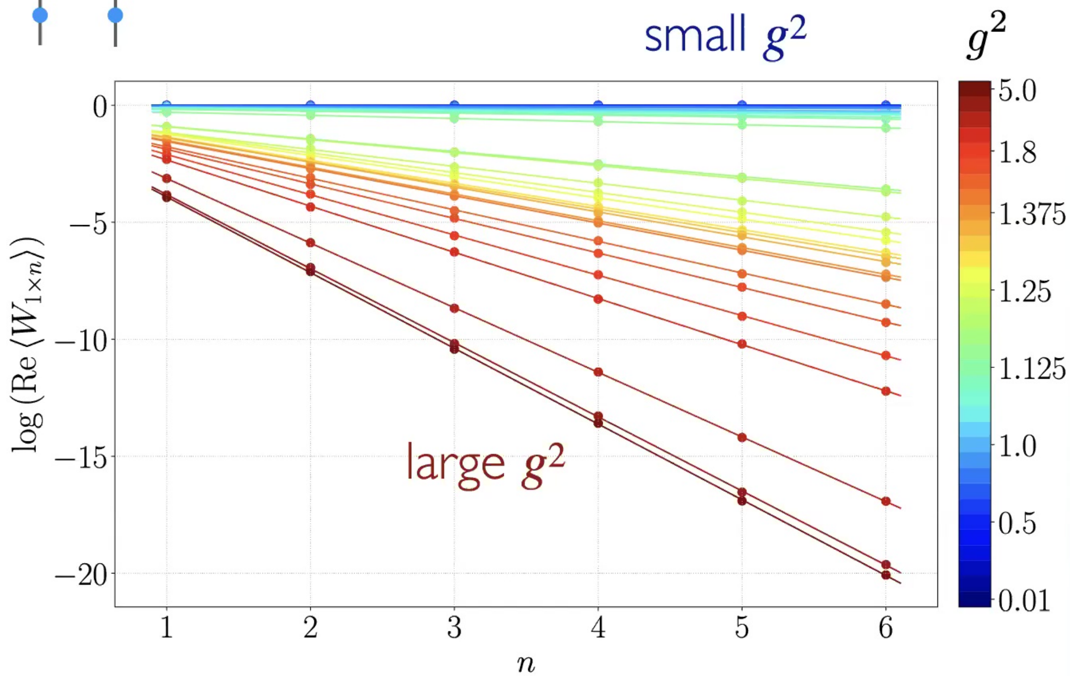


Robaina, MCB, Cirac, arXiv:2007.11630

WILSON LOOP

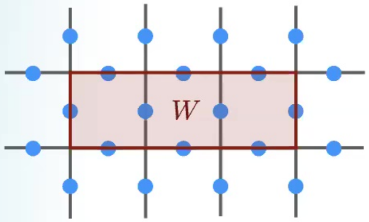


$W_{1\times n}$

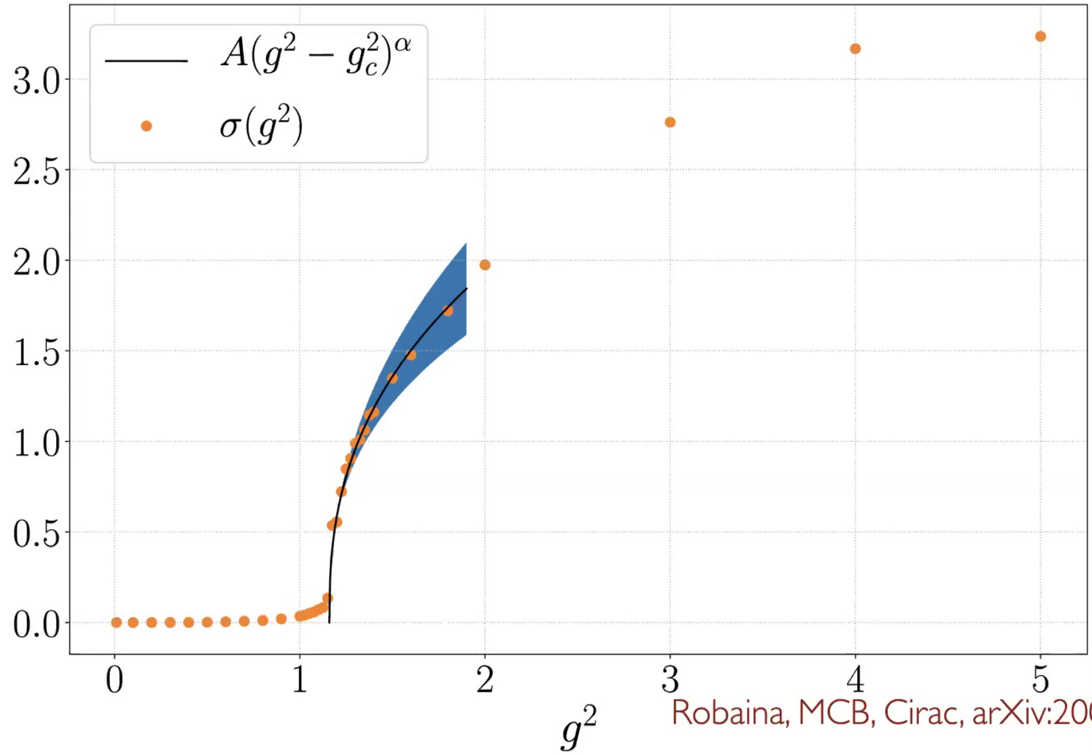


Robaina, MCB, Cirac, arXiv:2007.11630

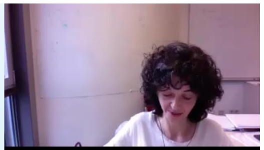
WILSON LOOP



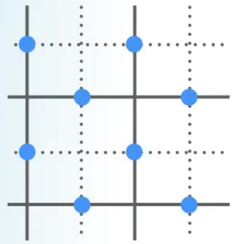
$W_{1\times n}$



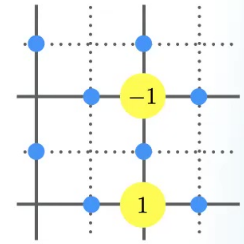
Robaina, MCB, Cirac, arXiv:2007.11630

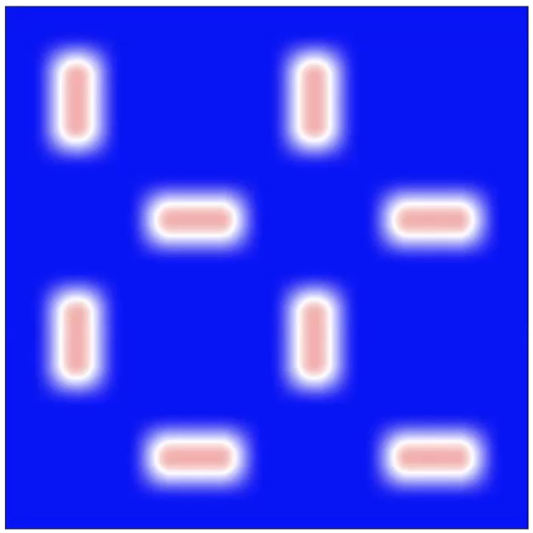


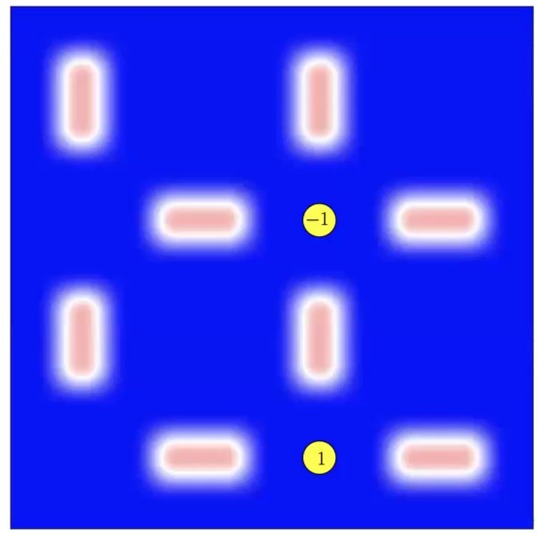
CONFINEMENT OF ELECTRIC FIELD



$$g^2 = 0.1$$







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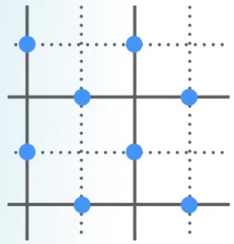
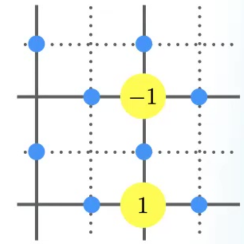
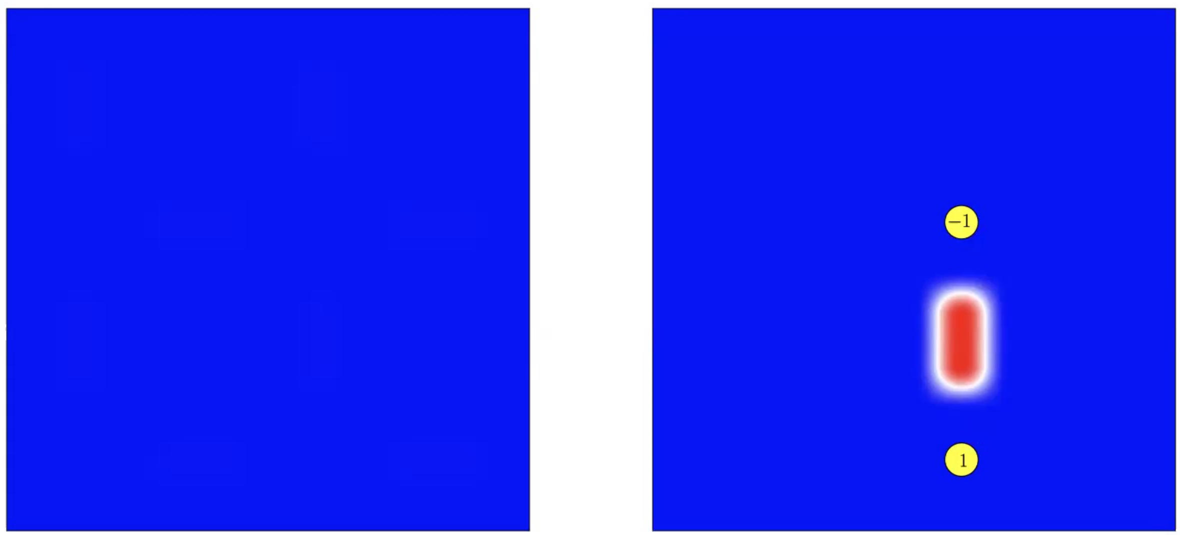
CONFINEMENT OF ELECTRIC FIELD



$$g^2 = 5$$





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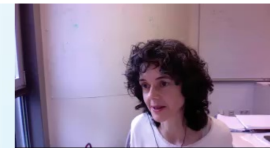

CONCLUSION

strategy to include plaquette terms in the most up-to-date (i)PEPS algorithms

first fully variational iPEPS study of a LGT

captures phases accurately with moderate bond dimension

can be applied to other cases

e.g. non-Abelian theories, dynamical fermions

Robaina, MCB, Cirac, arXiv:2007.11630