Title: Tensor networks for critical systems

Speakers: Frank Verstraete

Collection: Tensor Networks: from Simulations to Holography III

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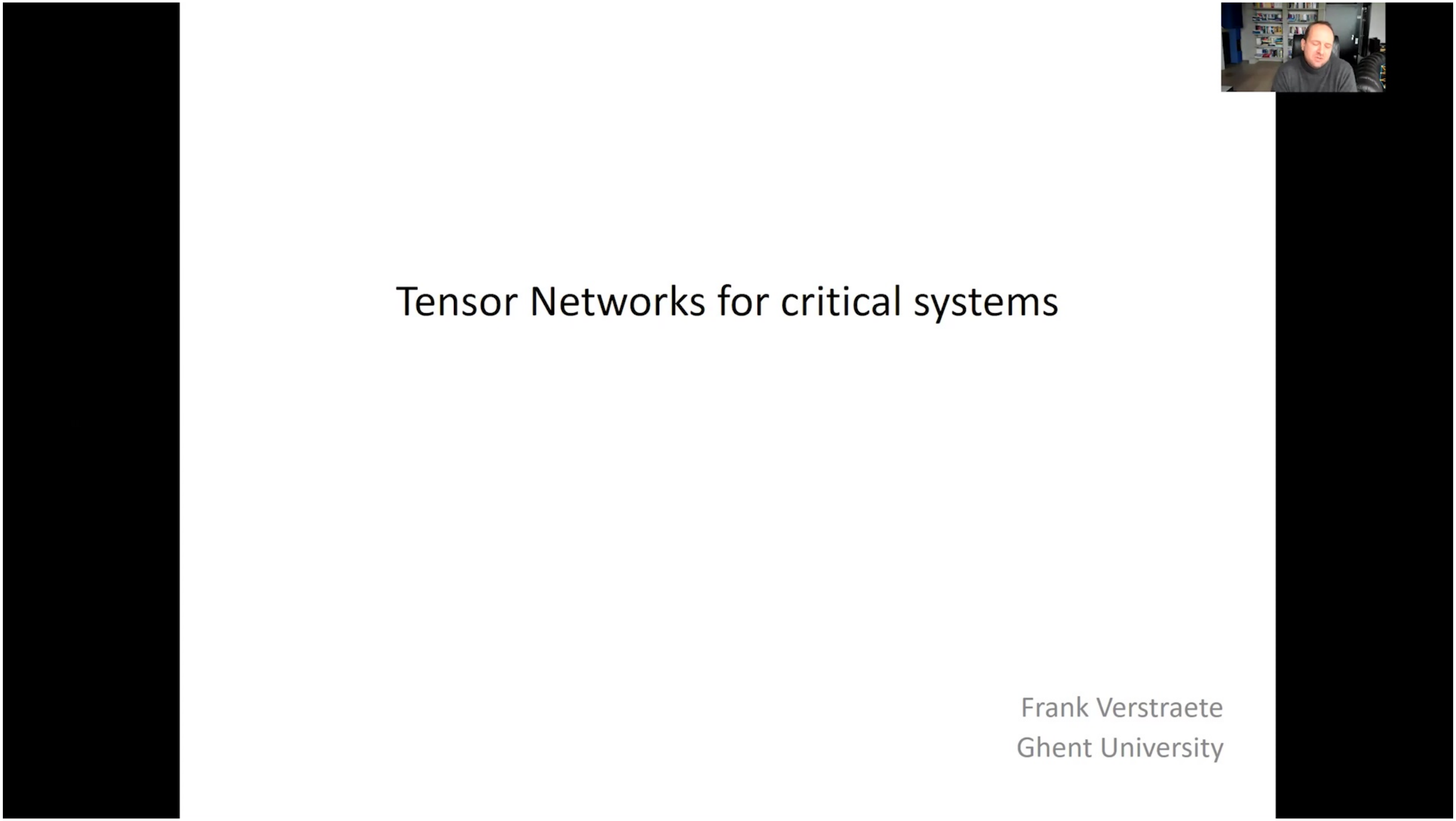
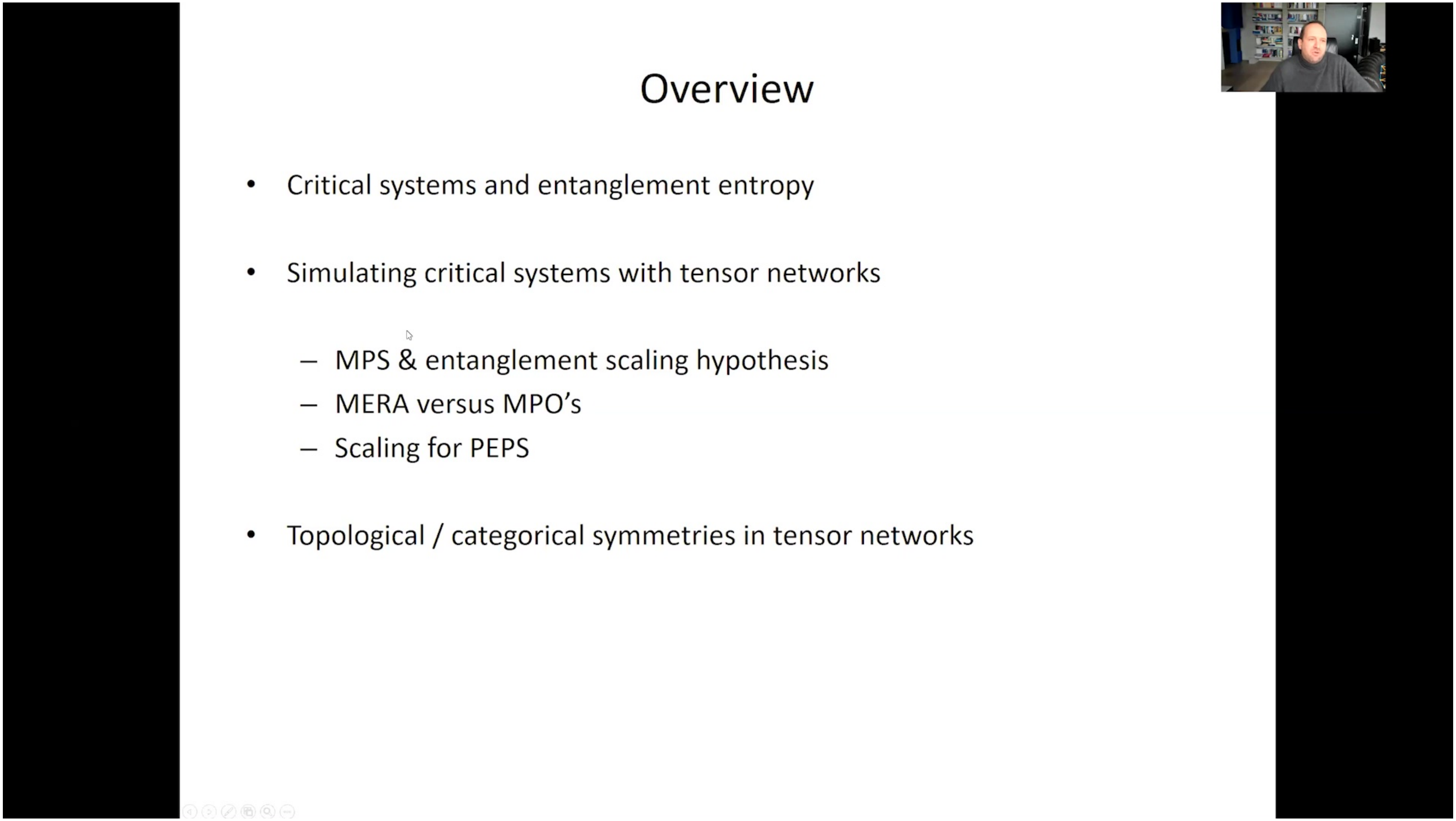
Abstract: In this talk I will give an overview of tensor network approaches to critical systems. I will discuss entanglement scaling laws, show how PEPS can simulate systems with Fermi surfaces, and present some results for simulating systems in the continuum.



Tensor Networks for critical systems

Frank Verstraete
Ghent University



Overview

- Critical systems and entanglement entropy
- Simulating critical systems with tensor networks
  - MPS & entanglement scaling hypothesis
  - MERA versus MPO's
  - Scaling for PEPS
- Topological / categorical symmetries in tensor networks

Critical systems and entanglement entropy

- Renyi entanglement entropy of a 1+1D CFT (Callan & Wilczek ‘94; Holzhey, Larsen & Wilczek ‘94; Vidal, Latorre, Rico & Kitaev ‘03; Calabrese & Cardy ’04):

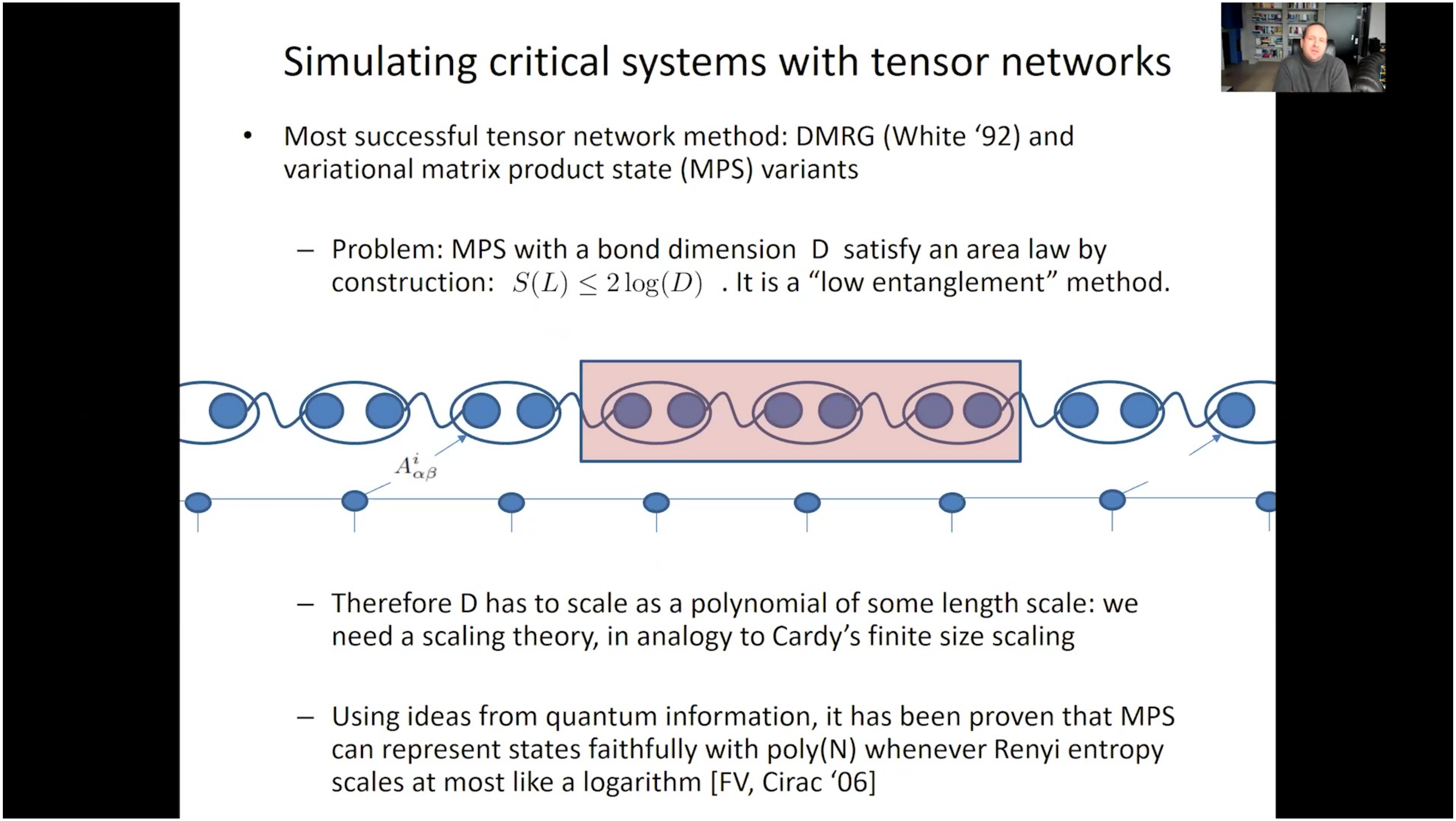
$$S_\alpha(\rho_I) = \frac{c}{6}(1 + 1/\alpha)\log_2(l/a) + O(1)$$

- Critical systems in 2D:
  - z=2 conformal critical point (Fradkin & Moore ‘06):

$$S = 2f_s(L/a) + \alpha c \log(L/a) + O(1)$$

  - systems with Fermi surface (Wolf ‘06; Gioev &Klich ‘06):

$$S \sim \frac{L^{d-1}\log L}{(2\pi)^{d-1}} \frac{1}{12} \int_{\partial\Omega} \int_{\partial\Gamma} |n_x \cdot n_p| dS_x \mathrm{d}S_p$$



Simulating critical systems with tensor networks

- Most successful tensor network method: DMRG (White '92) and variational matrix product state (MPS) variants
  - Problem: MPS with a bond dimension D satisfy an area law by construction: $S(L) \leq 2\log(D)$. It is a "low entanglement" method.
  - Therefore D has to scale as a polynomial of some length scale: we need a scaling theory, in analogy to Cardy's finite size scaling
  - Using ideas from quantum information, it has been proven that MPS can represent states faithfully with poly(N) whenever Renyi entropy scales at most like a logarithm [FV, Cirac '06]

- Founding DMRG paper (White ‘92):

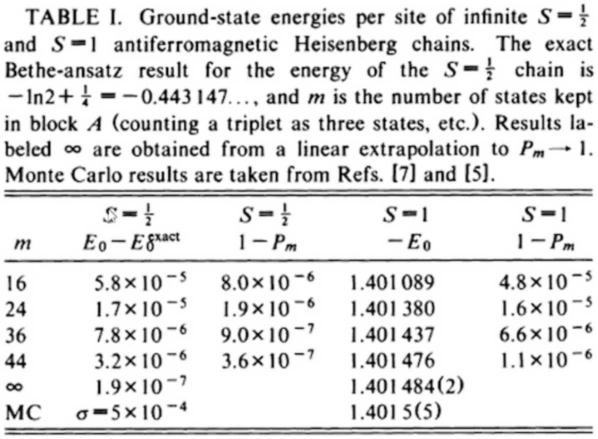
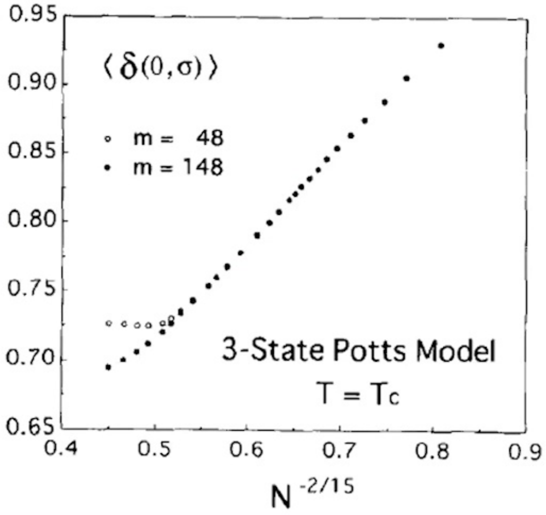


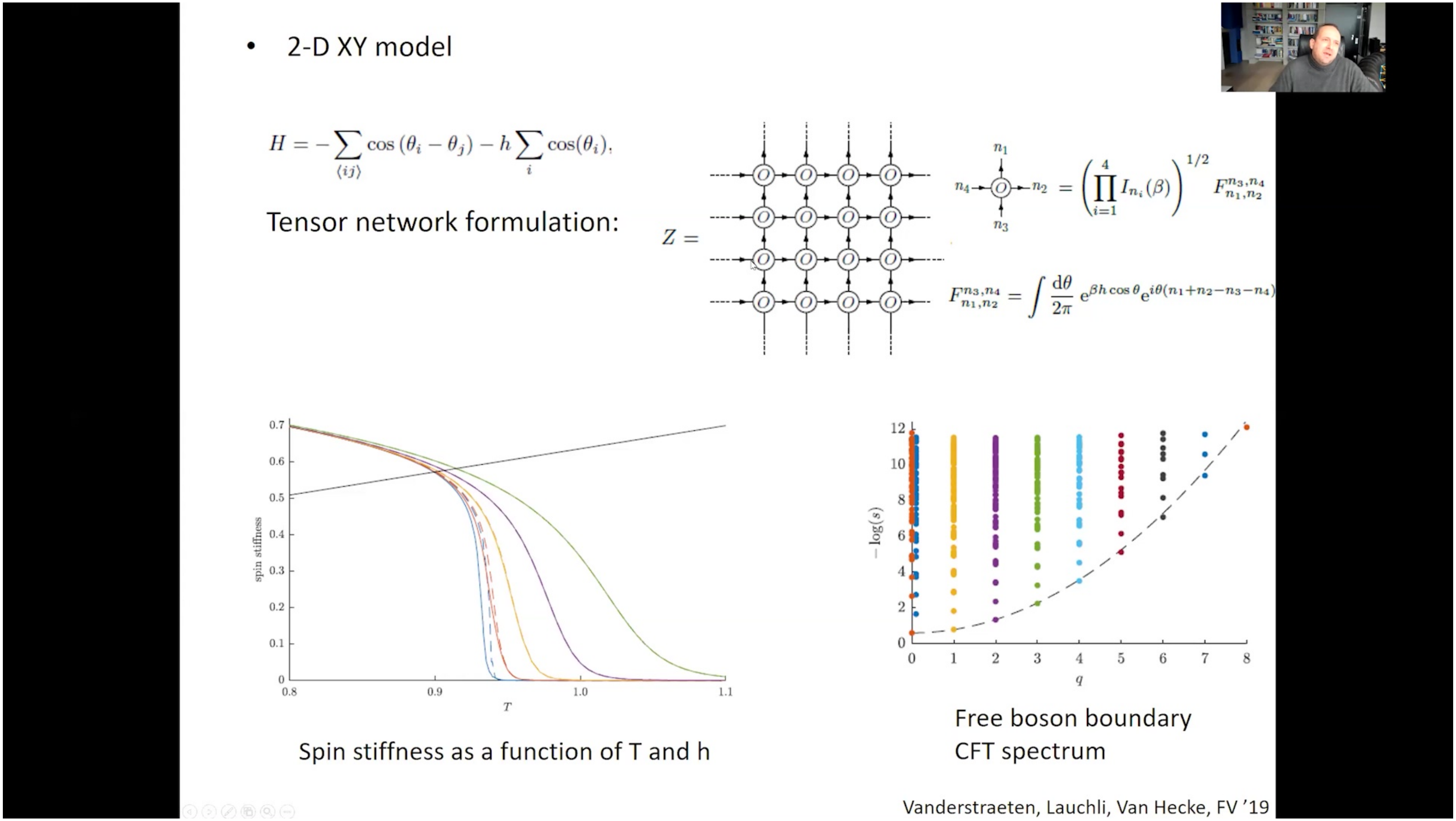
TABLE I. Ground-state energies per site of infinite $S=\frac{1}{2}$ and $S=1$ antiferromagnetic Heisenberg chains. The exact Bethe-ansatz result for the energy of the $S=\frac{1}{2}$ chain is $-\ln 2+\frac{1}{4}=-0.443\,147\ldots$, and m is the number of states kept in block A (counting a triplet as three states, etc.). Results labeled ∞ are obtained from a linear extrapolation to $P_m \rightarrow 1$. Monte Carlo results are taken from Refs. [7] and [5].

	$S=\frac{1}{2}$	$S=\frac{1}{2}$	$S=1$	$S=1$
m	$E_0-E_0^{\text{exact}}$	$1-P_m$	$-E_0$	$1-P_m$
16	5.8×10^{-5}	8.0×10^{-6}	1.401 089	4.8×10^{-5}
24	1.7×10^{-5}	1.9×10^{-6}	1.401 380	1.6×10^{-5}
36	7.8×10^{-6}	9.0×10^{-7}	1.401 437	6.6×10^{-6}
44	3.2×10^{-6}	3.6×10^{-7}	1.401 476	1.1×10^{-6}
∞	1.9×10^{-7}		1.401 484(2)	
MC	$\sigma=5\times10^{-4}$		1.401 5(5)	

- Nishino, Okunishi, Kikuchi ’96:

$$M(N)=N^{-(d-2+\eta)/2}g(\xi(m)/N)$$





- 2-D XY model

$$H = -\sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) - h \sum_i \cos(\theta_i),$$

Tensor network formulation:

$$Z =$$

$$\left(\prod_{i=1}^{4} I_{n_i}(\beta) \right)^{1/2} F^{n_3,n_4}_{n_1,n_2}$$

$$F^{n_3,n_4}_{n_1,n_2} = \int \frac{\mathrm{d}\theta}{2\pi}\, \mathrm{e}^{\beta h \cos\theta} \mathrm{e}^{i\theta(n_1+n_2-n_3-n_4)}$$

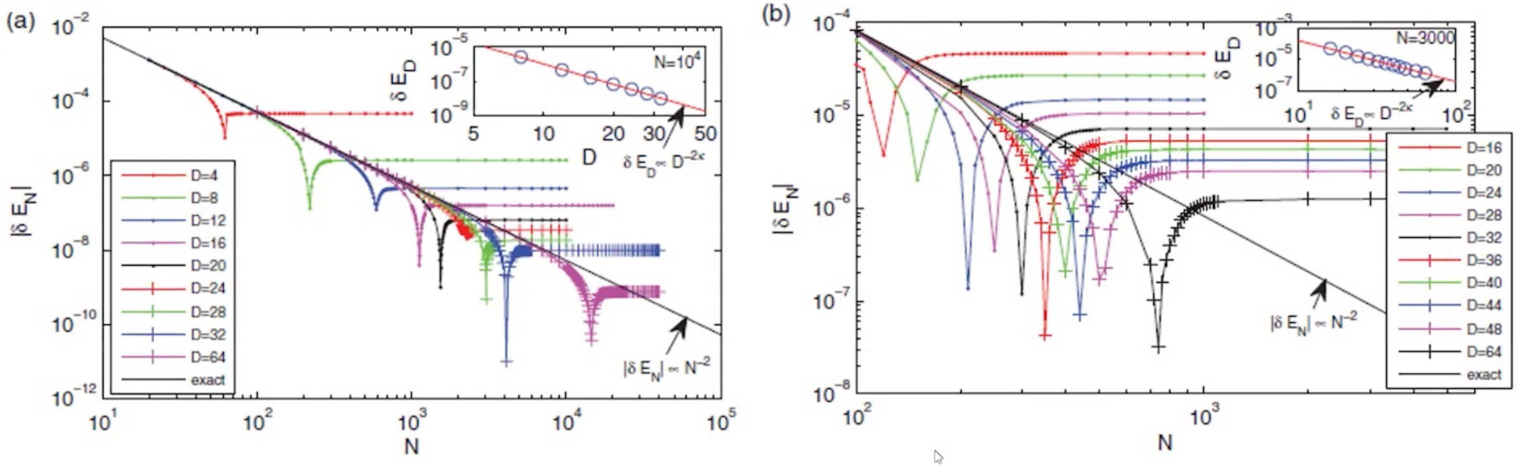
Spin stiffness as a function of T and h

Free boson boundary CFT spectrum

Vanderstraeten, Lauchli, Van Hecke, FV '19

Finite size vs. finite entanglement scaling

- Tensor network simulation of critical Ising and Heisenberg model with PBC



- Bond dimension plays role of finite T, and opens up a gap in the system

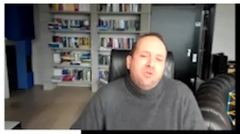
$$E_0(\xi_\epsilon) = E_0(\infty) + \frac{A}{\xi_\epsilon^2}$$

$$E_0(\xi_D) = E_0(\infty) + \frac{\beta}{\xi_D} P_r(b,D)$$

$$\xi(D) \propto D^\kappa \qquad \kappa = \frac{6}{c\left(1+\sqrt{\frac{12}{c}}\right)}$$

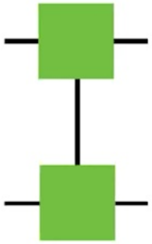
Pirvu, Tagliacozzo, FV, Vidal ‘12
Pollmann, Mukerjee, Turner, Moore ‘09

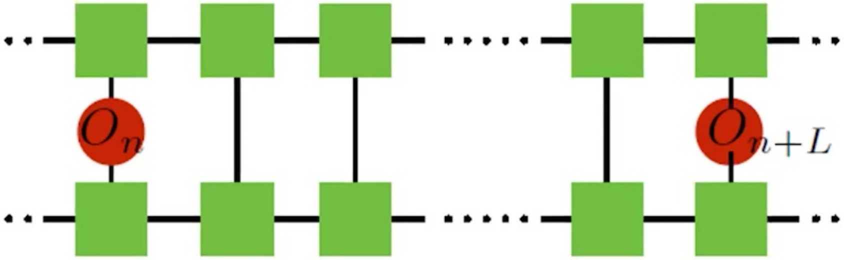
Length scales in the MPS formulation



Rams, Czarnik & Cincio '18

MPS
transfer matrix T:





$$|T| = \exp^{-\epsilon_i}$$

$$(0, \epsilon_1, \epsilon_2, \ldots)$$

Zauner et al. '15

$D \; < \; D'$

mass gap:

$$\epsilon_1 = m = 1/\xi$$

entanglement compression scale:

$$\delta = \epsilon_2 - \epsilon_1$$

$$\delta = \sum_{i=1} c_i \epsilon_i \quad (\sum_{i=1} c_i = 0)$$

Finite size vs. finite entanglement scaling

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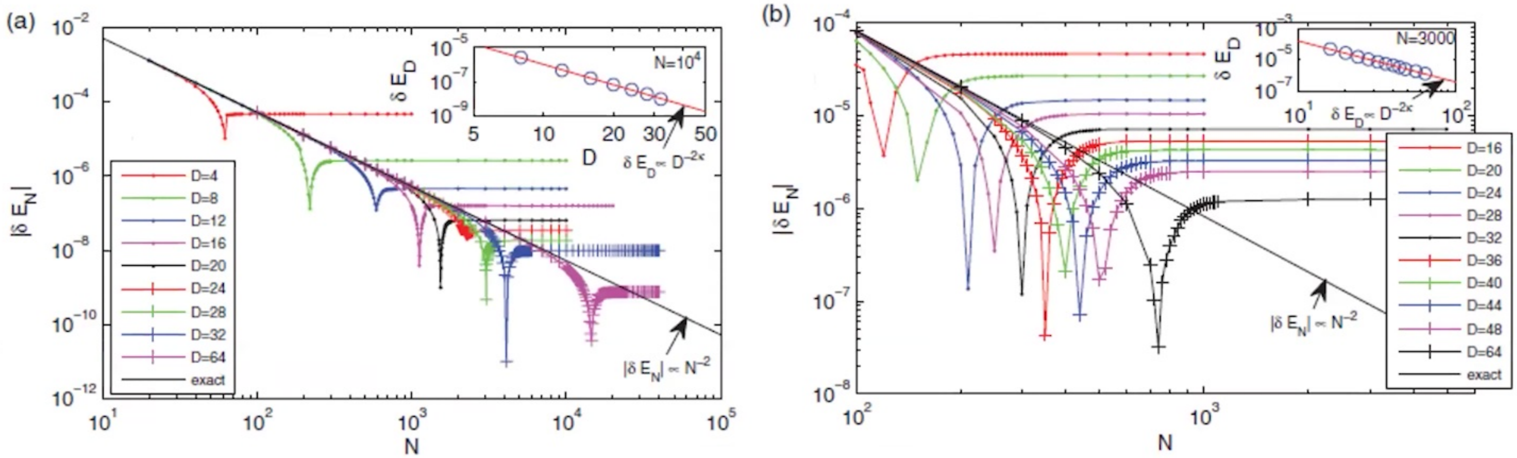

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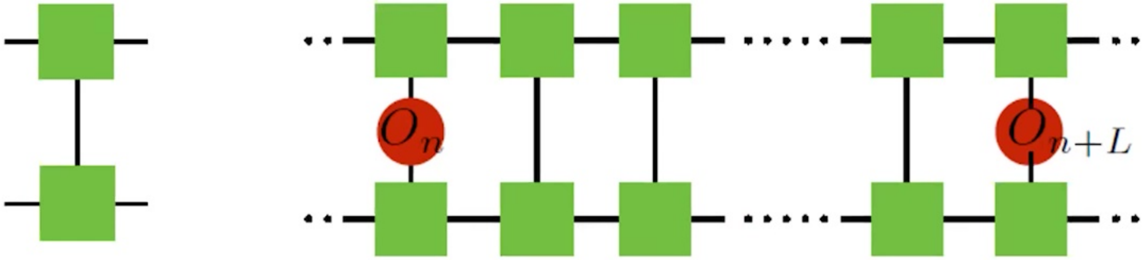
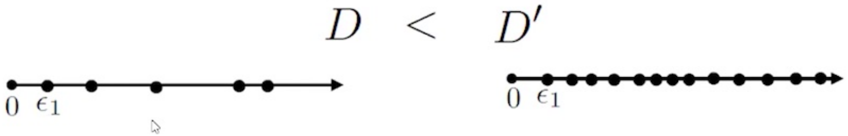


$$|T| = \exp^{-\epsilon_i}$$

$$(0, \epsilon_1, \epsilon_2, \dots)$$

Zauner et al. '15



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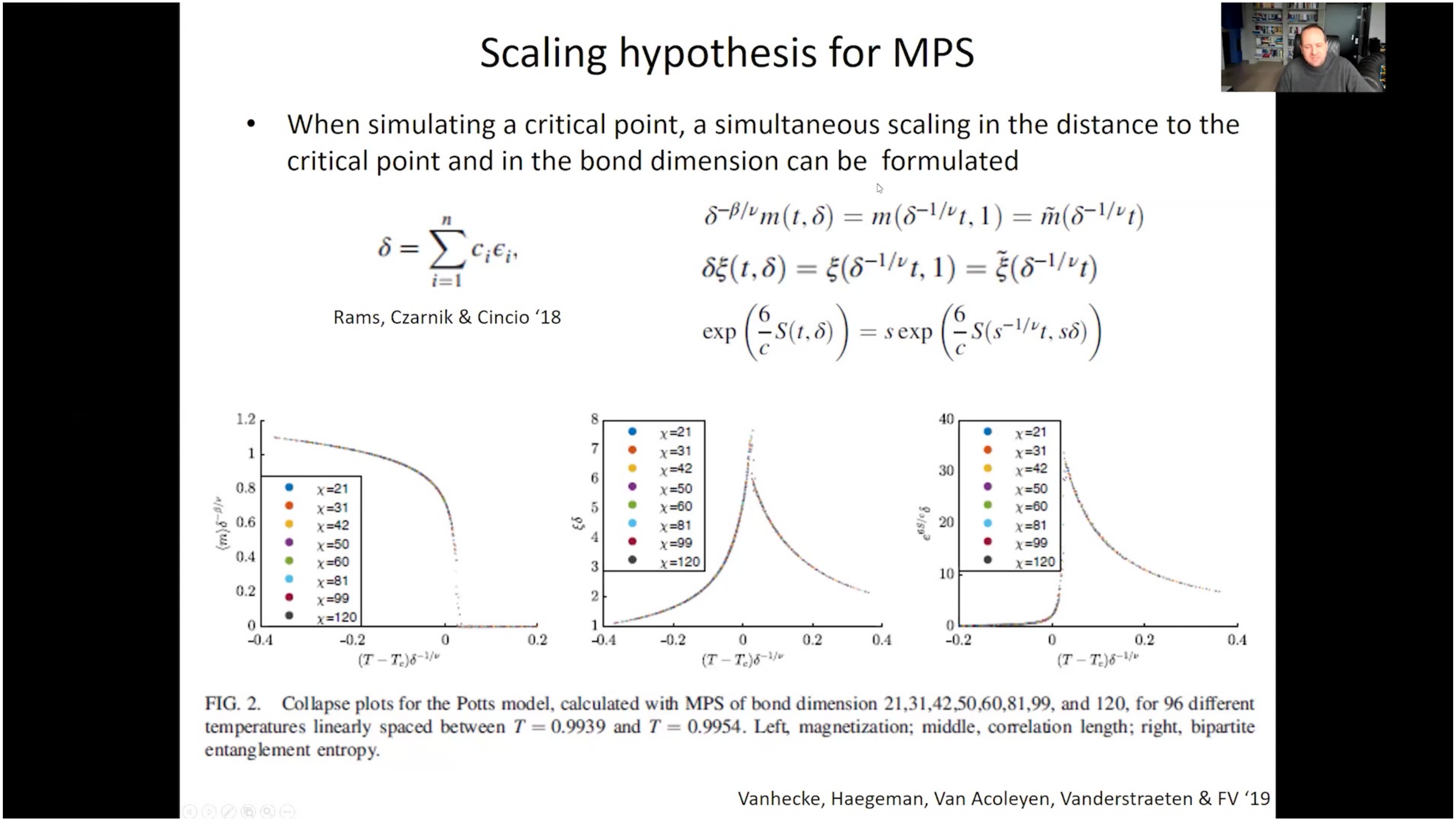


Scaling hypothesis for MPS

- When simulating a critical point, a simultaneous scaling in the distance to the critical point and in the bond dimension can be formulated

$$\delta = \sum_{i=1}^{n} c_i \epsilon_i,$$

Rams, Czarnik & Cincio '18

$$\delta^{-\beta/\nu} m(t,\delta) = m(\delta^{-1/\nu} t, 1) = \tilde{m}(\delta^{-1/\nu} t)$$

$$\delta \xi(t,\delta) = \xi(\delta^{-1/\nu} t, 1) = \tilde{\xi}(\delta^{-1/\nu} t)$$

$$\exp\left(\frac{6}{c} S(t,\delta)\right) = s \exp\left(\frac{6}{c} S(s^{-1/\nu} t, s\delta)\right)$$

FIG. 2. Collapse plots for the Potts model, calculated with MPS of bond dimension 21,31,42,50,60,81,99, and 120, for 96 different temperatures linearly spaced between $T = 0.9939$ and $T = 0.9954$. Left, magnetization; middle, correlation length; right, bipartite entanglement entropy.

Vanhecke, Haegeman, Van Acoleyen, Vanderstraeten & FV '19

What about scaling of MPS for field theories?

gapped spin systems : $\Lambda_{UV} = 1, \Lambda_{IR} < m$ ⟶ Exact

critical spin systems: $\Lambda_{UV} = 1, \Lambda_{IR} \to 0$

gapped QFTs: $\Lambda_{UV} \to \infty, \Lambda_{IR} < m$

critical QFTs: $\Lambda_{UV} \to \infty, \Lambda_{IR} \to 0$

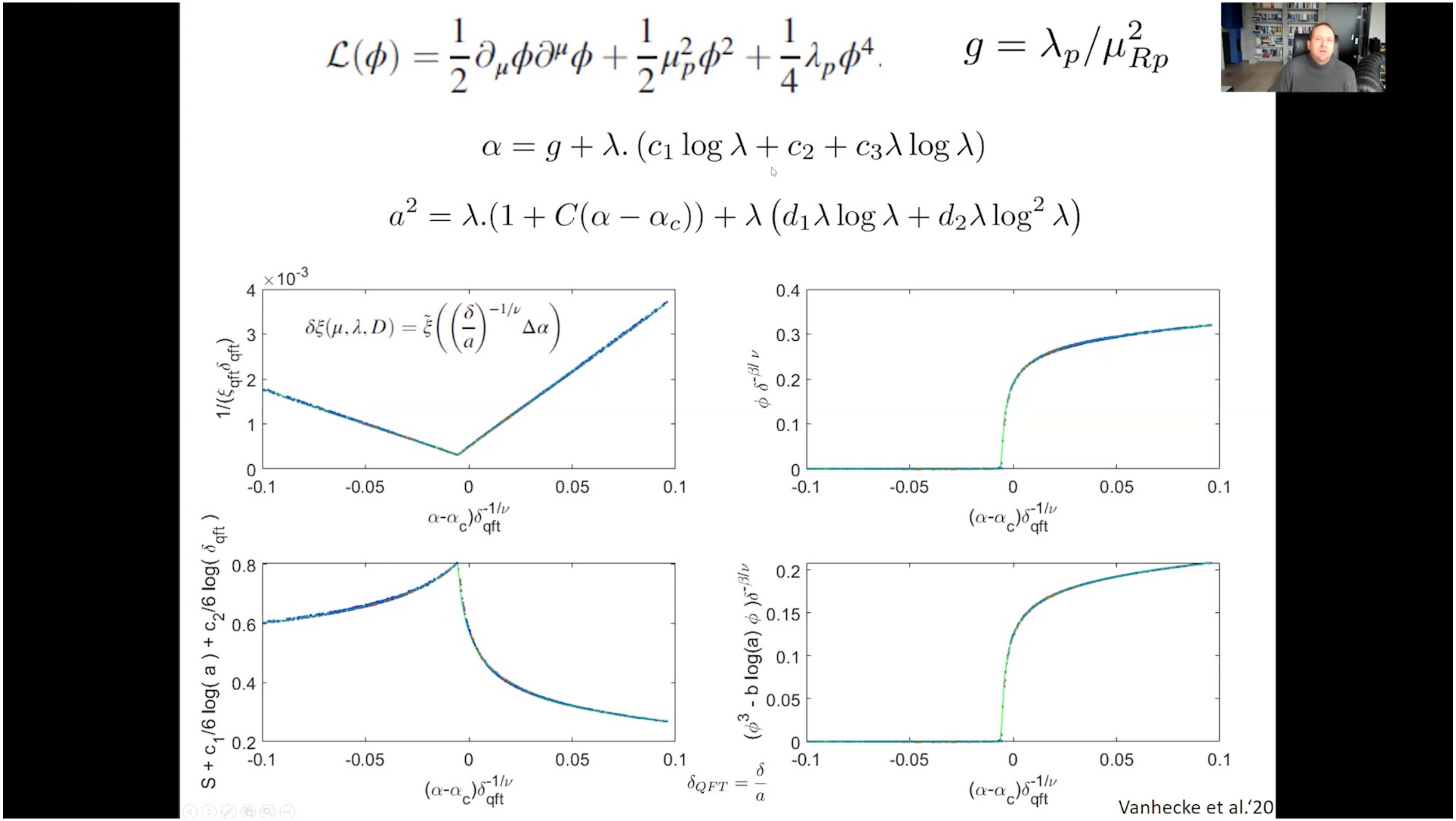
⟶ Extrapolation, scaling theory

- Let us look at $\lambda\phi^4$ to see how the two scales manifest themselves in the entanglement degrees of freedom

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu_p^2\phi^2 + \frac{1}{4}\lambda_p\phi^4.$$

  - Double scaling regime: entanglement scaling + continuum (lattice parameter) should lead to both a c=1 contribution from UV AND a c=1/2 contribution from IR

Vanhecke et al.‘19



$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu_p^2\phi^2 + \frac{1}{4}\lambda_p\phi^4. \qquad g = \lambda_p/\mu_{Rp}^2$$

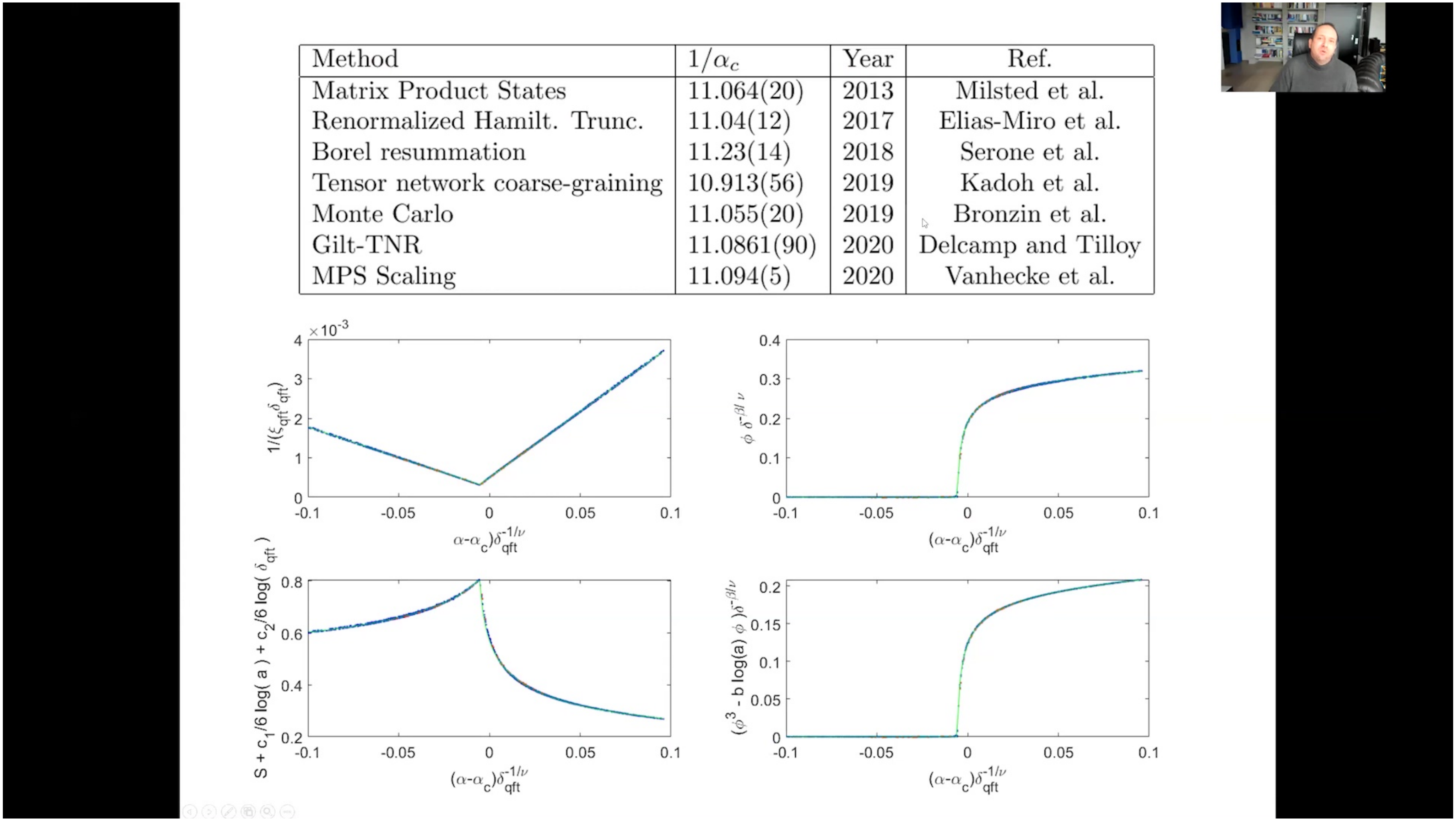
$$\alpha = g + \lambda.\left(c_1 \log\lambda + c_2 + c_3\lambda\log\lambda\right)$$

$$a^2 = \lambda.(1 + C(\alpha - \alpha_c)) + \lambda\left(d_1\lambda\log\lambda + d_2\lambda\log^2\lambda\right)$$

$$\delta\xi(\mu,\lambda,D) = \tilde{\xi}\left(\left(\frac{\delta}{a}\right)^{-1/\nu}\Delta\alpha\right)$$

$$\delta_{QFT} = \frac{\delta}{a}$$

Vanhecke et al.'20



Method	$1/\alpha_c$	Year	Ref.
Matrix Product States	11.064(20)	2013	Milsted et al.
Renormalized Hamilt. Trunc.	11.04(12)	2017	Elias-Miro et al.
Borel resummation	11.23(14)	2018	Serone et al.
Tensor network coarse-graining	10.913(56)	2019	Kadoh et al.
Monte Carlo	11.055(20)	2019	Bronzin et al.
Gilt-TNR	11.0861(90)	2020	Delcamp and Tilloy
MPS Scaling	11.094(5)	2020	Vanhecke et al.

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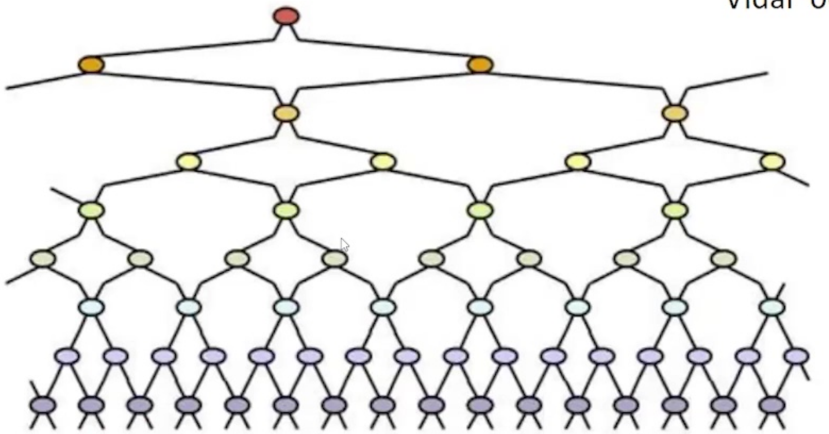
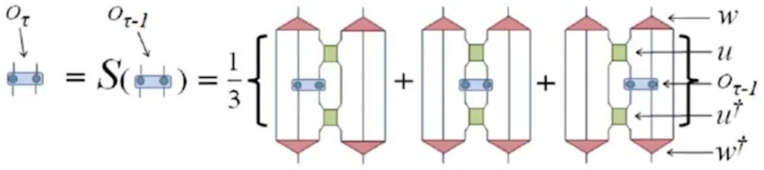
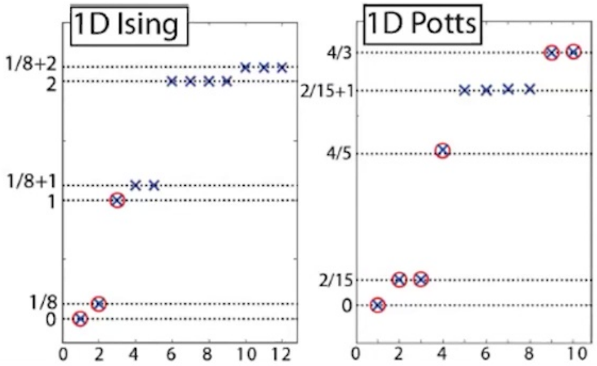
The Multiscale Entanglement Renormalization Ansatz

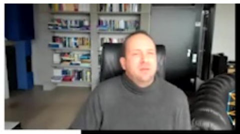
Vidal '06; Evenbly & Vidal '08



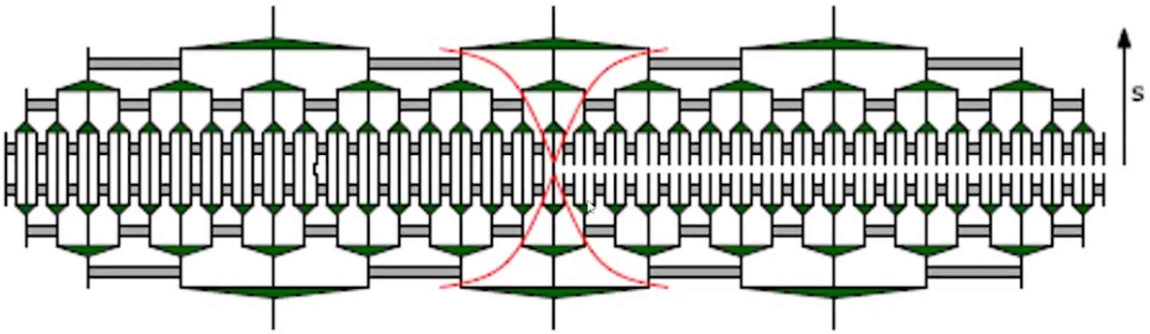
- Quantum circuit description of (critical) states having logarithmic scaling of the entanglement: RG in the Schrodinger picture
- Scaling exponents as eigenvalues of transfer matrices in scale space:

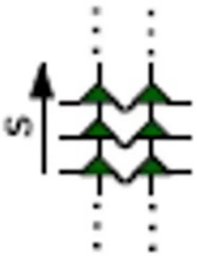






Entanglement scaling in MERA

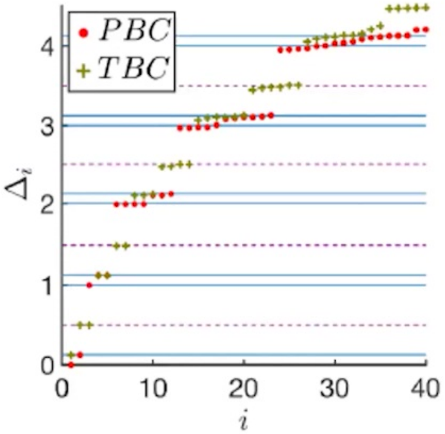




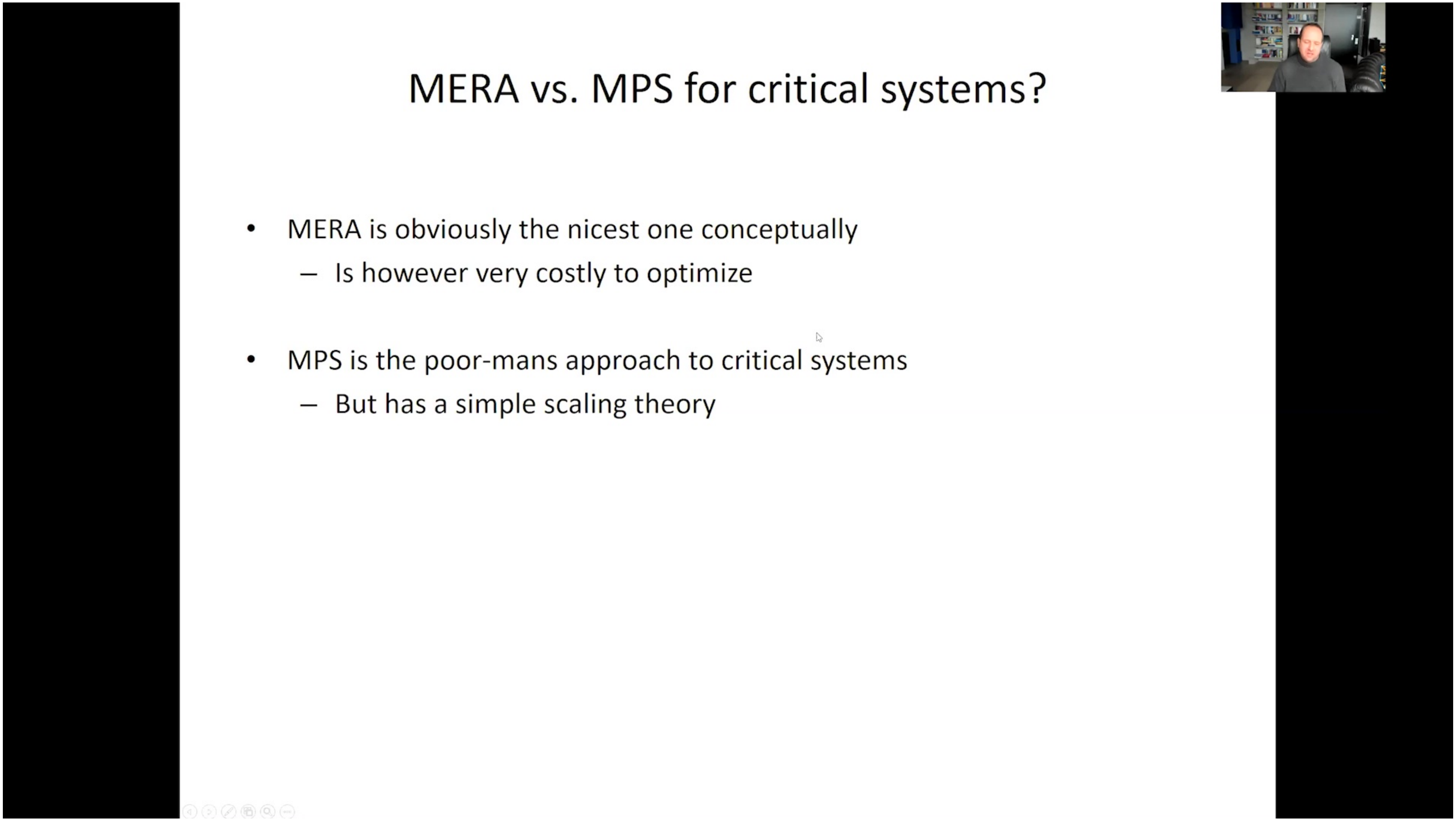
- What is the meaning of the finite bond dimension in the MERA?
- Entanglement structure is equivalent as the one of the modular Hamiltonian at $T = \log(3)/2\pi$, which becomes a translational invariant in scale space:

$$\rho_{\text{scale}} = \exp^{-\frac{2\pi}{\ln(3)}\bar{H}}$$

- The bond dimension of MERA can therefore be related to that of a Matrix Product Operator approximation of a Gibbs state at finite T
- Suggests new algorithms for MERA by relating isometries to tensor of MPO

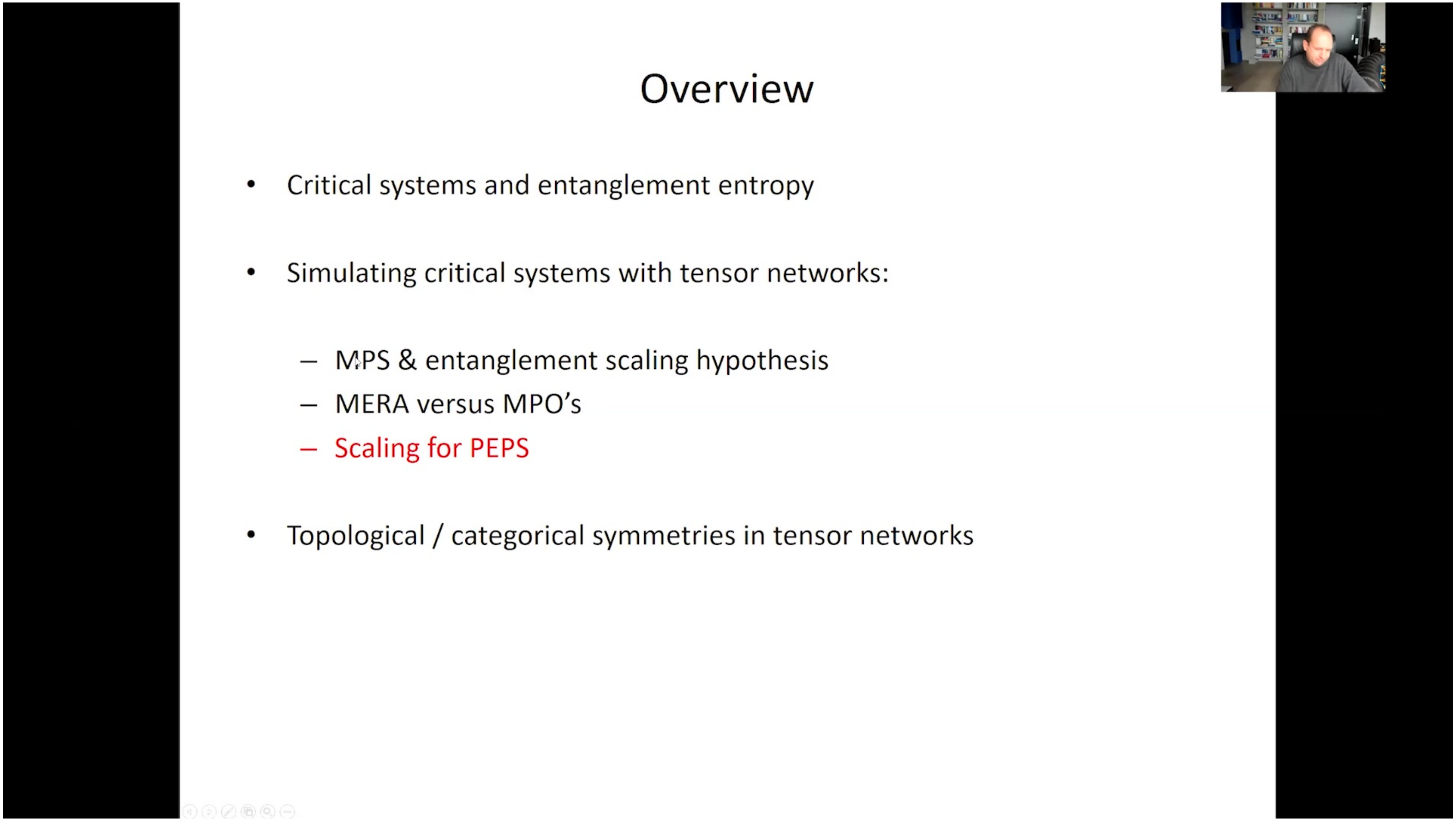


Czech, Evenbly, Lamprou, McCandlish, Qi, Sully, Vidal (2016)
Van Acoleyen, Hallam, Bal, Hauru , Haegeman, FV '20



MERA vs. MPS for critical systems?

- MERA is obviously the nicest one conceptually
  - Is however very costly to optimize
- MPS is the poor-mans approach to critical systems
  - But has a simple scaling theory



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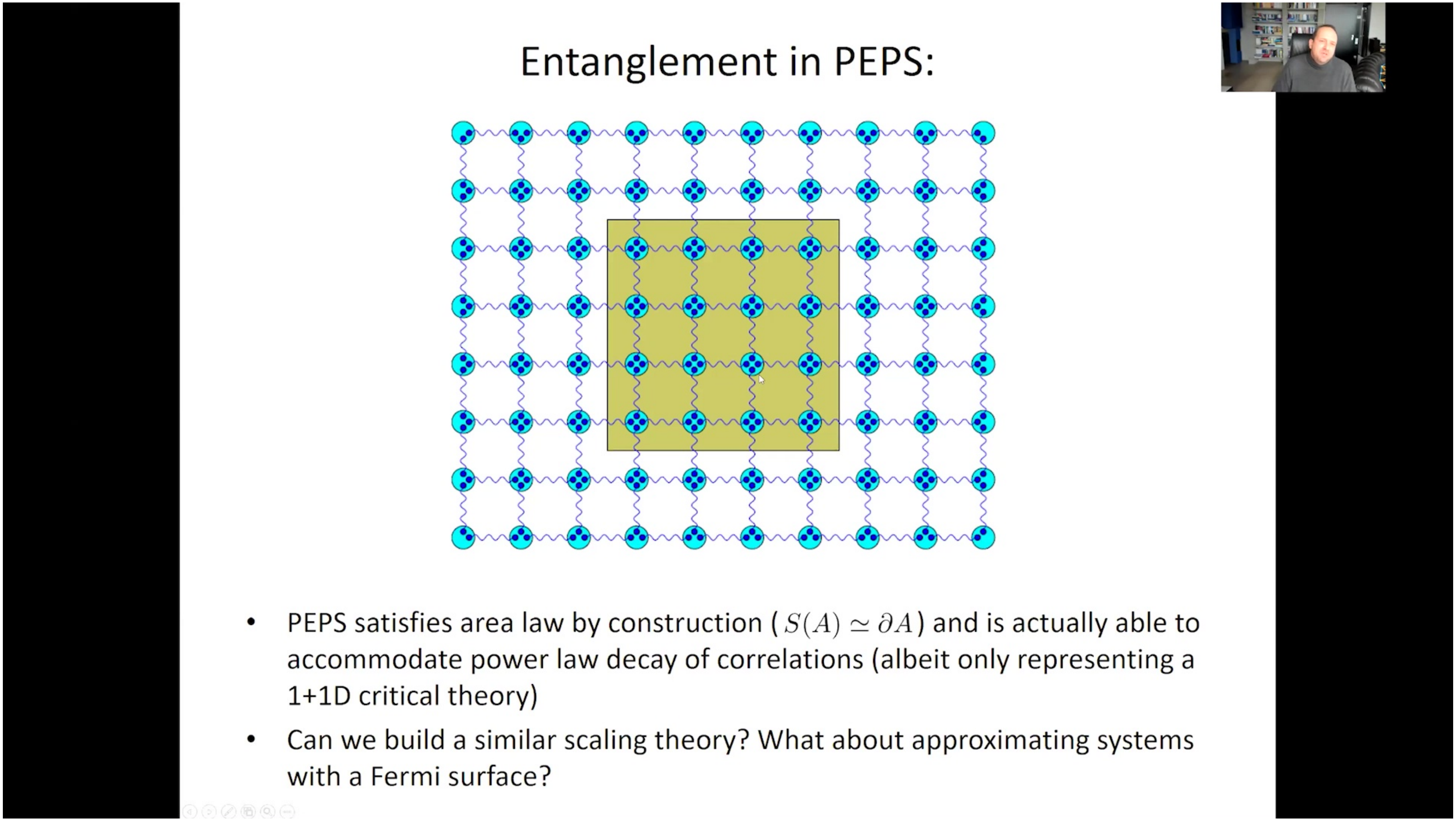
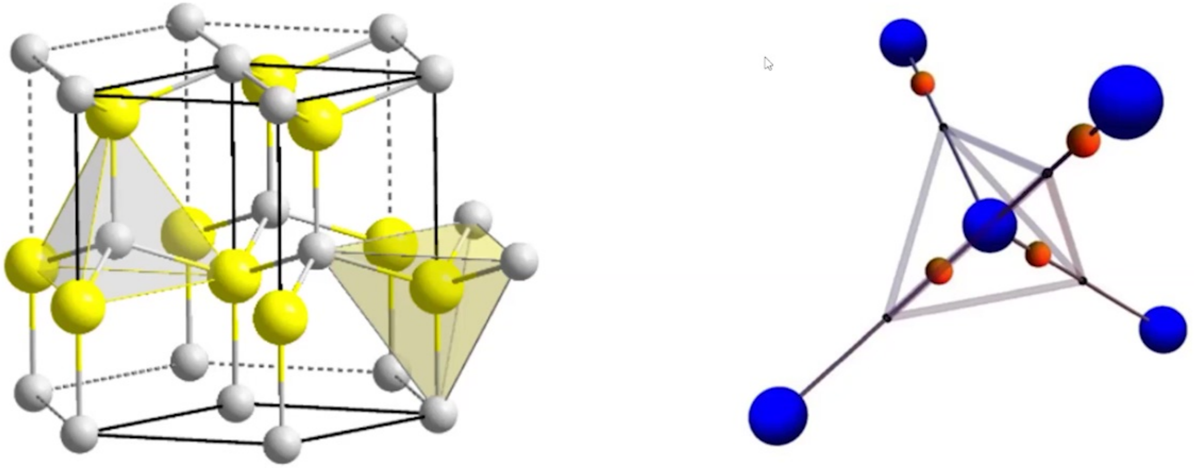


Entanglement in PEPS:

- PEPS satisfies area law by construction ($S(A) \simeq \partial A$) and is actually able to accommodate power law decay of correlations (albeit only representing a 1+1D critical theory)
- Can we build a similar scaling theory? What about approximating systems with a Fermi surface?

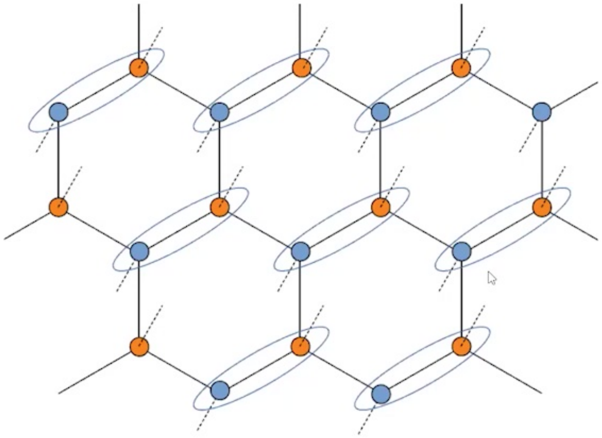
U(1) Coulomb phase: ice at T=0

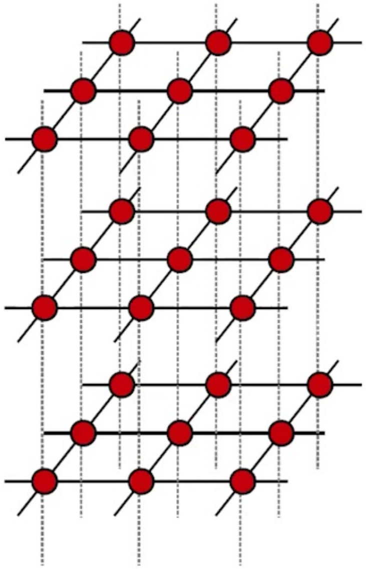


Tensor network for spin ice

$$T^{\text{ice}}_{i,j,k,l} = \begin{cases} 1, & \text{two indices have value 2} \\ 0, & \text{otherwise} \end{cases}$$

O_{hex}

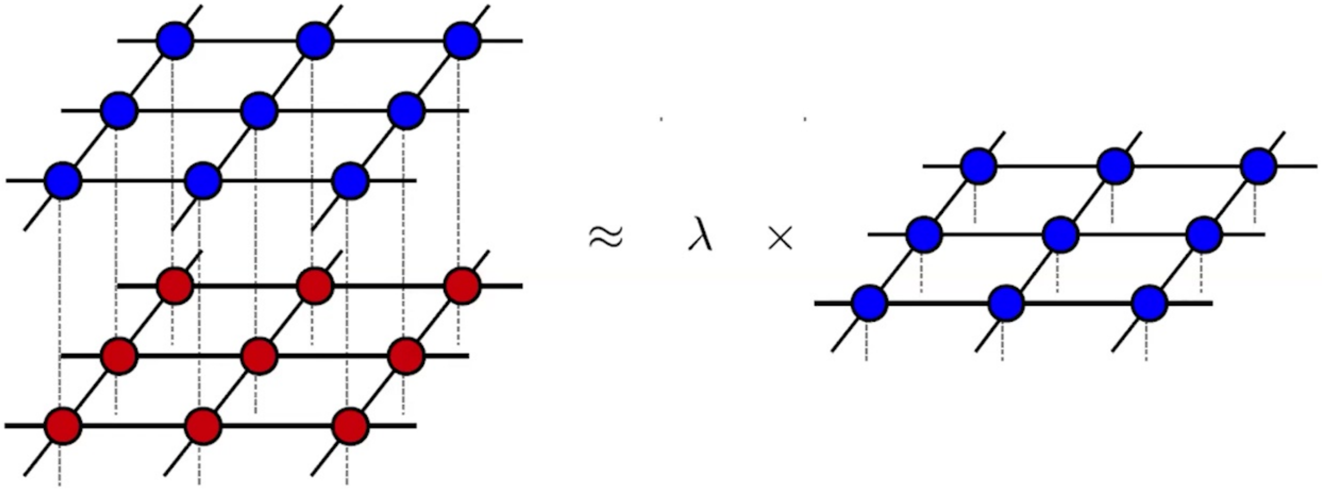


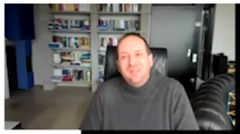


Diamond Ice: repeat the PEPO O_{hex} shifted by 1 sublattice shift
Hexagonal Ice Ih : multiply O_{hex} with its transpose

Free energy can then be obtained as an eigenvalue problem of the 2D transfer matrix of cubic lattice; both types of ice give rise to the same variational problem if we assume Z2 invariance of PEPS by rotation over pi

PEPS: finding eigenvectors of 2-D transfer matrices





- Residual entropy of 3D ice partition function

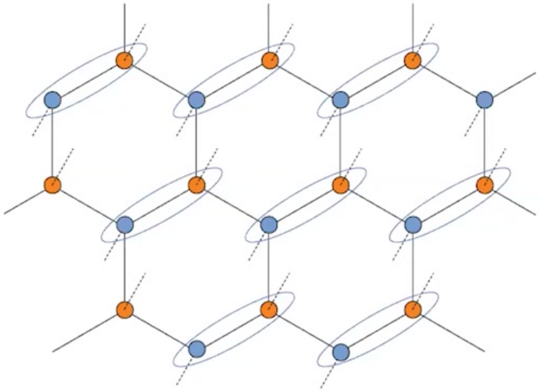


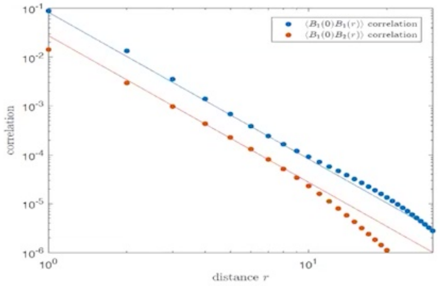
TABLE II. The residual entropies for ice I_h as computed from a mean-field approach, series expansion, multicanonical Monte Carlo, and numerical integration using Monte Carlo, compared to our variational PEPS results.

Pauling [13]	Mean field	1.5
Nagle [15]	Series expansion	1.50685(15)
Berg *et al.* [19]	Multicanonical	1.507117(35)
Herrero *et al.* [16]	Num. integration	1.50786(12)
Kolafa [17]	Num. integration	1.5074660(36)
PEPS	$D = 2$	1.50735
	$D = 3$	1.507451
	$D = 4$	1.507456

  - Coulomb phase description:

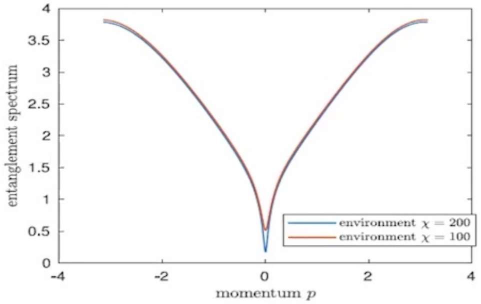
$$B_j(\vec{x}) = \begin{cases} +1, & \text{hydrogen on type-}A\text{ site} \\ -1, & \text{hydrogen on type-}B\text{ site} \end{cases}$$

$$\langle B_i(\vec{x}) B_j(0) \rangle = \frac{1}{4\pi K} \frac{3x_i x_j - |\vec{x}|^2 \delta_{ij}}{|\vec{x}|^5}$$



Extrapolated Stiffness: K = 0.967

  - Entanglement spectrum:



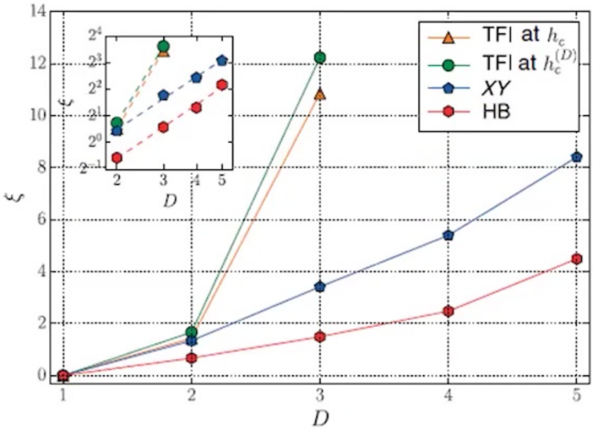
Vanderstraeten, Vanhecke, FV '18

Scaling theory of PEPS

- Rader & Lauchli '18 : O(N) model, N=1,2,3

$$e(\xi) = e(\infty) - \left[\alpha_{\rm NLSM}^{\rm (iPEPS)}\left(\frac{N-1}{2}\right)v\right]\frac{1}{\xi^3} + \mathcal{O}\left(\frac{1}{\xi^4}\right)$$

$$\frac{m^2(\xi)}{m^2(\infty)} = 1 + \left[\mu_{\rm NLSM}^{\rm (iPEPS)}\left(\frac{N-1}{2}\right)\frac{v}{\rho_s}\right]\frac{1}{\xi} + \mathcal{O}\left(\frac{1}{\xi^2}\right)$$

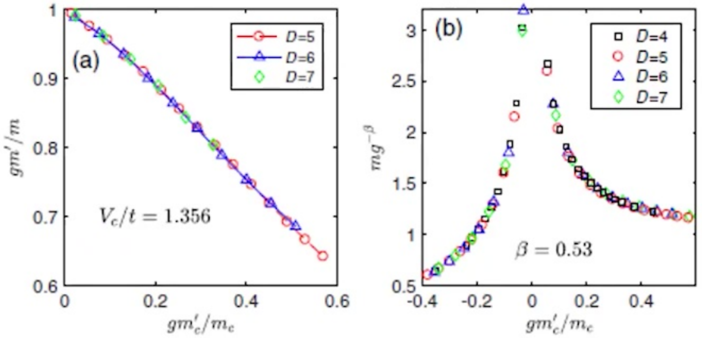


- Corboz, Czarnik, Kapteijns &Tagliacozzo:
  Spinless fermions on honeycomb lattice

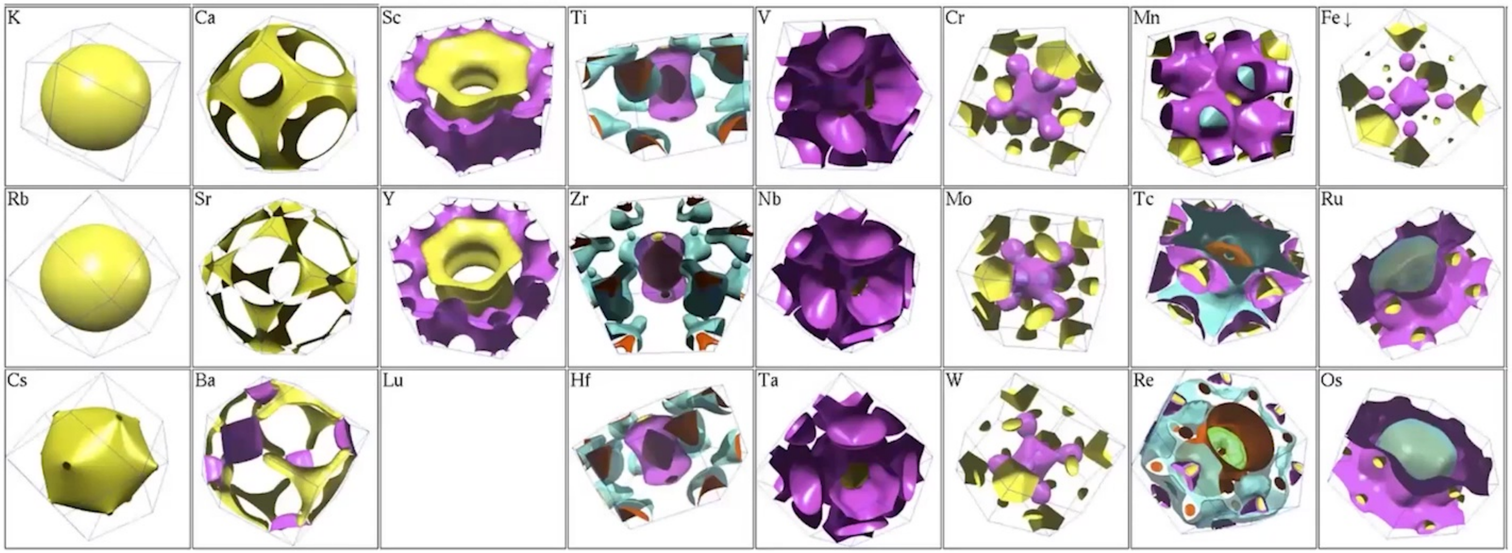
$$\hat{H} = -t\sum_{\langle \mathbf{i},\mathbf{j}\rangle}[\hat{c}_{\mathbf{i}}^{\dagger}\hat{c}_{\mathbf{j}} + \text{H.c.}] + V\sum_{\langle \mathbf{i},\mathbf{j}\rangle}\hat{n}_{\mathbf{i}}\hat{n}_{\mathbf{j}}.$$

Binder cumulants for derivative
of CDW order parameter $m = |n_A - n_B|$

$$m'(g,D) = \xi_D^{-(\beta-1)/\nu}\mathcal{M}'(g\xi_D^{1/\nu}).$$



What about systems with a Fermi surface?



http://www.phys.ufl.edu/fermisurface/

- Violation of area law:

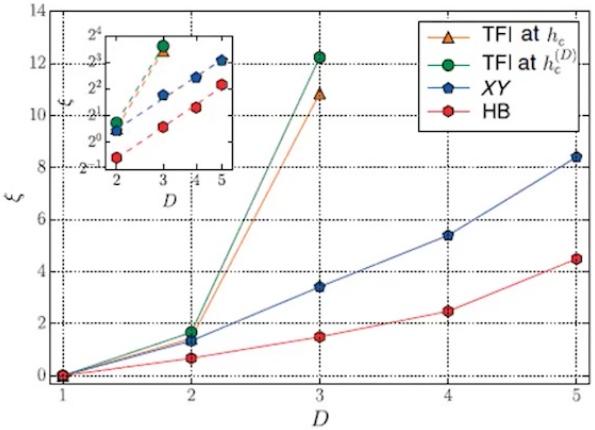
$$S \sim \frac{L^{d-1}\log L}{(2\pi)^{d-1}}\frac{1}{12}\int_{\partial\Omega}\int_{\partial\Gamma}|n_x \cdot n_p| dS_x \mathrm{d}S_p$$

Scaling theory of PEPS

- Rader & Lauchli '18 : O(N) model, N=1,2,3

$$e(\xi) = e(\infty) - \left[\alpha_{\mathrm{NLSM}}^{(\mathrm{iPEPS})}\left(\frac{N-1}{2}\right)v\right]\frac{1}{\xi^3} + \mathcal{O}\left(\frac{1}{\xi^4}\right)$$

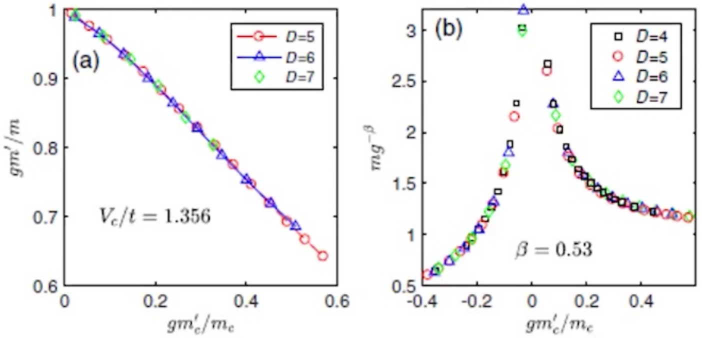
$$\frac{m^2(\xi)}{m^2(\infty)} = 1 + \left[\mu_{\mathrm{NLSM}}^{(\mathrm{iPEPS})}\left(\frac{N-1}{2}\right)\frac{v}{\rho_s}\right]\frac{1}{\xi} + \mathcal{O}\left(\frac{1}{\xi^2}\right)$$

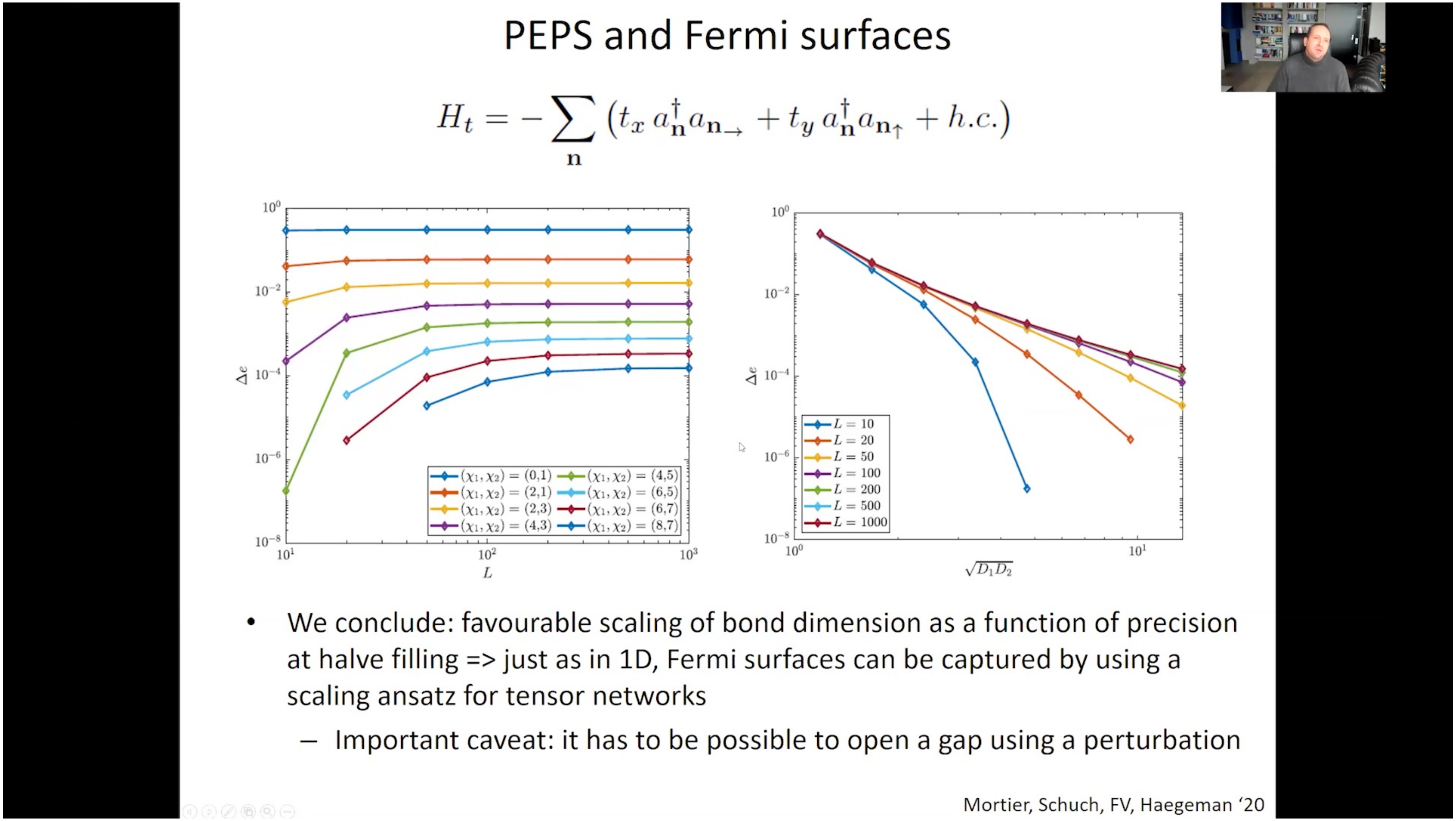


- Corboz, Czarnik, Kapteijns &Tagliacozzo:
  Spinless fermions on honeycomb lattice $\hat{H} = -t\sum_{\langle \mathbf{i},\mathbf{j}\rangle}[\hat{c}_{\mathbf{i}}^{\dagger}\hat{c}_{\mathbf{j}} + \mathrm{H.c.}] + V\sum_{\langle \mathbf{i},\mathbf{j}\rangle}\hat{n}_{\mathbf{i}}\hat{n}_{\mathbf{j}}.$

Binder cumulants for derivative
of CDW order parameter $m = |n_A - n_B|$

$$m'(g,D) = \xi_D^{-(\beta-1)/\nu}\mathcal{M}'(g\xi_D^{1/\nu}).$$





PEPS and Fermi surfaces

$$H_t = -\sum_{\mathbf{n}} \left(t_x \, a_{\mathbf{n}}^\dagger a_{\mathbf{n}_\rightarrow} + t_y \, a_{\mathbf{n}}^\dagger a_{\mathbf{n}_\uparrow} + h.c. \right)$$

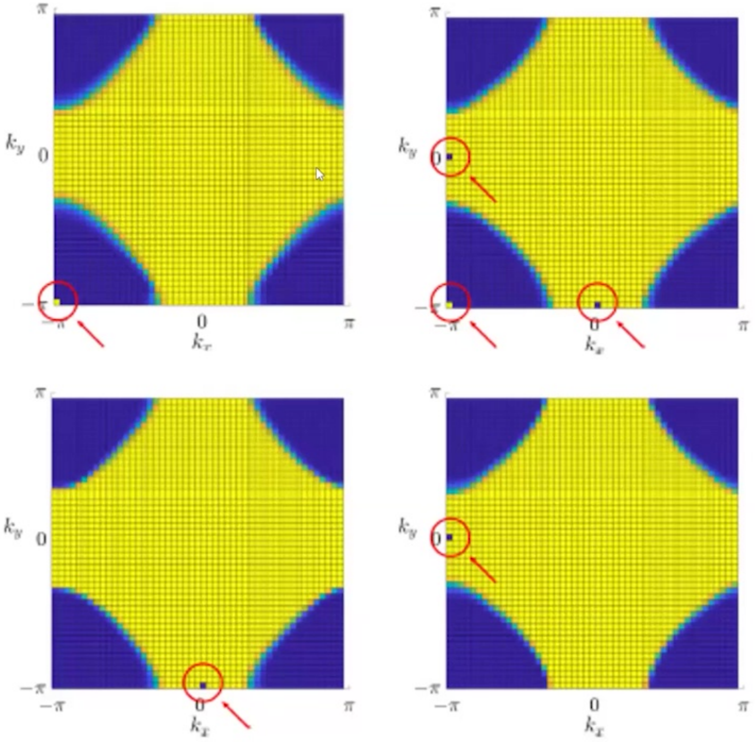
- We conclude: favourable scaling of bond dimension as a function of precision at halve filling => just as in 1D, Fermi surfaces can be captured by using a scaling ansatz for tensor networks
  - Important caveat: it has to be possible to open a gap using a perturbation

Mortier, Schuch, FV, Haegeman '20

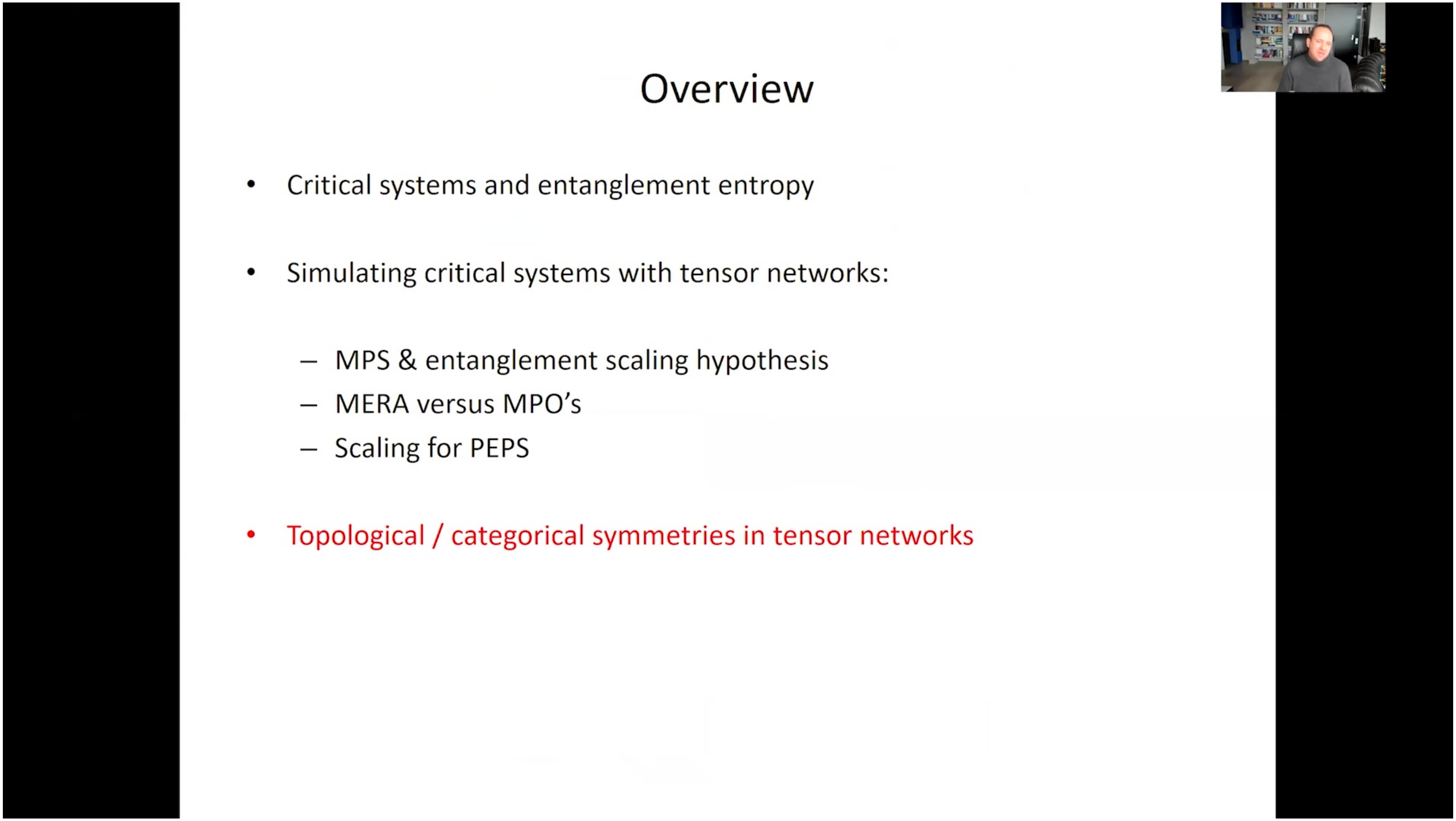
Topological Obstructions for PEPS

- Free fermionic PEPS cannot deal with systems with a nontrivial Chern number

$$H_t = -t \sum_{\mathbf{n}} \left(a_{\mathbf{n}}^\dagger a_{\mathbf{n}_\rightarrow} + a_{\mathbf{n}}^\dagger a_{\mathbf{n}_\uparrow} + h.c. \right) - \mu \sum_{\mathbf{n}} a_{\mathbf{n}}^\dagger a_{\mathbf{n}} - \Delta \sum_{\mathbf{n}} \left(a_{\mathbf{n}}^\dagger a_{\mathbf{n}_\rightarrow}^\dagger + i a_{\mathbf{n}}^\dagger a_{\mathbf{n}_\uparrow}^\dagger + h.c. \right)$$

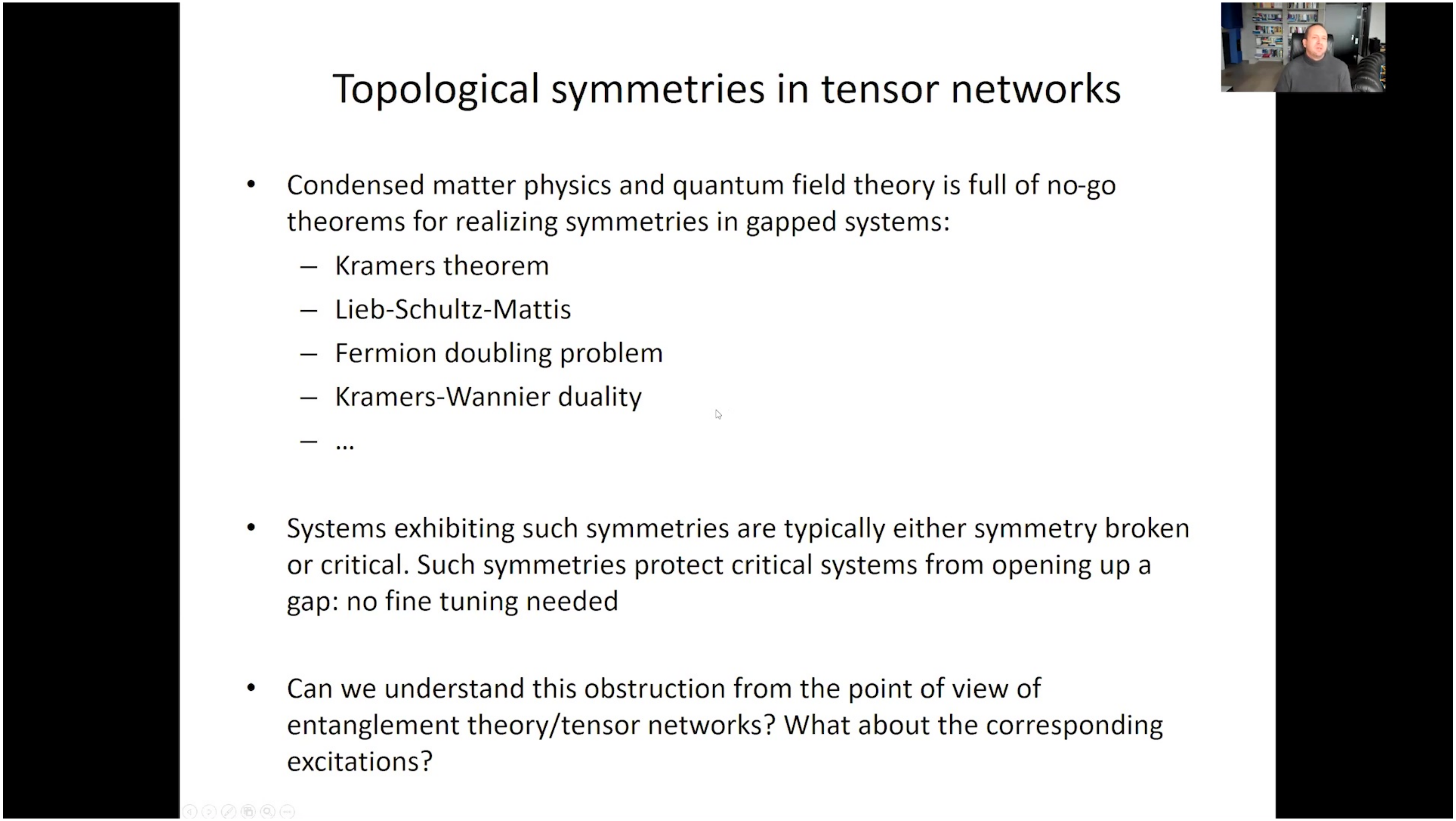


Mortier, Schuch, FV, Haegeman '20



Overview

- Critical systems and entanglement entropy
- Simulating critical systems with tensor networks:
  - MPS & entanglement scaling hypothesis
  - MERA versus MPO's
  - Scaling for PEPS
- Topological / categorical symmetries in tensor networks



Topological symmetries in tensor networks

- Condensed matter physics and quantum field theory is full of no-go theorems for realizing symmetries in gapped systems:
  - Kramers theorem
  - Lieb-Schultz-Mattis
  - Fermion doubling problem
  - Kramers-Wannier duality
  - …

- Systems exhibiting such symmetries are typically either symmetry broken or critical. Such symmetries protect critical systems from opening up a gap: no fine tuning needed

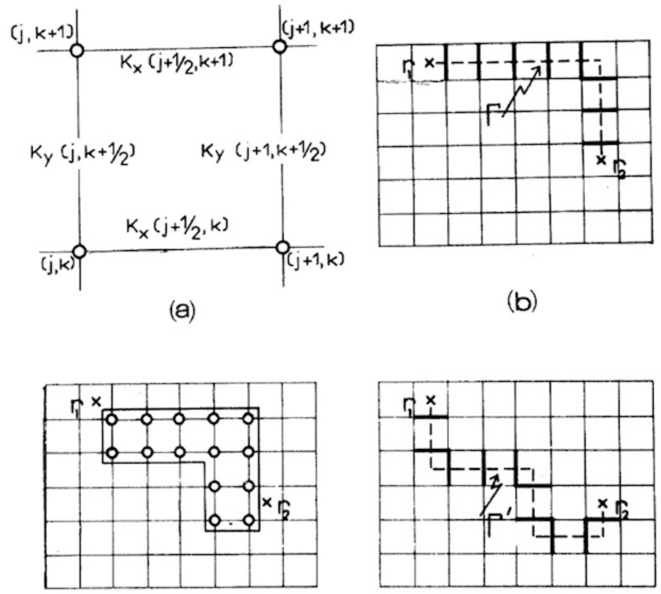
- Can we understand this obstruction from the point of view of entanglement theory/tensor networks? What about the corresponding excitations?

PHYSICAL REVIEW B VOLUME 3, NUMBER 11 1 JUNE 1

Determination of an Operator Algebra for the Two-Dimensional Ising Model

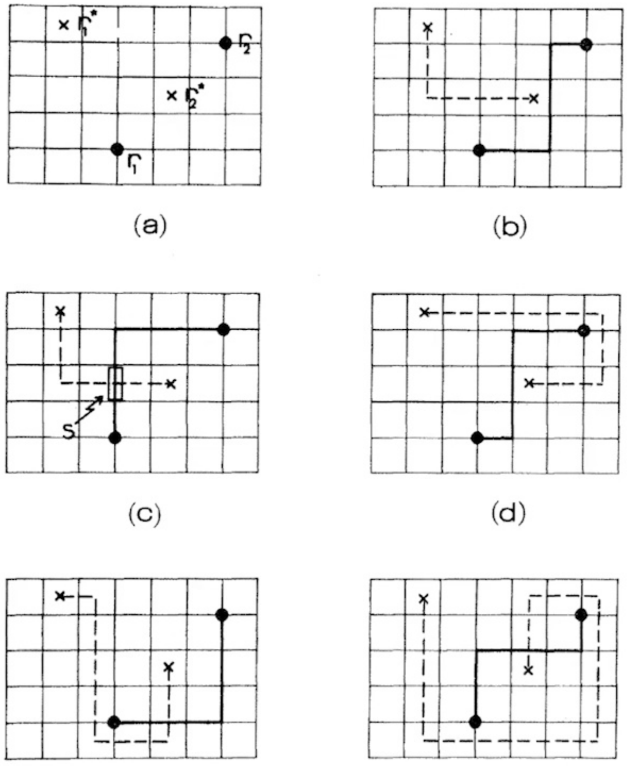
Leo P. Kadanoff and Horacio Ceva
Department of Physics, Brown University, Providence, Rhode Island 02912

(Received 18 November 1970)



(a) (b)

(c) (d)

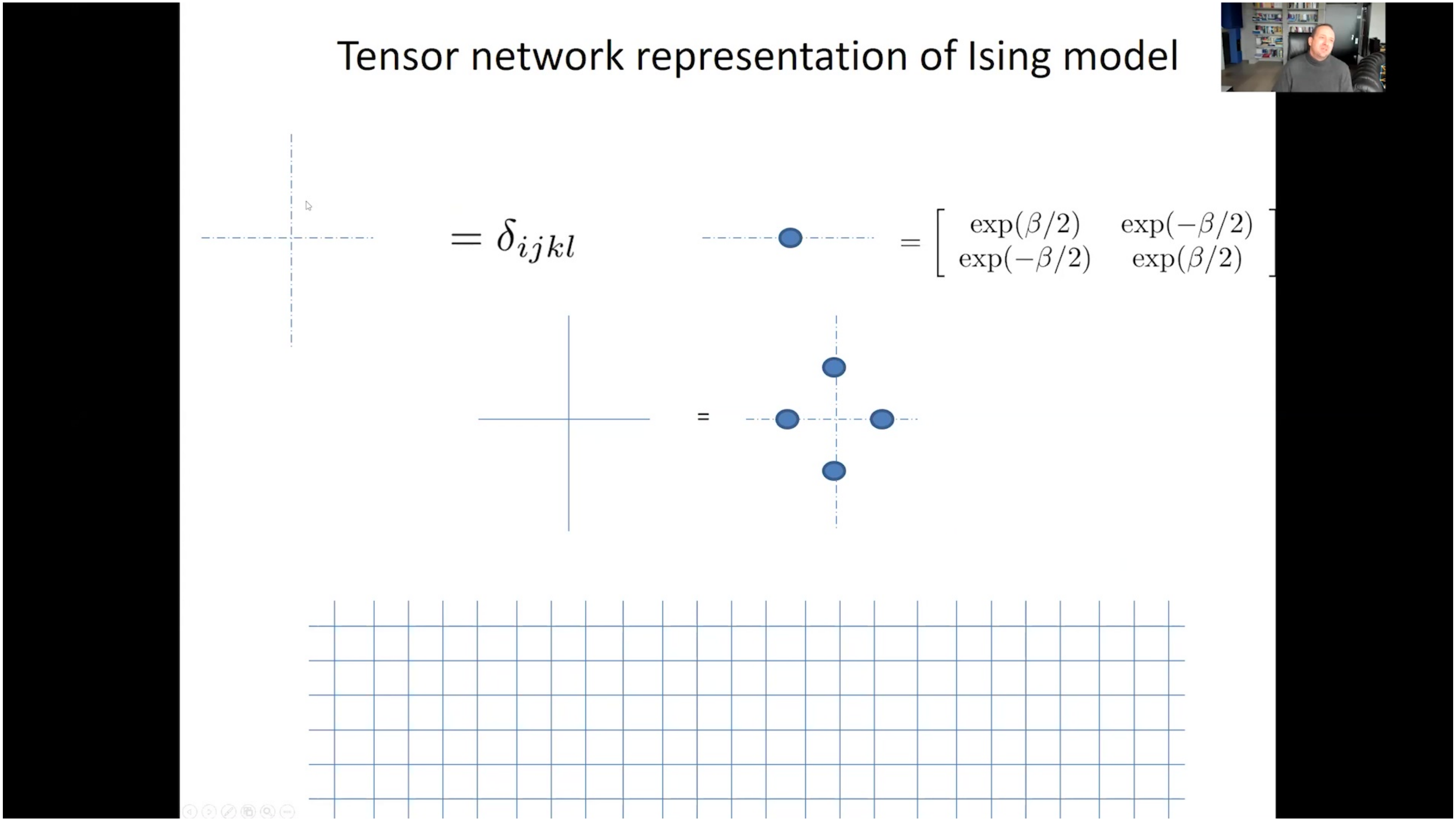


(a) (b)

(c) (d)

(e) (f)

- Kadanoff & Ceva: there is a NONLOCAL order parameter in the symmetric phase (string of …XXXXX…..) which anticommutes with the LOCAL order parameter in the symmetry broken phase (Z).
  - Both order parameters have to vanish at the critical point, but a new symmetry as a combination of the two emerges, which is highly nontrivial as they anticommute => "topological symmetry"
  - Not a symmetry in the strict sense, as it is not unitary
- How can we understand this from the point of view of tensor networks? Can we generalize the Kadanoff-Ceva construction to other models?
- Lots of works in this direction: Fuchs, Runkel, Schweigert '00-'10; Petkova, Zuber '01; … ; Aasen, Mong, Fendley '16; Buican, Gromov '17; Ji, Wen '20; …

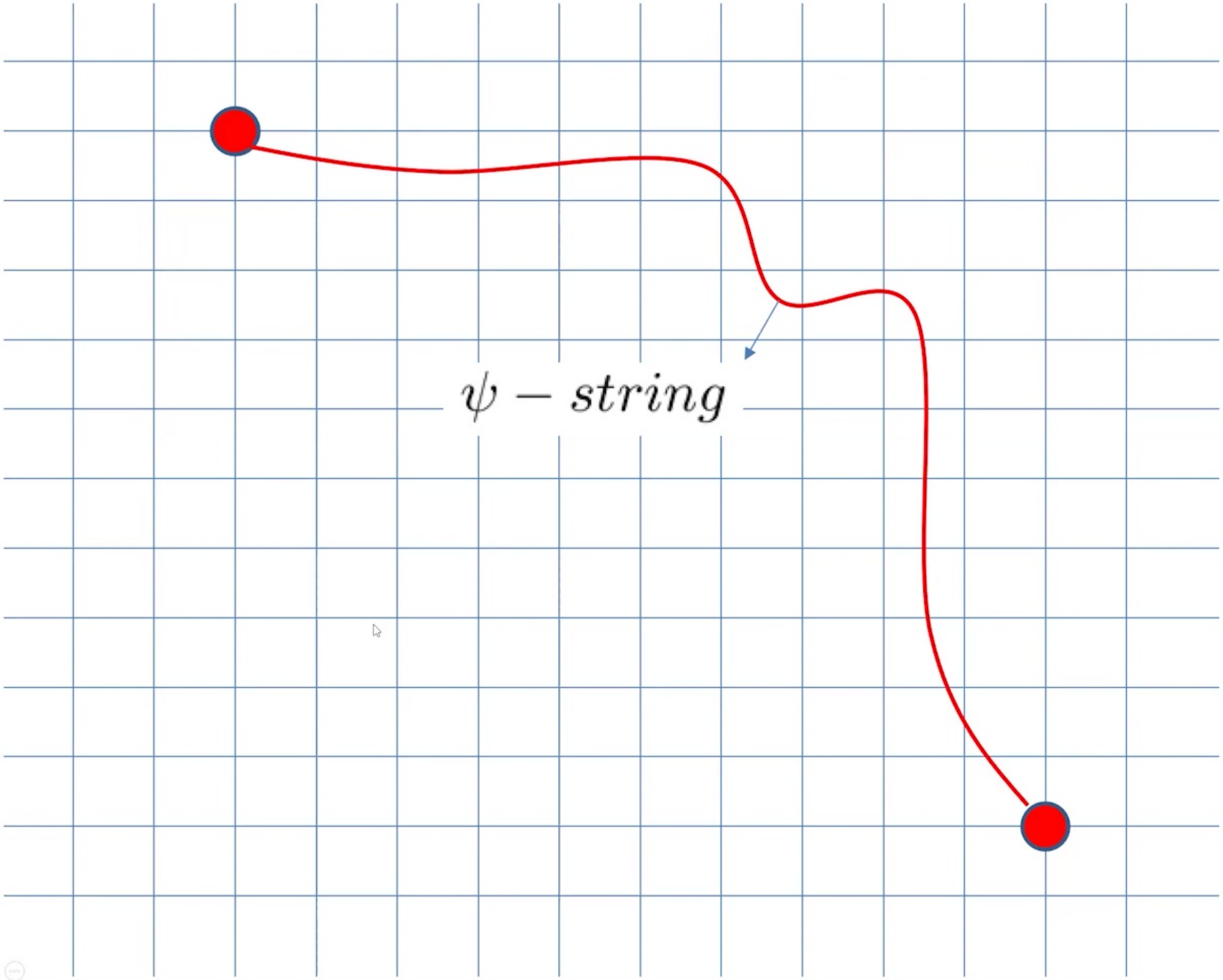


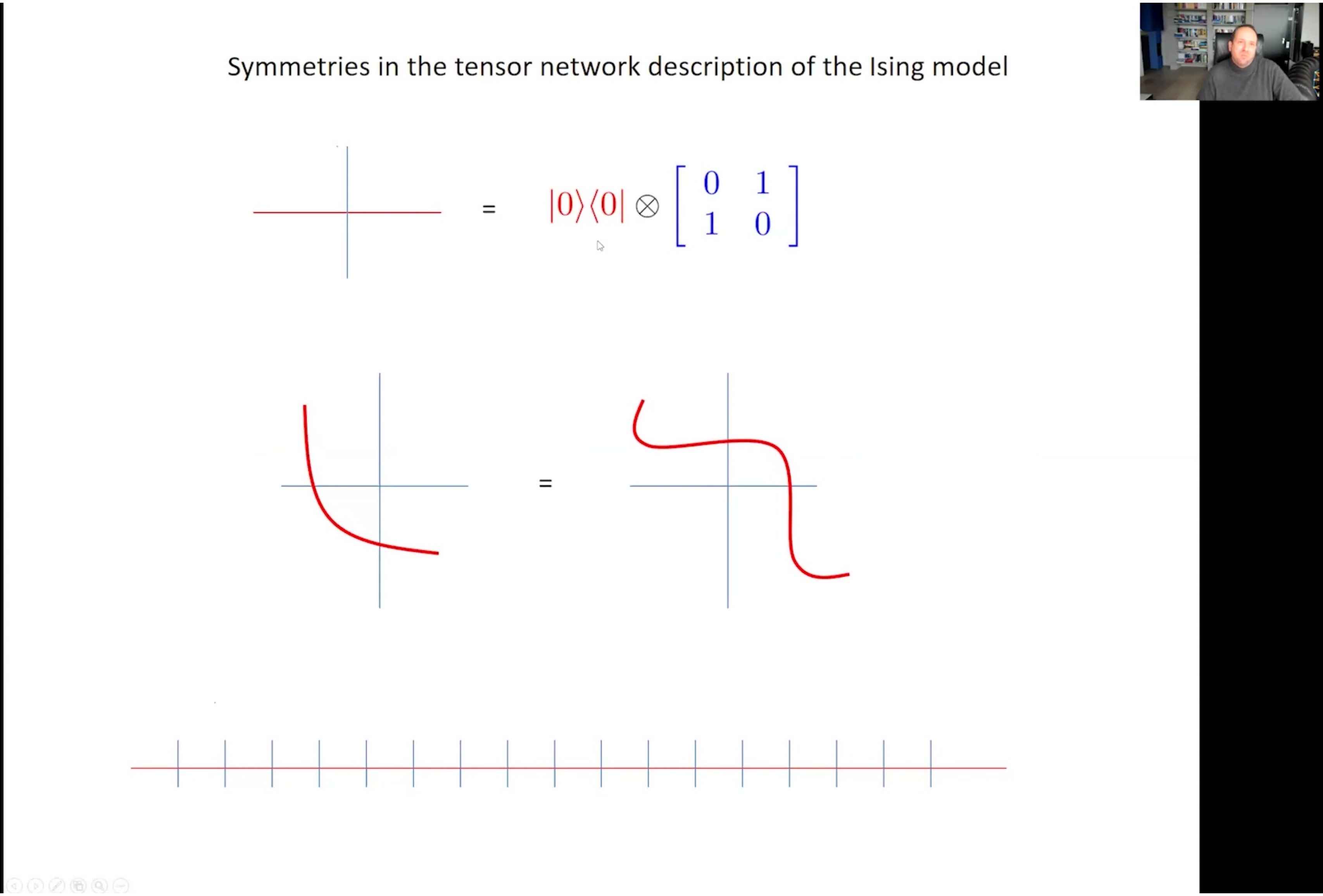
Tensor network representation of Ising model

$= \delta_{ijkl}$

$= \begin{bmatrix} \exp(\beta/2) & \exp(-\beta/2) \\ \exp(-\beta/2) & \exp(\beta/2) \end{bmatrix}$

Excitations of disorder type

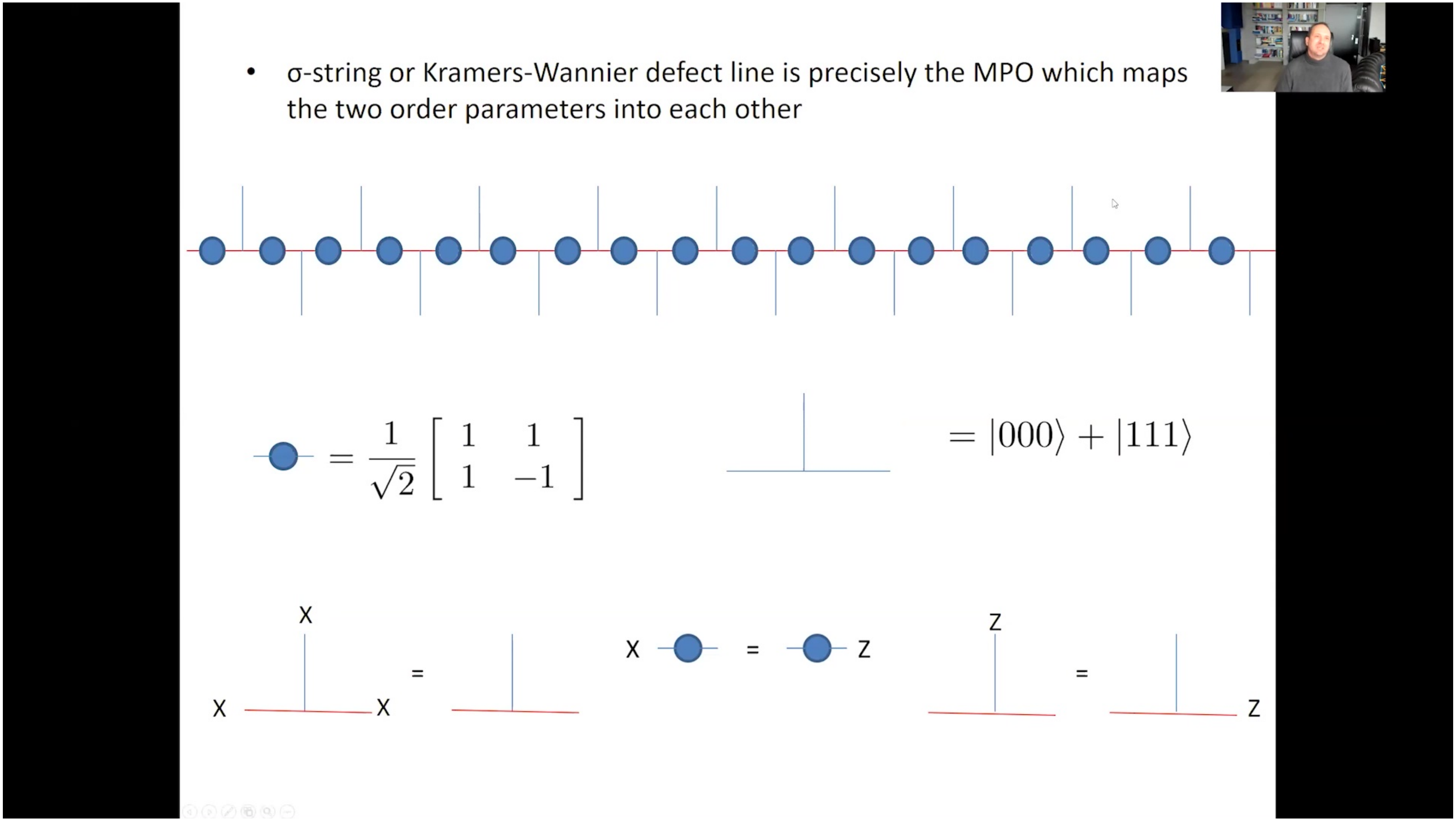




Symmetries in the tensor network description of the Ising model

$$= \ |0\rangle\langle 0| \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$=$$



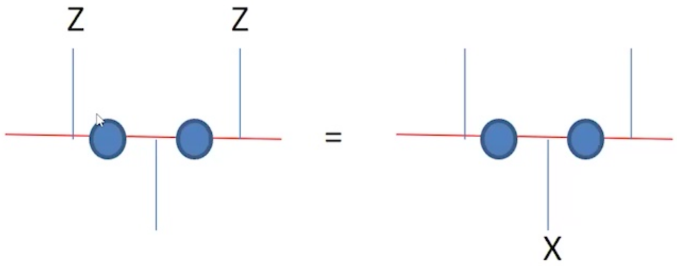
- σ-string or Kramers-Wannier defect line is precisely the MPO which maps the two order parameters into each other

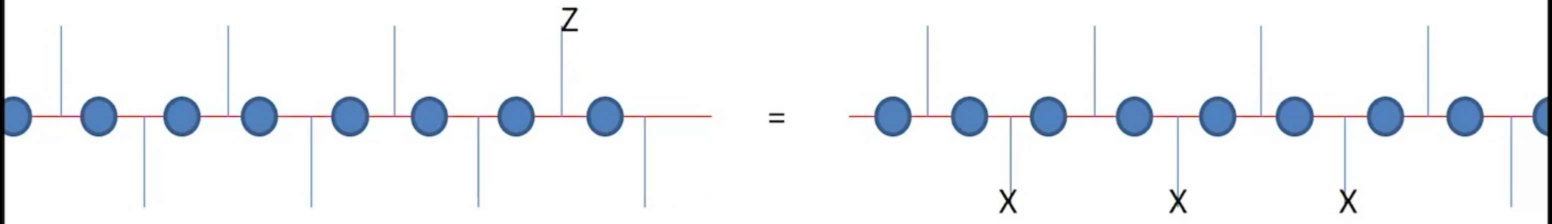
$$= \frac{1}{\sqrt{2}}\left[\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array}\right]$$

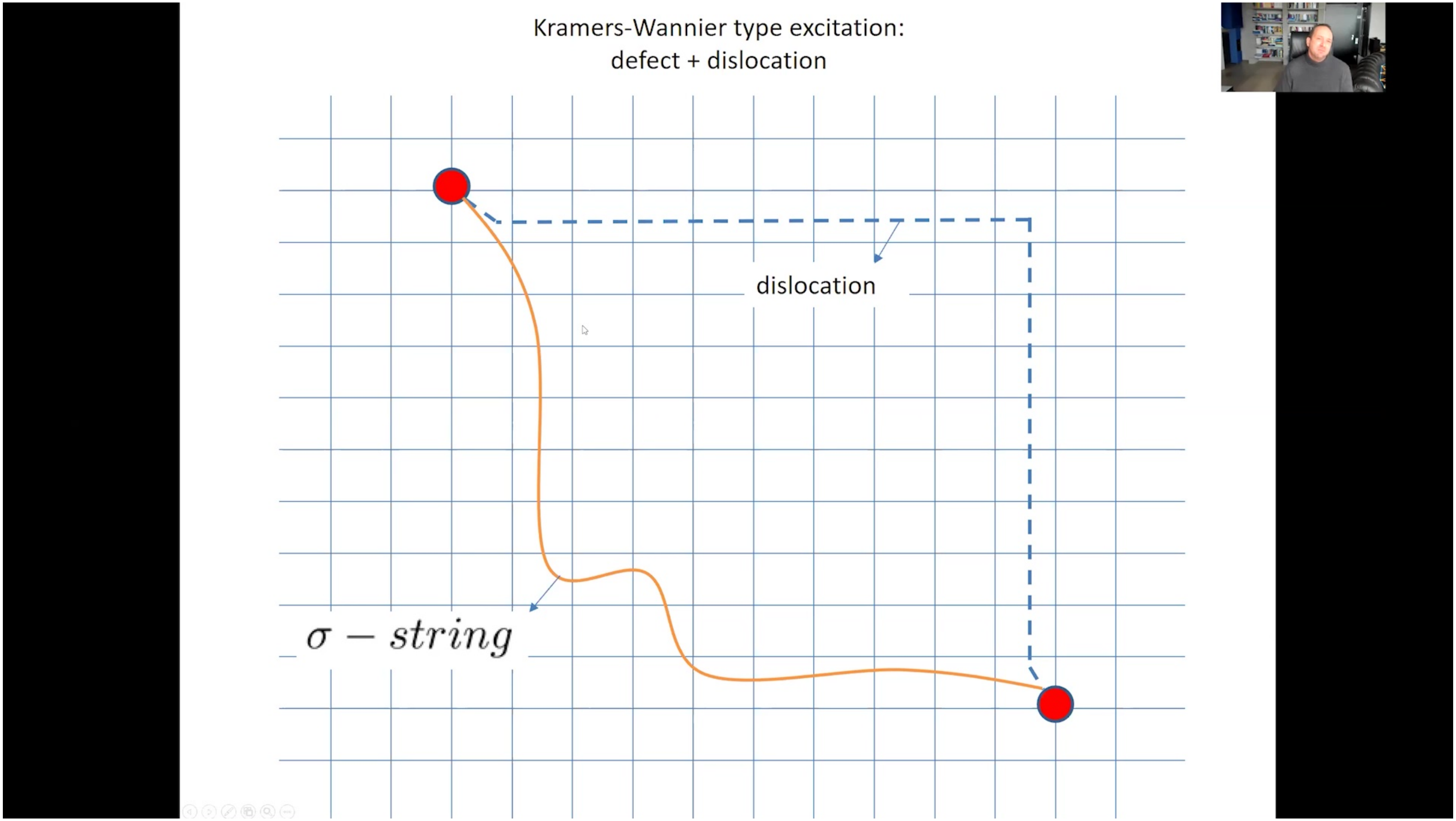
$$= |000\rangle + |111\rangle$$

- This σ-MPO maps the Ising model to its dual (and hence critical Ising to itsel

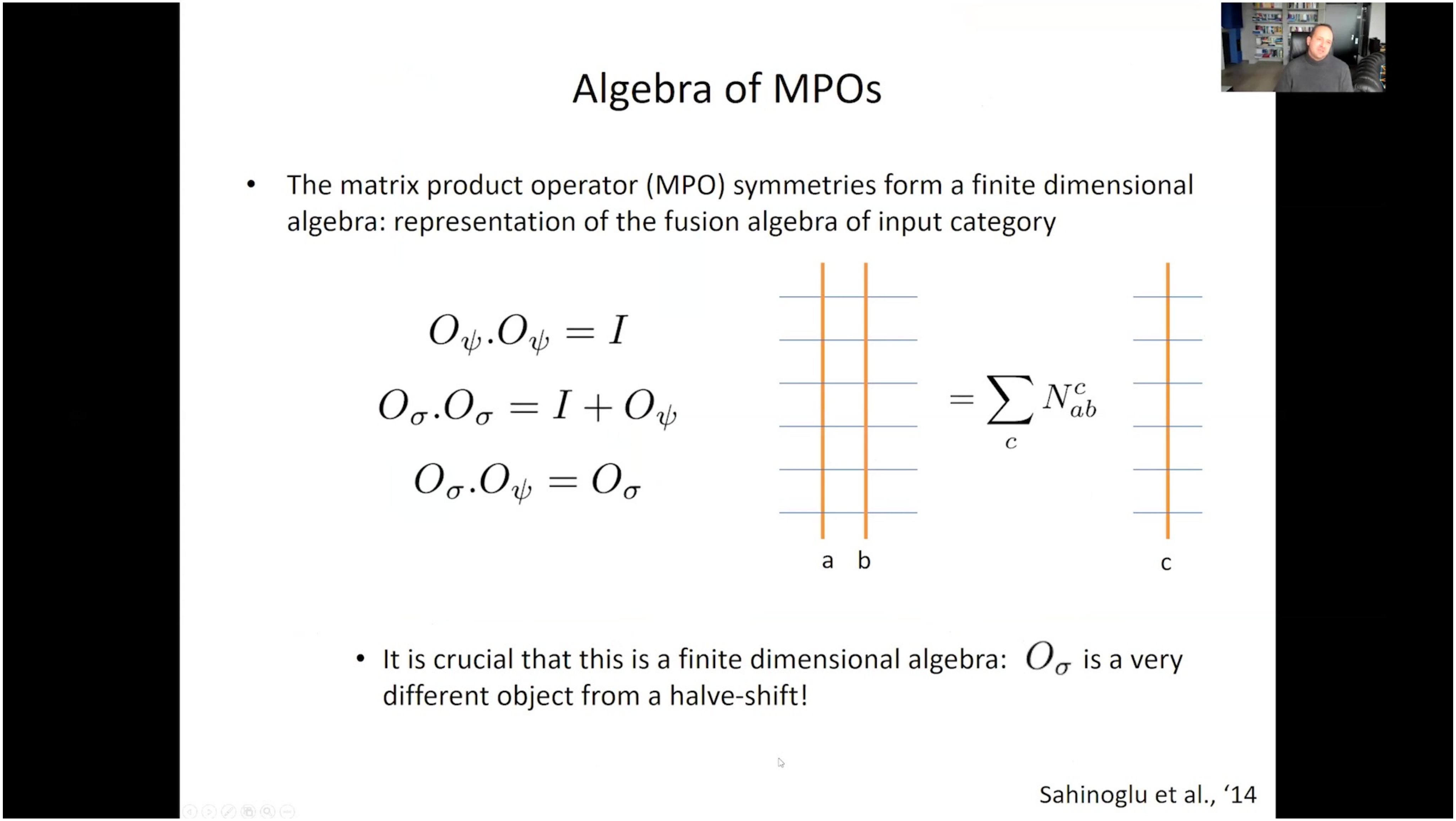
$$H = \sum_i Z_i Z_{i+1} + \lambda X_i \leftrightarrow \sum_i \lambda Z_i Z_{i+1} + X_i$$







Kramers-Wannier type excitation:
defect + dislocation



Algebra of MPOs

- The matrix product operator (MPO) symmetries form a finite dimensional algebra: representation of the fusion algebra of input category

$$O_\psi . O_\psi = I$$

$$O_\sigma . O_\sigma = I + O_\psi$$

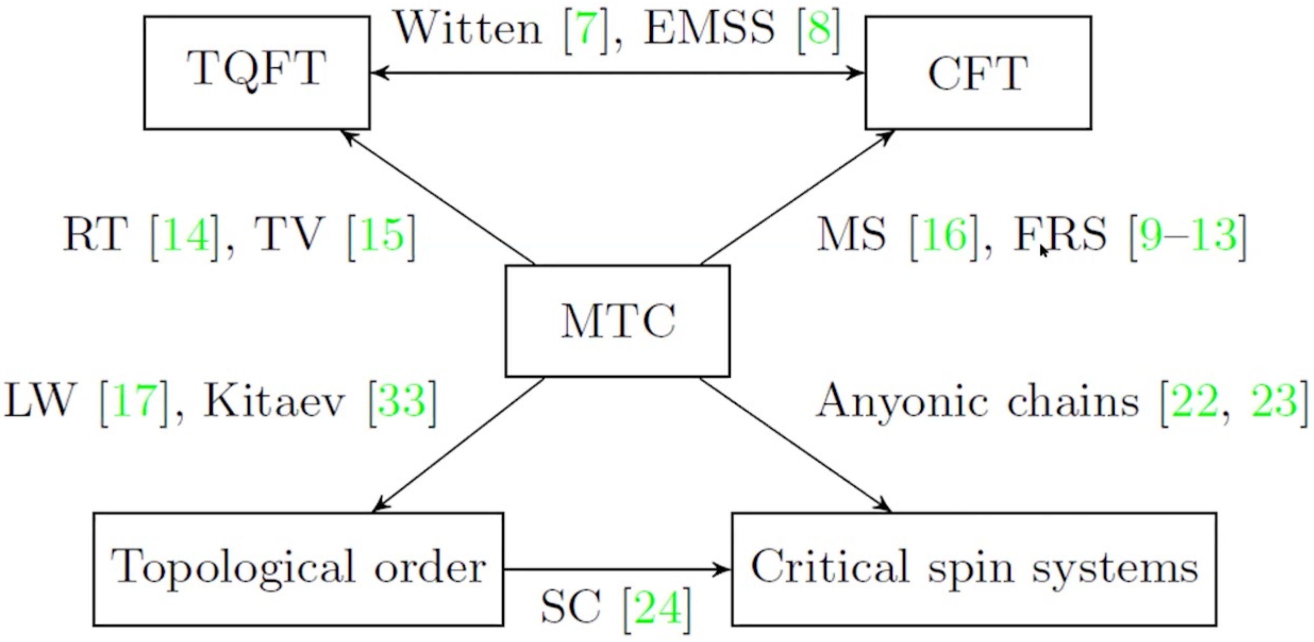
$$O_\sigma . O_\psi = O_\sigma$$

$$= \sum_c N_{ab}^c$$

- It is crucial that this is a finite dimensional algebra: O_σ is a very different object from a halve-shift!

Sahinoglu et al., '14

Categorical connection between TFT and CFT



TQFT

Witten [7], EMSS [8]

CFT

RT [14], TV [15]

MS [16], FRS [9–13]

MTC

LW [17], Kitaev [33]

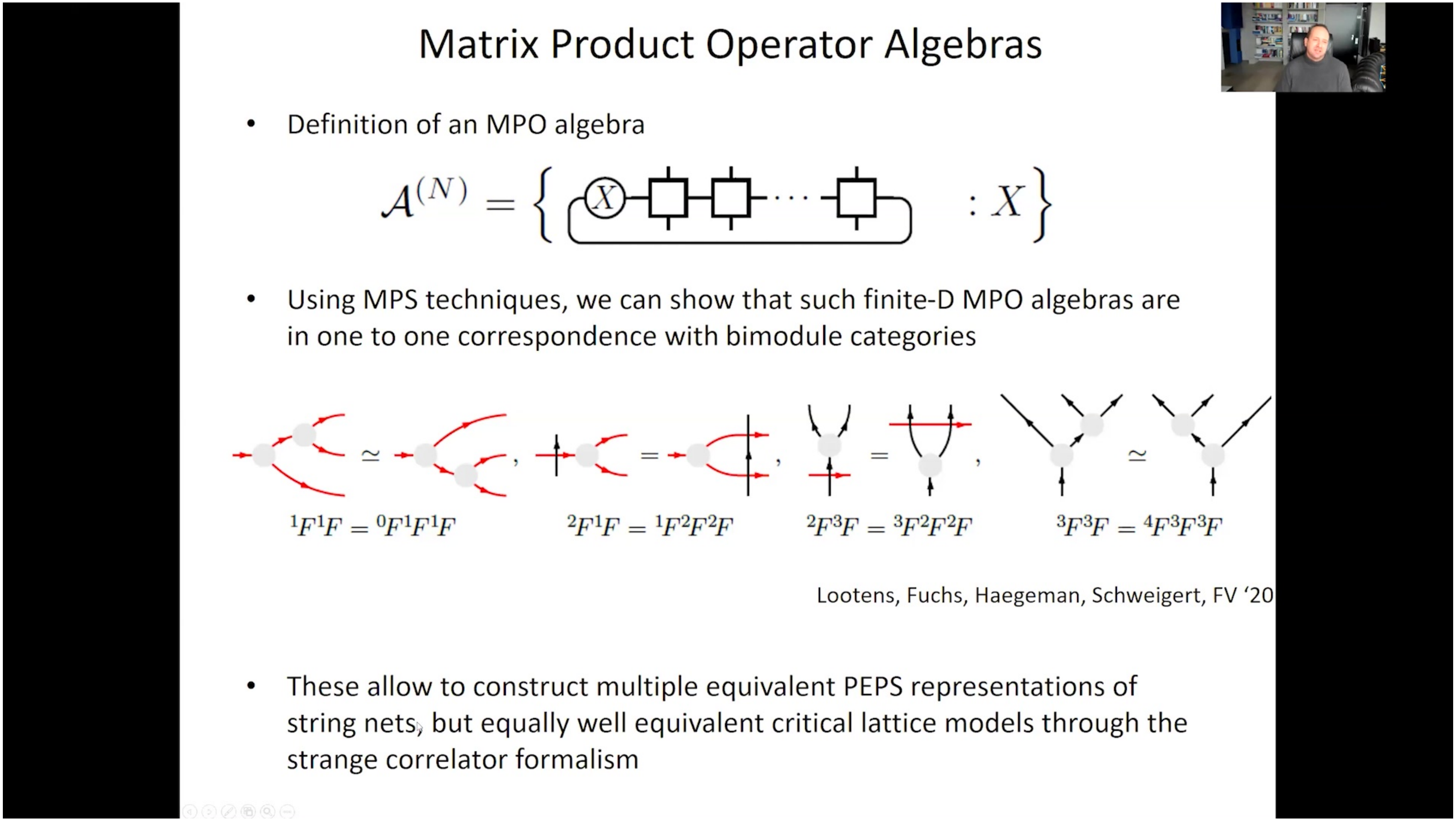
Anyonic chains [22, 23]

Topological order

SC [24]

Critical spin systems

Lootens, Vanhove, FV '19



Matrix Product Operator Algebras

- Definition of an MPO algebra

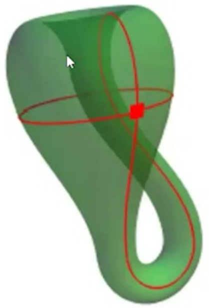
- Using MPS techniques, we can show that such finite-D MPO algebras are in one to one correspondence with bimodule categories

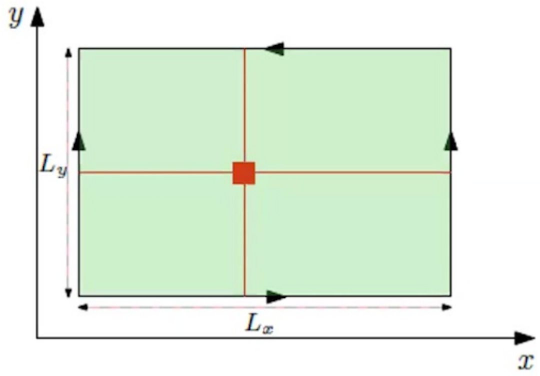
${}^1F^1F = {}^0F^1F^1F$ ${}^2F^1F = {}^1F^2F^2F$ ${}^2F^3F = {}^3F^2F^2F$ ${}^3F^3F = {}^4F^3F^3F$

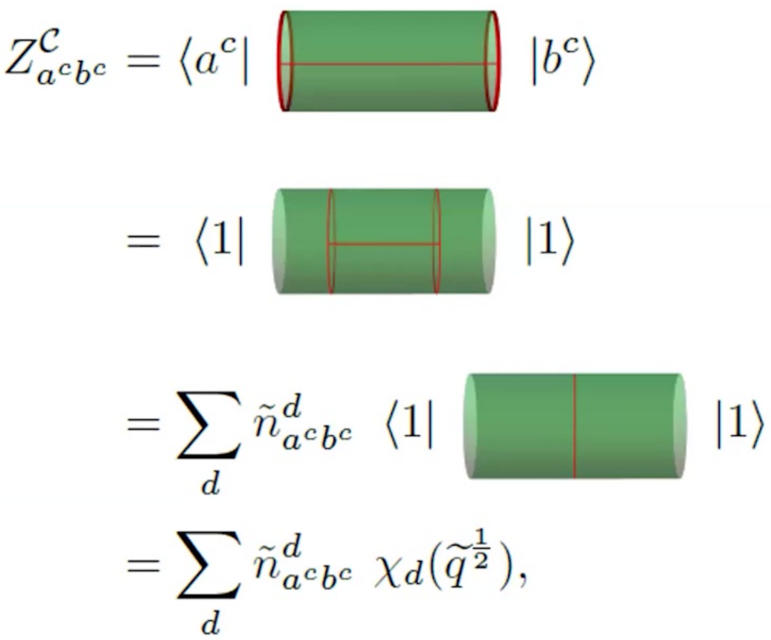
Lootens, Fuchs, Haegeman, Schweigert, FV ‘20

- These allow to construct multiple equivalent PEPS representations of string nets, but equally well equivalent critical lattice models through the strange correlator formalism

Application: extracting single characters of CFT partition functions using tensor networks







$$Z^{\mathcal{C}}_{a^c b^c} = \langle a^c| \qquad |b^c\rangle$$

$$= \langle 1| \qquad |1\rangle$$

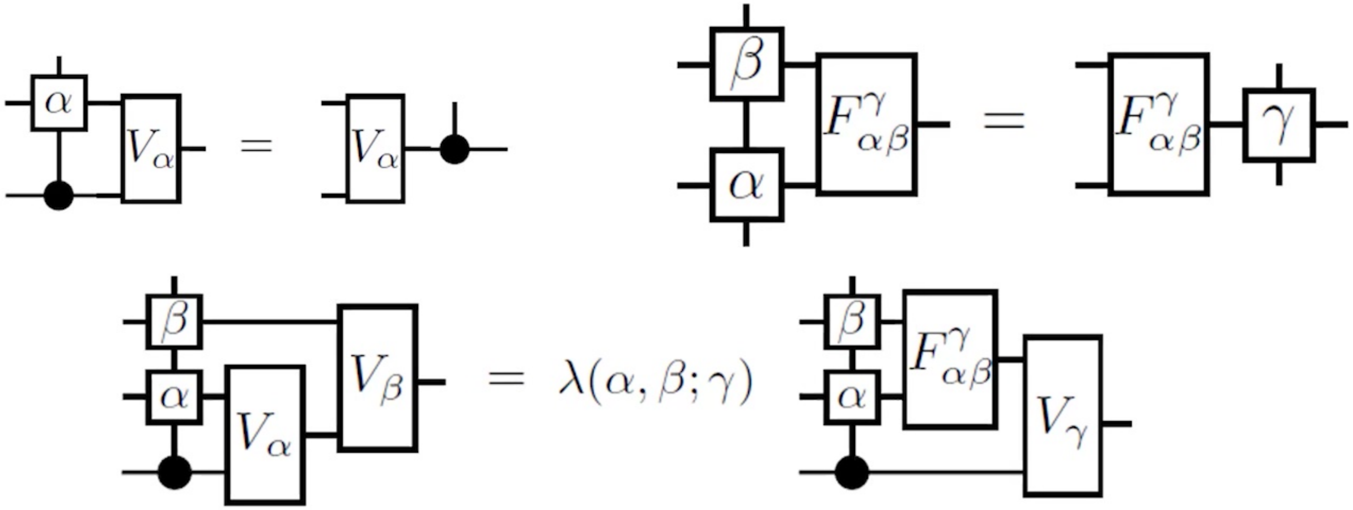
$$= \sum_d \tilde{n}^d_{a^c b^c} \; \langle 1| \qquad |1\rangle$$

$$= \sum_d \tilde{n}^d_{a^c b^c} \; \chi_d(\widetilde{q}^{\frac{1}{2}}),$$

Vanhove, Tu, Lootens, FV ‘20

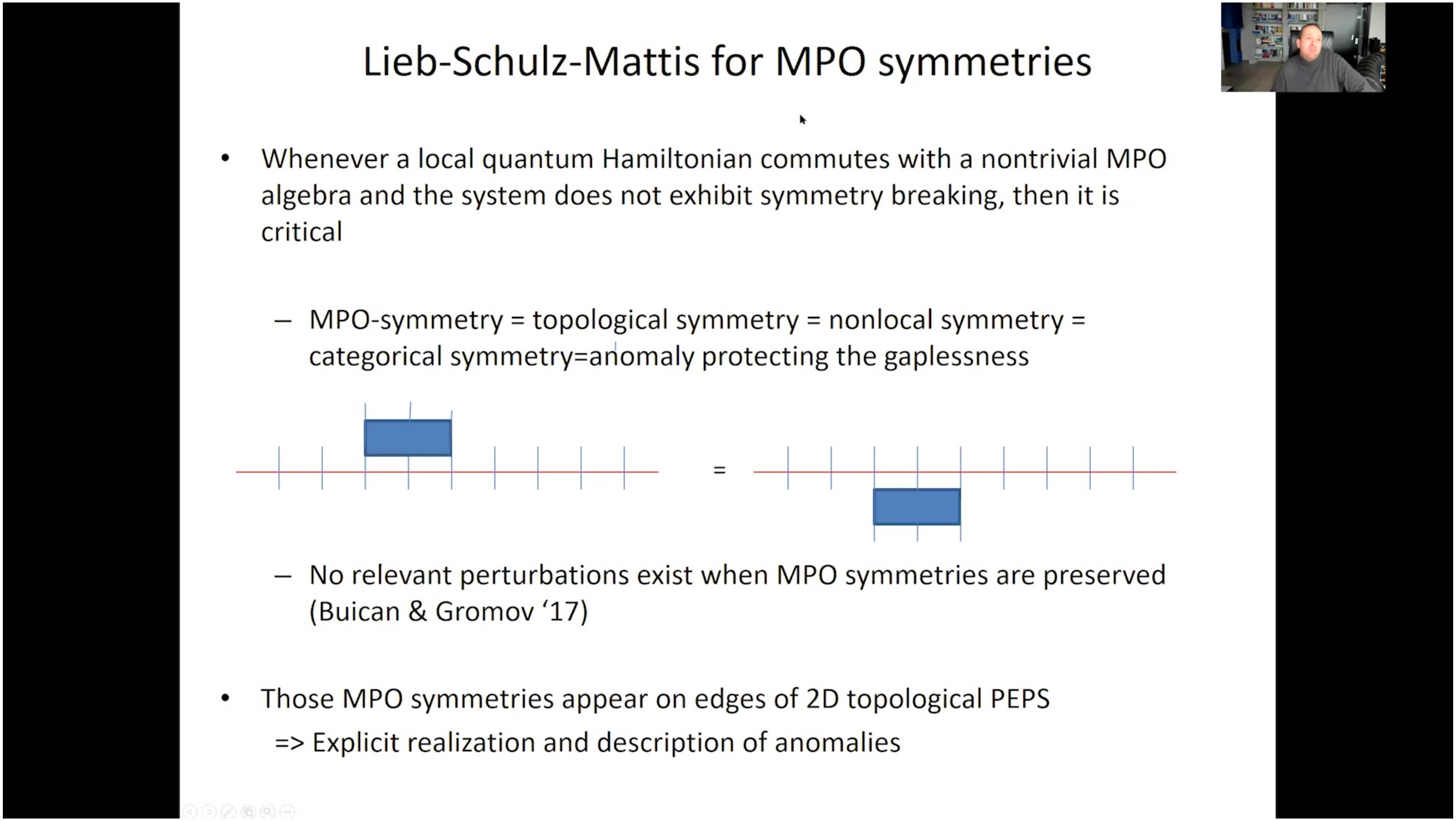
No MPS can exhibit a MPO symmetry

- Key theorem: an MPS cannot exhibit such an MPO symmetry => only critical or (spatial) symmetry broken systems can exhibit MPO symmetries



$$\frac{\lambda(\alpha, \beta; a).\lambda(a, \gamma; b)}{\lambda(\beta, \gamma; c).\lambda(\alpha, c; b)} = F^{\alpha\beta\gamma}_{bac}$$

Chen, Gu, Liu, Wen ‘13
Cirac, Perez-Garcia, Schuch, FV ‘20



Lieb-Schulz-Mattis for MPO symmetries

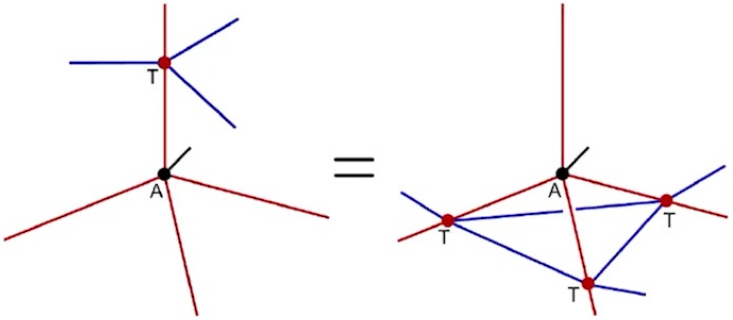
- Whenever a local quantum Hamiltonian commutes with a nontrivial MPO algebra and the system does not exhibit symmetry breaking, then it is critical
  - MPO-symmetry = topological symmetry = nonlocal symmetry = categorical symmetry=anomaly protecting the gaplessness

  - No relevant perturbations exist when MPO symmetries are preserved (Buican & Gromov ‘17)
- Those MPO symmetries appear on edges of 2D topological PEPS

  => Explicit realization and description of anomalies

Three dimensions

- Instead of “pulling through” symmetry, we have to consider tetrahedral symmetries:

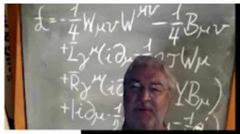


  - For group case: see e.g. A Bullivant, C. Delcamp (’19,’20)

- Challenges:
  - Fusion rules of the PEPO operators?
  - Identify critical point by enhanced PEPO symmetries?
    - Exact critical exponents using PEPO algebras?
  - Construct chiral topologically ordered tensor network descriptions by studying the edge physics?

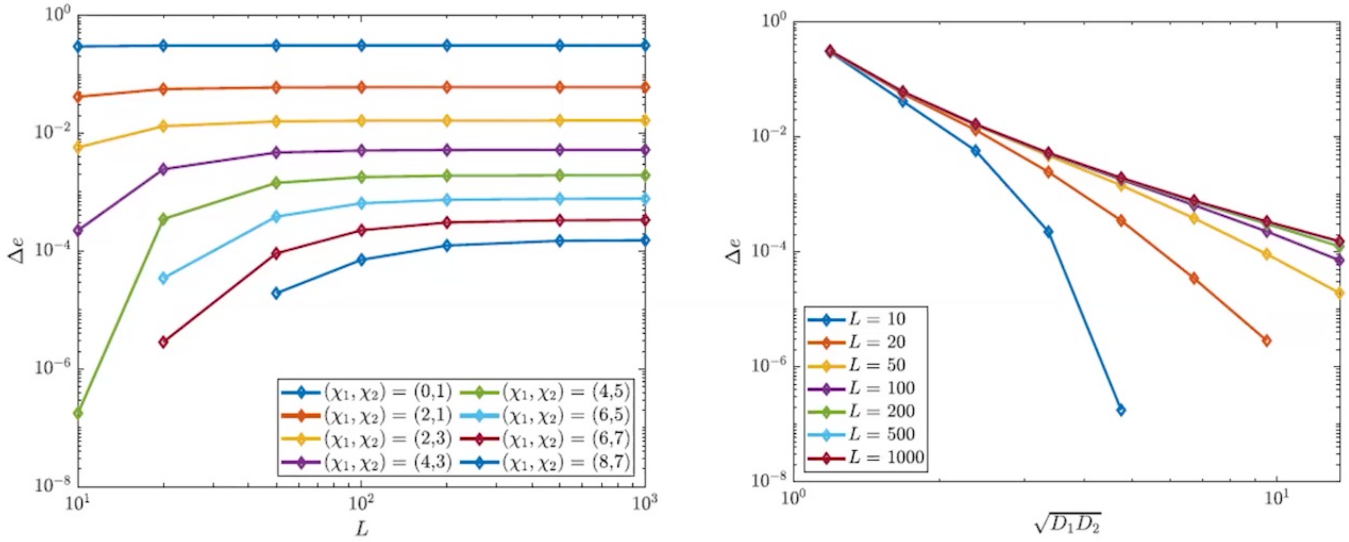
Summary

- Tensor networks provide an entanglement-scaling perspective on describing and simulating critical theories.
  - We talked about entanglement scaling theory for MPS, MERA and PEPS, including systems with Fermi surfaces
  - We discussed topological obstructions to representing critical theories with MPS
    - Challenge: do the same for PEPS. Membrane operators, chiral phases as anomalies on Walker-Wang type models, …
    - What about continuous MPS?



PEPS and Fermi surfaces

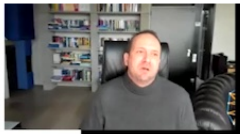
$$H_t = -\sum_{\mathbf{n}} \left(t_x \, a^\dagger_{\mathbf{n}} a_{\mathbf{n}_\rightarrow} + t_y \, a^\dagger_{\mathbf{n}} a_{\mathbf{n}_\uparrow} + h.c. \right)$$



- We conclude: favourable scaling of bond dimension as a function of precision at halve filling => just as in 1D, Fermi surfaces can be captured by using a scaling ansatz for tensor networks
  - – Important caveat: it has to be possible to open a gap using a perturbation

Mortier, Schuch, FV, Haegeman ‘20

What about scaling of MPS for field theories?



gapped spin systems : $\Lambda_{UV} = 1, \Lambda_{IR} < m$ ⟶ Exact

critical spin systems: $\Lambda_{UV} = 1, \Lambda_{IR} \to 0$

gapped QFTs: $\Lambda_{UV} \to \infty, \Lambda_{IR} < m$

critical QFTs: $\Lambda_{UV} \to \infty, \Lambda_{IR} \to 0$

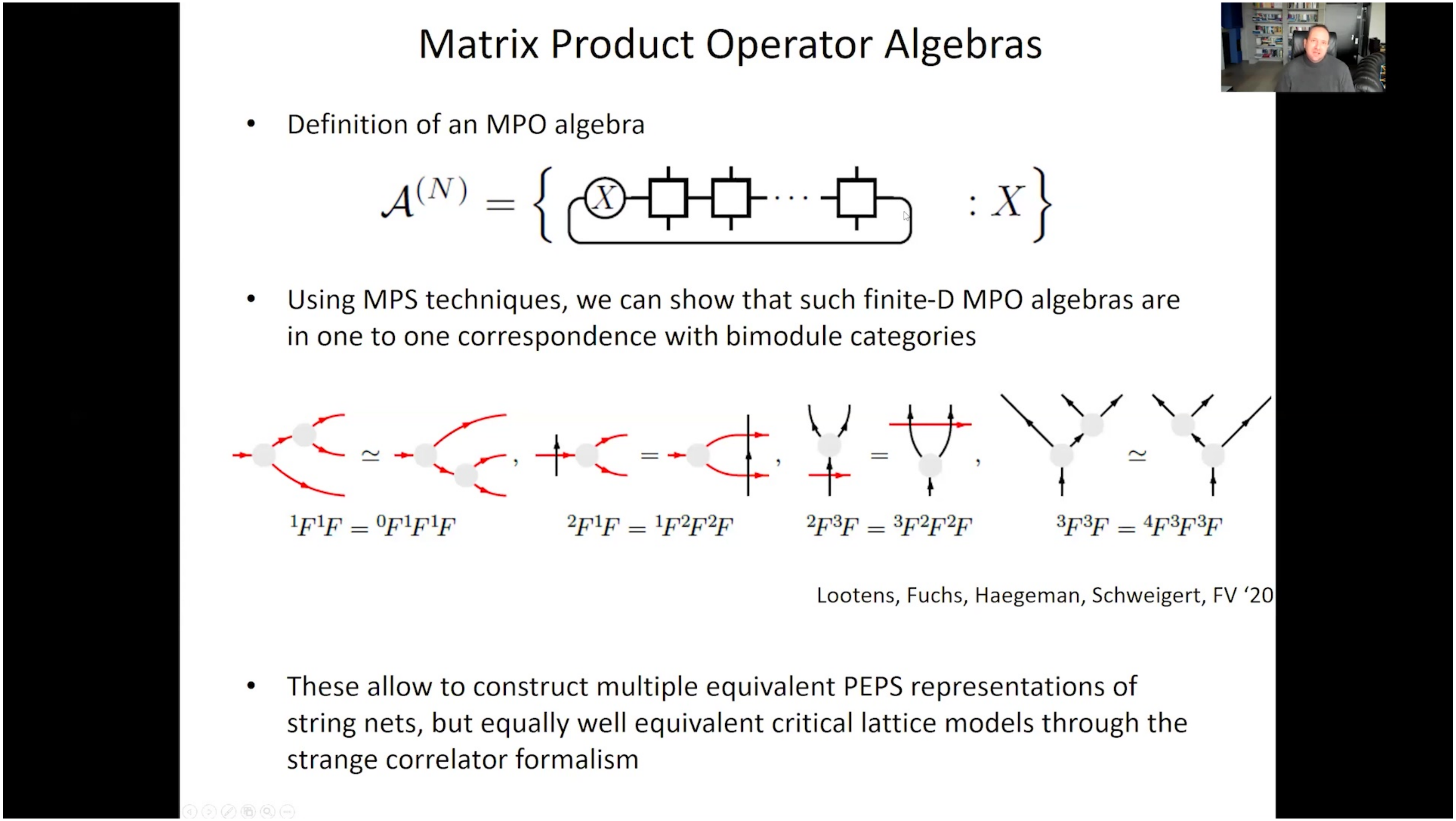
⟶ Extrapolation, scaling theory

- Let us look at $\lambda\phi^4$ to see how the two scales manifest themselves in the entanglement degrees of freedom

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu_p^2\phi^2 + \frac{1}{4}\lambda_p\phi^4.$$

  - Double scaling regime: entanglement scaling + continuum (lattice parameter) should lead to both a c=1 contribution from UV AND a c=1/2 contribution from IR

Vanhecke et al.'19



Matrix Product Operator Algebras

- Definition of an MPO algebra

$$\mathcal{A}^{(N)} = \left\{ \qquad : X \right\}$$

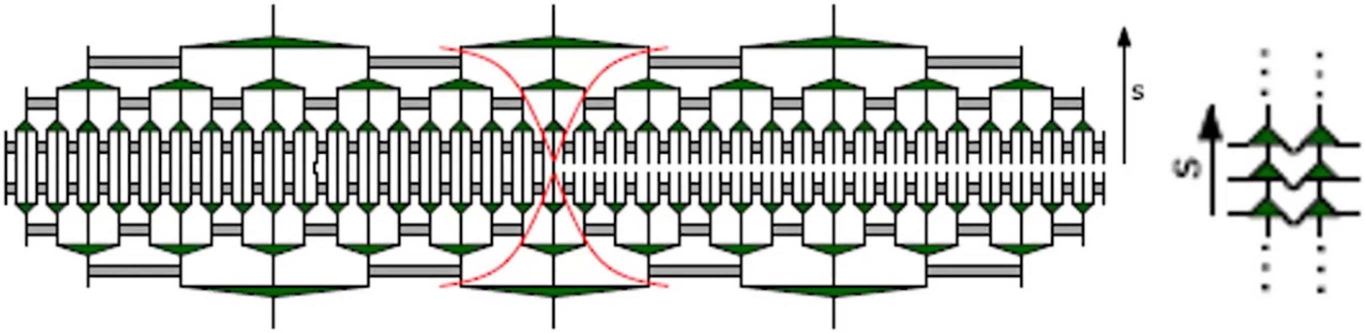
- Using MPS techniques, we can show that such finite-D MPO algebras are in one to one correspondence with bimodule categories

$${}^{1}F\,{}^{1}F = {}^{0}F\,{}^{1}F\,{}^{1}F \qquad {}^{2}F\,{}^{1}F = {}^{1}F\,{}^{2}F\,{}^{2}F \qquad {}^{2}F\,{}^{3}F = {}^{3}F\,{}^{2}F\,{}^{2}F \qquad {}^{3}F\,{}^{3}F = {}^{4}F\,{}^{3}F\,{}^{3}F$$

Lootens, Fuchs, Haegeman, Schweigert, FV '20

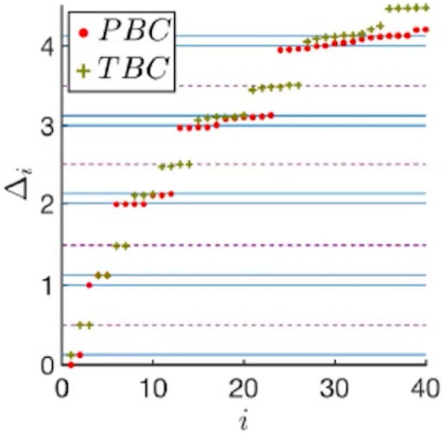
- These allow to construct multiple equivalent PEPS representations of string nets, but equally well equivalent critical lattice models through the strange correlator formalism

Entanglement scaling in MERA



- What is the meaning of the finite bond dimension in the MERA?
- Entanglement structure is equivalent as the one of the modular Hamiltonian at $T = \log(3)/2\pi$, which becomes a translational invariant in scale space:

$$\rho_{\text{scale}} = \exp^{-\frac{2\pi}{\ln(3)}\bar{H}}$$



- The bond dimension of MERA can therefore be related to that of a Matrix Product Operator approximation of a Gibbs state at finite T
- Suggests new algorithms for MERA by relating isometries to tensor of MPO

Czech, Evenbly, Lamprou, McCandlish, Qi, Sully, Vidal (2016)
Van Acoleyen, Hallam, Bal, Hauru , Haegeman, FV '20