

Title: Analytic calculation of the power spectrum covariance: speedup by four orders of magnitude

Speakers: Digvijay Wadekar

Series: Cosmology & Gravitation

Date: November 10, 2020 - 11:00 AM

URL: <http://pirsa.org/20110007>

Abstract: In order to infer cosmological parameters from galaxy survey data, we typically use summary statistics such as the power spectrum and we need an accurate estimate of their covariance matrix. The traditional process of obtaining the covariance involves simulating thousands of mocks. I will present an analytic approach for the covariance matrix which is more than four orders of magnitude faster than mocks and show its validation with an analysis of the BOSS DR12 data. Furthermore, our analytic approach is free of sampling noise which makes it useful for upcoming surveys like DESI and Euclid. Towards the end, I will change gears and talk about some recent work on the assembly bias of neutral hydrogen.

&nbs;

# Analytic calculation of power spectrum covariance

*(Jay) Digvijay Wadekar*

New York University



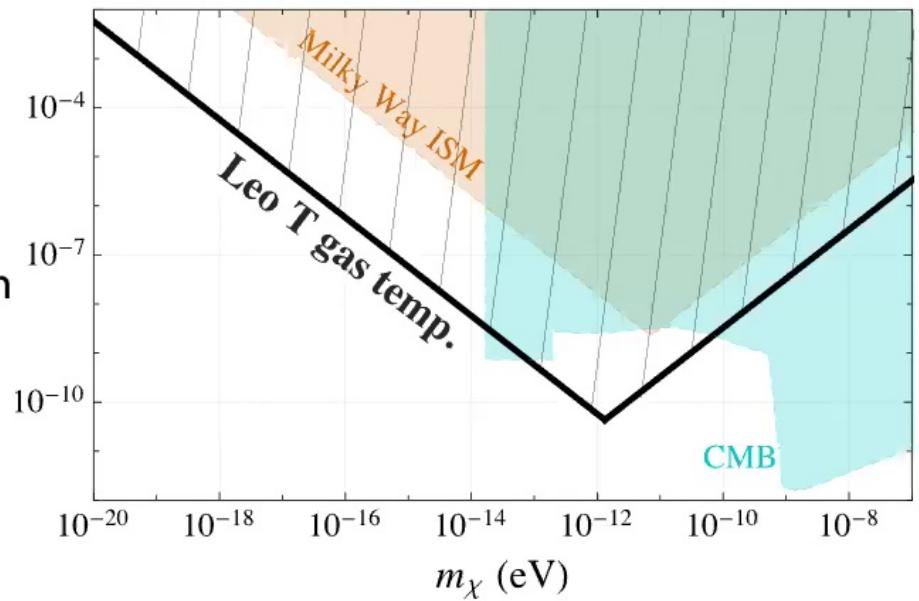
DW, Roman Scoccimarro (arXiv 1910.02914)  
DW, Misha Ivanov, Roman Scoccimarro (arXiv:2009.00622)

# Research overview

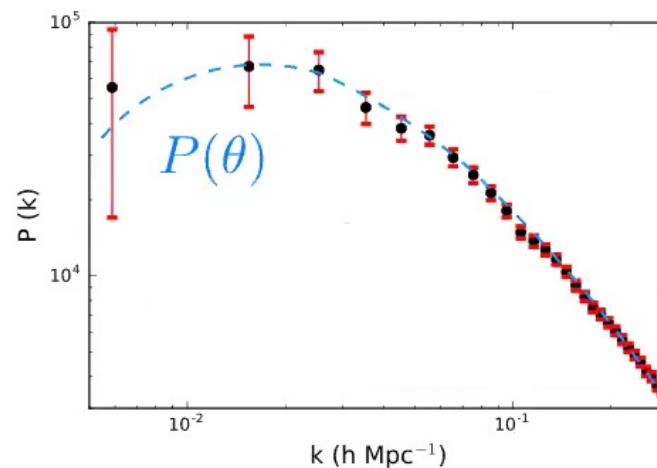
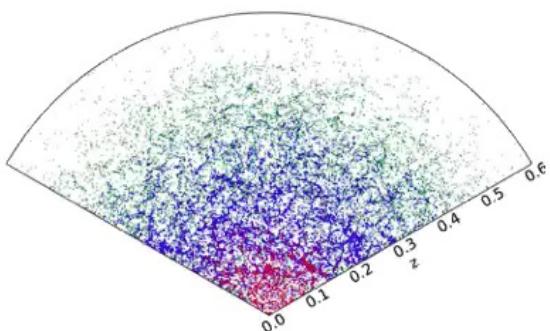
- Dwarf galaxy observations to constrain alternatives to cold dark matter (CDM)

DW & Farrar 19  
Farrar et al. 19

Ultra-light  
dark photon  
coupling



# Galaxy power spectrum covariance

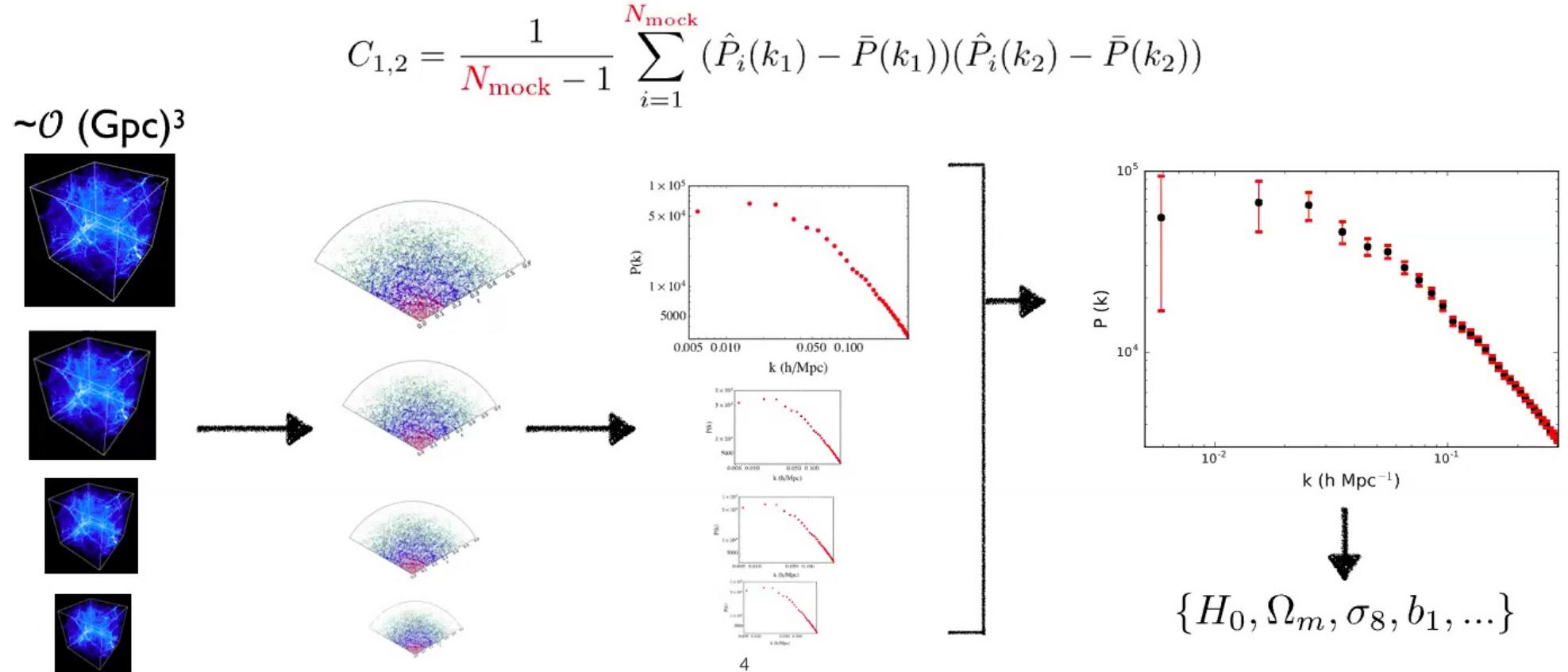


$$\left\{ \begin{array}{l} H_0 \\ \Omega_m \\ \sigma_8 \\ b_1 \\ \dots \end{array} \right\}$$

$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2} \sqrt{\det C(\theta)}} \exp \left[ -\frac{1}{2} (P_d - P(\theta))^T C(\theta)^{-1} (P_d - P(\theta)) \right]$$

# Covariance from mock catalogs

- Need to simulate mock surveys ( $\sim$  thousands)



# Covariance from mock catalogs

- As survey volume increases, mock catalogs become tougher to simulate (DESI, LSST, Euclid and others)
- Dependence of covariance on cosmology and bias parameters is computationally prohibitive

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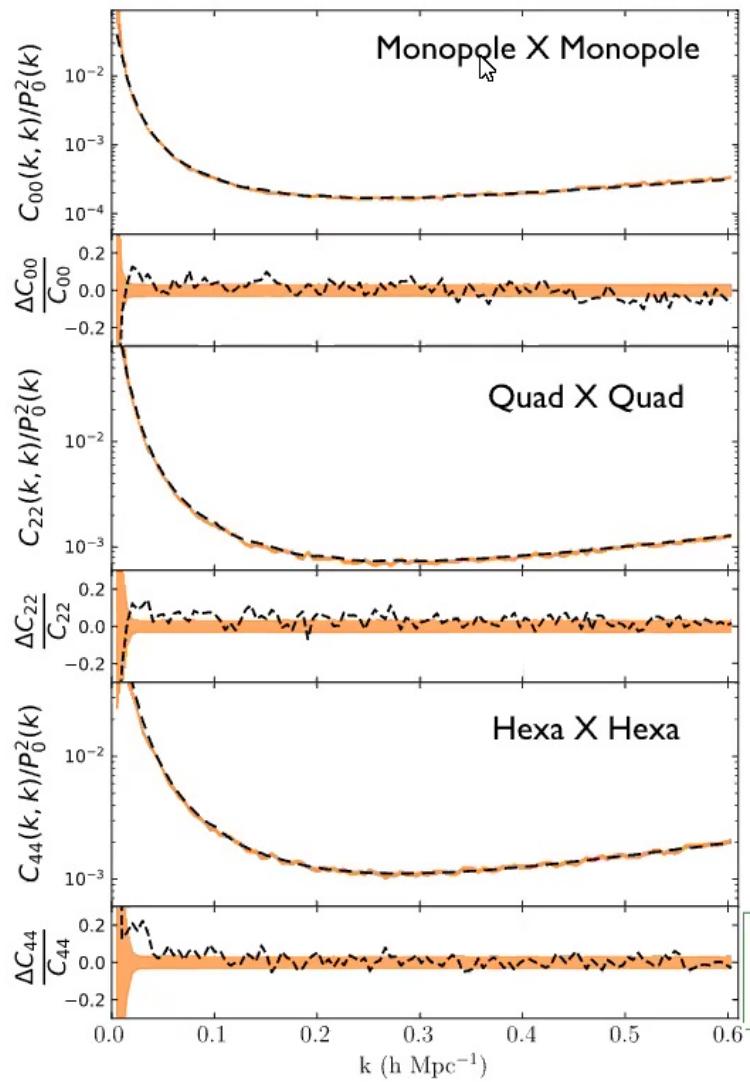
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- Mocks suffer from sampling noise
  - Need to artificially inflate constraints

# Results

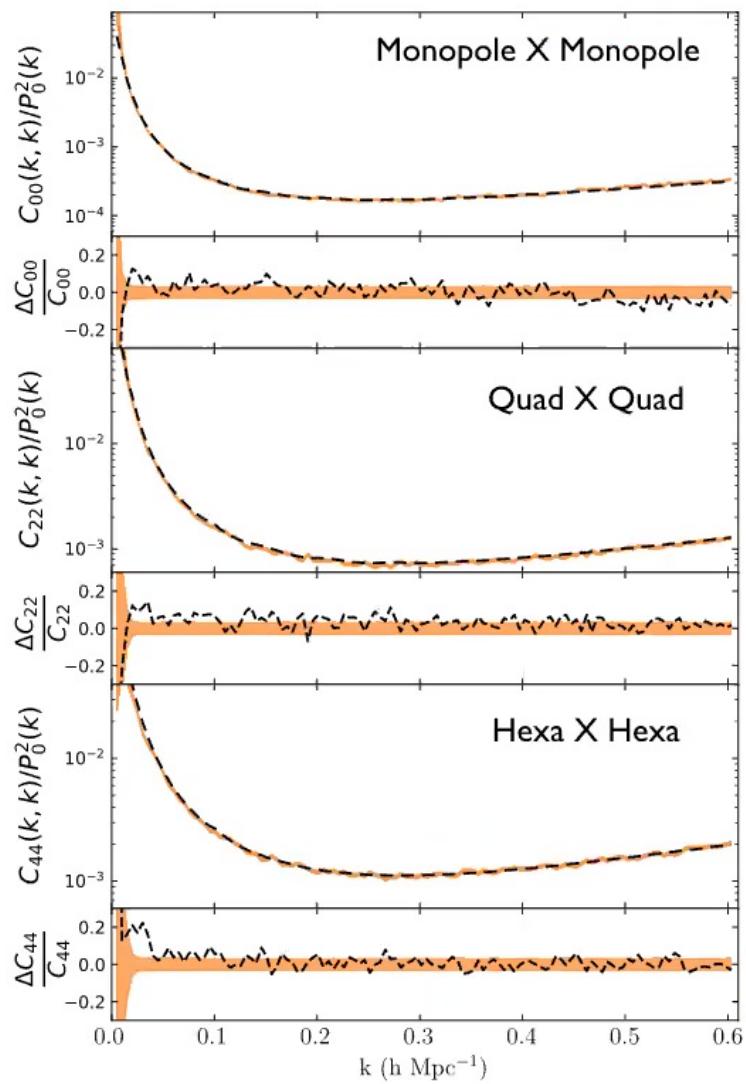


----- ***Our analytic method***

— Patchy Mocks  
(state-of-the-art mocks used for  
SDSS BOSS parameter estimation)

DW & Scoccimarro 19  
(Cova-PT public code)

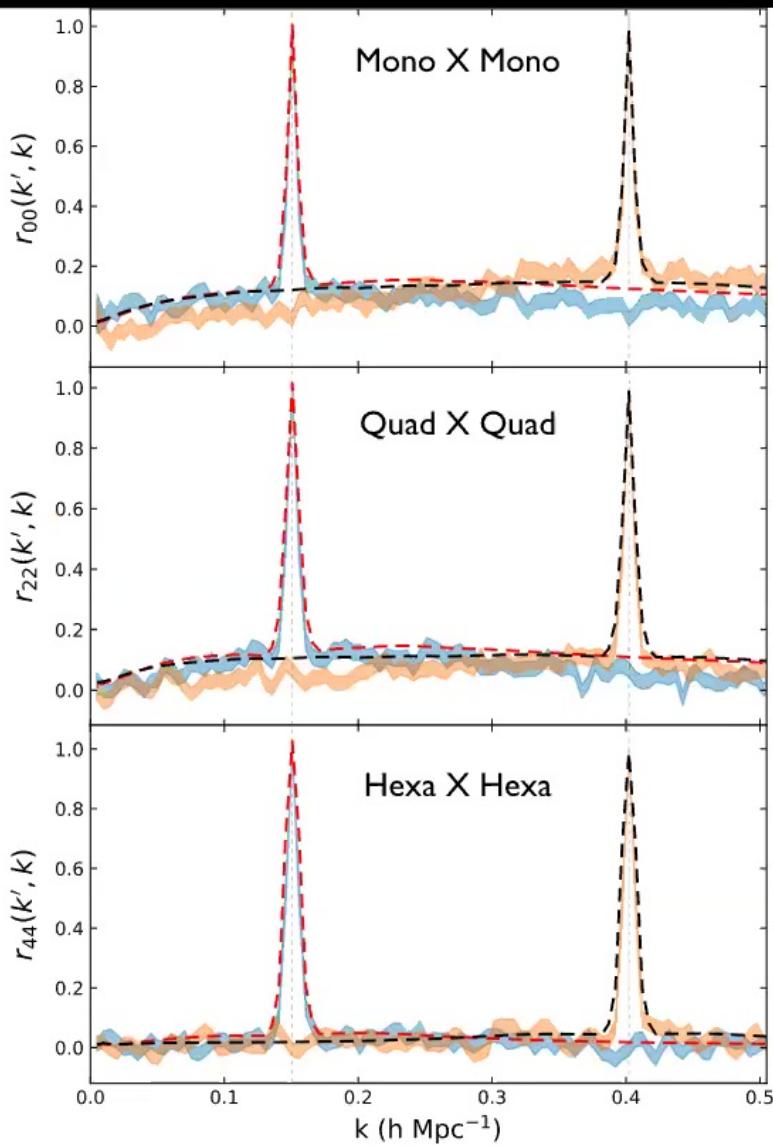
# Results



----- **Our analytic method  
(≤ MINUTE)**

—— Patchy Mocks (**MONTHS**)  
(state-of-the-art mocks used for  
SDSS BOSS parameter estimation)

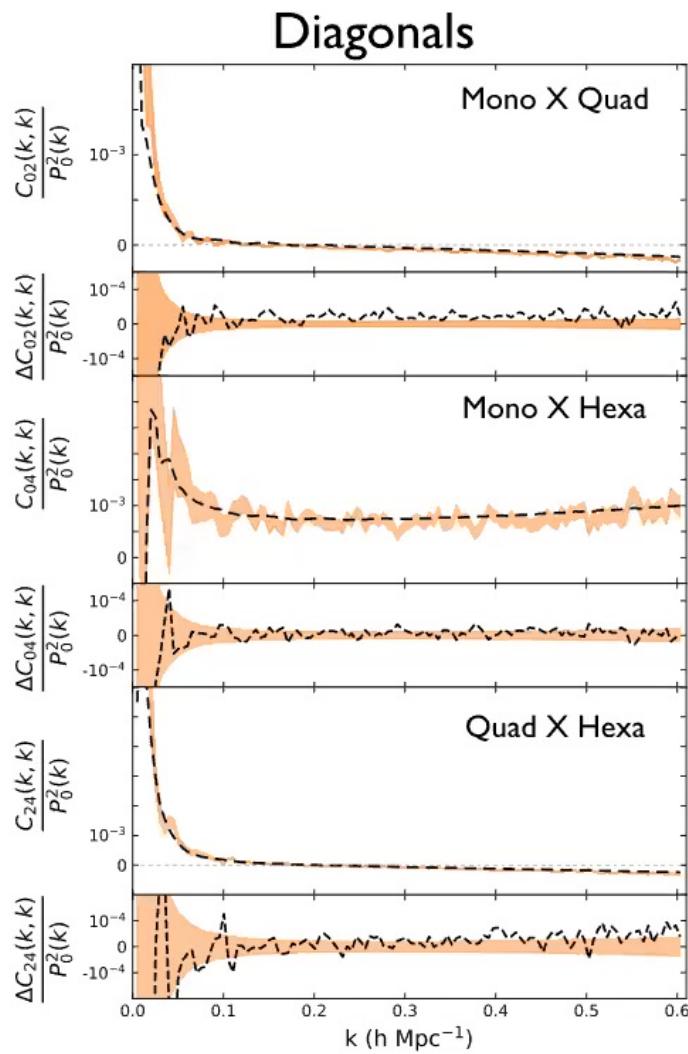
DW & Scoccimarro 19



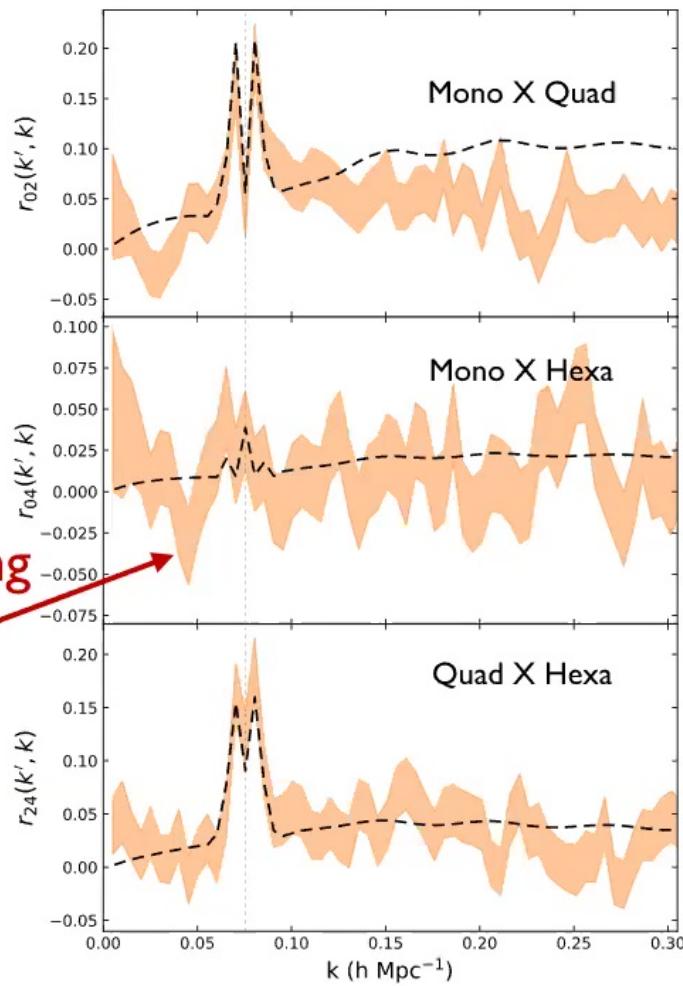
## Results (off-diagonals)

(2 rows compared in fig)

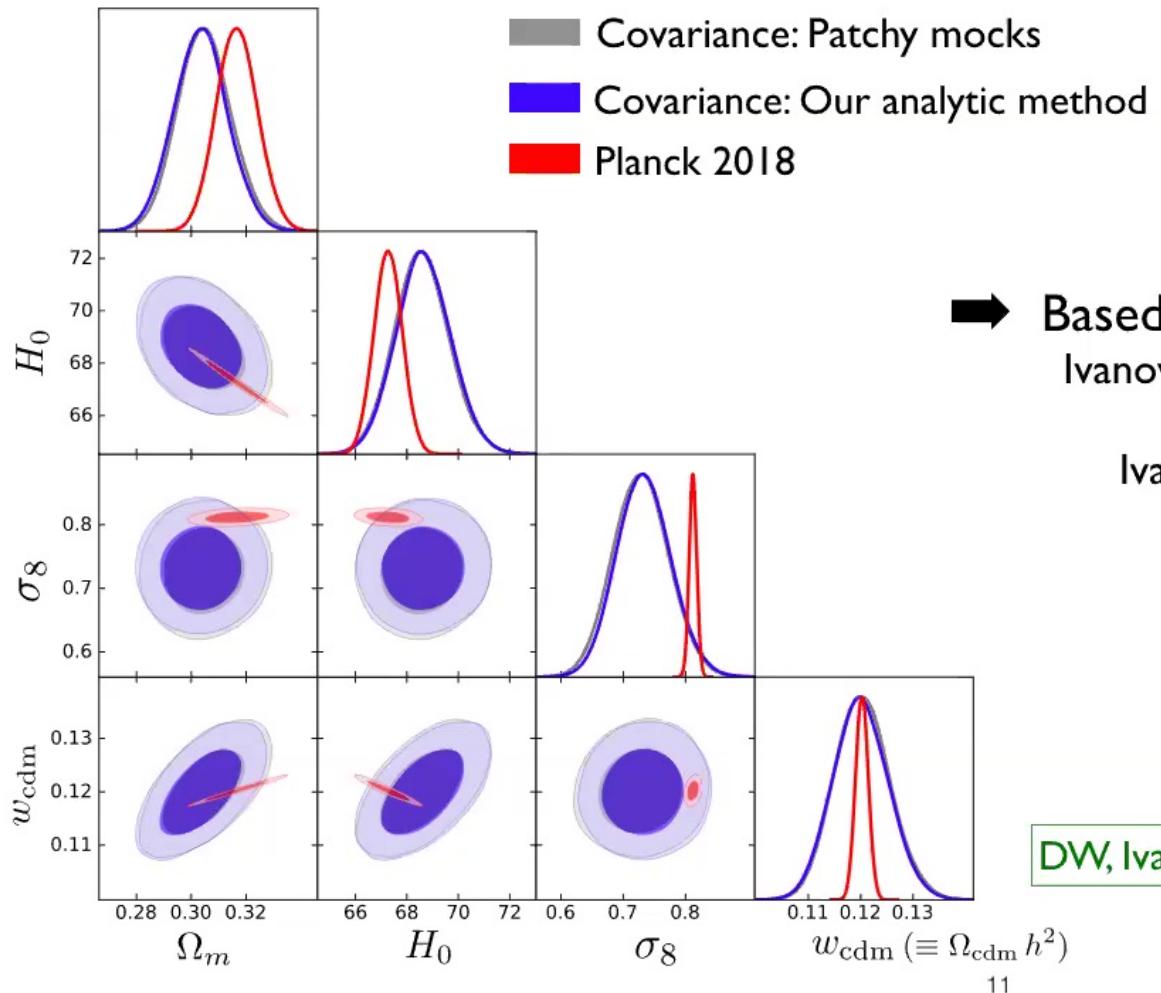
## Cross-covariance



## Row of matrix



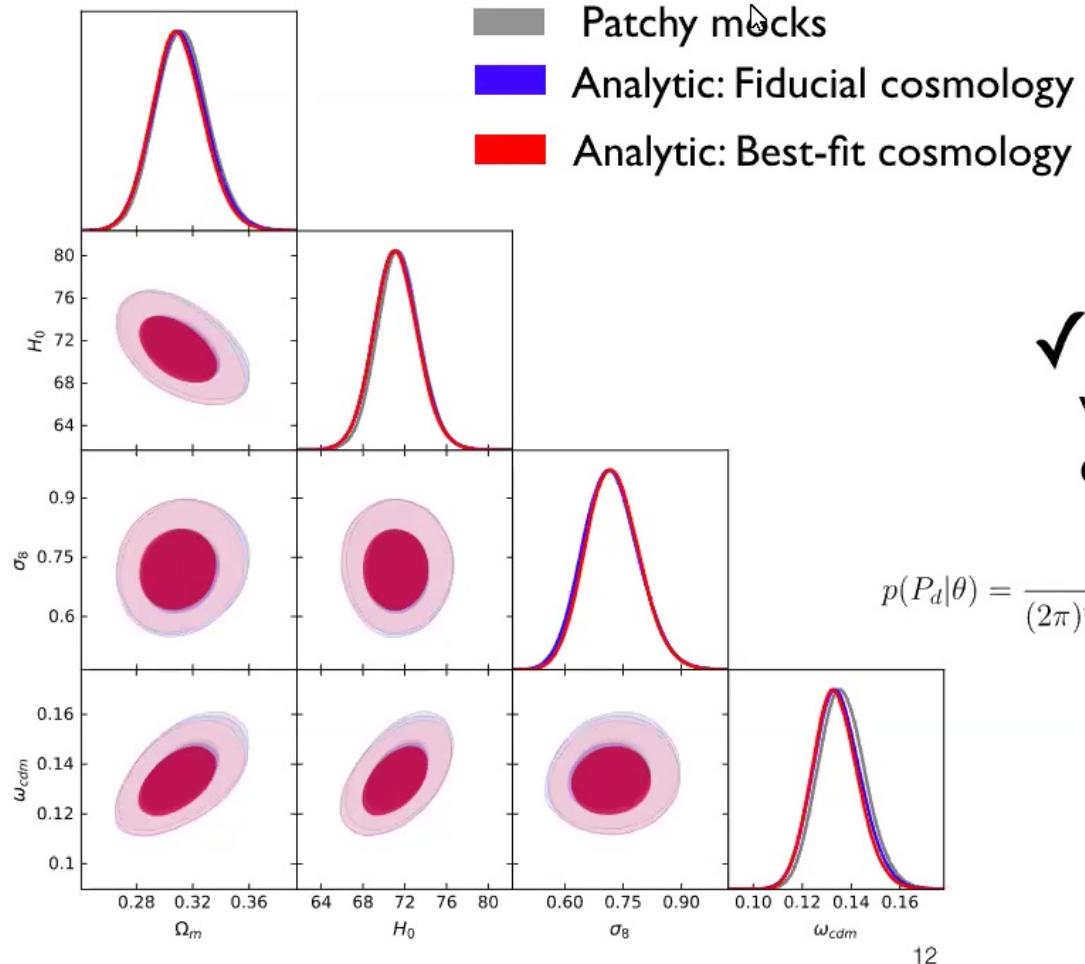
## Results: BOSS DR12 full-shape analysis



→ Based on BOSS analysis pipeline of  
Ivanov, Simonovic, Zaldarriaga, JCAP 20  
Philcox et al. 2020  
Ivanov et al 2020 (CLASS-PT)

DW, Ivanov & Scoccimarro, 2020

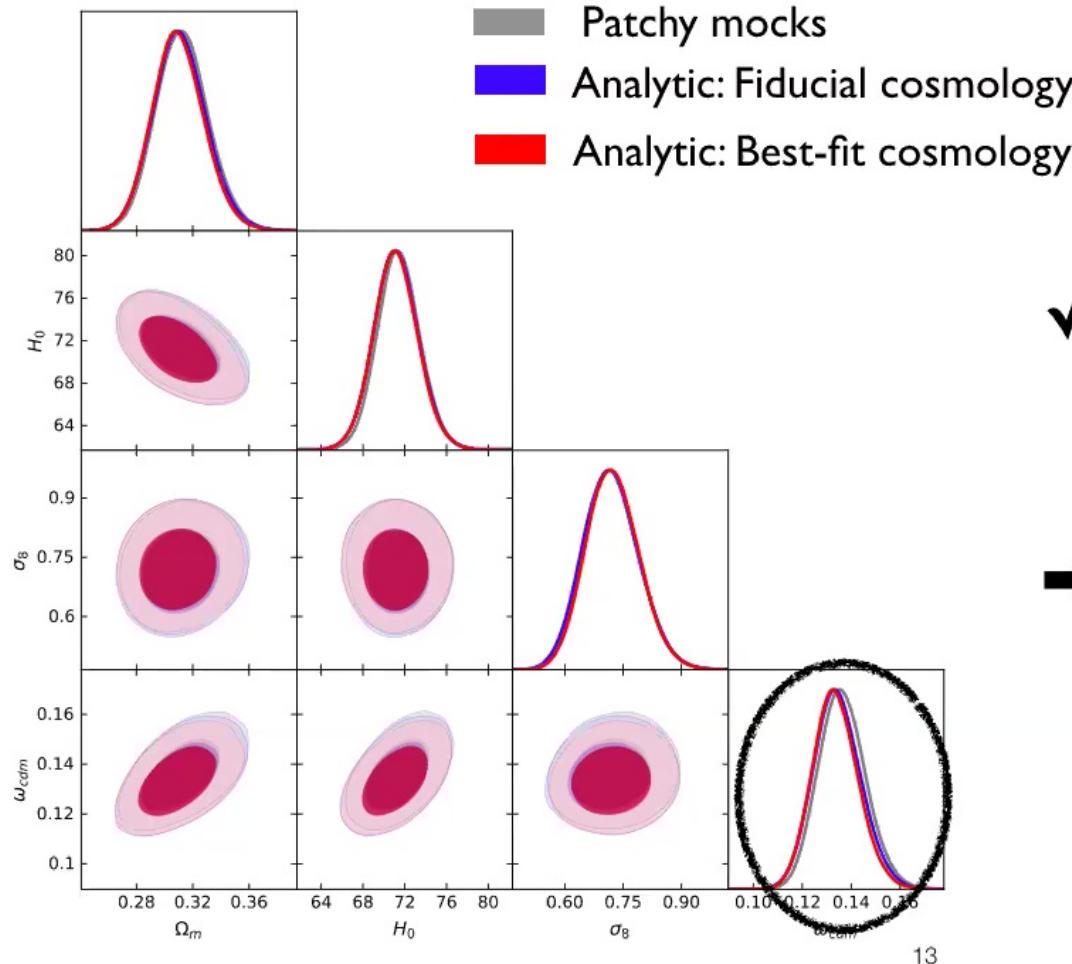
# Case study: BOSS sample of NGC high-z



✓ BOSS results are robust  
w.r.t change in cosmology of  
covariance matrix

$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2} \sqrt{\det C(\theta)}} \exp \left[ -\frac{1}{2}(P_d - P(\theta))^T C(\theta)^{-1} (P_d - P(\theta)) \right]$$

## Case study: BOSS sample of NGC high-z

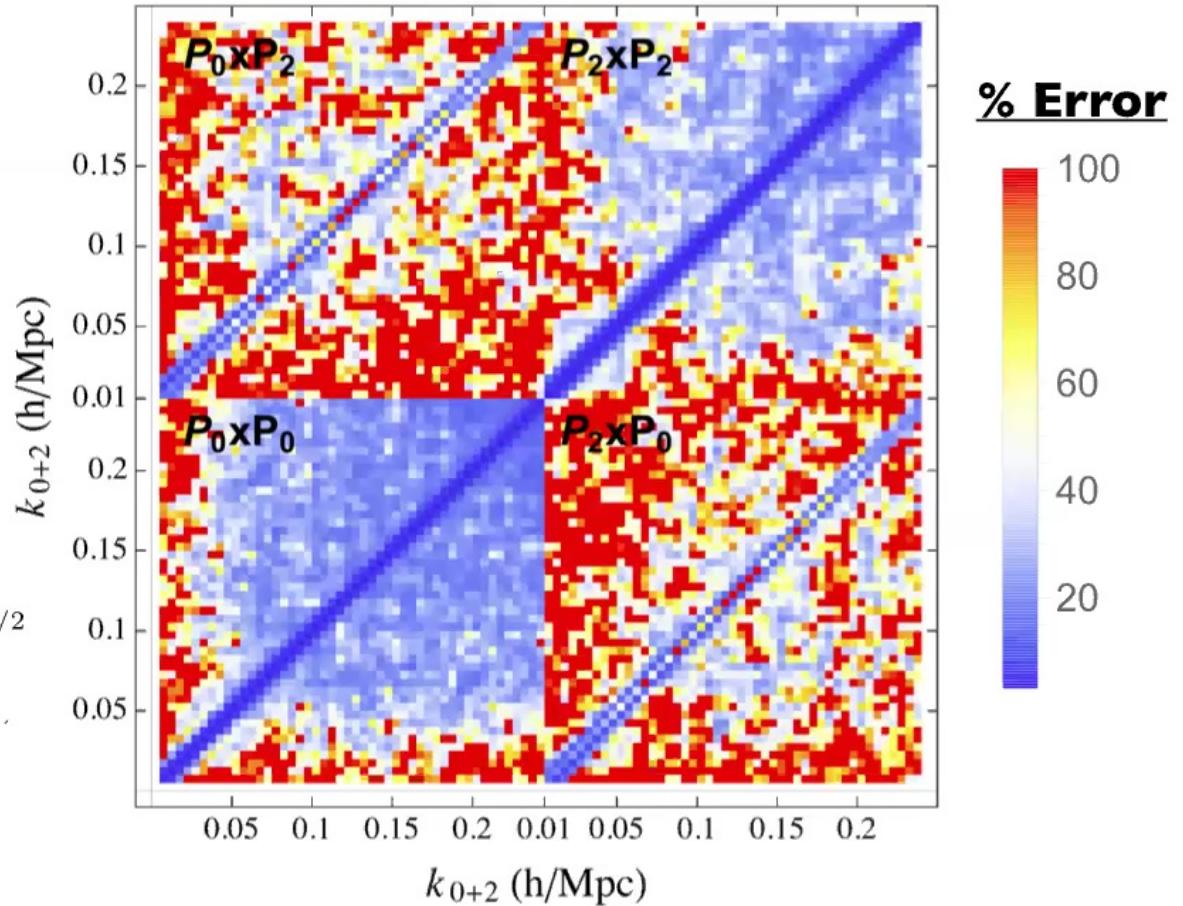


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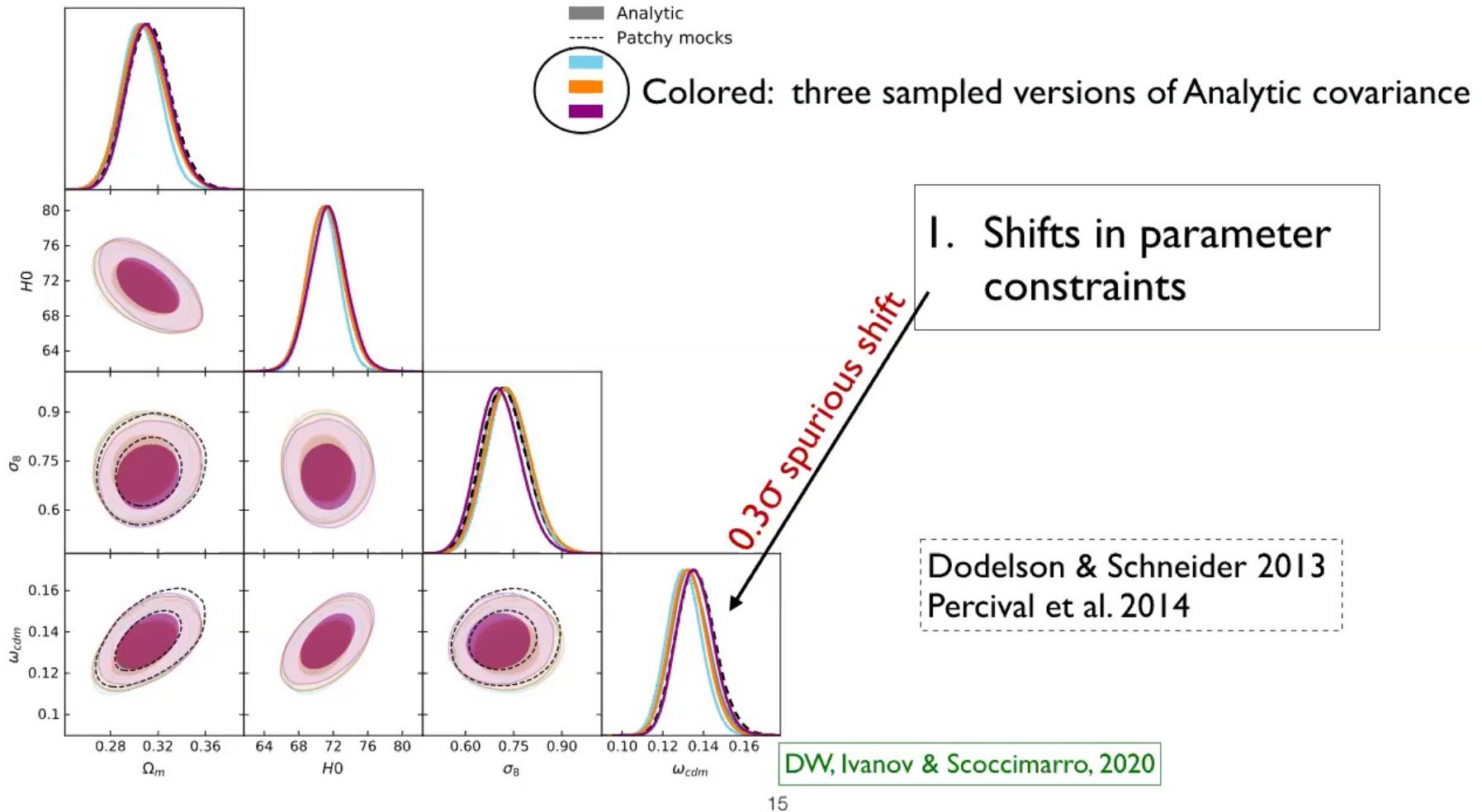
→ Small shifts ( $0.2\sigma$ )  
because of sampling noise

## Large sampling noise in off-diagonals & cross-covariance from 2048 Patchy mocks

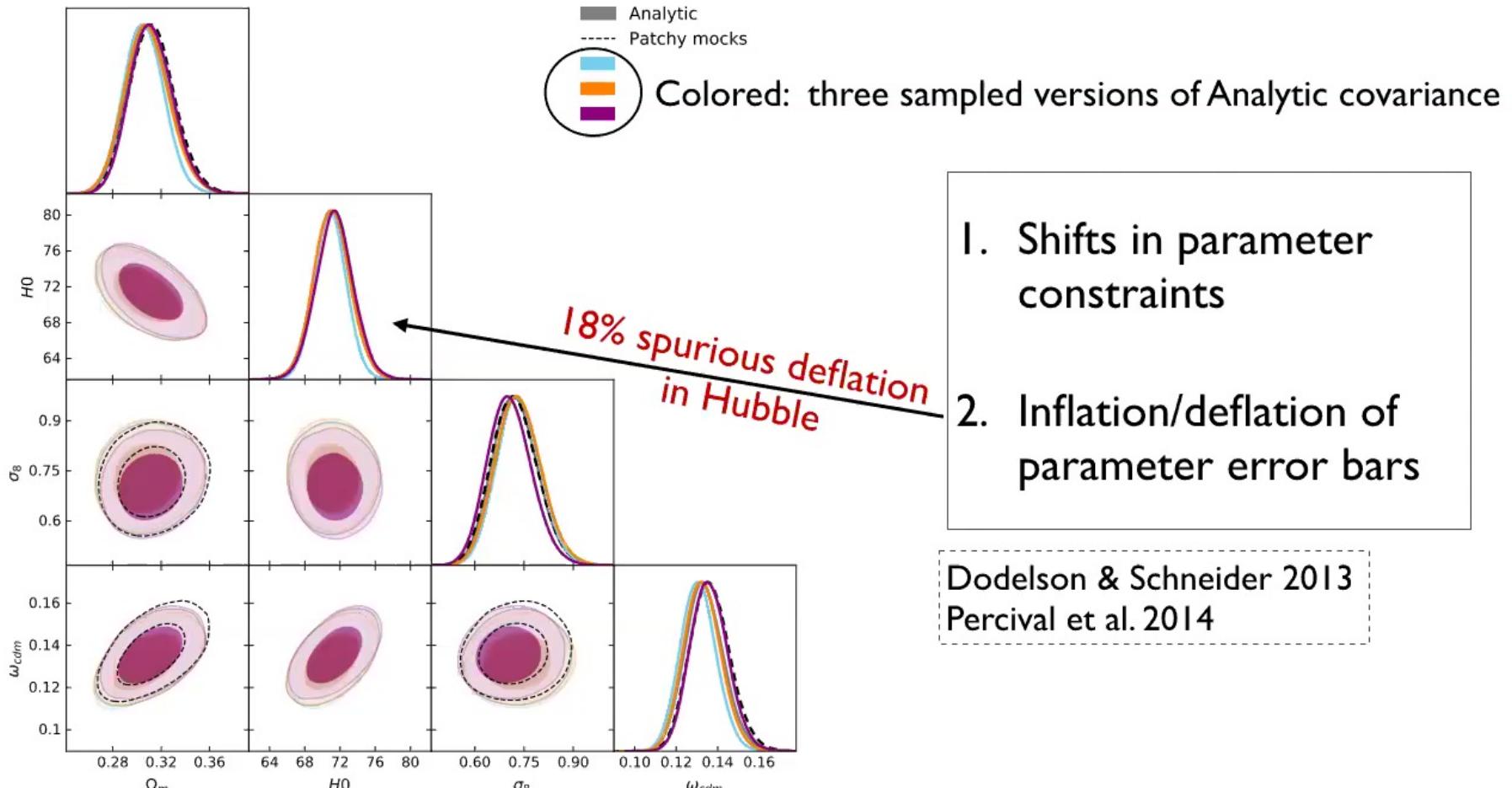
$$\frac{\Delta \mathbf{C}_{\ell_1 \ell_2}(k_i, k_j)}{\mathbf{C}_{\ell_1 \ell_2}(k_i, k_j)} \sim \left[ \frac{1}{N_m} \frac{\mathbf{C}_{\ell_1 \ell_1}(k_i, k_i) \mathbf{C}_{\ell_2 \ell_2}(k_j, k_j)}{\mathbf{C}_{\ell_1 \ell_2}^2(k_i, k_j)} \right]^{1/2}$$



# Sampling noise in covariance from 2048 mocks



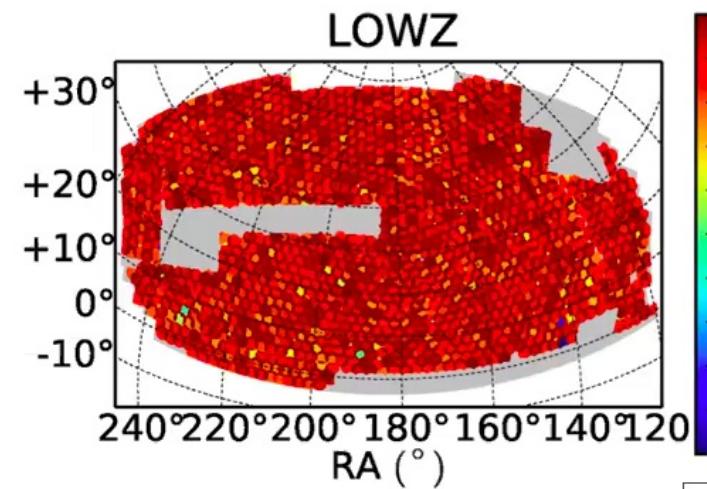
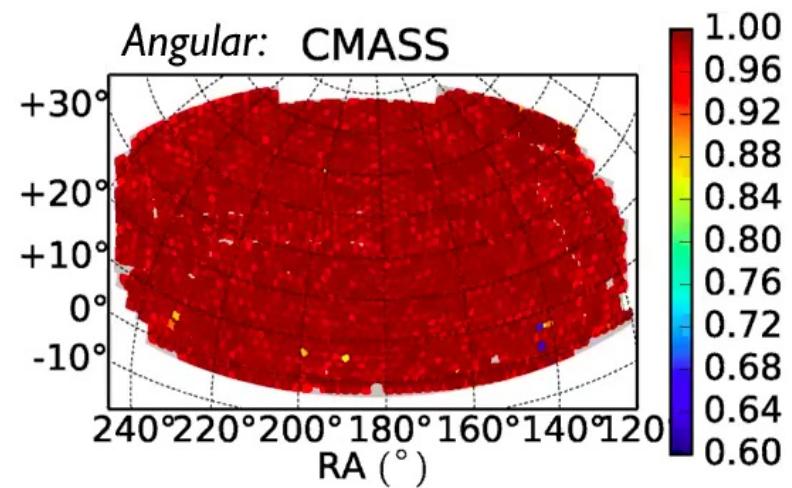
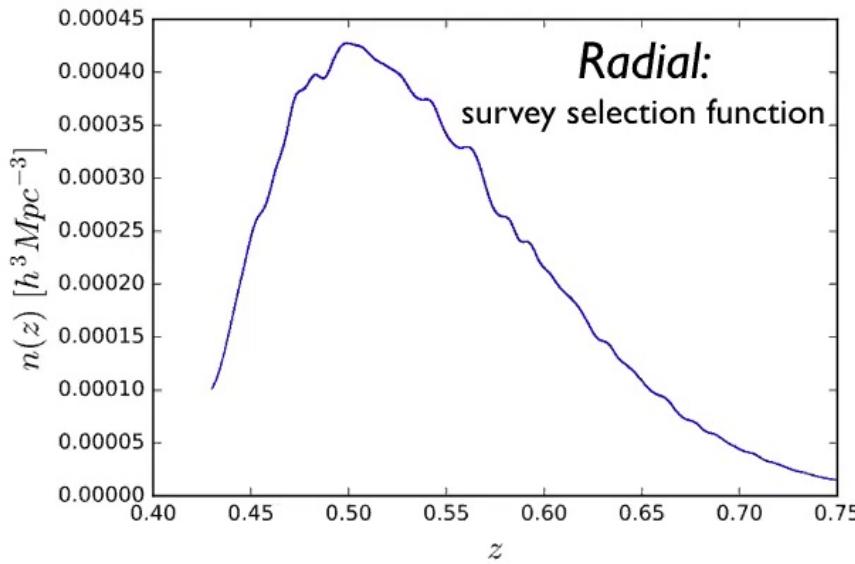
# Sampling noise in covariance from 2048 mocks



What are the challenges to analytically calculate the covariance?

## Challenge I: Highly non-trivial survey window

$$\delta_W(\mathbf{x}) \equiv W(\mathbf{x})\delta(\mathbf{x})$$



Reid et al. 2015

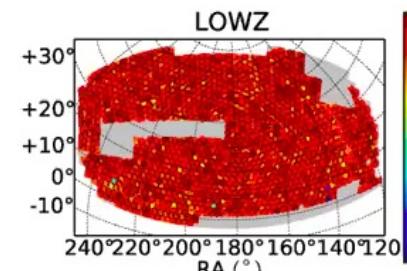
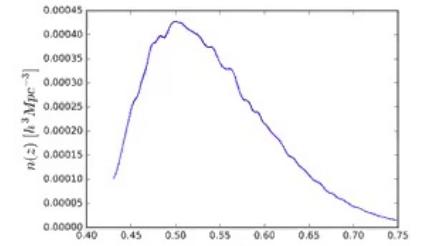
# Survey window enters covariance

$$C(k_1, k_2) = \langle \hat{P}(k_1) \hat{P}(k_2) \rangle - \langle \hat{P}(k_1) \rangle \langle \hat{P}(k_2) \rangle$$

$$\begin{aligned} \langle \hat{P}(k_1) \hat{P}(k_2) \rangle &= \frac{1}{V^2} \int_{\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2} \langle \delta_W(\mathbf{k}_1) \delta_W(-\mathbf{k}_1) \delta_W(\mathbf{k}_2) \delta_W(-\mathbf{k}_2) \rangle \\ &= \frac{1}{V^2} {}_2 \int_{\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \mathbf{p}_1, \mathbf{p}'_1, \mathbf{p}_2, \mathbf{p}'_2} W(\mathbf{k}_1 - \mathbf{p}_1) W(-\mathbf{k}_1 - \mathbf{p}'_1) W(\mathbf{k}_2 - \mathbf{p}_2) W(-\mathbf{k}_2 - \mathbf{p}'_2) \\ &\quad \times \langle \delta(\mathbf{p}_1) \delta(\mathbf{p}'_1) \delta(\mathbf{p}_2) \delta(\mathbf{p}'_2) \rangle \end{aligned}$$



18 dimensional integral



## Solution: separate clustering and window terms

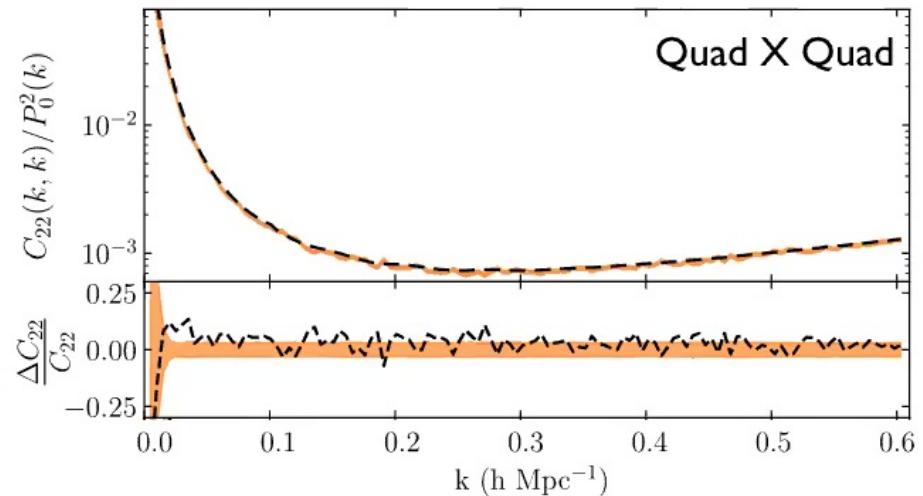
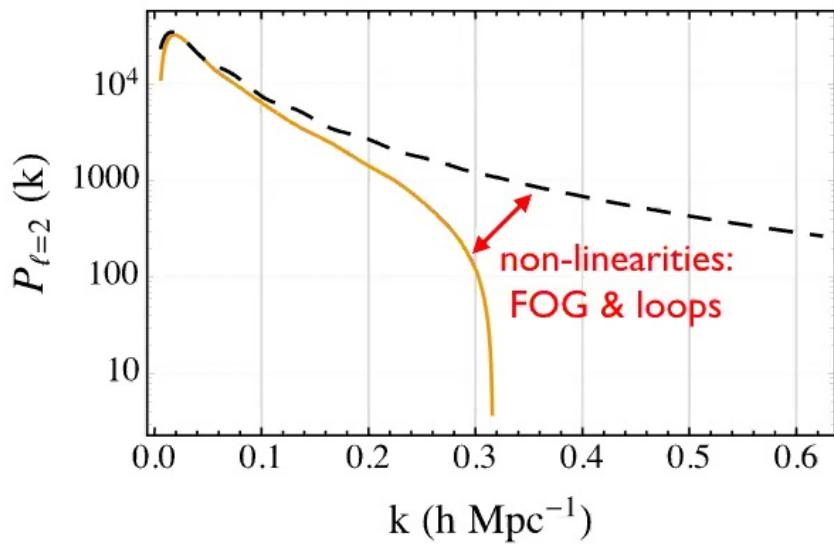
Contains all dependence on cosmology and bias parameters

$$C_{\ell_1 \ell_2}^G(k_1, k_2) \simeq \sum_{\ell'_1, \ell'_2} P_{\ell'_1}(k_2) P_{\ell'_2}(k_1) \left\{ \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{I_{22}^2} \int_{\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \mathbf{x}_1, \mathbf{x}_2} W_{22}(\mathbf{x}_1) W_{22}(\mathbf{x}_2) e^{-i(\mathbf{x}_1 - \mathbf{x}_2) \cdot (\mathbf{k}_1 - \mathbf{k}_2)} \right. \\ \left. \times \mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_1) \mathcal{L}_{\ell'_1}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_1) \mathcal{L}_{\ell'_2}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_2) [\mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_2) + \mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_1)] \right\}$$

{ Computed from survey  
random catalog }

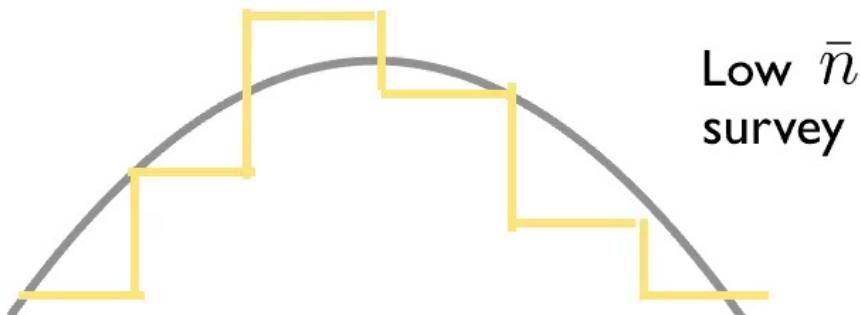
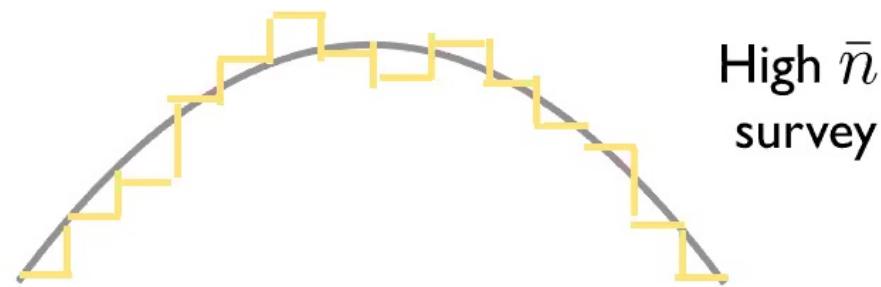
See also:  
Li et al. 19

## Challenge II: Analytic modeling in the non-linear regime



Analytic covariance works  
very well at high- $k$ .  
WHY!?

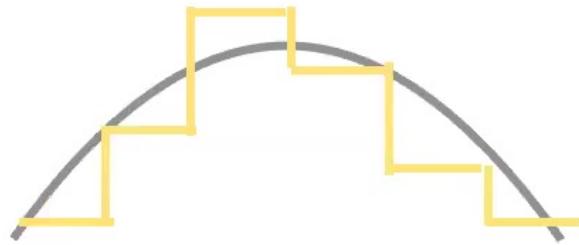
# Why does analytic work in the non-linear regime?



- Poisson fluctuations dominate the error bars at small scales:
  - ✓ Can be well modeled analytically

# Why does analytic work in the non-linear regime?

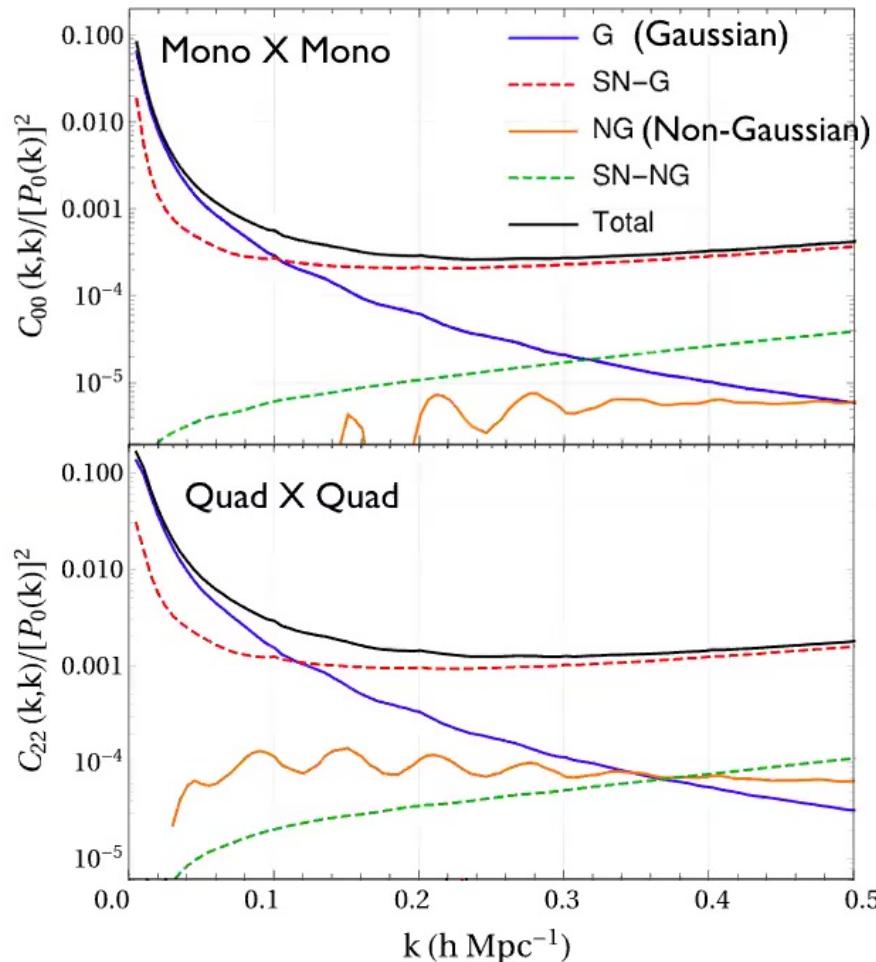
Shot noise (dashed)  
dominates at high-k



$$C(k_1, k_2) = \langle \delta(k_1)\delta(-k_1)\delta(k_2)\delta(-k_2) \rangle - \langle \delta(k_1)\delta(-k_1) \rangle \langle \delta(k_2)\delta(-k_2) \rangle$$

$$C^G(k_1, k_2) \simeq 2 \langle \delta(k_1)\delta(-k_2) \rangle \langle \delta(k_2)\delta(-k_1) \rangle$$

$$C^{NG}(k_1, k_2) = \langle \delta(k_1)\delta(-k_2)\delta(k_2)\delta(-k_1) \rangle_c$$

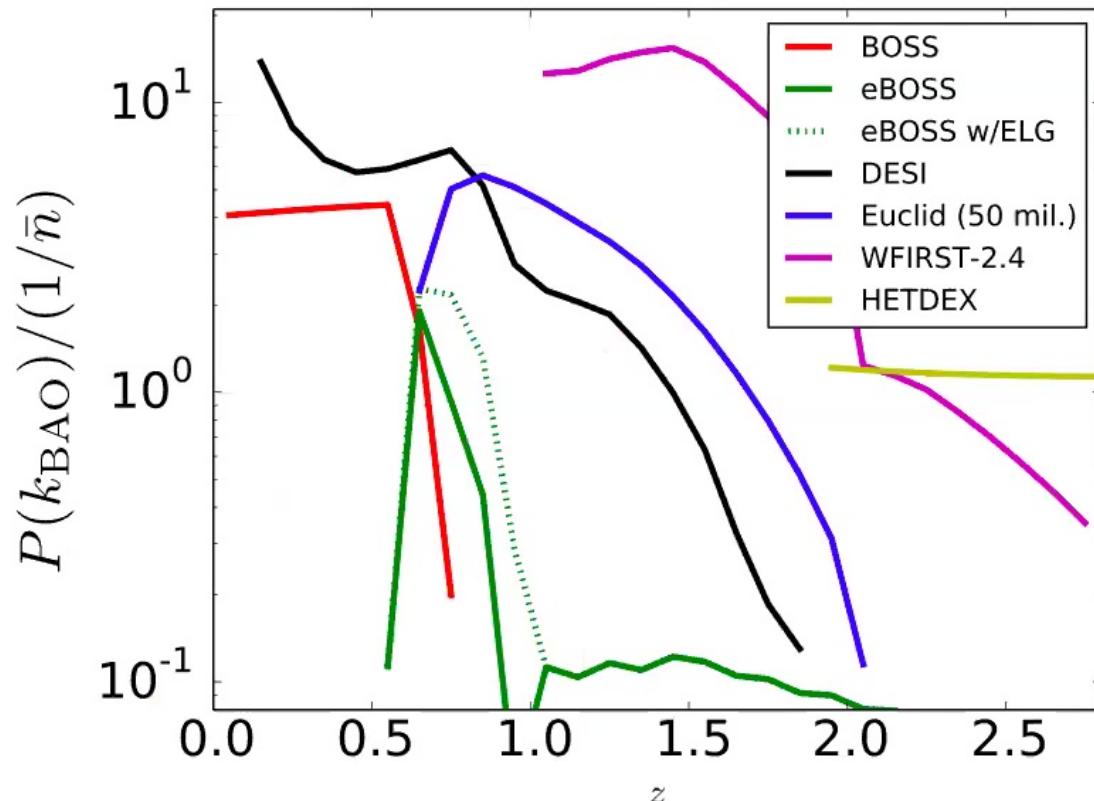


## Analytic cov. should work at small scales for upcoming surveys

- Shot noise level of upcoming surveys is comparable to BOSS



✓ Shot noise will dominate covariance for upcoming surveys



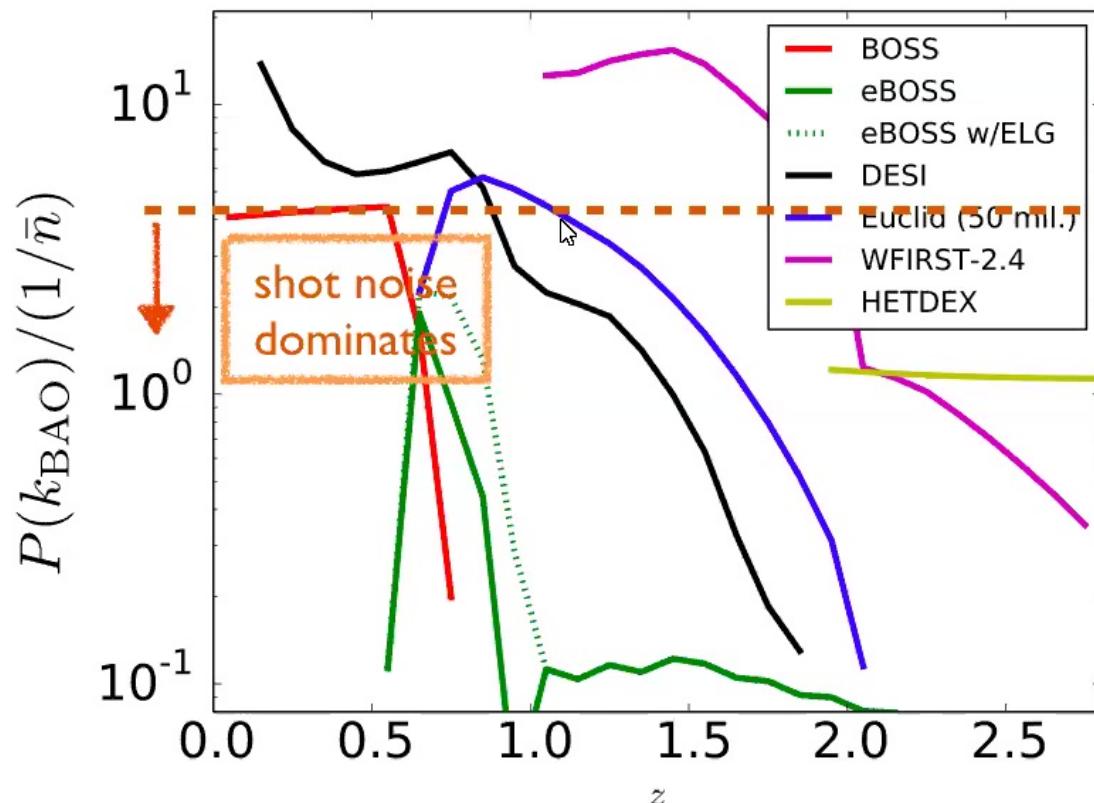
Font-Ribera et al.  
2014

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Font-Ribera et al.  
2014

## Analytic covariance is crucial for going beyond a 2-point analysis

- Number of mock simulations:  
 $\mathcal{O}(1000)$
- For low sampling noise:  
size of data  $\ll$  no. of vector mocks

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- ✓ Number of k-bins in power spectrum  $\sim 100$
- Number of triangles in bispectrum (3-pt)  $\sim 6000$ 
  - Bottleneck for BOSS  
(Gil-Marin et al 17 could only use  $\sim 800$  triangles)
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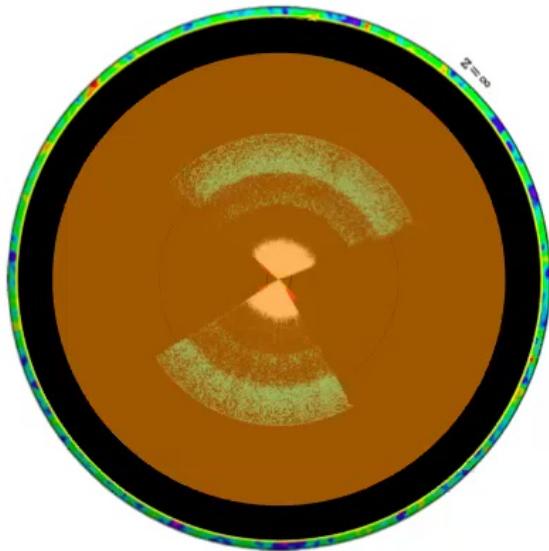
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(Gil-Marin et al 17 could only use  $\sim 800$  triangles)
  - ➡ Also important for other areas with high-dimensionality of the covariance matrix:  
for e.g., 3x2pt analysis in photometric surveys  
or combining cluster counts with correlation fns
- Number of mock simulations:  
 $O(1000)$
  - For low sampling noise:  
size of data  $\ll$  no. of vector mocks

# Neutral hydrogen (HI) from dark matter with machine learning and symbolic regression

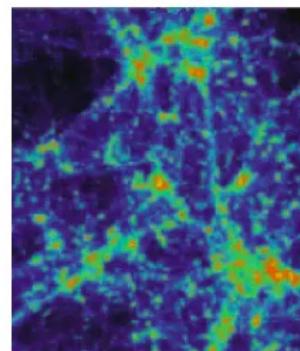
DW, Paco Villaescusa-Navarro,  
Shirley Ho & Laurence Perrault-Levasseur  
(aXiv:2007.10340 & aXiv:2011.xxxx)

## Emulation of hydro sims for future surveys

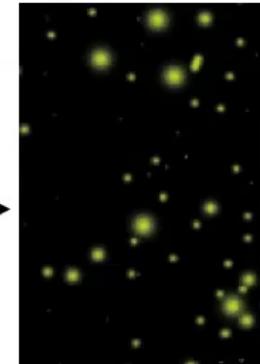


- Volume of upcoming surveys:  
 $\sim 0$  ( $100 \text{ Gpc}^3$ )
- Hydro sims are expensive:  
 $\sim 10$  million CPU hours  
for ( $0.001 \text{ Gpc}^3$ )

N-body (DM)



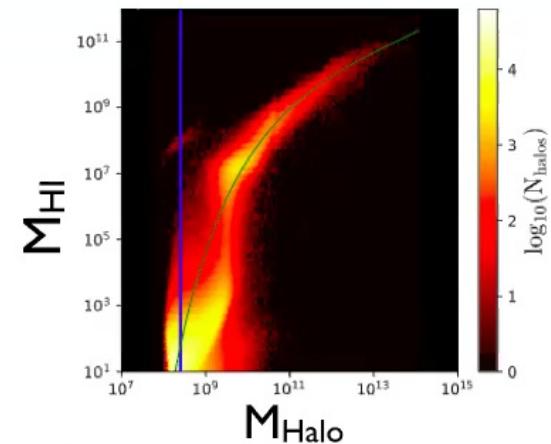
HI (neutral hydrogen)



Quickly

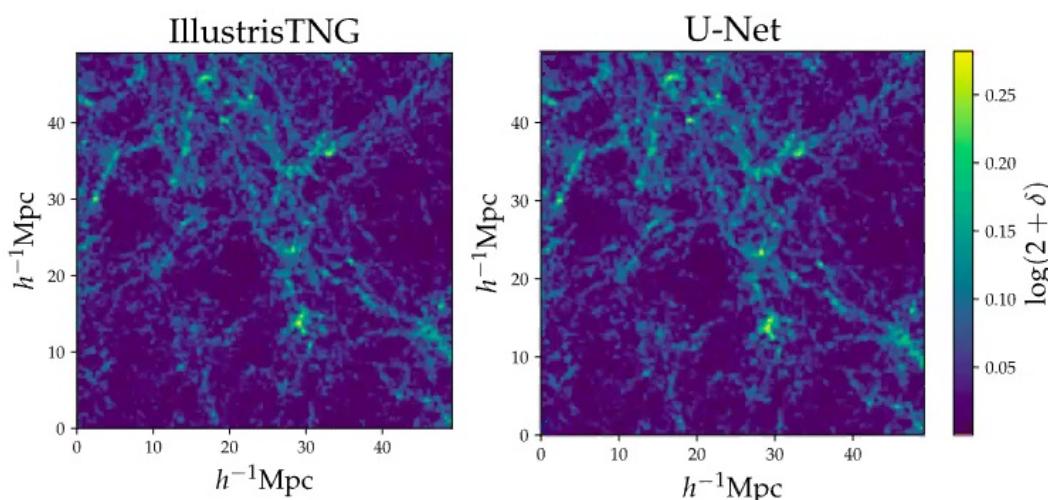
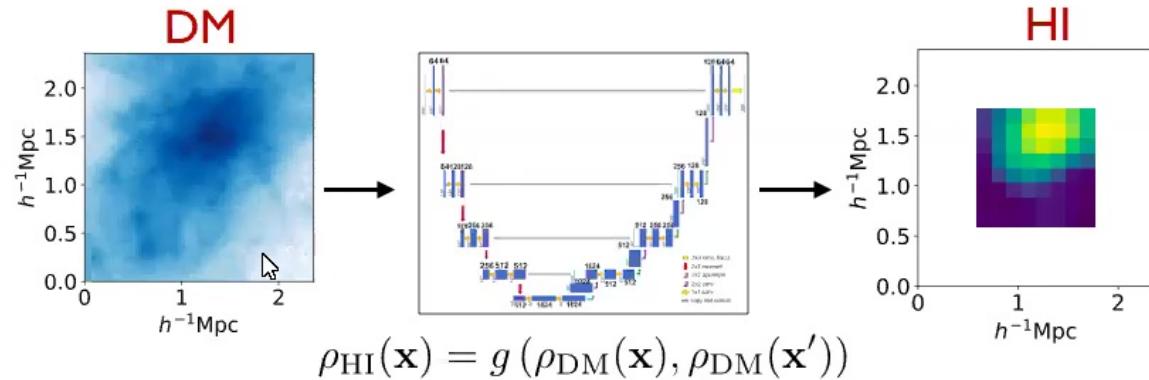
HOD:  
(Halo  
Occupation  
Distribution)

- Identify DM halos
- Fill HI using:  
 $M_{\text{HI}} = f(M_{\text{halo}})$
- assembly history,  
environmental  
info. neglected

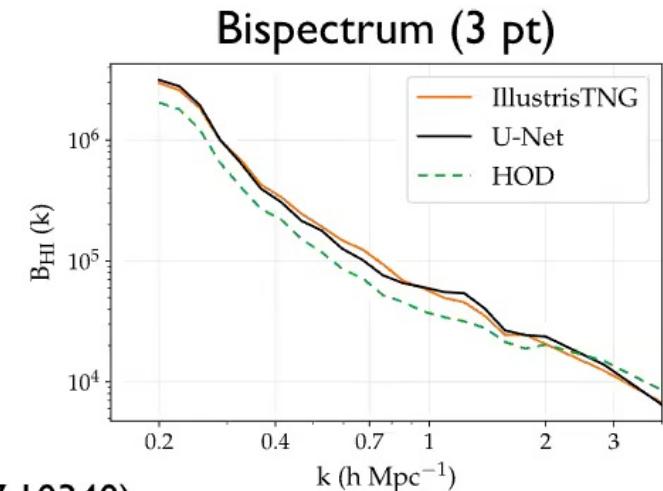
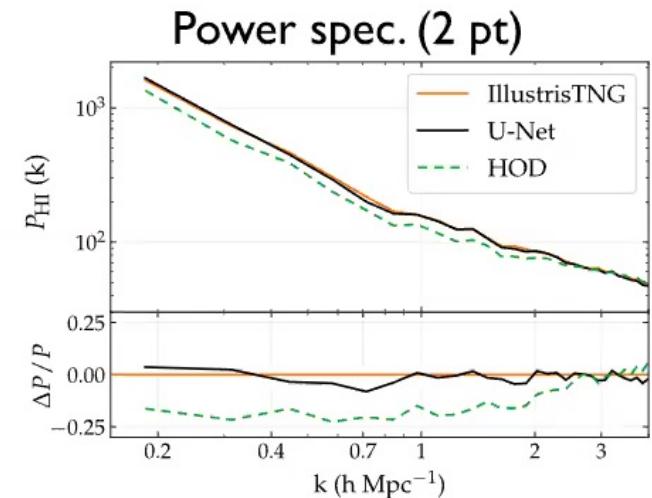


Villaescusa-Navarro et al. 2019

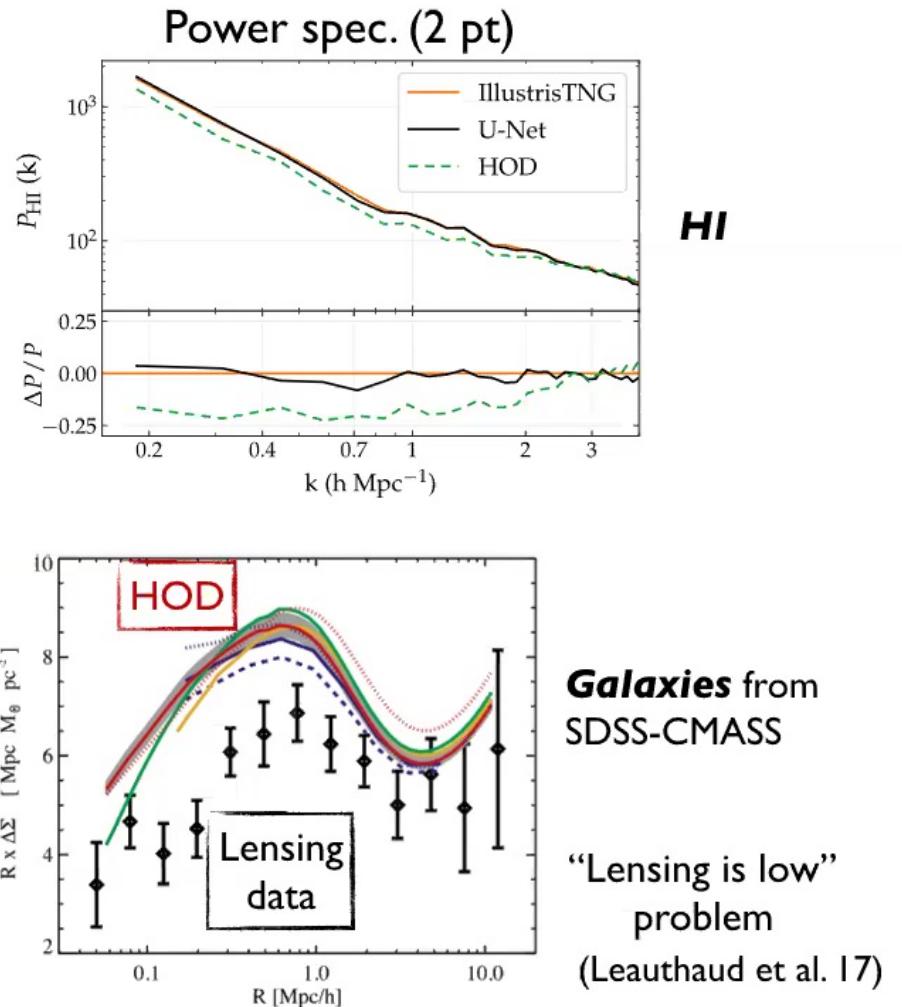
# Neural networks as emulators



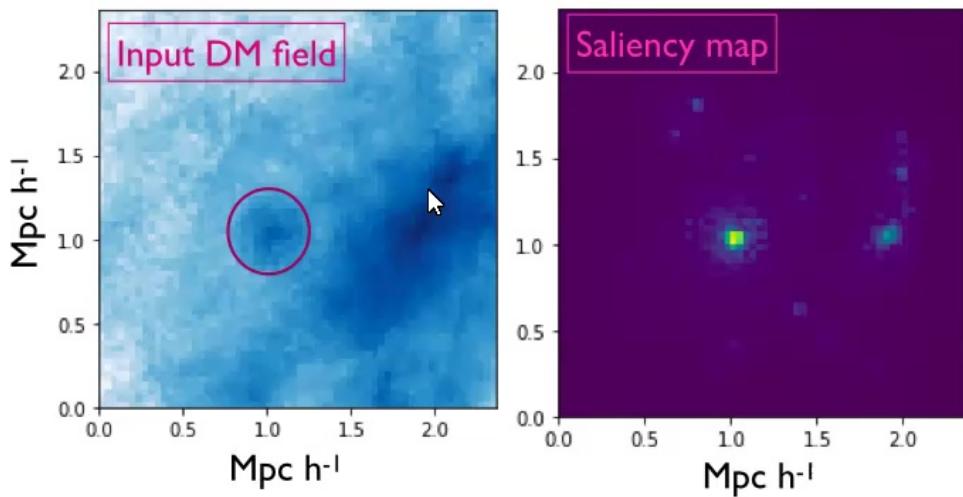
(arXiv:2007.10340)



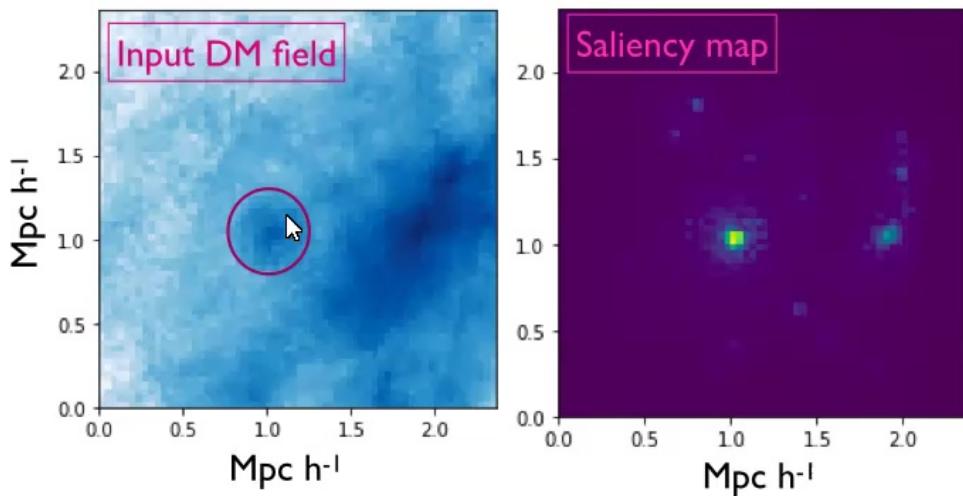
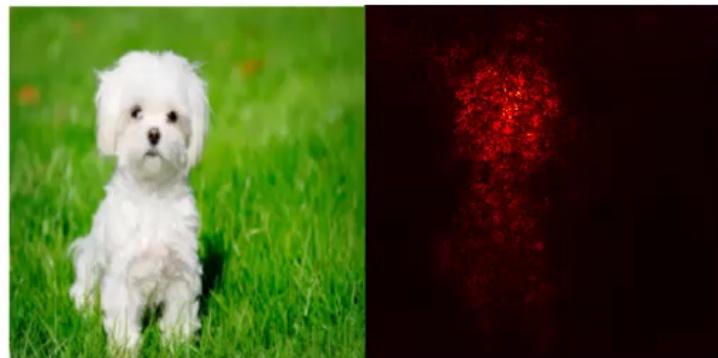
# Can we interpret what the network has learnt?



Network has learnt to include env. info



## Network has learnt to include env. info



- Network lowers  $M_{HI}$  in a cluster-like environment (ram pressure stripping)

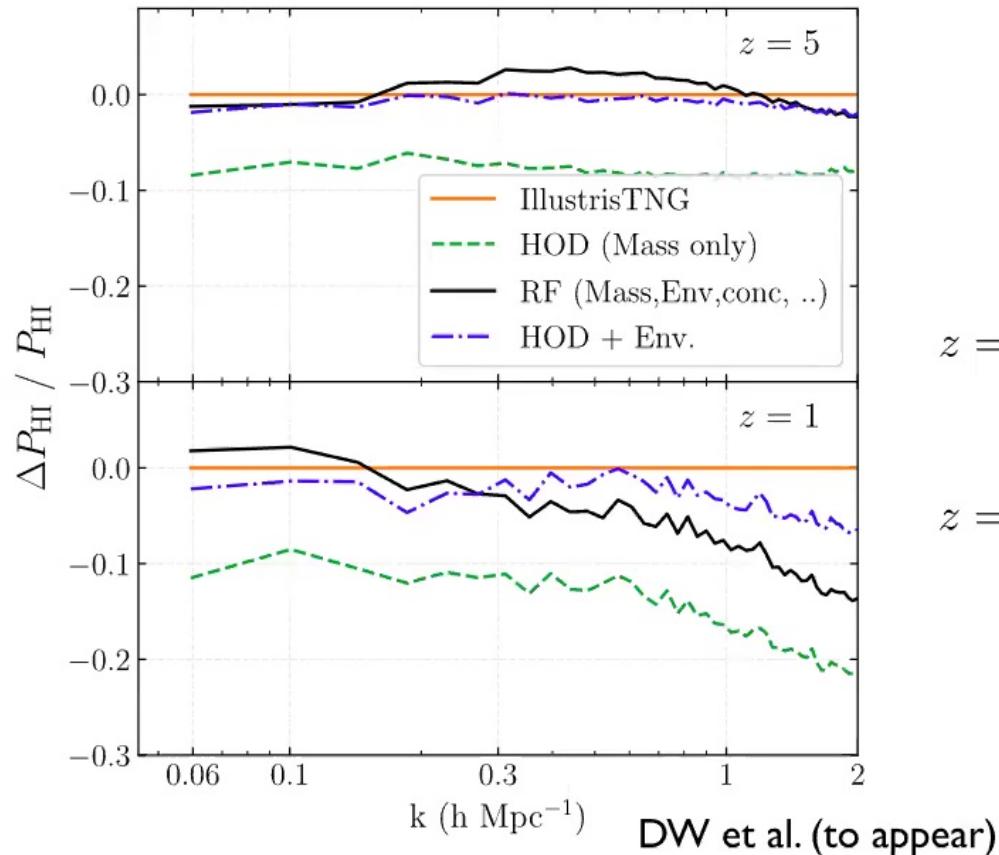


## Modeling the halo HI mass with symbolic regression

$$\text{HI mass of halo} = f(\text{halo mass}, ?)$$

- Halo env. overdensity ( $R$ )
- Env. anisotropy ( $R$ )
- Halo concentration
- Halo spin
- Halo assembly history
- Halo shape
- Velocity dispersion anisotropy
- ....., .....

# Results: Modeling the halo HI mass with symbolic regression



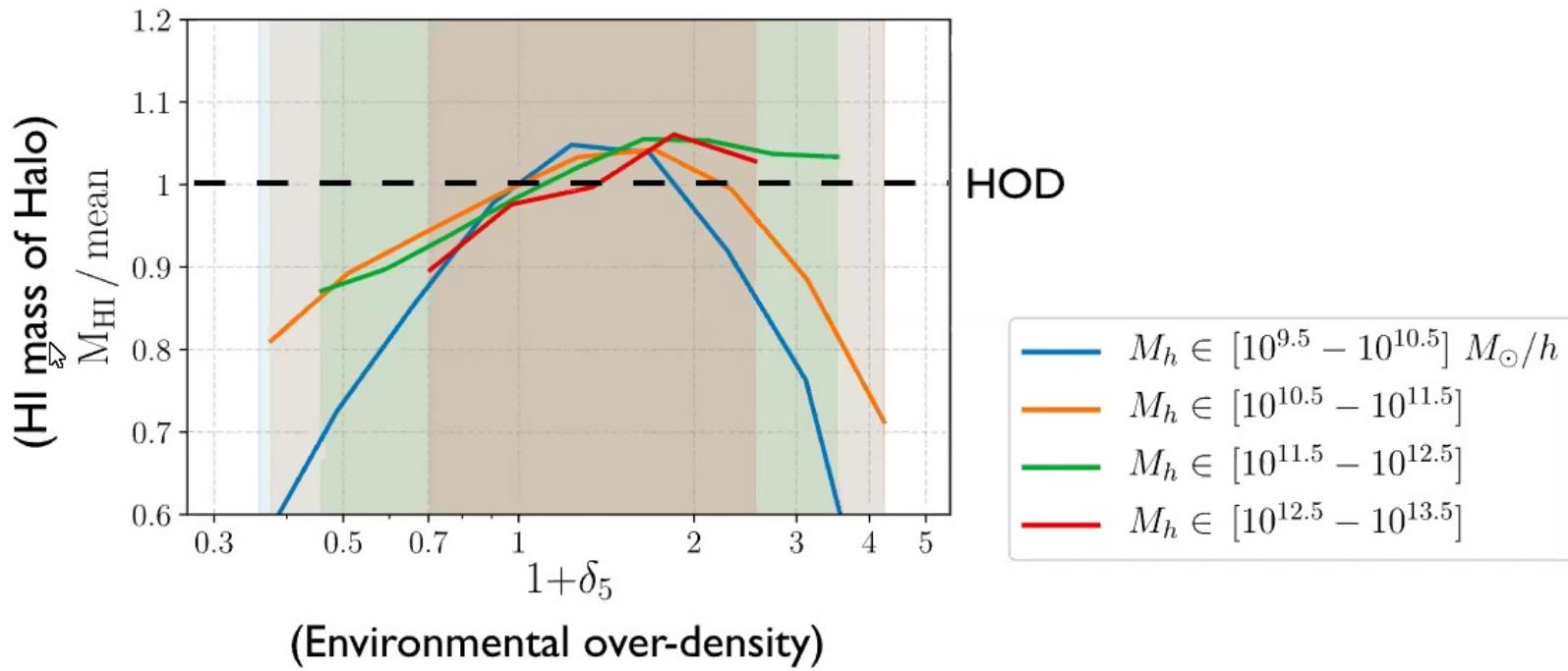
**Env. overdensity  
at 0.5 Mpc**      **Env. anisotropy**

$$z = 5 : \frac{M_{\text{HI}}}{M_{\text{HOD}}} = 0.95 + \alpha'_{0.5} \delta'_{0.5} (\alpha'_{0.5} + \delta'_{0.5})$$

$$z = 1 : \frac{M_{\text{HI}}}{M_{\text{HOD}}} = 0.8 + 1.4 \alpha'_{0.5} m_{10} - 0.6 (\alpha'^2_{0.5} m_{10}^2 + \alpha'_{0.5} \delta'_5)$$

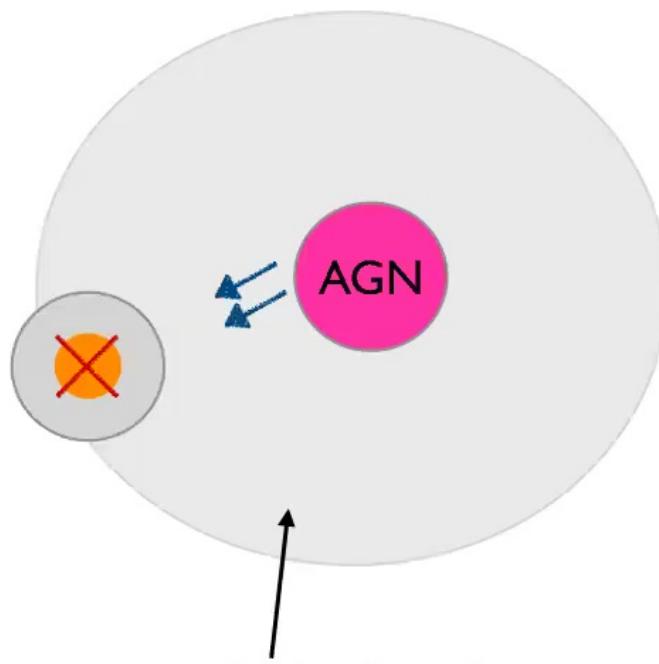
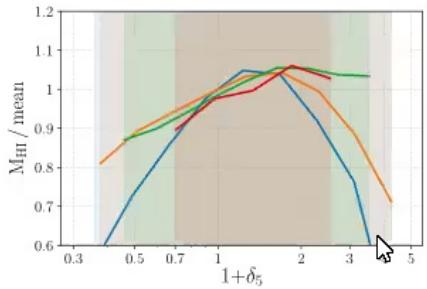
<https://github.com/MilesCranmer/PySR>

# Understanding the effect of halo environment on HI: Don't baryons just follow dark matter?



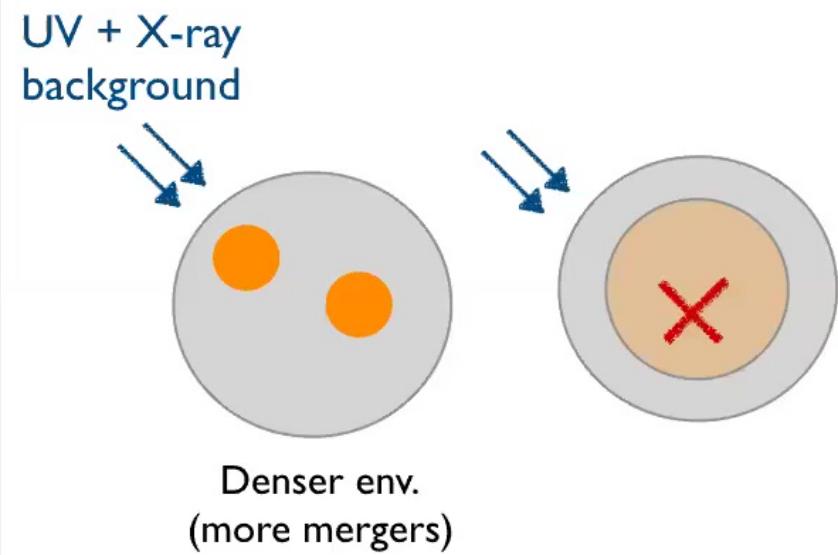
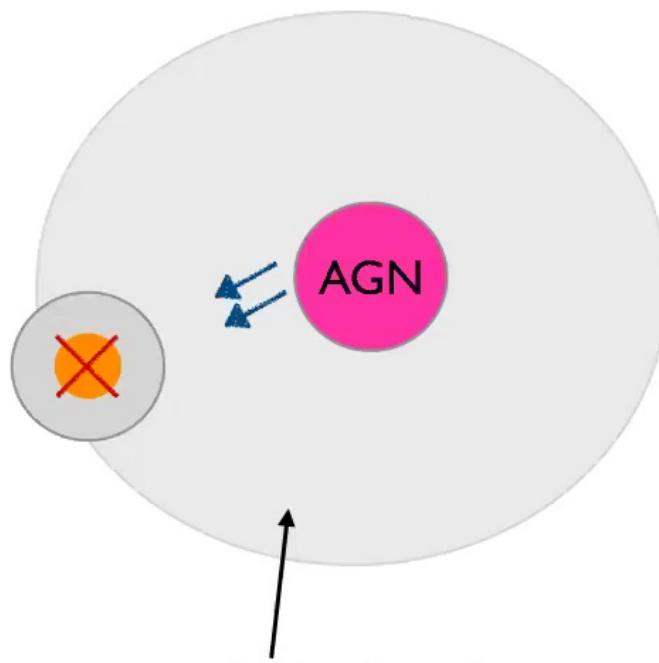
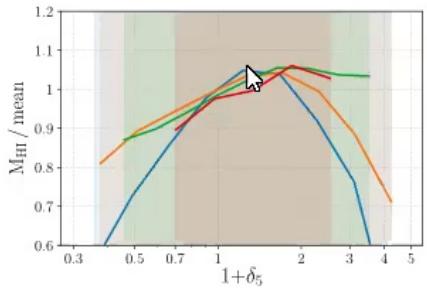
DW et al. (to appear)

# Understanding the effect of halo environment on HI: Don't baryons just follow dark matter?



Denser env.: ionized medium  
(ram pressure stripping)

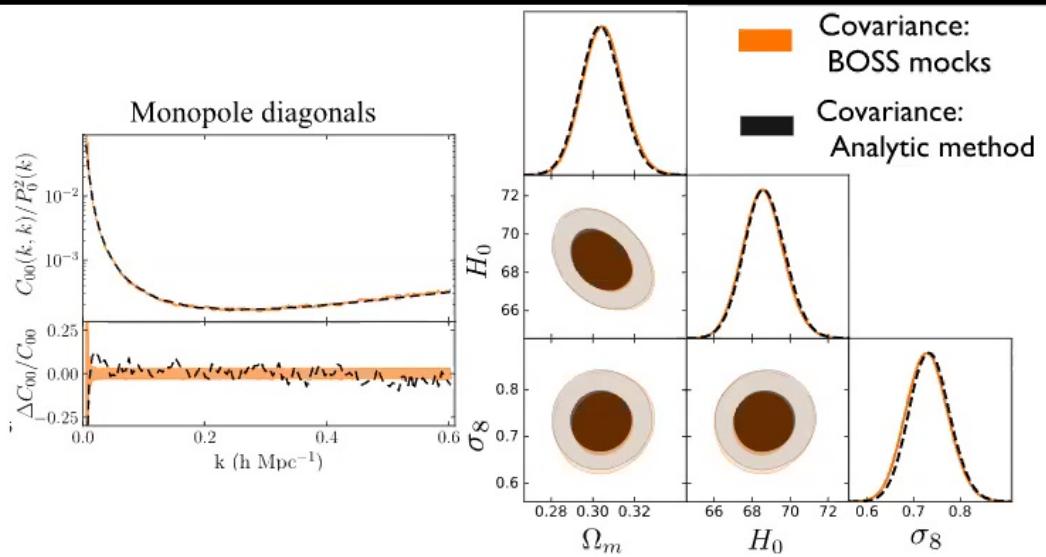
# Understanding the effect of halo environment on HI: Don't baryons just follow dark matter?



# Summary

★ Analytic covariance is an excellent alternative to mock simulations for upcoming spectroscopic surveys

1. Very good agreement with the state-of-the-art mocks up to non-linear scales
2. Immense computational speedup ( $\sim 10^4$ )
3. No sampling noise effects



# Summary

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1. Very good agreement with the state-of-the-art mocks up to non-linear scales
2. Immense computational speedup ( $\sim 10^4$ )
3. No sampling noise effects

★ Symbolic regression can be used to model assembly bias from hydro sims. and guide its detection in survey data

