Title: Wave dark matter

Speakers: Lam Hui

Series: Colloquium

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Abstract: A dark matter candidate lighter than about 30 eV exhibits wave behavior in a typical galactic environment. Examples include the QCD axion as well as other axion-like-particles. We review the particle physics motivations, and discuss experimental and observational implications of the wave dynamics, including interference substructures, vortices, soliton condensation and black hole hair.

Wave dark matter

Lam Hui
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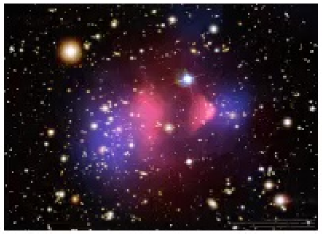
Work done with

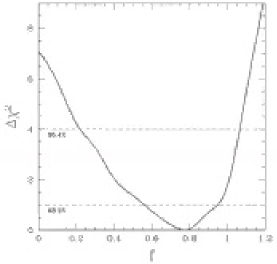
Jerry Ostriker, Scott Tremaine, Edward Witten 1610.08297
Xinyu Li, Greg Bryan 1810.01915
Dan Kabat, Xinyu Li, Luca Santoni, Sam Wong 1904.12803
Austin Joyce, Michael Landry, Xinyu Li 2004.01188
Xinyu Li, Tomer Yavetz

Rich evidence for dark matter - from its gravitational effects

- Dynamical measurements.

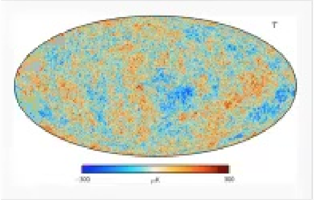
- Gravitational lensing measurements.



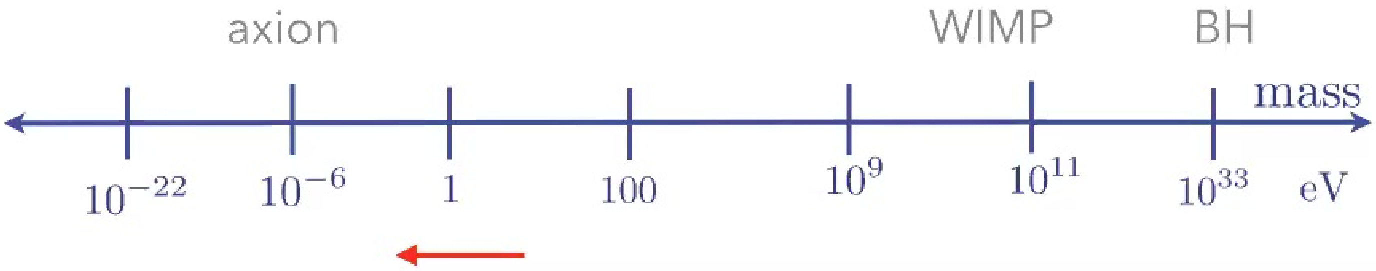


Hoekstra, Yee, Gladders

- Growth of perturbations.



We have rich evidence for the existence of DM, but remain ignorant about its basic properties e.g. mass:

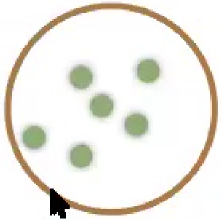


What we do know: mass density in solar neighborhood is $0.3\ \mathrm{GeV/cm^3}$

Question: at what mass is the interparticle separation < de Broglie wavelength? (1/mv)

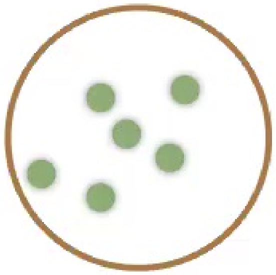
wave regime $m < 30\,\mathrm{eV}$

$$1/mv \sim 10^{-3}\,\mathrm{cm} \quad \text{for} \quad m = 10\,\mathrm{eV}$$
$$10^{4}\,\mathrm{cm} \quad \text{for} \quad m = 10^{-6}\,\mathrm{eV}$$
$$100\,\mathrm{pc} \quad \text{for} \quad m = 10^{-22}\,\mathrm{eV}$$



bosonic

Let's discuss:



Particle physics motivations

Wave dynamics and phenomenology

Astrophysical implications (ultra-light DM)

Experimental implications (light DM)

$$1/mv \sim 10^{-3}\,\mathrm{cm} \quad \text{for} \quad m = 10\,\mathrm{eV}$$

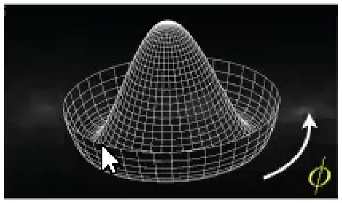
$10^{4}\,\mathrm{cm}$ for $m = 10^{-6}\,\mathrm{eV}$ QCD axion

$100\,\mathrm{pc}$ for $m = 10^{-22}\,\mathrm{eV}$ Fuzzy DM (Hu, Barkana, Gruzinov)

Particle physics motivations

- A natural candidate for a light (scalar) particle is a pseudo-Nambu-Goldstone boson.

  A well known example is the QCD axion (Peccei, Quinn; Weinberg; Wilczek; Kim; Shifman, Vainshtein, Zakharov, Zhitnitsky; Dine, Fischler, Srednicki; Preskill, Wise, Wilczek; Abbott, Sikivie).

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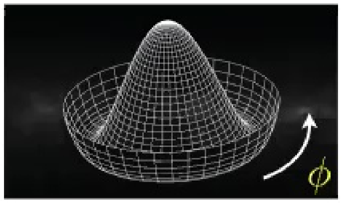

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Footnote on ultra -light version

mass $m \leftarrow 10^{-22}\,\mathrm{eV} \rightarrow$

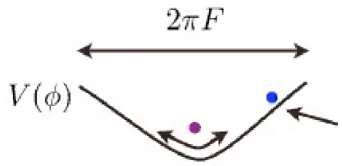
Fuzzy dark matter (FDM)
Hu, Barkana, Gruzinov
Amendola, Barbieri

- Consider an angular field (a pseudo Nambu-Goldstone) of periodicity $2\pi F$ i.e. an axion-like field with a potential from non-perturbative effects (not QCD axion).

$$\mathcal{L} \sim -\frac{1}{2}(\partial\phi)^2 - \Lambda^4(1 - \cos[\phi/F]) \qquad m \sim \Lambda^2/F$$

(candidates: Arvanitaki et al.
Svrcek, Witten)

- Relic abundance matches dark matter abundance (mis-alignment mechanism).



$\phi \sim F$ at early times until $H \sim m$

$$\Omega_{\mathrm{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2}$$

(Preskill, Wise, Wilczek; Abbot, Sikivie; Dine, Fischler, with constant m)

Dynamics of wave dark matter:

- Ignoring self-interactions $\longrightarrow -\Box\phi + m^2\phi = 0$

  Non-relativistic limit $\longrightarrow i\dot{\psi} = \left[-\frac{\nabla^2}{2m} + m\Phi_{\rm grav.}\right]\psi$

$$\phi = \frac{1}{\sqrt{2m}}\left[\psi e^{-imt} + \psi^* e^{imt}\right]$$

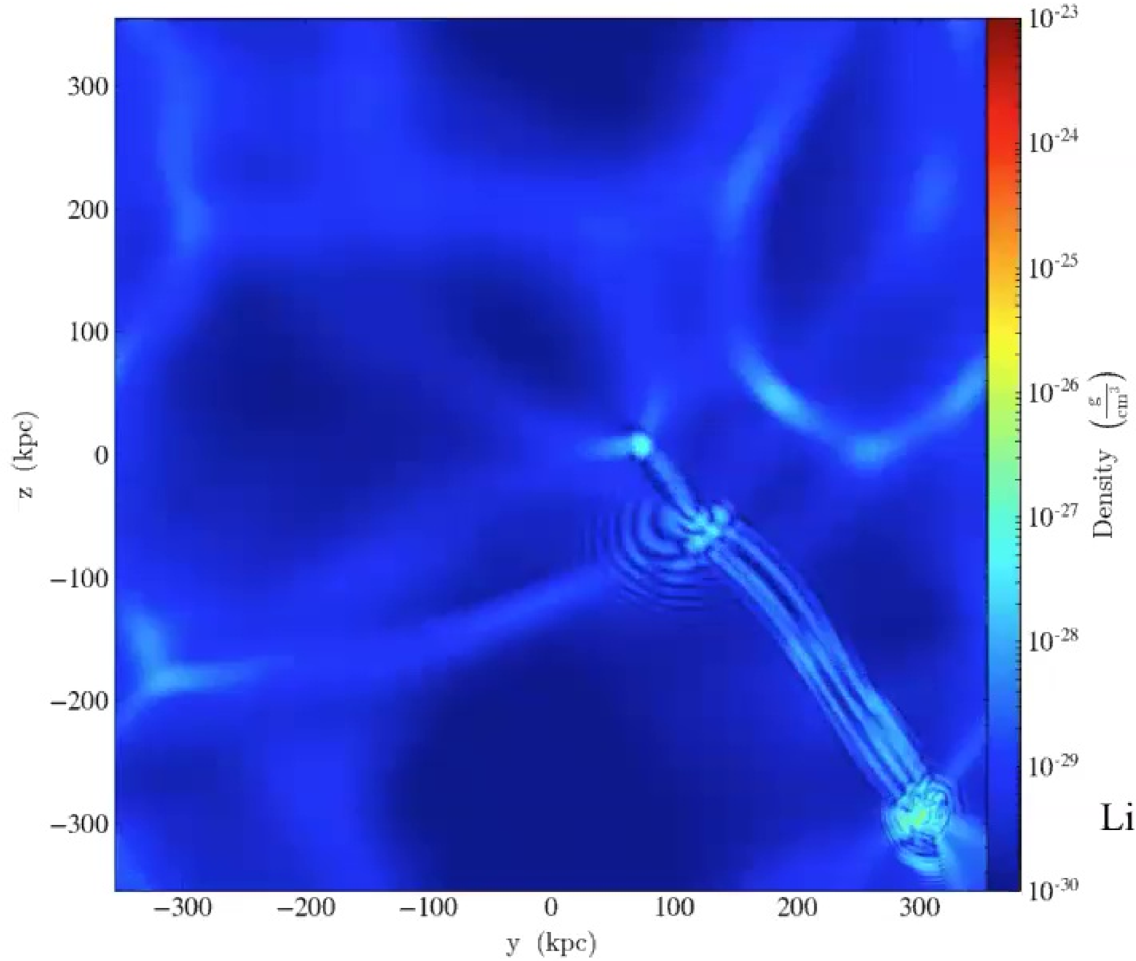
- An alternative viewpoint: ψ as a (classical) fluid. $\psi = \sqrt{\rho/m}\, e^{i\theta}$ i.e. $\rho = m\,|\psi|^2$

  mass conservation $\dot{\rho} + \nabla\cdot\rho v = 0$ where $v = \frac{1}{m}\nabla\theta$

  superfluid
  (see also Berezhiani, Khoury)

  Euler equation $\dot{v} + v\cdot\nabla v = -\nabla\Phi_{\rm grav.} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$

Wave effects in a cosmological simulation

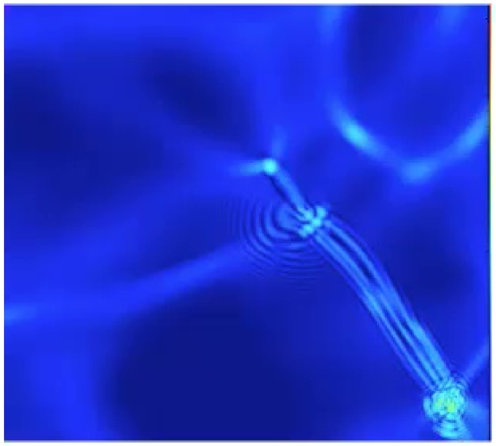


Li, LH, Bryan

See Schive, Chiueh, Broadhurst; Veltmaat, Niemeyer; Schwabe, Niemeyer, Engels; Mocz et al.; Nori, Baldi; Kendall, Easther

Wave effects from light/ultra-light DM:

- dynamical friction
- evaporation of sub-halos by tunneling
- interference
- tidal streams and gravitational lensing
- Lyman-alpha forest
- direct detection
- detection by pulsar timing array
- vortices (and walls)
- black hole hair
- soliton oscillations



Vortices

- Consider again fluid formulation: $\psi = \sqrt{\rho/m}\, e^{i\theta}$

$$\dot{\rho} + \nabla \cdot \rho v = 0 \quad \text{where} \quad v = \frac{1}{m}\nabla\theta$$

$$\dot{v} + v \cdot \nabla v = -\nabla\Phi_{\rm grav.} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

- Naively, vorticity cannot exist, because the velocity field is a gradient flow. In addition, one might think Kelvin's theorem should hold i.e. no vorticity is generated if there's no vorticity to begin with.

- The loophole: where $\rho = 0$. Note: such complete destructive interference can only occur in the late universe when O(1) fluctuations are present. No vortices in the early universe.

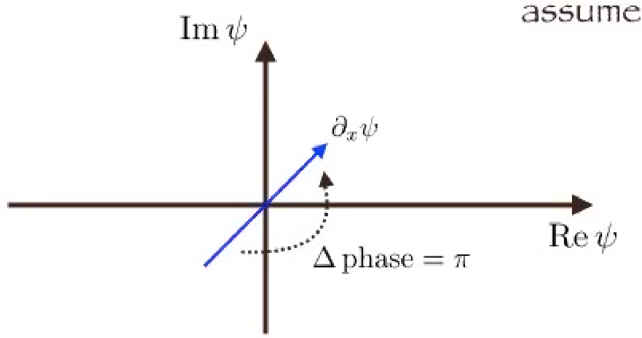
- The phenomenon of vortices is well understood in condensed matter physics.

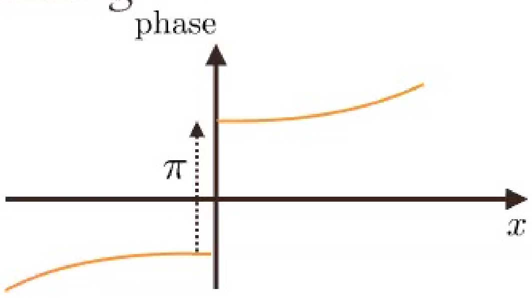
- A simpler example first: a wall defect in 1D

Consider ψ in one spatial dimension. Suppose it vanishes at some point, say x=0.

$$\psi(x) \sim \cancel{\psi(0)} + x\,\partial_x\psi\Big|_0 + \ldots$$

assumed non-vanishing





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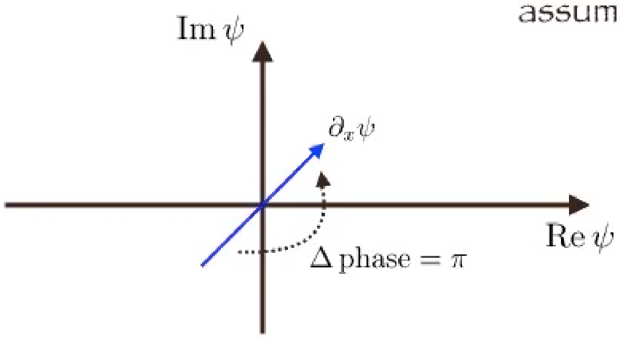
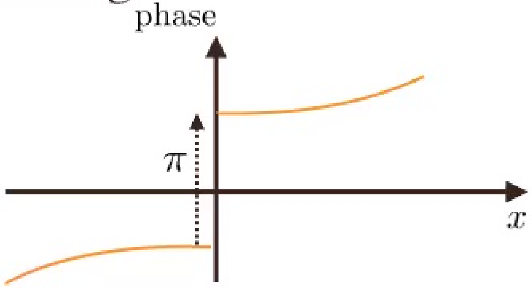
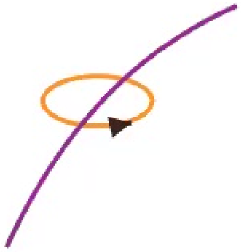




- Vortex

Argument generalizes to higher dimensions. In 3D, vanishing of both real & imaginary parts implies intersection of 2 surfaces i.e. a line/string defect -> vortex.

$$\psi(\vec{x}) \sim \cancel{\psi(0)} + \vec{x}\cdot\vec{\partial}\psi\big|_0 + \ldots$$

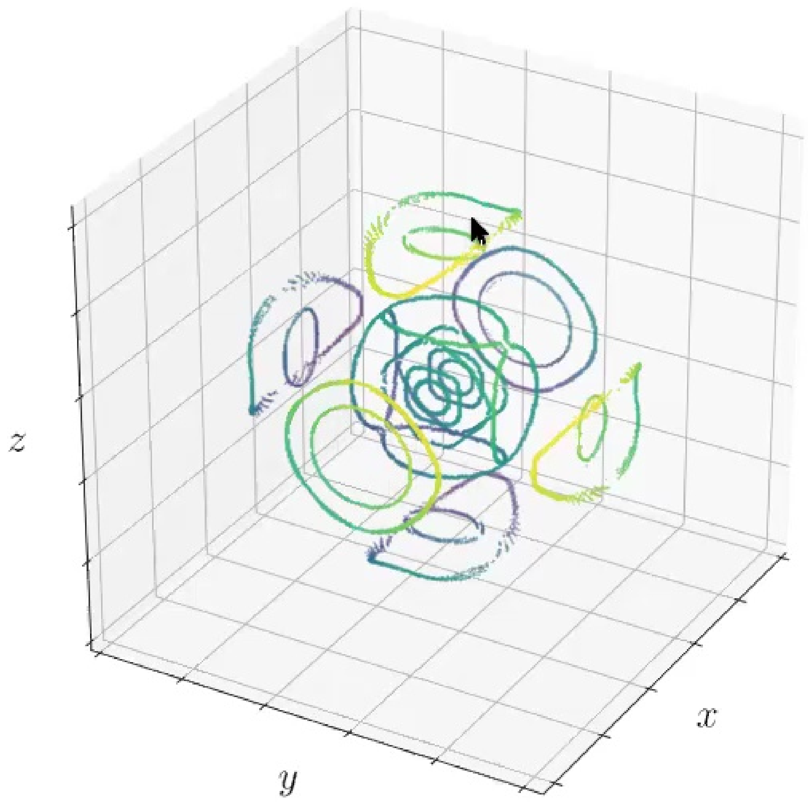


phase wraps by 2π (or $2\pi n$)

$$\oint \vec{v}\cdot d\vec{\ell} = 2\pi n/m$$

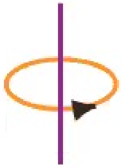
$$\rho \propto r^{2n} \quad \& \quad v \propto 1/r$$

Note: this is not the usual axion string.

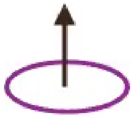

z
y
x

Simple solutions of the free equation: $i\,\partial_t\psi = -\frac{\nabla^2}{2m}\psi$

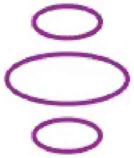
$$\psi = x + iy$$



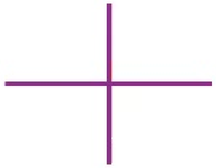
$$\psi = x^2 + y^2 - R^2 + 2i(-Rz + t/m)$$

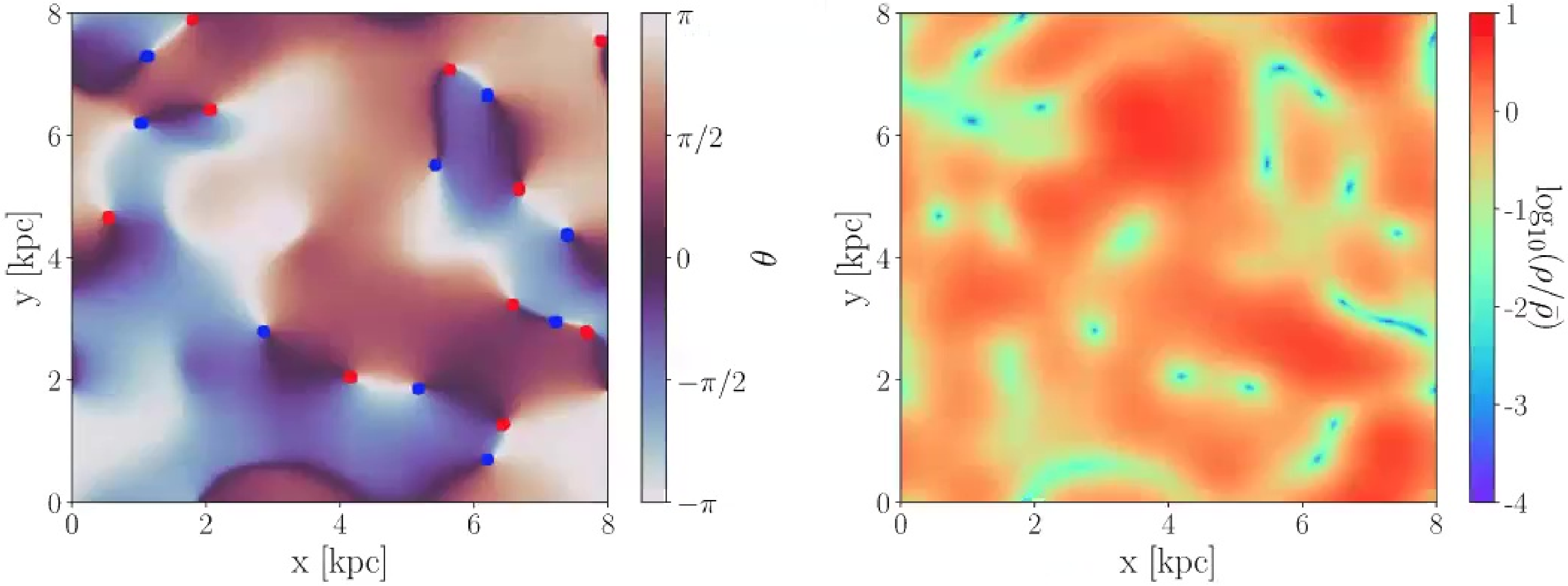


$$\psi = x^2 + y^2 + z^2 - R^2 + i(-2Rz + 3t/m)$$



$$\psi = (x + iy)(y + iz) - t/m$$





A 2D example built from a superposition of waves with random phases
Vortices appear and disappear in pairs.

Additional comments:

- Should defects be rare? No - roughly one vortex ring per de Broglie volume. Can compute this analytically for a model halo composing of a superposition of waves with random phases: essentially looking for zero-crossing.
  Note: this holds even if the halo has no net angular momentum.
- Smaller rings move faster: $v \sim \dfrac{1}{mR}$. Curved segments also move faster.
- Vortices (and interference substructures) are transient phenomena. Coherent time scale is de Broglie time $1/mv^2$ (million years for ultra-light). Vortices can't arbitrarily appear or disappear - Kelvin's theorem.
- Angular momentum eigenstates have vortices, though angular momentum does not require vortices (e.g. can always add s-wave with large amplitude).
- See condensed matter literature.

destructive interference

destructive interference

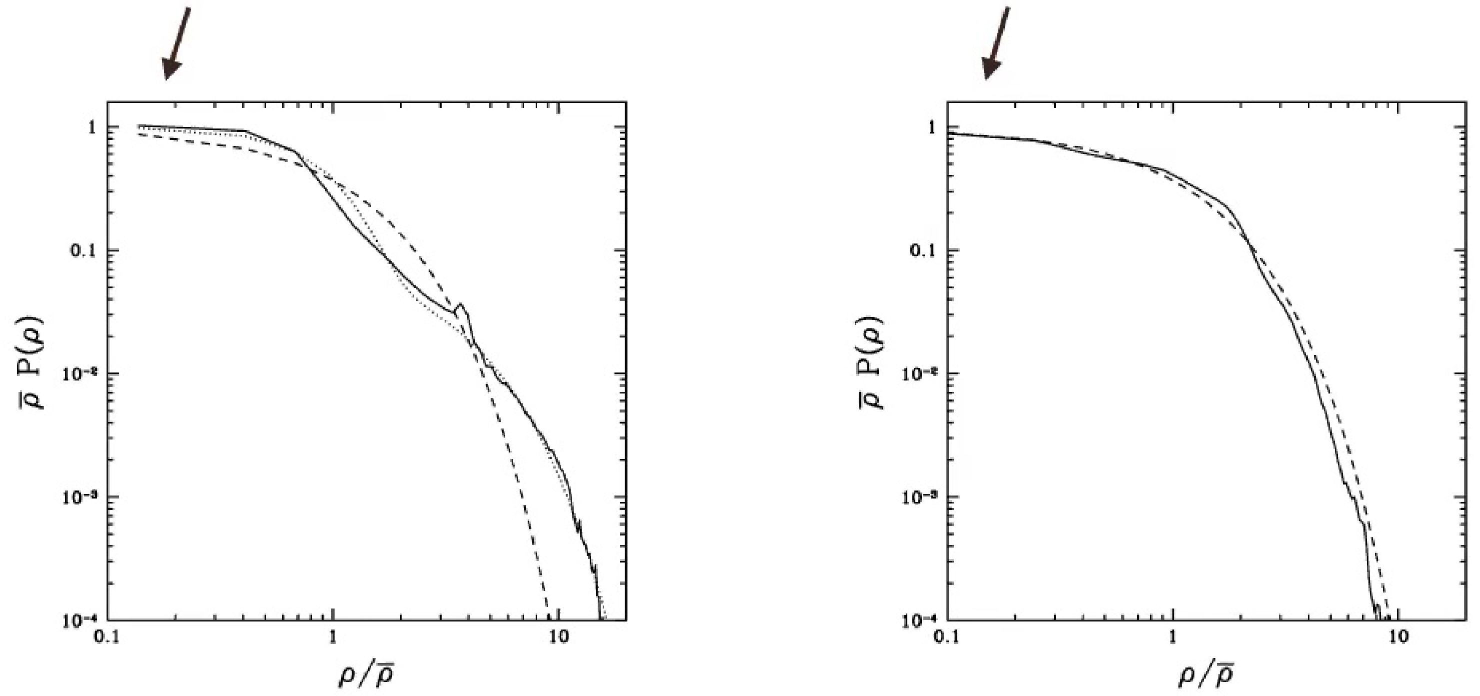
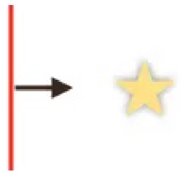


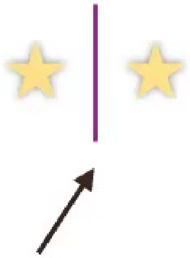
Figure 5: The one-point probability distribution of density: $P(\rho)d\rho$ gives the probability that the density ρ takes the values within the interval $d\rho$. The solid lines are measured from numerical wave simulations of two halos that form from mergers of smaller seed halos and gravitational collapse. The left panel is from a simulation where the initial seed halos are distributed uniformly, and the right panel is from a simulation where the initial seed halos are distributed randomly. The dashed line in each panel shows the analytic prediction from the random phase halo model: $\bar{\rho}P(\rho) = e^{-\rho/\bar{\rho}}$. The dotted line on the left panel is $\bar{\rho}P(\rho) = 0.9\,e^{-1.06(\rho/\bar{\rho})^2} + 0.1\,e^{-0.42(\rho/\bar{\rho})}$. See [72] for details.

Observational signatures (for ultralight DM):

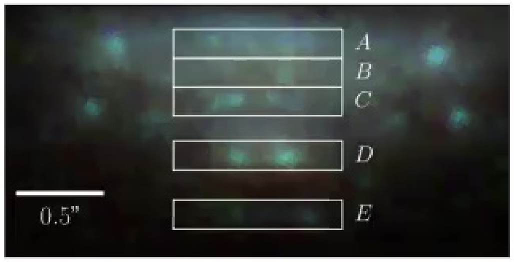


- Gravitational lensing by a vortex can lead to 10^{-4} arcsec displacement of distant sources in 10^5 years. (Mishra-Sharma, Van Tilburg, Weiner)

- In lensing events with extreme magnification (> 100), interference substructure can lead to fluctuations at the 10 percent level.



critical line



Daí et al.: strongly lensed arc

(See also: Dalal, Kochanek; Alexander et al.; Chan et al.; Broadhurst et al.)

- Heating of tidal streams. (See Amorisco, Loeb)

Experimental implications (light DM e.g. QCD axion):

$$\mathcal{L} \sim \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial_\mu \phi}{f} \bar{\Psi} \gamma^5 \gamma^\mu \Psi$$

Reviews: Sikivie 2003
Graham et al. 2015, Marsh 2016

- Coupling to EM

ADMX (cavity) - photon from axion in magnetic field	ϕ^2
ABRACADABRA - magnetic flux from axion in magnetic field	$\dot{\phi}$
ADBC - rotation of polarization of photon propagating in axion	$\Delta\phi$

- Coupling to spin $\hat{H} \sim \vec{\nabla}\phi \cdot \hat{\sigma}$

CASPEr - spin precession like in NMR	$\vec{\nabla}\phi$
Eot-Wash - torsional spin pendulum	$\vec{\nabla}\phi$

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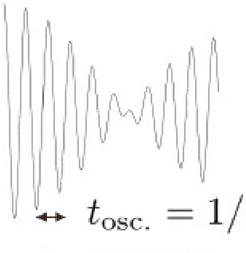
-

$$\phi \sim \underbrace{\psi}_{\text{slow}} \underbrace{e^{-imt}}_{\text{fast}} + \psi^* e^{imt}$$



$t_{\text{osc.}} = 1/m \sim 10^{-9}\,\text{s}$

$t_{\text{coherent}} = 1/(m\,v^2) \sim 10^{-3}\,\text{s}$

Derevianko; Foster, Rodd, Safdi; Centers et al.

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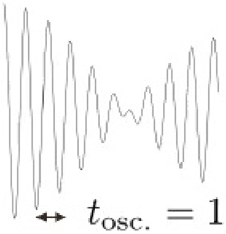
  slow

  fast



$\leftrightarrow$ $t_{\mathrm{osc.}} = 1/m \sim 10^{-9}\,\mathrm{s}$

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Derevianko; Foster, Rodd, Safdi;
Centers et al.

- Measure correlation functions e.g.

  $\langle \phi(t)^2 \phi(t')^2 \rangle - \langle \phi^2 \rangle^2 \sim [|t-t'|/t_{\mathrm{coherent}}]^{-3} + \mathrm{osc.}$ (or even space-time correlations).

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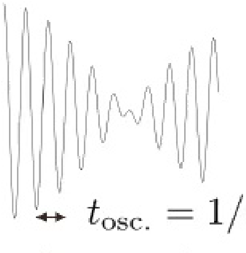
slow ↗ ↖ fast



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Derevianko; Foster, Rodd, Safdi;
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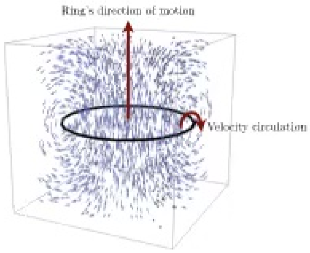
$\langle\phi(t)^2\phi(t')^2\rangle - \langle\phi^2\rangle^2 \sim [|t-t'|/t_{\rm coherent}]^{-3} + {\rm osc.}$ (or even space-time correlations).

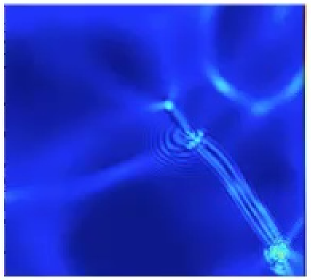
- At vortices $\phi = 0$ but $\vec{\nabla}\phi \neq 0$.
- Phase of oscillation might be interesting: $\phi \sim |\psi|\cos(mt-\theta)$.

Summary

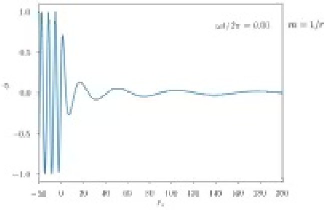
- For dark matter particle lighter than about 30 eV (e.g. axion/axion-like-particle), wave interference phenomena are unavoidable. One implication is:



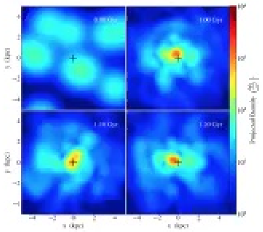
vortices.



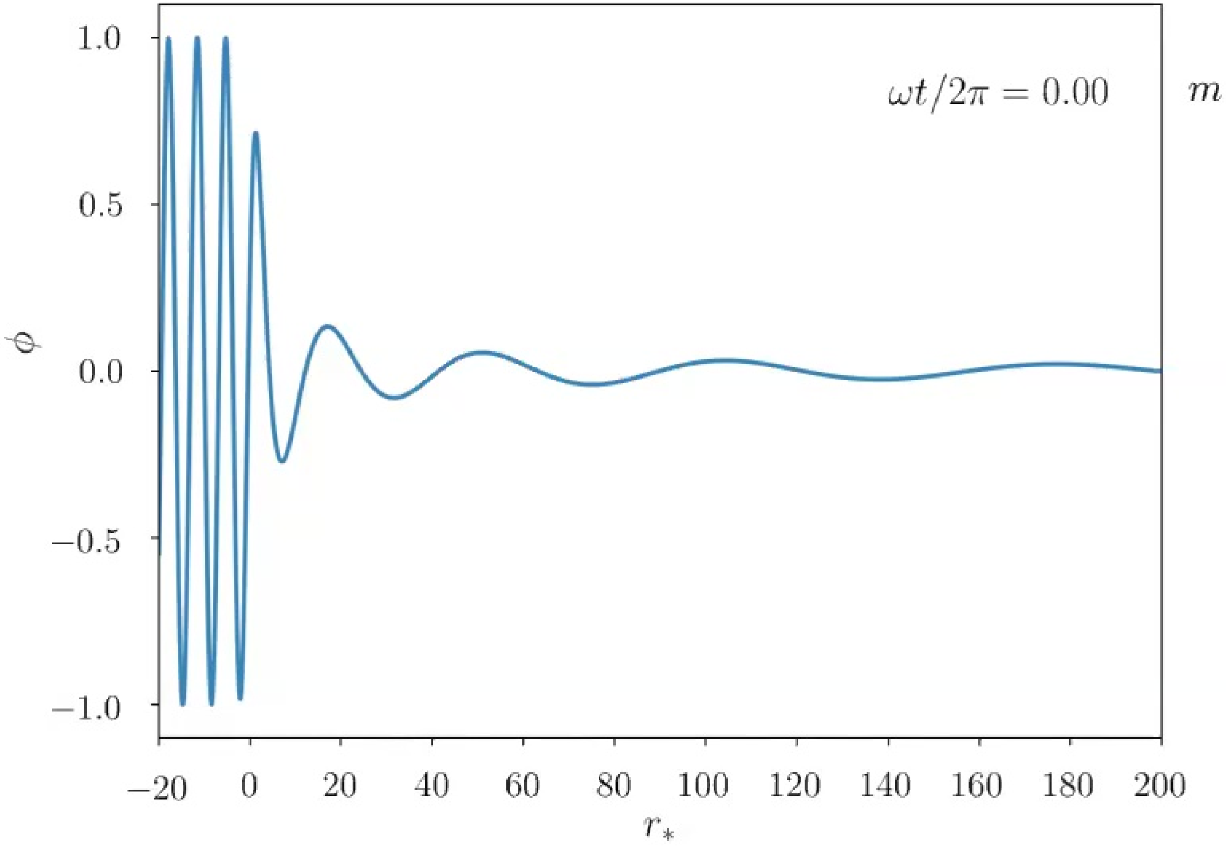
- Such axion dark matter gives black hole hair (or more mundanely: accretion flow).



- Wave interference gives rise to soliton oscillations and random walk.



$$(-\Box + m^2)\phi = 0 \quad \text{in Schwarz. bgd.}$$



$m = 1/r_{\text{Schwarz.}}$

$$r_* = r + \ln(r-1) \qquad (r_{\text{Schwarz.}} = 1)$$

similar pile – up occurs for $m > 10^{-3}\, r_{\text{Schwarz.}}^{-1}$