Title: Decoherent Quench Across Quantum Phase Transitions

Speakers: Yi-Zhuang You
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Abstract: Monitored quantum dynamics has attracted much research attention recently. Environmental monitoring typically leads to the decoherence of a quantum system. We explore the effect of energy-level decoherence in quench dynamics. In particular, we consider the linear quench across quantum critical phase transitions under the influence of decoherence. Due to the critical slowing down, the system will necessarily fall out of equilibrium in the vicinity of the critical point within a time scale known as the freeze-out time. The freeze-out time will scale with the quench rate following the Kibble-Zurek scaling. In the presence of decoherence, we found a new scaling behavior differed from the Kibble-Zurek scaling. We demonstrated our findings in the critical quench between 2D Chern insulator and trivial insulator. We show that the new scaling behavior can be justified from the behavior of Hall conductivity across the quantum quench.

Decoherent Quench Across Quantum Phase Transitions

Yi-Zhuang You
(UCSD)



Wei-Ting Kuo Daniel Arovas
(UCSD)

Smitha Vishveshwara
(UIUC)

Quantum Matter Frontier Seminars
Nov 2020

Motivation

- Exploring the effect of measurement in quantum dynamics
- Evolution of quantum states in quantum mechanics
  - Unitary evolution (coherent)

$$-\mathrm{i}\partial_t|\Psi\rangle = H|\Psi\rangle$$

  - Measurement (incoherent)

$$|\Psi\rangle \to \frac{P|\Psi\rangle}{\sqrt{\langle\Psi|P|\Psi\rangle}}$$

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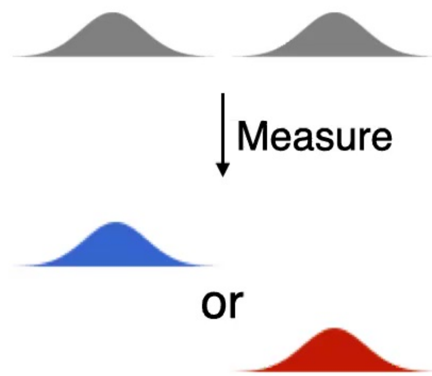

- Measurement - system interacting with environment
- Environmental monitoring effect
  → decoherence, dissipation …

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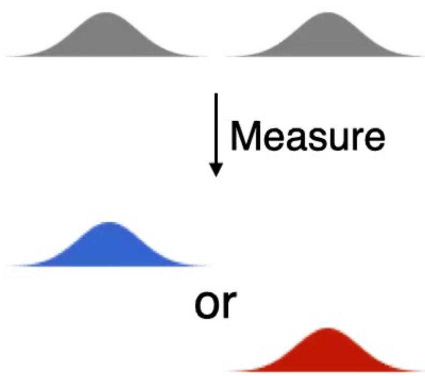

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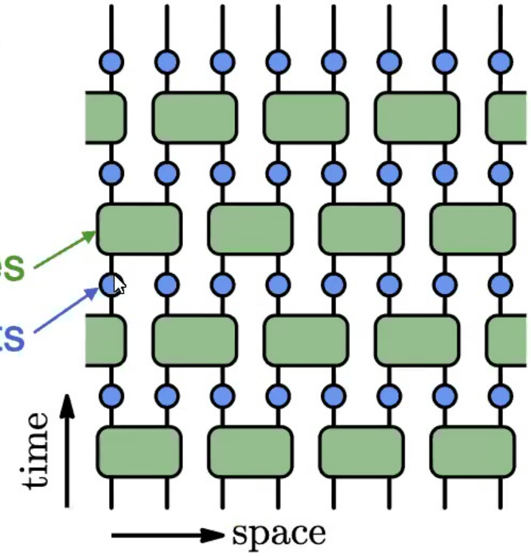
- Exploring the effect of measurement in quantum dynamics
  - Measurements in quantum circuits drives entanglement transitions

    Li, Chen, Fisher 1808.06134; Skinner, Ruhman, Nahum 1808.05953; Chan, Nandkishore, Pretko, Smith 1808.05949 …



  - Measurements can stabilize different quantum phases

    Lavasani, Alavirad, Barkeshli 2004.07243; Sang, Hsieh 2004.09509…

What are being measured are local terms of a Hamiltonian.
→ As if environment is monitoring the energy of the system.

Open Quantum Systems

- How system interacts with environment?

$$H = H_{\text{sys}} + H_{\text{env}} + H_{\text{int}}$$

Closed System

$H_{\text{int}} = 0$

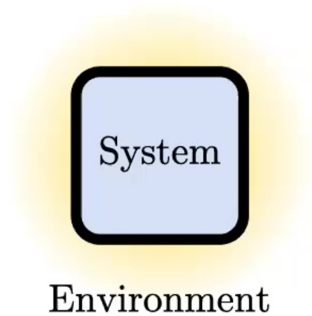
Open System $H_{\text{int}} \neq 0$

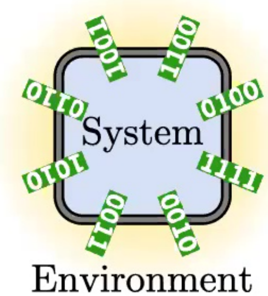
“Partially” Open

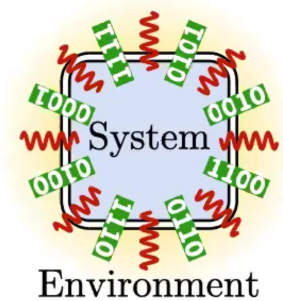
$[H_{\text{int}}, H_{\text{sys}}] = 0$

“Fully” Open

$[H_{\text{int}}, H_{\text{sys}}] \neq 0$







Closed System:
- ✗ Information flow
- ✗ Energy flow
- Coherent

“Partially” Open:
- ✓ Information flow
- ✗ Energy flow
- Decoherence

“Fully” Open:
- ✓ Information flow
- ✓ Energy flow
- Dissipation

Decoherence in Energy Basis

- One apparent scenario for $[H_{\rm int}, H_{\rm sys}] = 0$ is

$$H_{\rm int} = H_{\rm sys} \otimes A_{\rm env}$$

Some random Hermitian operator in environment

- Continuous indirect measurement
  - Environment starts with ancilla state $|\phi\rangle$
  - Evolves jointly with system under $H_{\rm int}$ for short time
  - Environment collapse to measurement basis $|i\rangle$
- Effect on the system density matrix

$$\rho_{\rm sys} \to \sum_i K_i \rho_{\rm sys} K_i^\dagger$$

Kraus operator: $K_i = \langle i| e^{-{\rm i}\epsilon H_{\rm int}} |\phi\rangle$

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$$\rho_{\rm sys} \to \rho_{\rm sys} - {\rm i}\epsilon\langle\phi|A|\phi\rangle[H_{\rm sys}, \rho_{\rm sys}] - \frac{1}{2}\epsilon^2\langle\phi|A^2|\phi\rangle[H_{\rm sys}, [H_{\rm sys}, \rho_{\rm sys}]] + \cdots$$

Assuming: $\mathop{\mathbb{E}}_{A,\phi}\langle\phi|A|\phi\rangle \to 0 \qquad \mathop{\mathbb{E}}_{A,\phi}\langle\phi|A^2|\phi\rangle = \dfrac{2\gamma\,\delta t}{\epsilon^2}$

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- Lindblad equation (omitting "sys" subscript)

$$\partial_t \rho(t) = -\mathrm{i}[H(t), \rho(t)] - \gamma[H(t), [H(t), \rho(t)]]$$

  - Lindblad operator = Hamiltonian: describing decoherence in the energy basis
  - Decoherence strength γ
- Toy Model (two levels with an energy gap Δ)

$$H = \begin{bmatrix} 0 & 0 \\ 0 & \Delta \end{bmatrix} \qquad \rho = \begin{bmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{bmatrix}$$

$$\partial_t \rho_{01} = \mathrm{i}\Delta\rho_{01} - \gamma\Delta^2\rho_{01} \quad \Rightarrow \quad \rho_{01} \sim e^{-\gamma\Delta^2 t} e^{\mathrm{i}\Delta t}$$

  - Decoherence time $\boxed{\tau_{\mathrm{dec}} \sim \frac{1}{\gamma\Delta^2}}$

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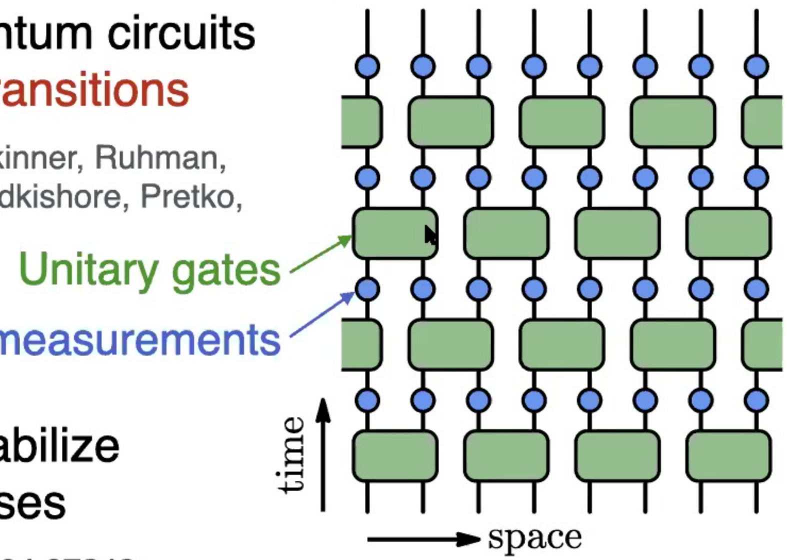

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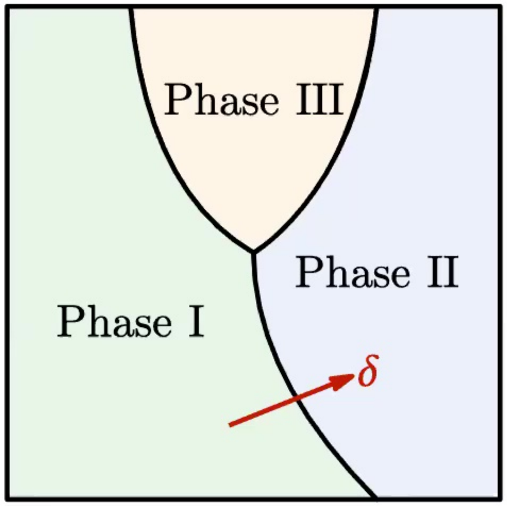
Decoherent Quantum Quench

- Quantum quench through continuous phase transition



- Continuously tuning parameter δ with time

$$H(t) = \underbrace{H_{\mathrm{CFT}}}_{\text{Critical theory}} + \delta(t)\underbrace{H_{\mathrm{pert}}}_{\text{Relevant perturbation}}$$

Linearize: $\delta(t) \to t/\tau$

Quench rate $1/\tau$

Decoherent Critical Quench

- Quantum quench through continuous phase transition

$$H(t) = H_{\text{CFT}} + \delta(t) H_{\text{pert}} \qquad \text{Linearize: } \delta(t) = t/\tau$$

  - Correlation length $\xi \sim \delta^{-\nu} \sim (t/\tau)^{-\nu}$
  - Excitation gap $\Delta \sim \xi^{-z} \sim (t/\tau)^{\nu z}$

- Two relevant time-scales of quantum dynamics
  - Correlation time: how fast system responses coherently

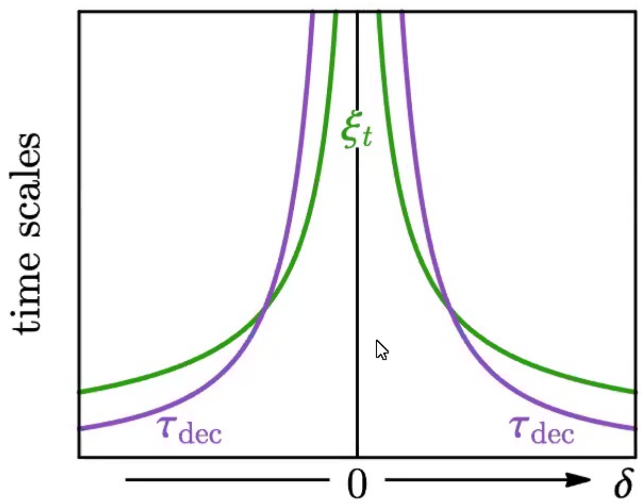
$$\xi_t \sim \frac{1}{\Delta} \sim \left(\frac{t}{\tau}\right)^{-\nu z}$$

  - Decoherence time: how fast system decohere to energy eigenstates

$$\tau_{\text{dec}} \sim \frac{1}{\gamma \Delta^2} \sim \frac{1}{\gamma}\left(\frac{t}{\tau}\right)^{-2\nu z}$$

Decoherent Critical Quench

- Two relevant time-scales of quantum dynamics
  - Correlation time: $\xi_t \sim (t/\tau)^{-\nu z}$
  - Decoherence time: $\tau_{\mathrm{dec}} \sim \gamma^{-1}(t/\tau)^{-2\nu z}$



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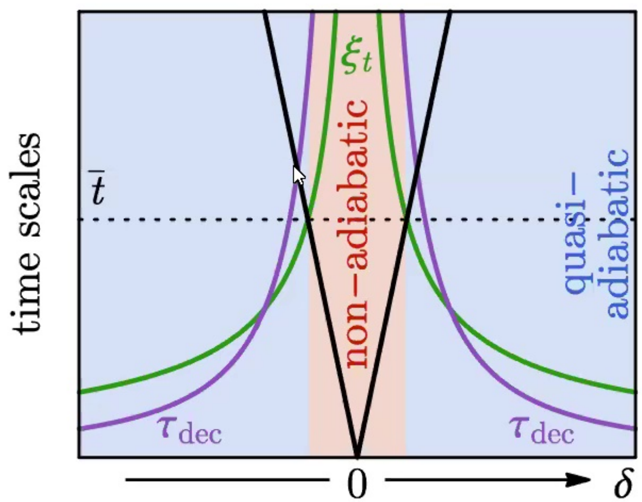

- Slow quench $\tau \gg \gamma^{1+1/\nu z}$

$$\bar{t} \sim \tau^{\frac{\nu z}{1+\nu z}} \qquad \bar{\xi} \sim \tau^{\frac{\nu}{1+\nu z}}$$

Kibble-Zurek Scaling

Decoherent Critical Quench

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  - Correlation time: $\xi_t \sim (t/\tau)^{-\nu z}$
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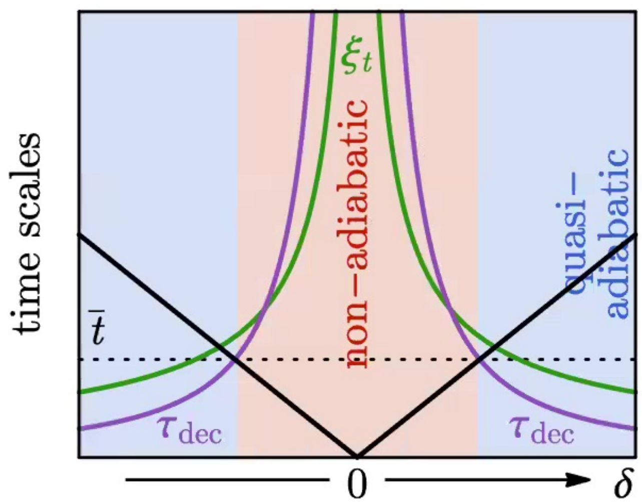


- Slow quench $\tau \gg \gamma^{1+1/\nu z}$

  $\bar{t} \sim \tau^{\frac{\nu z}{1+\nu z}}$ $\quad \bar{\xi} \sim \tau^{\frac{\nu}{1+\nu z}}$

  Kibble-Zurek Scaling

- Fast quench $\tau \ll \gamma^{1+1/\nu z}$

  $\bar{t} \sim (\gamma^{-1}\tau^{2\nu z})^{\frac{1}{1+2\nu z}}$

  $\bar{\xi} \sim (\gamma\tau)^{\frac{\nu}{1+2\nu z}}$

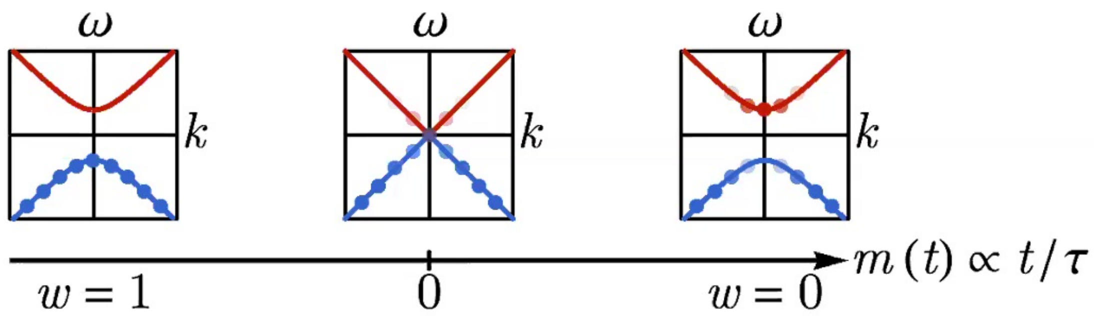
  (Different scaling under strong decoherence)

Chern Insulator Transition

- (2+1)D spinless fermion + time dependent band structure

$$H(t) = \frac{1}{2} \sum_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}} \, \boldsymbol{h}_{\boldsymbol{k}}(t) \cdot \boldsymbol{\sigma} \, c_{\boldsymbol{k}}$$

Caio,Cooper, Bhaseen 2015; Hu, Zoller, Budike 2016; D'Alessio, Rigol 2016; Wilson, Song, Refael 2016; Wang, Zhang, Chen, Yu, Zhai 2016 …



- Band topological number (winding of $\hat{\boldsymbol{h}}_{\boldsymbol{k}} \equiv \boldsymbol{h}_{\boldsymbol{k}}/|\boldsymbol{h}_{\boldsymbol{k}}|$)

$$w = \frac{1}{4\pi} \int \mathrm{d}^2\boldsymbol{k} \, \hat{\boldsymbol{h}}_{\boldsymbol{k}} \cdot (\partial_{k_x} \hat{\boldsymbol{h}}_{\boldsymbol{k}} \times \partial_{k_y} \hat{\boldsymbol{h}}_{\boldsymbol{k}})$$

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- Density matrix (free fermion)

$$\rho(t) = \prod_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}}|0\rangle\rho_{\boldsymbol{k}}(t)\langle 0|c_{\boldsymbol{k}}$$

$$\rho_{\boldsymbol{k}}(t) = \frac{1+\boldsymbol{n}_{\boldsymbol{k}}(t)\cdot\boldsymbol{\sigma}}{2}$$

Pseudo spin: $\boldsymbol{n}_{\boldsymbol{k}}(t) = \mathrm{Tr}\,\rho(t)c^\dagger_{\boldsymbol{k}}\boldsymbol{\sigma}c_{\boldsymbol{k}}$

- Dynamic Equation

$$\partial_t\rho(t) = -\mathrm{i}[H(t),\rho(t)] - \gamma[H(t),[H(t),\rho(t)]]$$

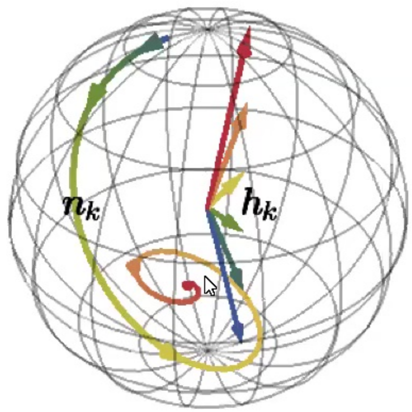
$$\Rightarrow \quad \partial_t\boldsymbol{n}_{\boldsymbol{k}} = \boldsymbol{h}_{\boldsymbol{k}}\times\boldsymbol{n}_{\boldsymbol{k}} + \gamma\boldsymbol{h}_{\boldsymbol{k}}\times(\boldsymbol{h}_{\boldsymbol{k}}\times\boldsymbol{n}_{\boldsymbol{k}})$$

Pseudo-Spin Dynamics

- Dynamic Equation

$$\partial_t \boldsymbol{n_k} = \boldsymbol{h_k} \times \boldsymbol{n_k} + \gamma \boldsymbol{h_k} \times (\boldsymbol{h_k} \times \boldsymbol{n_k})$$

Weak decoherence
$\gamma \ll \tau^{1/2}$



Pseudospin precession
(Quantum dynamics)

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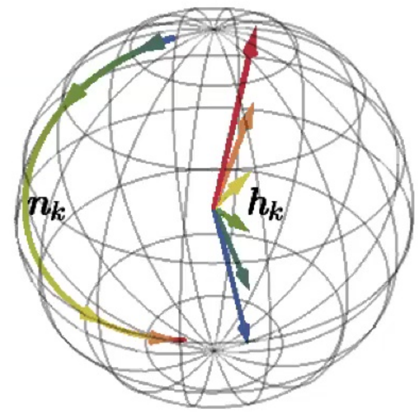
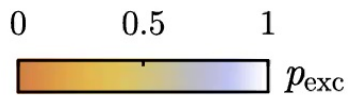
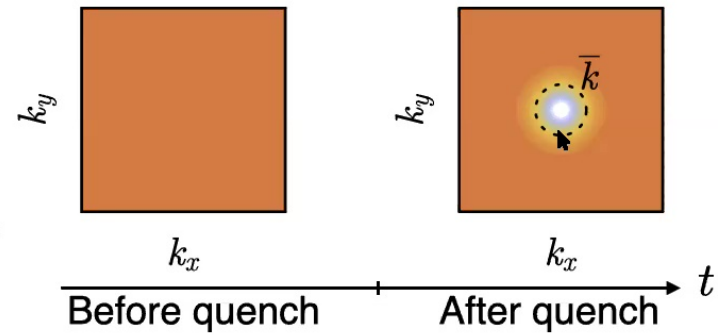


Zeno effect
(Decoherent dynamics)

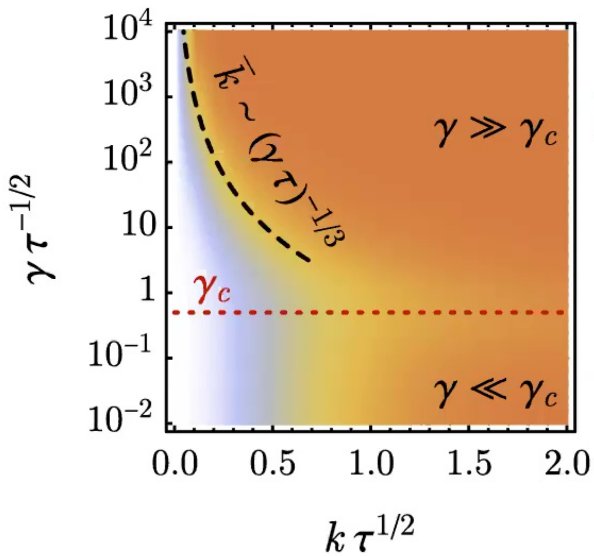
Excitation after Quench

- Momentum space picture of excitation density





- Scaling behavior



$$\bar{k} \sim \bar{\xi}^{-1} \sim \begin{cases} \tau^{-\frac{\nu}{1+\nu z}} & \gamma \ll \gamma_c \\ (\gamma\tau)^{-\frac{\nu}{1+2\nu z}} & \gamma \gg \gamma_c \end{cases}$$

$\nu = z = 1$ (free fermion CFT)

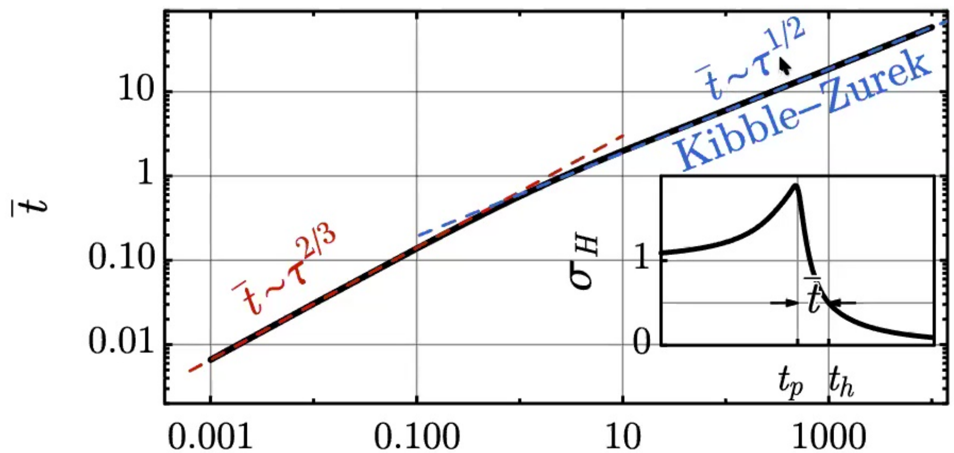
$$\bar{k} \sim \begin{cases} \tau^{-1/2} & \gamma \ll \tau^{1/2} \\ (\gamma\tau)^{-1/3} & \gamma \gg \tau^{1/2} \end{cases}$$

Probing the Frozen Time

- Time scale associated to the change of Hall conductance

Peak: $\sigma_H(t) \sim \pm \ln(t/\bar{t})$

Tail: $\sigma_H(t) \sim \pm (t/\bar{t})^{-2}$



$\tau \ll \gamma^2$

Strong decoherence

$\tau \gg \gamma^2$

Weak decoherence

Summary

- We study the critical quench dynamics in the presence of energy-basis decoherence

$$\partial_t \rho(t) = -\mathrm{i}[H(t), \rho(t)] - \gamma[H(t), [H(t), \rho(t)]]$$

- Different scaling behavior emerges in the strong decoherence regime compared to Kibble-Zurek

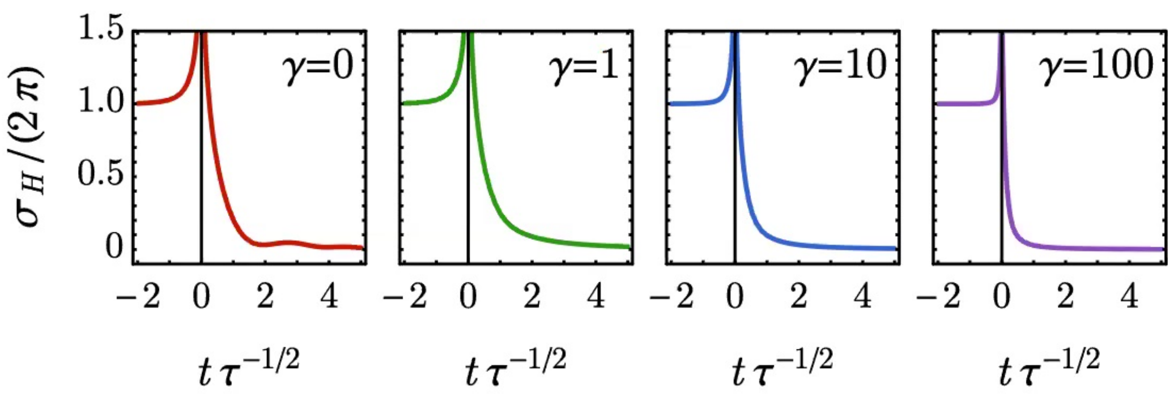
$$\bar{t} \sim \left(\gamma^{-1}\tau^{2\nu z}\right)^{\frac{1}{1+2\nu z}} \qquad \bar{\xi} \sim \left(\gamma\tau\right)^{\frac{\nu}{1+2\nu z}}$$

- These behaviors tested in free fermion topological transitions
- General result applicable for other transitions (bosonic SPT transition, DQCP …), which could possibly be probed by the topological response across the transition.

Hall Conductance

- Hall conductance for non-equilibrium system (via linear response theory)

$$\sigma_H(t) = \int_{-T/2}^{T/2} \mathrm{d}t' \mathrm{i}t' \, \mathrm{Tr}\left([J_x(t), J_y(t+t')]\rho(t)\right)$$
$$= \int \mathrm{d}^2\boldsymbol{k} \frac{\boldsymbol{n_k} \cdot (\partial_{k_x}\boldsymbol{h_k} \times \partial_{k_y}\boldsymbol{h_k})}{2|\boldsymbol{h_k}|^2}$$



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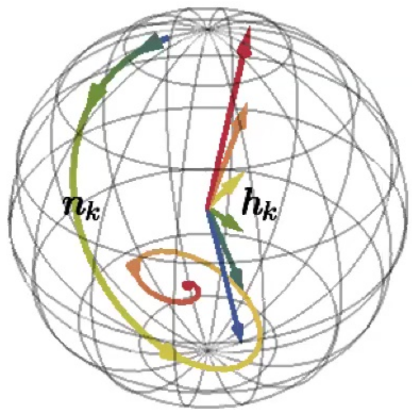

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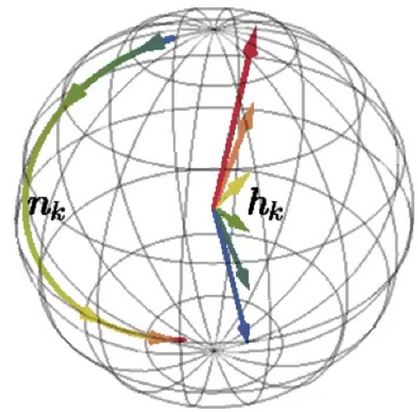

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