

Title: Symmetry and information flow in quantum circuits with measurements

Speakers: Ehud Altman

Date: November 23, 2020 - 12:30 PM

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Abstract: Quantum circuits, relevant for quantum computing applications, present a new kind of many-body problem. Recently it was discovered that the quantum state evolved by random unitary gates, interrupted by occasional local measurements undergoes a phase transition from a highly entangled (volume law) state at small measurement rate to an area law state above a critical rate. I will review the current understanding of this transition from the statistical mechanics and the information perspectives. I will then argue that a circuit with intrinsic symmetries admits more phases, which represent distinct patterns of protection and flow of quantum information. These states can be studied and classified by mapping to an effective ground state problem of a Hamiltonian with enlarged effective symmetry. I will give two simple examples to illustrate these ideas: (i) a circuit with intrinsic Z_2 spin symmetry; (ii) A circuit with Gaussian Majorana fermion gates showing a surprising Kosterlitz-Thouless transition in the entanglement content.

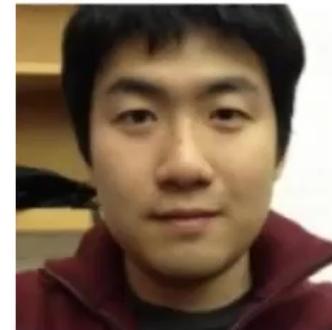
SYMMETRY AND INFORMATION FLOW IN QUANTUM CIRCUITS WITH MEASUREMENTS

Y. Bao, S. Choi, EA, Phys. Rev. B 101, 104301 (2020)
S. Choi, Y. Bao, XL Qi, EA, Phys. Rev. Lett. 125, 030505 (2020)
Y. Bao, S. Choi, EA, arXiv:2012:XXXX

Yimu Bao



Soonwon Choi

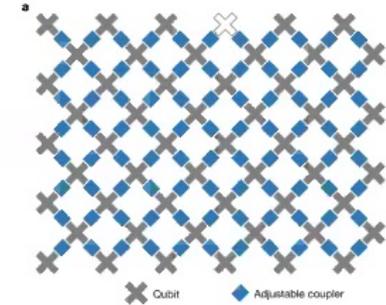
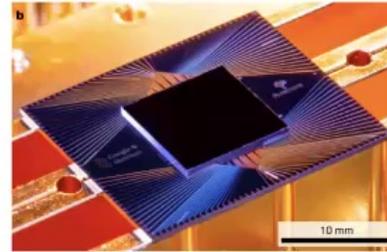


GORDON AND BETTY
MOORE
FOUNDATION

Huge progress in building controllable quantum circuits

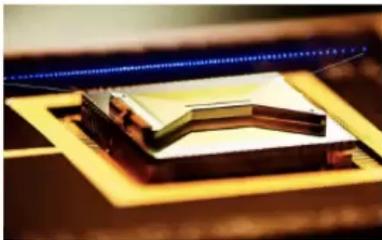
Google quantum supremacy demonstration with SC quantum circuits

Arute et. al. (Martinis, Roushan group) Nature 2019



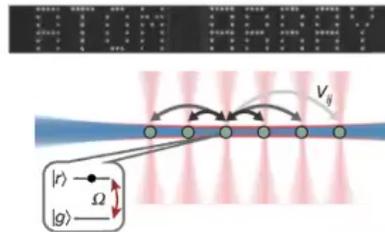
Other platforms:

Trapped ions



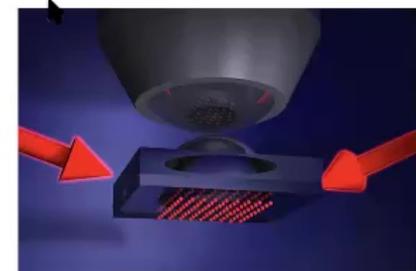
Monroe group @ UMD

Rydberg atoms on Tweezers



Lukin group @ Harvard

Ultra cold atoms



Griener group @ Harvard

Beside promise of quantum computing these systems present new paradigms for many body physics:
New emergent phenomena from interplay of entanglement, quantum measurements?



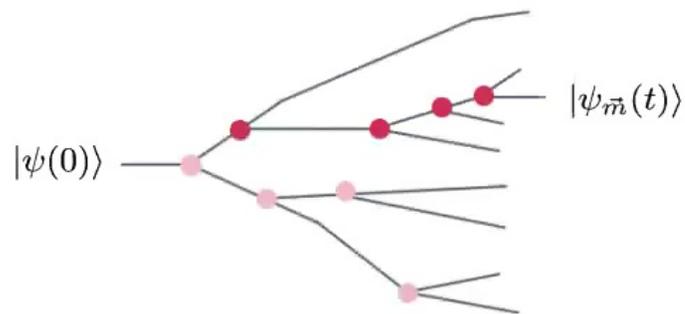
Background

1. Measurement induced phase transitions in quantum circuits

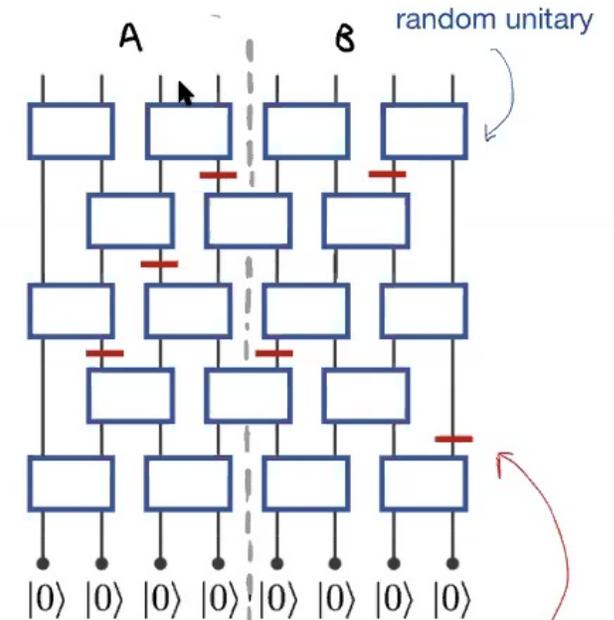
Skinner, Ruhman, Nahum PRX 2019; Li, Chen, Fisher PRB 2018, Chan et. al. PRB 2019, ...

Note: the entanglement in question is that of a pure state undergoing to stochastic evolution due to the measurements. That is, the entanglement conditioned on a sequence of random measurement outcomes

$$\vec{m} = \{m_1, m_2, m_3, \dots\}$$



$$\langle S_A \rangle_u = \langle \sum_{\vec{m}} p_{\vec{m}} S_{A, \vec{m}} \rangle_u$$



Measure qubit with probability p

Project on measurement result:

$$|\psi\rangle \mapsto \frac{\hat{P}_\mu |\psi\rangle}{\sqrt{\langle \psi | \hat{P}_\mu | \psi \rangle}} \text{ with prob. } \langle \hat{P}_\mu \rangle$$



How many measurements does it take to collapse a quantum state?

$$\frac{1}{\sqrt{2}} |\text{cat sitting}\rangle + \frac{1}{\sqrt{2}} |\text{cat running}\rangle$$

Are all entangled states fragile like Schrodinger's cat?

NO! Highly entangled states encode information in nonlocal coefficients:

$$|\Psi\rangle = c_1 \left| \begin{array}{c} \uparrow \uparrow \uparrow \\ \uparrow \uparrow \uparrow \\ \uparrow \uparrow \uparrow \end{array} \right\rangle + c_2 \left| \begin{array}{c} \uparrow \uparrow \uparrow \\ \uparrow \uparrow \downarrow \\ \uparrow \uparrow \downarrow \end{array} \right\rangle + \dots + c_{2^N} \left| \begin{array}{c} \downarrow \downarrow \downarrow \\ \downarrow \downarrow \downarrow \\ \downarrow \downarrow \downarrow \end{array} \right\rangle$$

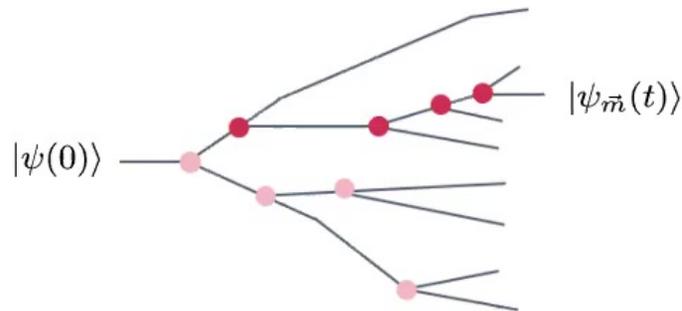
Measuring a few local qubits does not reveal much of this information.

Background

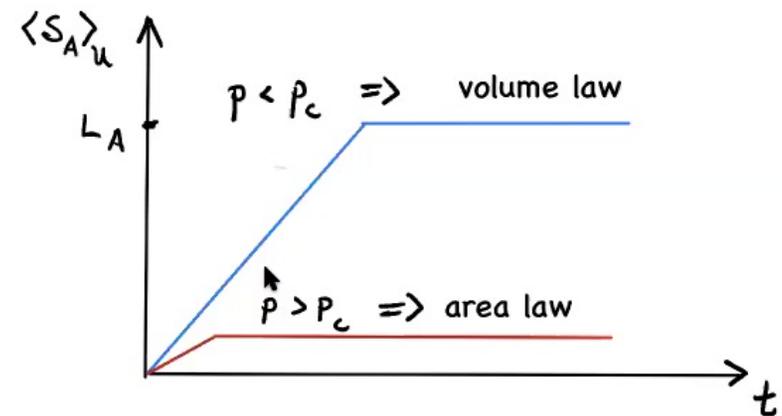
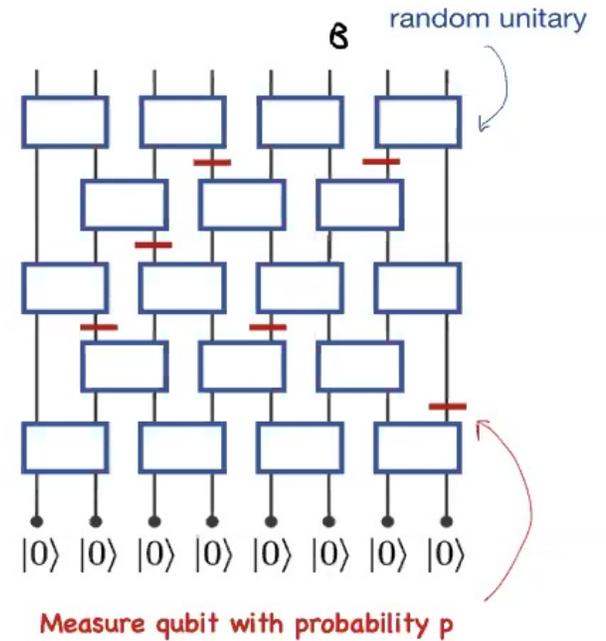
1. Measurement induced phase transitions in quantum circuits

Skinner, Ruhman, Nahum PRX 2019; Li, Chen, Fisher PRB 2018, Chan et. al. PRB 2019, ...

$$\vec{m} = \{m_1, m_2, m_3, \dots\}$$



$$\langle S_A \rangle_u = \left\langle \sum_{\vec{m}} p_{\vec{m}} S_{A, \vec{m}} \right\rangle_u$$



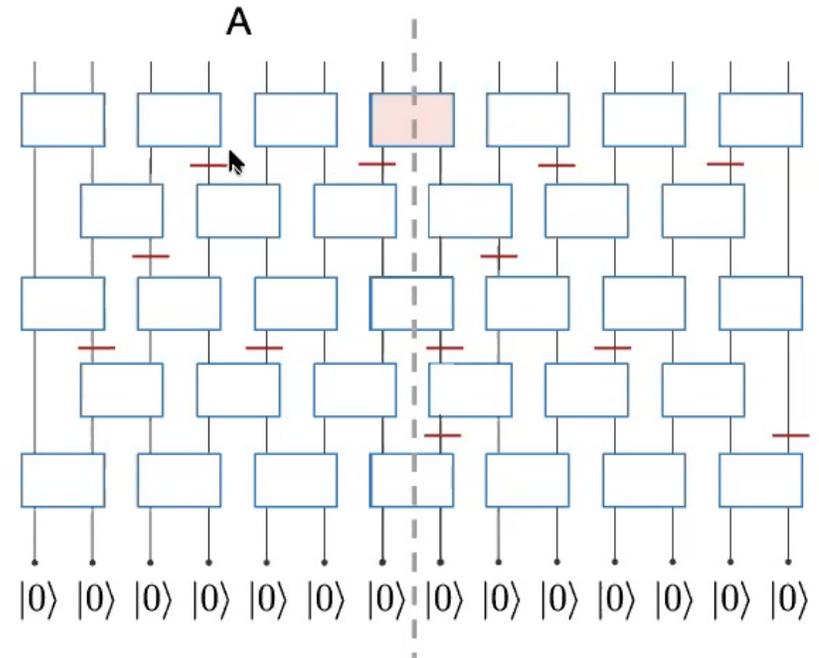
The transition is not a simple competition between entangling unitary gates and disentangling measurements !

A naïve equation for change of the entanglement entropy in one time step:

$$\Delta S_A \stackrel{?}{\sim} \log 2 - N p_{\text{meas}}$$

Entanglement increases by $O(1)$ due to one unitary gate

but can decrease by an extensive amount due to measurements



Is the volume law phase unstable?

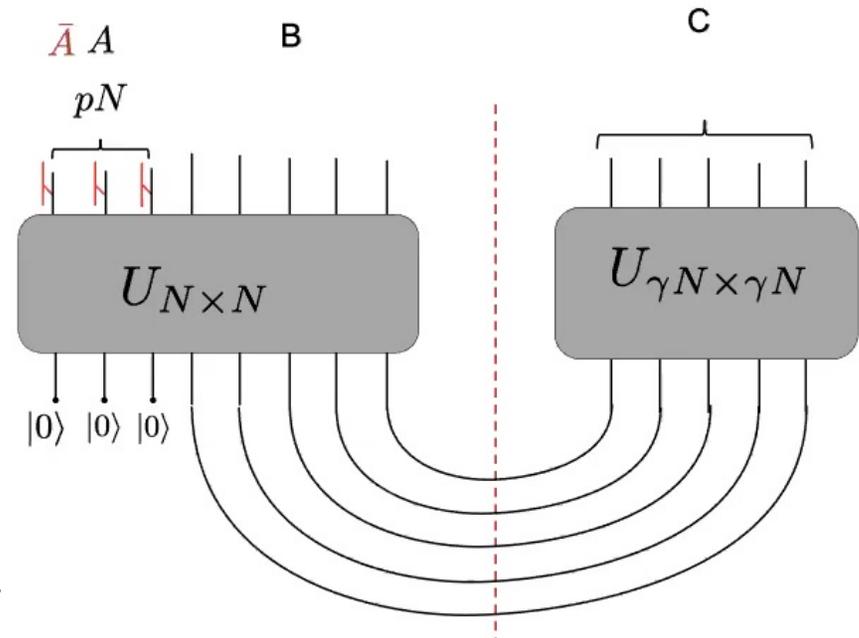
A. Chan *et al*, arXiv:1808.05949v1



Background

2. Understanding the phases in terms of quantum information flow in the circuit.

Do pN measurements necessarily reduce the entanglement by an extensive amount ?



Volume law = information remains hidden from measurement
The circuit has a finite quantum channel capacity.

Natural error correction: quantum information protected by scrambling



Choi, Bao, Qi and EA, PRL 2020; Gullans and Huse PRX 2020

Background

2. Understanding the phases in terms of quantum information flow in the circuit.

Do pN measurements necessarily reduce the entanglement by an extensive amount?

No! Decoupling inequality: if $p < 1 - \gamma$

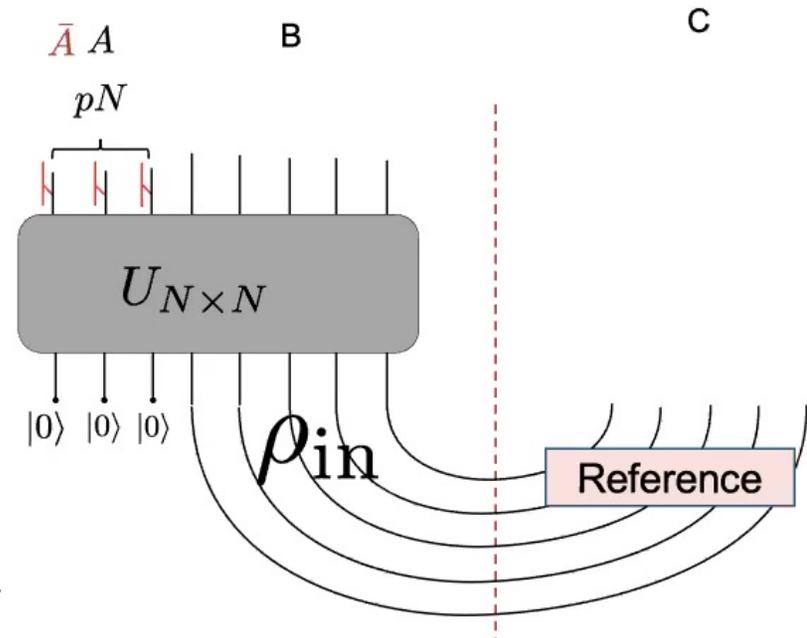
$$\text{i.e. } A + C < \frac{1}{2}(A + \bar{A} + B + C)$$

Then measuring pN qubits does not change S up to corrections exponentially small in N

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Transition in
Quantum Channel Capacity per qubit



Purification
Phase Transition

Choi, Bao, Qi and EA, PRL 2020; Gullans and Huse PRX 2020

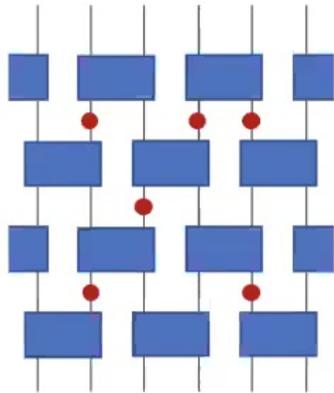


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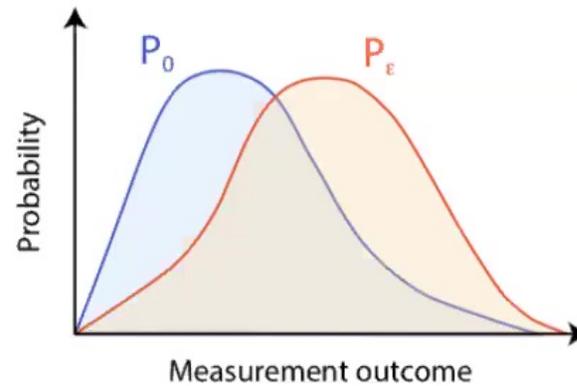
Measurement perspective:

Does the distribution of measurement outcomes betray a small difference in initial state ?



$$|\psi_0\rangle \text{ or } |\psi_\epsilon\rangle = e^{i\epsilon X} |\psi_0\rangle$$

Choi, Bao Qi and EA PRL 2020;

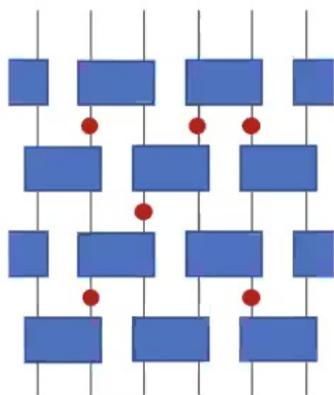


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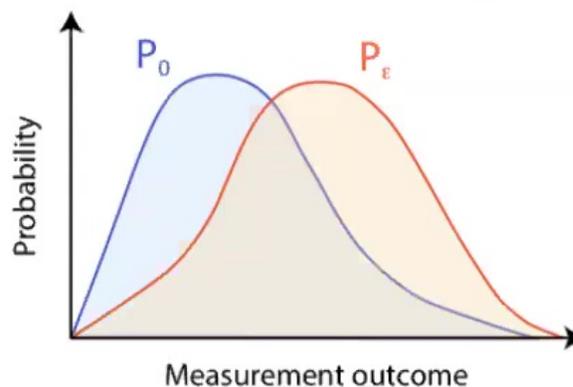
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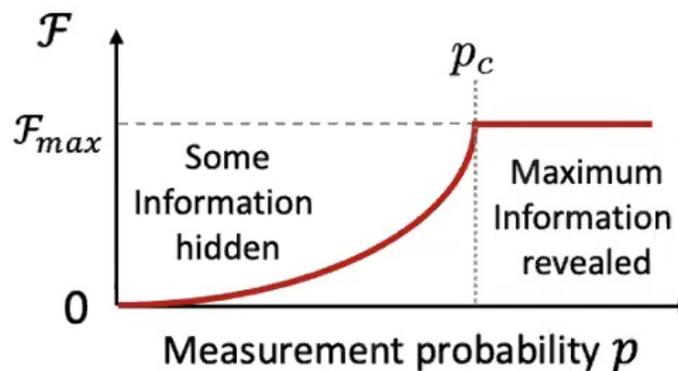
$$|\psi_0\rangle \text{ or } |\psi_\epsilon\rangle = e^{i\epsilon X} |\psi_0\rangle$$

KL divergence between the two distributions:



$$\mathcal{D}_{KL}(P_0||P_\epsilon) = \frac{1}{2} \mathcal{F} \epsilon^2$$

Fisher information



- No need for entanglement measurements
- Only classical record of measurement outcomes

Choi, Bao Qi and EA PRL 2020;

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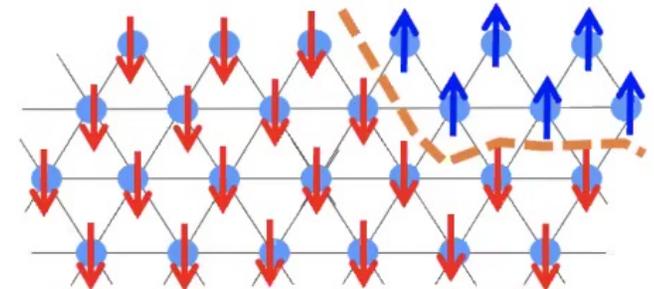
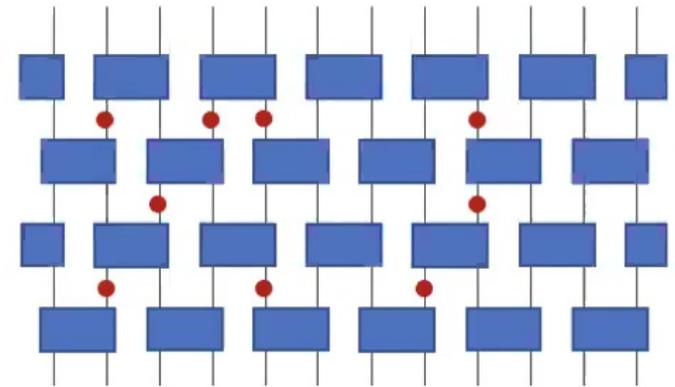
Choi, Bao Qi and EA PRL 2020; Gullans and Huse arXiv:1905.05195

3. Mapping to equilibrium stat-mech problem

Volume law = ferromagnetic phase. Linear domain wall free energy

Bao Choi and EA PRB 2020; Jian, You, Vasseur and Ludwig PRB 2020

$$S^{(2)}(A)$$



Questions for this talk

1. Can we have more phases than just area / volume law ?
2. How does the physical symmetry of the circuit, i.e. unitaries and measurements, determine the phases that can be realized in it?
3. What is the meaning of these states in terms of information flow and scrambling in the circuit? What are their sharp signatures?

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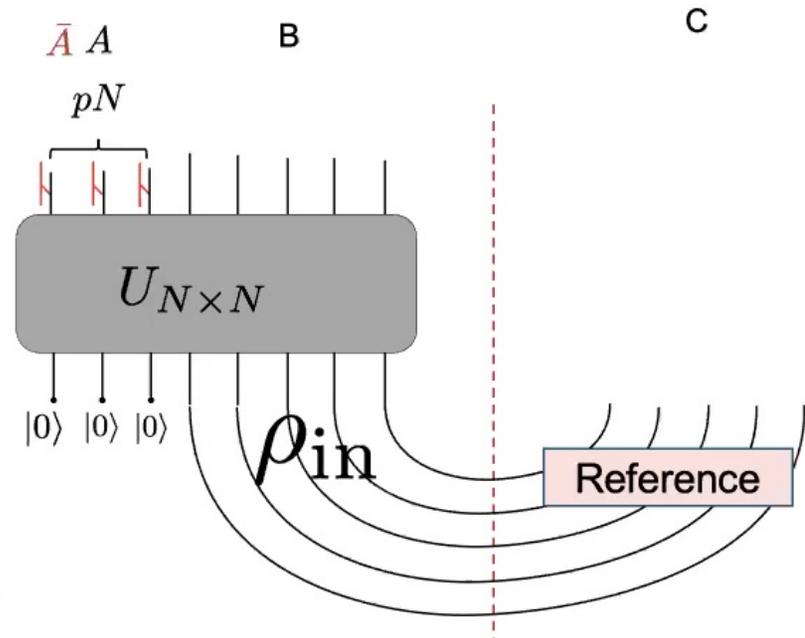
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Transition in
Quantum Channel Capacity per qubit

=

Purification
Phase Transition

Choi, Bao, Qi and EA, PRL 2020; Gullans and Huse PRX 2020

Overview and outline

- Mapping quantum information-dynamics to ground states of an effective Hamiltonian

Classification of phases:

- Example 1: Spin model with Z_2 symmetry
 Z_2 enlarged to D_4 symmetry !
How broken symmetries translate to information
- Example 2: Gaussian fermion circuit
 Z_2 fermion parity enlarged to $U(1)$!
Measurement induced KT transition

Mapping quantum information dynamics to equilibrium stat-mech

Bao Choi and EA PRB 2020; Jian, You, Vasseur and Ludwig PRB 2020

1. Replica trick

Conditional entropy:

$$\begin{aligned}\langle S_A \rangle_u &= \left\langle \sum_{\vec{m}} p_{\vec{m}} S_{A, \vec{m}} \right\rangle_u \\ &= \langle S(A|M) \rangle_u \equiv \langle S_{AM} - S_M \rangle_u\end{aligned}$$



Mapping quantum information dynamics to equilibrium stat-mech

Bao Choi and EA PRB 2020; Jian, You, Vasseur and Ludwig PRB 2020

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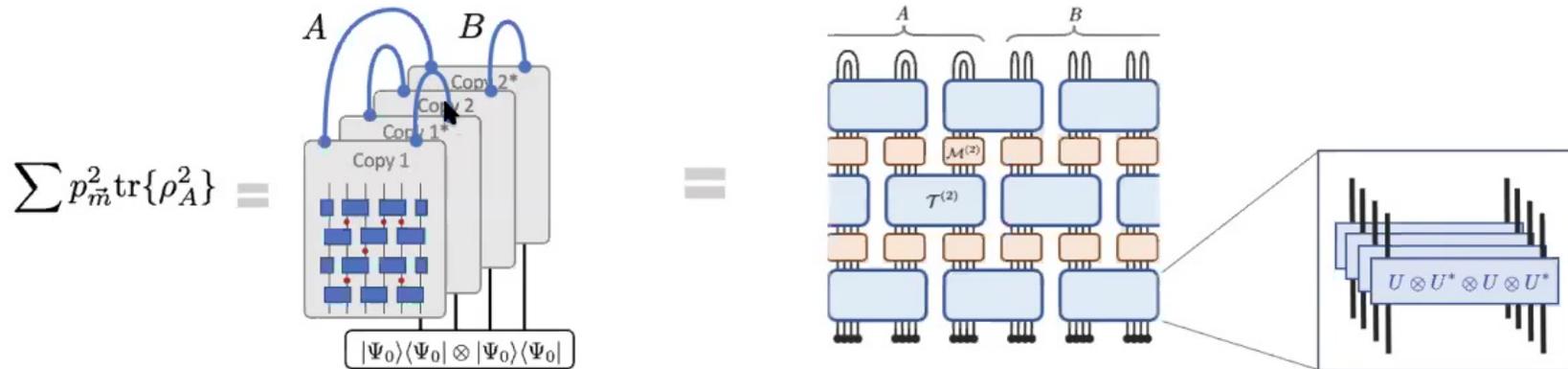
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Renyi conditional entropies:

$$S_A^{(n)} \equiv -\frac{1}{1-n} \log \left[\frac{\langle \sum_m p_m^n \text{tr}(\rho_A^n) \rangle_u}{\langle \sum_m p_m^n \rangle_u} \right] = \langle S^{(n)}(A|M) \rangle_u$$

Can be expressed as contraction of a tensor network

2. Tensor network representation (n=2 example)



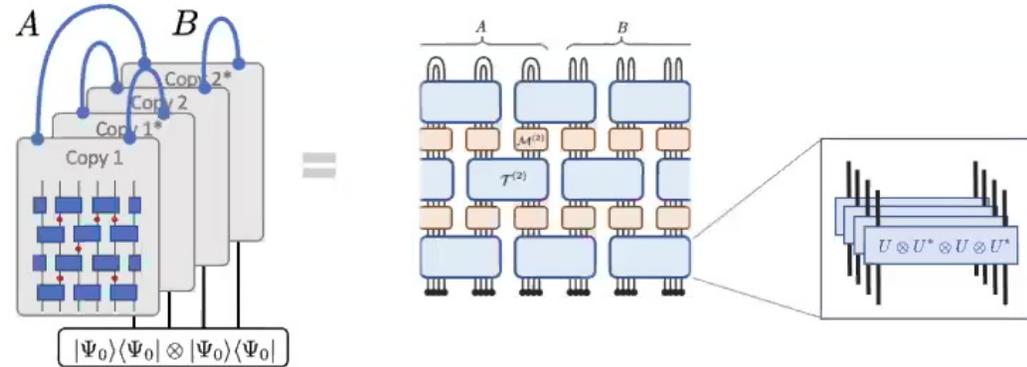
Mapping quantum information dynamics to equilibrium stat-mech

Bao Choi and EA PRB 2020; Jian, You, Vasseur and Ludwig PRB 2020

2. Tensor network representation (n=2 example)

$$S_A^{(2)} \equiv \log \left[\frac{\langle \sum_m p_m^2 \text{tr}(\rho_A^2) \rangle_U}{\langle \sum_m p_m^2 \rangle_U} \right]$$

$$\sum_m p_m^2 \text{tr}\{\rho_A^2\} =$$



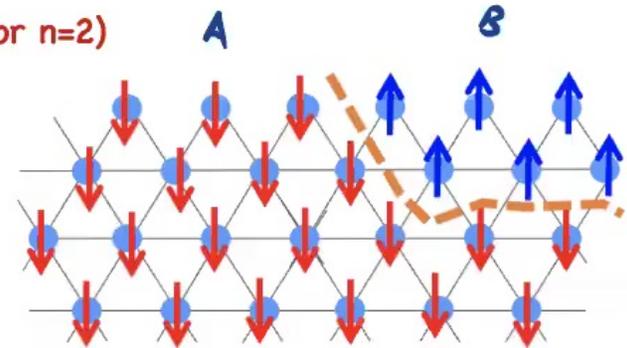
3. Average over Haar random unitary gates: mapping to Ising partition function (for n=2)

Averaging imposes pairing of forward and backward (ket/bra) trajectories on this unitary gate.

➔ Projects the Hilbert space on two states.

$$|\uparrow\rangle = \sum_{a,b} |a, a, b, b\rangle$$

$$|\downarrow\rangle = \sum_{a,b} |b, a, a, b\rangle$$



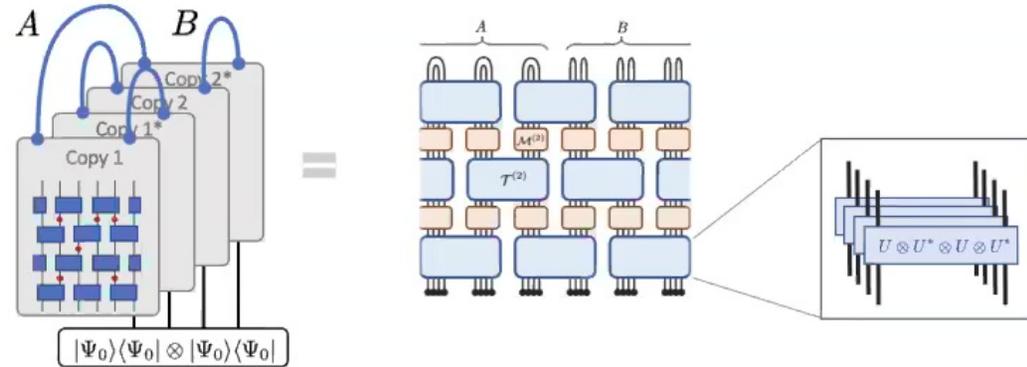
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$$\sum p_m^2 \text{tr}\{\rho_A^2\} =$$



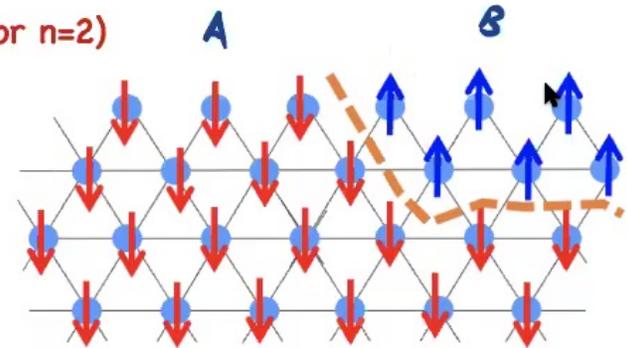
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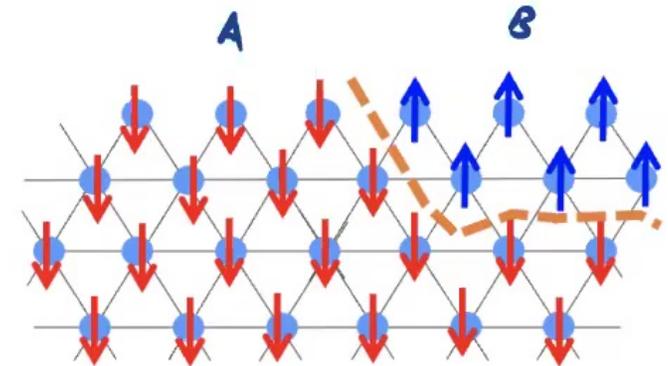
Entanglement entropy \rightarrow free energy of Ising domain wall: $S^{(2)}(A) = F_{dw}^{(2)} - F_0^{(2)}$

Volume law phase = ferromagnet $\Delta F_{dw}^{(2)} \sim L$

Area law phase = paramagnet $\Delta F_{dw}^{(2)} \sim \text{const}$

How the Ising symmetry is related to entanglement

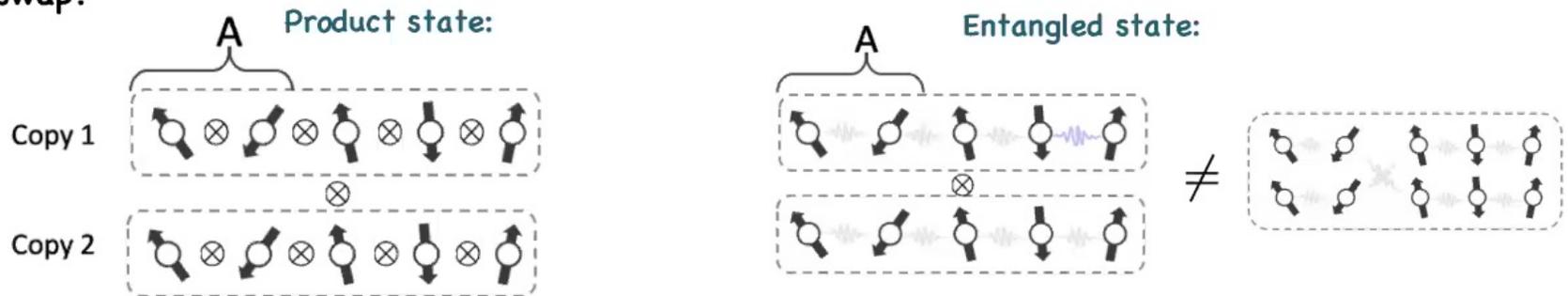
Ising symmetry = Permutation symmetry of **two** replica copies
 $\sigma \in \{e, \text{swap}\} = S_2$



Global swap symmetry:

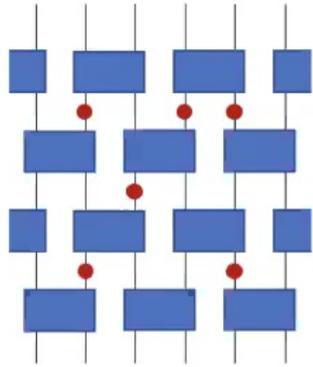
$$|\Psi\rangle = |\psi_{\text{copy 1}}\rangle \otimes |\psi_{\text{copy 2}}\rangle = |\psi_{\text{copy 2}}\rangle \otimes |\psi_{\text{copy 1}}\rangle$$

Subsystem swap?



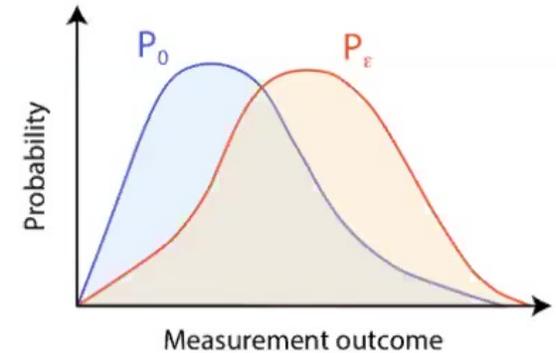
Invariant under sub-system swap

Distinguishability transition in the stat-mech model



Does the distribution of measurement outcomes
Betray a small difference in initial state ?

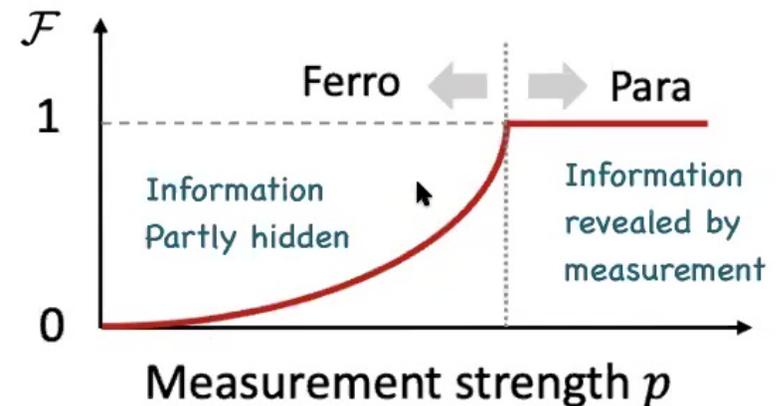
$$|\psi_0\rangle \text{ or } |\psi_\epsilon\rangle = e^{i\epsilon X} |\psi_0\rangle$$



Replica KL divergence:
$$D^{(n)}(\epsilon) \equiv \frac{1}{1-n} \log \left(\left\langle \sum_{\vec{m}} p_{\vec{m},0} p_{\vec{m},\epsilon}^{n-1} \right\rangle_U \right) - \frac{1}{1-n} \log \left(\left\langle \sum_{\vec{m}} p_{\vec{m},0}^n \right\rangle_U \right) = \frac{1}{2} \mathcal{F}^{(n)} \epsilon^2$$

Fisher information is related to the
spontaneous magnetization on perturbed site:

$$\mathcal{F}^{(2)} = 1 - \langle \sigma^z \rangle_{\text{bottom}}$$

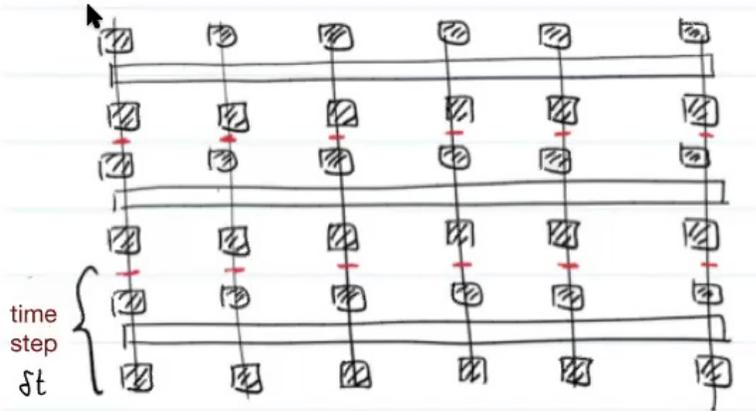


Continuous time models: mapping to quantum ground state problems

Single qubit gates: $\boxed{\text{hatched}} = e^{-i\theta_j \hat{Z}_j} \otimes V^* \otimes V \otimes V^*$

Entangling gates: $\boxed{\text{white}} = e^{-i\delta t J_{ij}^{\alpha\beta} \hat{\sigma}_i^{\alpha} \hat{\sigma}_j^{\beta}} \otimes U^* \otimes U \otimes U^*$
 $\langle (J_{ij}^{\alpha\beta})^2 \rangle = f_{\alpha\beta}(x_i - x_j)$

Measurements: $\dagger = \mathbb{1} - \underbrace{(\delta t \Gamma)^2}_{p^2} \sum_{\sigma} \hat{P}_{\sigma} \otimes \hat{P}_{\sigma} \otimes \hat{P}_{\sigma} \otimes \hat{P}_{\sigma}$



Continuous time models: mapping to quantum ground state problems

Single qubit gates: $\boxed{\text{hatched}} = e^{-i\theta_j \hat{Z}_j} \otimes V^* \otimes V \otimes V^*$

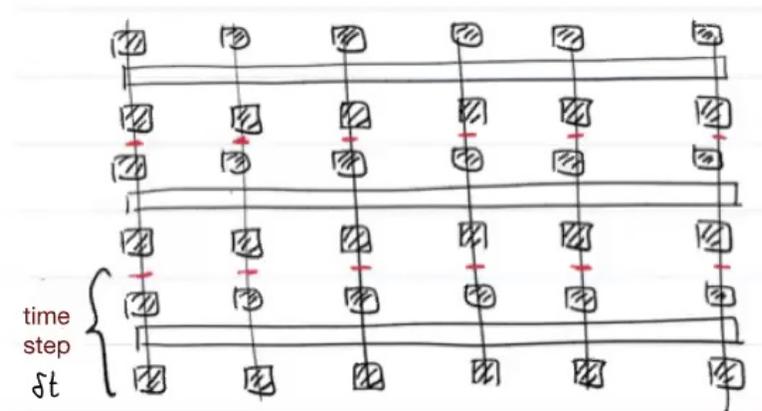
Entangling gates: $\boxed{\text{white}} = e^{-i\delta t J_{ij}^* \hat{\sigma}_i^* \hat{\sigma}_j^*} \otimes U^* \otimes U \otimes U^*$
 $= U$
 $\langle (J_{ij}^* \hat{\sigma}_i^* \hat{\sigma}_j^*)^2 \rangle = f_{\alpha\beta}(x_i - x_j)$

Measurements: $\boxed{\text{red cross}} = \mathbb{1} - \underbrace{(\delta t \Gamma)^2}_{p^2} \sum_{\sigma} \hat{P}_{\sigma} \otimes \hat{P}_{\sigma} \otimes \hat{P}_{\sigma} \otimes \hat{P}_{\sigma}$

Averaging the single site unitaries over $\theta_j \in [0, 2\pi]$ projects onto a reduced Hilbert space

$|m, s\rangle \quad m = -1, 0, 1 \quad (s = \pm 1)$

(s turns out to be fixed in the dynamics, leaving us with a local spin-1 Hilbert space)



Continuous time models: mapping to quantum ground state problems

Single qubit gates: $\text{[Hatched Box]} = e^{-i\theta_j \hat{Z}_j} \otimes V^* \otimes V \otimes V^*$

Entangling gates: $\text{[Crossed Box]} = e^{-i\delta t J_{ij}^{\alpha\beta} \hat{\sigma}_i^\alpha \hat{\sigma}_j^\beta} \otimes U^* \otimes U \otimes U^*$
 $\langle (J_{ij}^{\alpha\beta})^2 \rangle = f_{\alpha\beta}(x_i - x_j)$

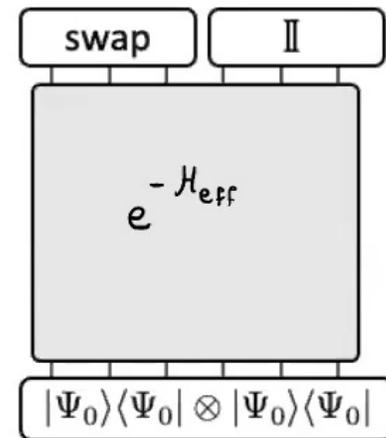
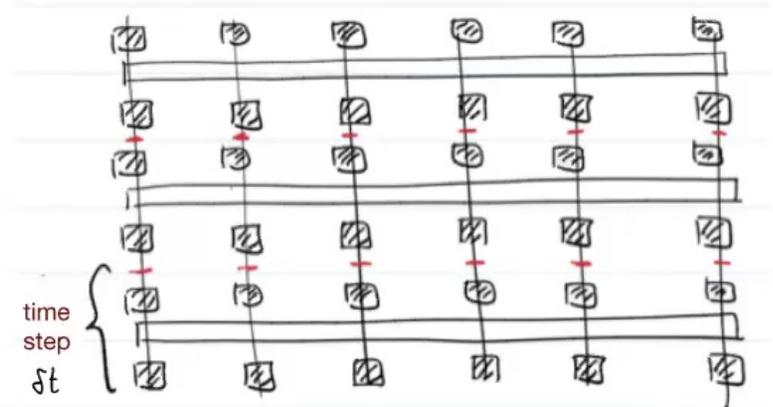
Measurements: $\text{[Red Plus Sign]} = \mathbb{1} - \underbrace{(\delta t \Gamma)^2}_{p^2} \sum_{\sigma} \hat{P}_\sigma \otimes \hat{P}_\sigma \otimes \hat{P}_\sigma \otimes \hat{P}_\sigma$

Averaging over the other unitaries and taking $\delta t \rightarrow 0$ we obtain effective imaginary time evolution with a spin-1 Hamiltonian H_{eff}

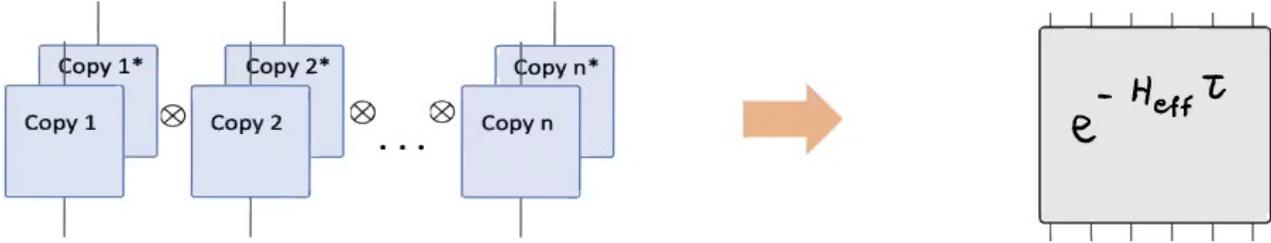
Most quantities of interest are obtained as an overlap with an appropriate boundary state.

$$e^{-S_A^{(2)}} = \frac{\langle \mathbb{I} | (\text{SWAP})_A e^{-H_{\text{eff}} \tau} | \Psi_{\text{in}} \rangle}{\langle \mathbb{I} | e^{-H_{\text{eff}} \tau} | \Psi_{\text{in}} \rangle} \approx \frac{\langle \mathbb{I} | (\text{SWAP})_A | \Psi_{\text{gs}} \rangle}{\langle \mathbb{I} | \Psi_{\text{gs}} \rangle}$$

$$|\mathbb{I}\rangle = (|11\rangle + |10\rangle)^{\otimes N}$$



Symmetry of H_{eff}



Internal symmetries

$$G$$

Permutation symmetry

$$S_n$$



$$[(G^{\otimes n} \times S_n) \times (G^{\otimes n} \times S_n)] \times \mathbb{Z}_2$$

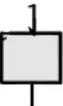
Larger than the original symmetry

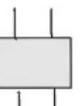
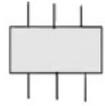


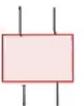
Example: circuit with Z_2 symmetry

Symmetry generator: $\mathcal{P} = \prod_{i=1}^N \hat{Z}_i$

Possible circuit elements:

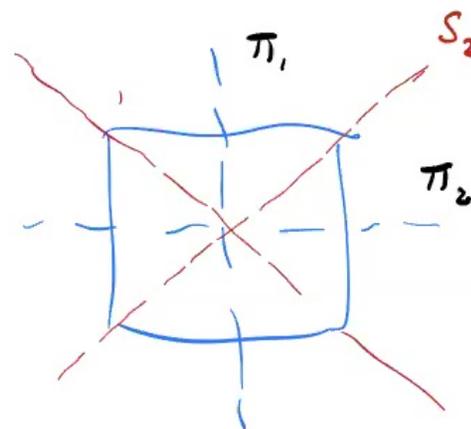
 = $e^{-h_i \hat{Z}_i}$

 = $e^{-iK_{ij} \hat{Z}_i \hat{Z}_j}$,  , ...

 = $e^{-iJ_{ij} \hat{X}_i \hat{X}_j}$

 = measure \hat{Z}_i

 = measure $X_i X_j$



- Swap symmetry: S_2
 - Parity symmetry: $Z_2 \times Z_2$
- $(Z_2 \times Z_2) \times S_2 = D_4$

- Full symmetry of dynamics: $(D_4 \times D_4) \times Z_2^*$

But reduced to just D_4 in the projected Hilbert space



Phases classified by symmetry of Heff

$$(Z_2 \times Z_2) \rtimes S_2 = D_4$$

	Area Law	Volume Law
Trivial/SPT	D_4	$Z_2 \times Z_2$
Broken symmetry	Z_2	Z_4
		Z_2, Z_2, Z_2
		\emptyset Fully broken

Detecting broken Z2 symmetry in the circuit

Measuring Z_2 parity variance on a segment:

$$\Pi_L \equiv \left\langle \sum_m p_m \langle \mathcal{P}_L \rangle^2 \right\rangle_U \sim \Pi_L^{(2)} \equiv \frac{\left\langle \sum_m p_m^2 \langle \mathcal{P}_L \rangle^2 \right\rangle_U}{\left\langle \sum_m p_m^2 \right\rangle_U}$$

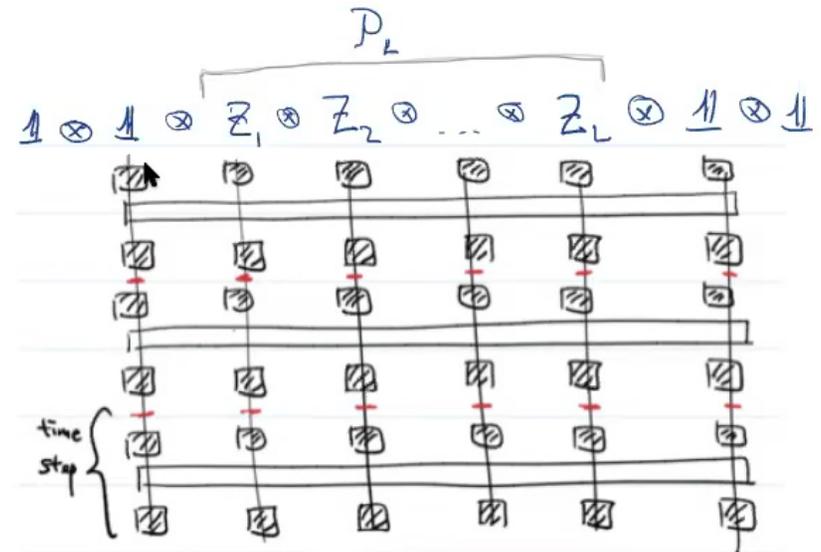
Symmetric phase: $\Pi_L \xrightarrow{L \rightarrow \infty} \text{const}$

Definite parity. Measurements reveal the parity

Broken symmetry phase: $\Pi_L \sim e^{-L/\xi}$

Indefinite parity. Information on parity is hidden from measurements

$\log(\Pi_L) \sim Z_2$ domain wall free energy



Detecting broken Z2 symmetry in the circuit

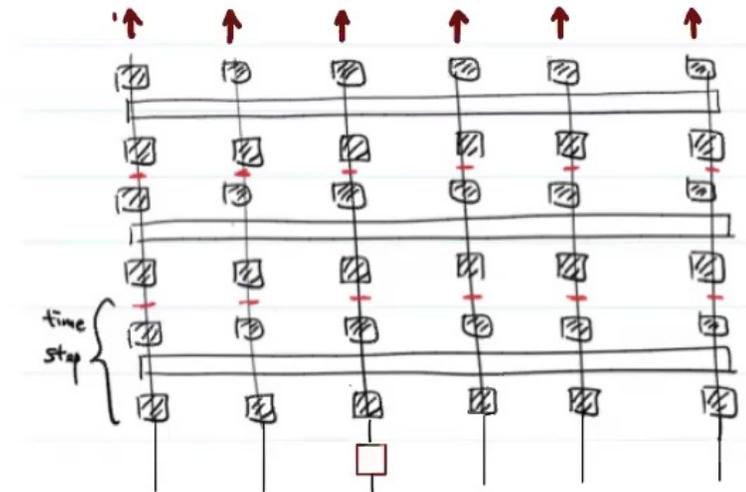
Measuring the Z2 order parameter:

Response of the measurement results to a symmetry breaking perturbation at $t=0$

$$\square = e^{i\theta X_i}$$

Difference between measurement entropies with and without the perturbation:

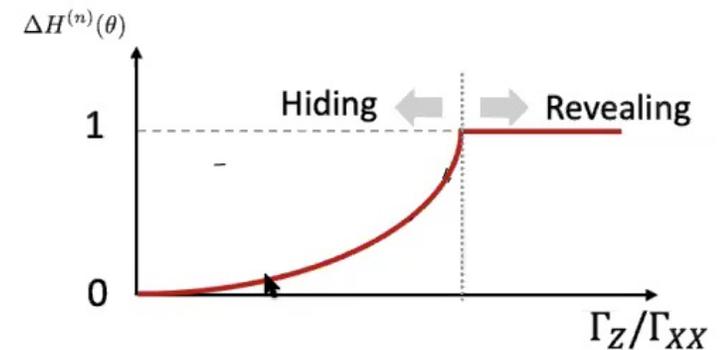
$$\Delta H^{(n)} = \frac{1}{1-n} \log \left(\frac{\langle \sum_m P_{m,\theta}^n \rangle_{\mathcal{U}}}{\langle \sum_m P_{m,0}^n \rangle_{\mathcal{U}}} \right) \quad \Delta H^{(2)} = \frac{\theta^2}{2} \left(1 - \langle M^{(2)}(\tau=0) \rangle \right)$$



Measures the magnetization at bottom boundary. Top boundary breaks the Z2 symmetry

Symmetric phase: Vanishing magnetization ΔH is maximal. Measurement reveal information on parity

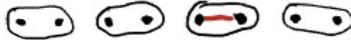
Broken symmetry phase: Magnetization reduces ΔH . Some information on initial parity is not revealed by the measurements



Free Majorana fermion circuit

Hopping:  = $e^{\delta t J_i \gamma_{i-1} \bar{\gamma}_i} \otimes U^* \otimes U \otimes U^*$

On site parity measurement:

 = measure $i \bar{\gamma}_j \gamma_j$ 

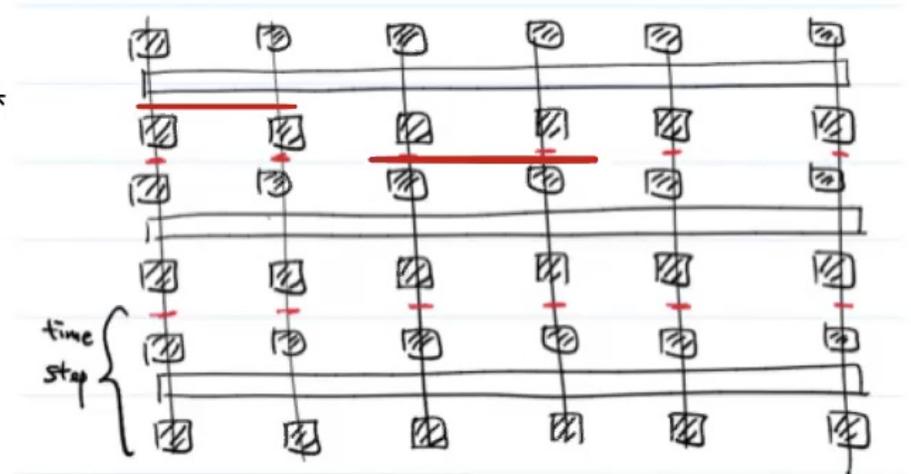
Bond parity measurement:

 = measure $i \gamma_{j-1} \bar{\gamma}_j$ 

Single site gate:

 = $e^{-\theta_j \bar{\gamma}_j \gamma_j} \otimes V^* \otimes V \otimes V^*$

Symmetry: Z2 fermion parity



Free Majorana fermion circuit

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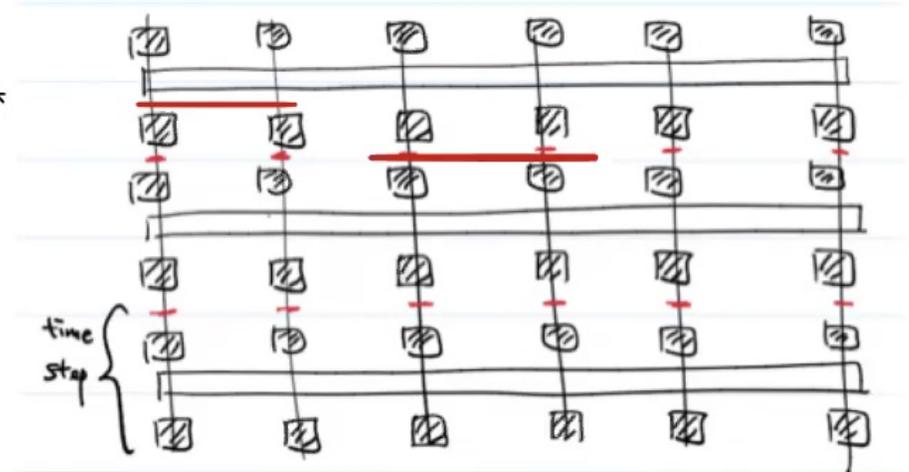
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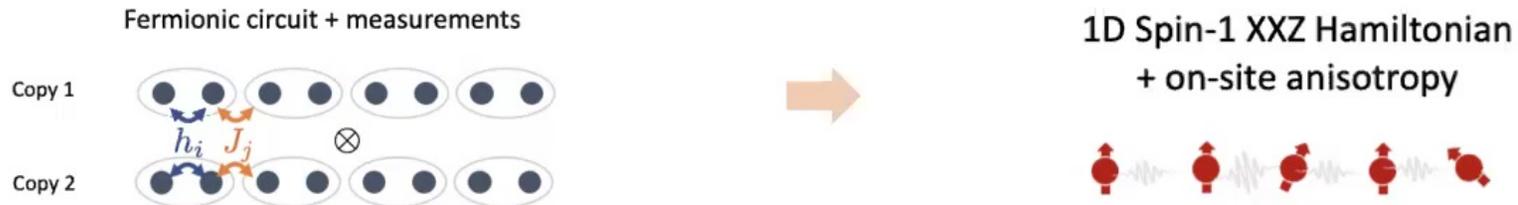
Symmetry: Z_2 fermion parity



Previous results:

1. Absence of volume law phase for any non vanishing measurement strength
X. Cao, A. Tilloy and A. De Luca, SciPost 2019
2. Numerical simulations indicated transition from a phase with $\log(L)$ entanglement to area law. This was done with complex fermions
O. Alberton, M. Buchhold, S. Diehl, arXiv:2005.09722

The effective spin-1 model



$$\mathcal{H}_Q = \sum_{j=1}^L (-J_b - \Gamma_b) (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + (-J_b + \Gamma_b) S_j^z S_{j+1}^z + \Gamma_s (S_j^z)^2$$

Hopping
Bond meas.
Site meas.

- $U(1)$ symmetry!
(From replica symmetry and Gaussianity)

Two identical (quadratic) Majorana chains

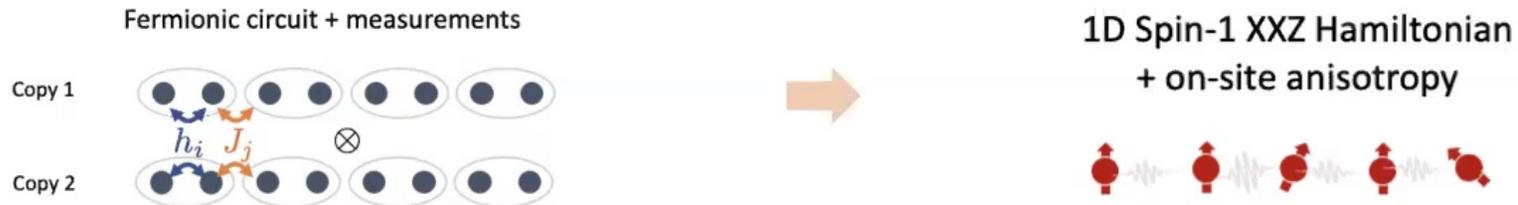
$$c_j^\dagger = \gamma_{j,1} - i \gamma_{j,2} \quad \bar{c}_j^\dagger = \bar{\gamma}_{j,1} - i \bar{\gamma}_{j,2}$$

The conserved charge:

$$\mathcal{N} = \sum_i (c_i^\dagger c_i + \bar{c}_i^\dagger \bar{c}_i)$$



The effective spin-1 model



$$\mathcal{H}_Q = \sum_{j=1}^L (-J_b - \Gamma_b) (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + (-J_b + \Gamma_b) S_j^z S_{j+1}^z + \Gamma_s (S_j^z)^2$$

↙
↖
↗

Hopping
Bond meas.
Site meas.

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Two identical (quadratic) Majorana chains

$$c_j^\dagger = \gamma_{j,1} - i \gamma_{j,2} \quad \bar{c}_j^\dagger = \bar{\gamma}_{j,1} - i \bar{\gamma}_{j,2}$$

The conserved charge: $\mathcal{N} = \sum_i (c_i^\dagger c_i + \bar{c}_i^\dagger \bar{c}_i)$

- $SU(2)$ symmetry when no measurement
(Replica + time reversal symmetries)

Two identical chains + time reversed chains
= sinfull fermions



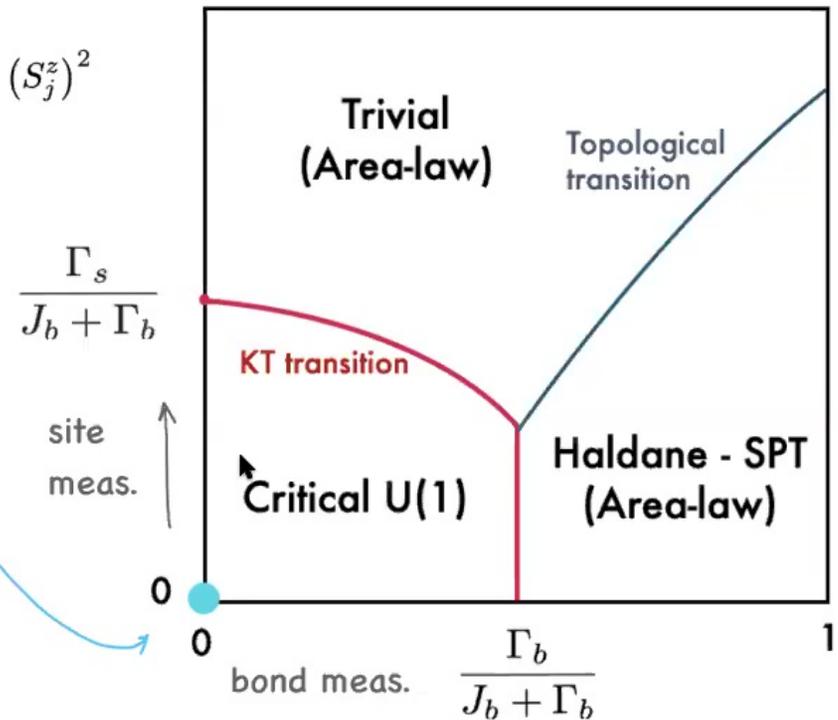
Phase diagram

$$\mathcal{H}_Q = \sum_{j=1}^L (-J_b - \Gamma_b) (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + (-J_b + \Gamma_b) S_j^z S_{j+1}^z + \Gamma_s (S_j^z)^2$$

↑ hopping
↑ bond meas.
↑ site meas.

No measurements = su(2) symmetric ferromagnet
 ⇒ volume law

Any non-vanishing measurement rate breaks the su(2) symmetry down to U(1) destroying the order.
 ⇒ No volume law phase.



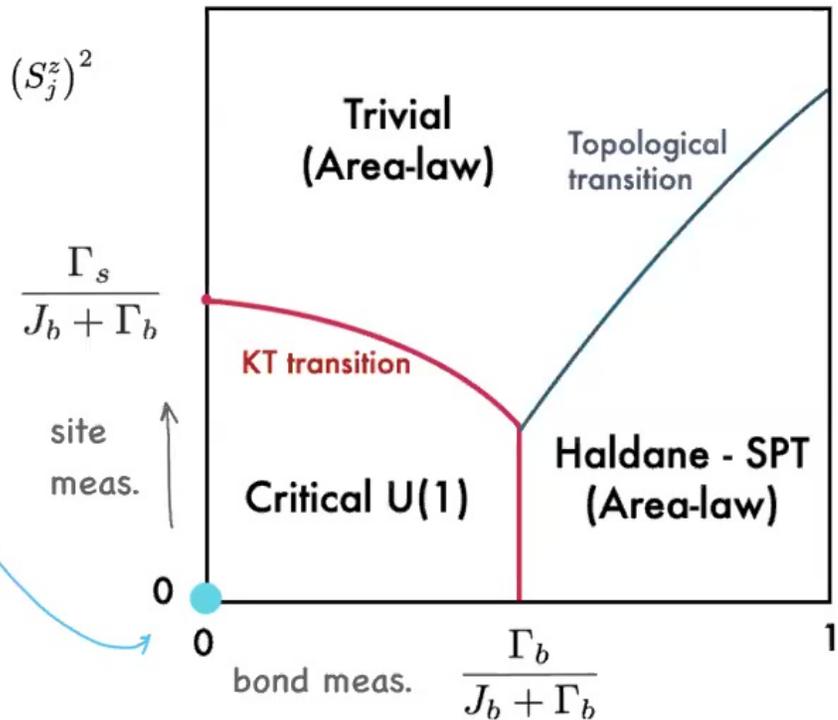
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↑ site meas.

No measurements = su(2) symmetric ferromagnet
 ⇒ volume law

Any non-vanishing measurement rate breaks the su(2) symmetry down to U(1) destroying the order.
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This explains the critical phase with $S \sim \log(L)$ observed in recent numerics (Alberton, Buchhold, Diehl arXiv:2005.09722).
 But contrary to what was postulated there, we see this doesn't require charge conservation. The U(1) symmetry is emergent.

The topological transition is closely related to that discussed for a measurement only model by Nahum and Skinner arXiv:1911.11169



Signatures of the phases

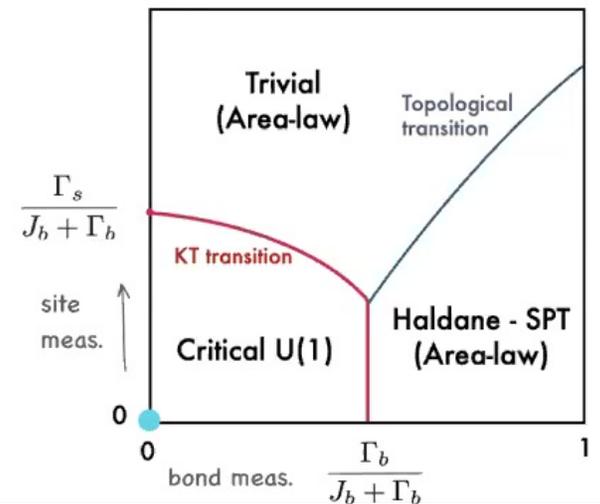
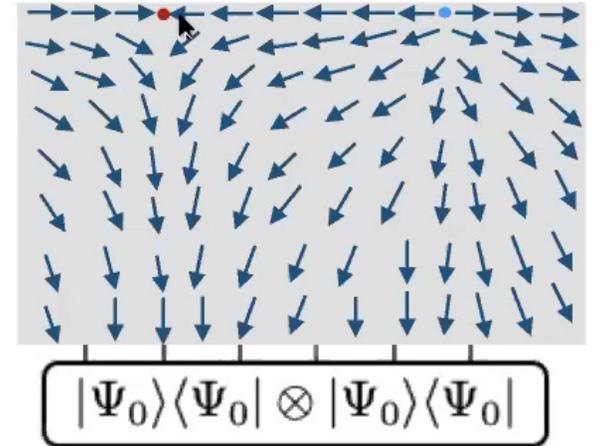
Parity variance on a segment:

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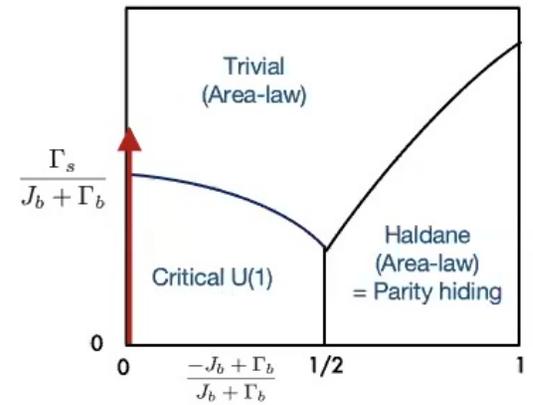
Creates vortex anti vortex pair on the boundary:

$$\Pi_L^{(2)} \rightarrow \prod_{j \in L} e^{i\pi S_j^z} \rightarrow e^{i\hat{\phi}(x_0+L)} e^{-i\hat{\phi}(x_0)}$$

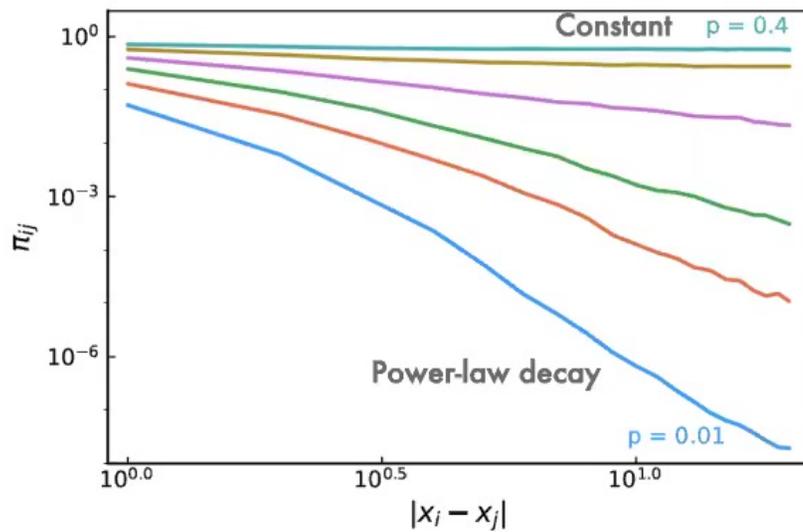
$$\frac{1}{\langle I | \Psi_0 \rangle} \langle I | e^{i\hat{\phi}(x_0+L)} e^{-i\hat{\phi}(x_0)} | \Psi_0 \rangle \rightarrow \begin{cases} \left(\frac{1}{x}\right)^{K/2} & \text{Critical phase} \\ \text{const} & \text{Trivial area law} \\ e^{-x/\xi} & \text{SPT area law} \end{cases}$$



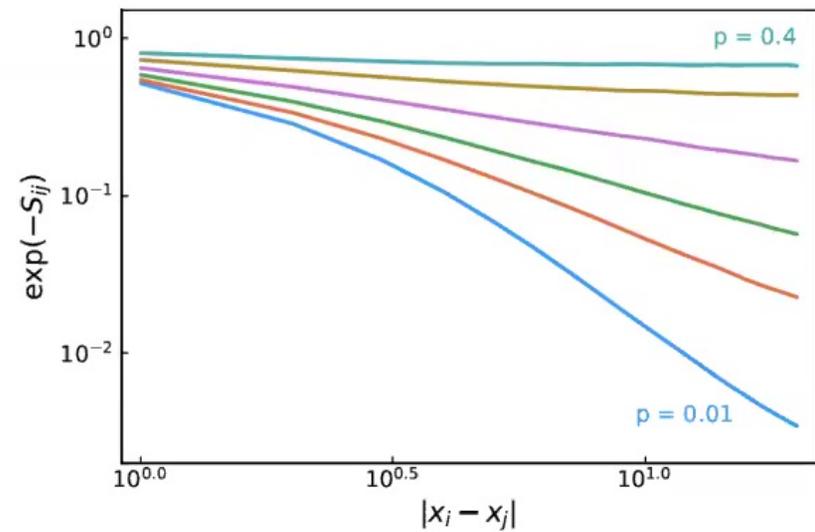
Numerical simulation of gaussian Majorana model



Parity correlation

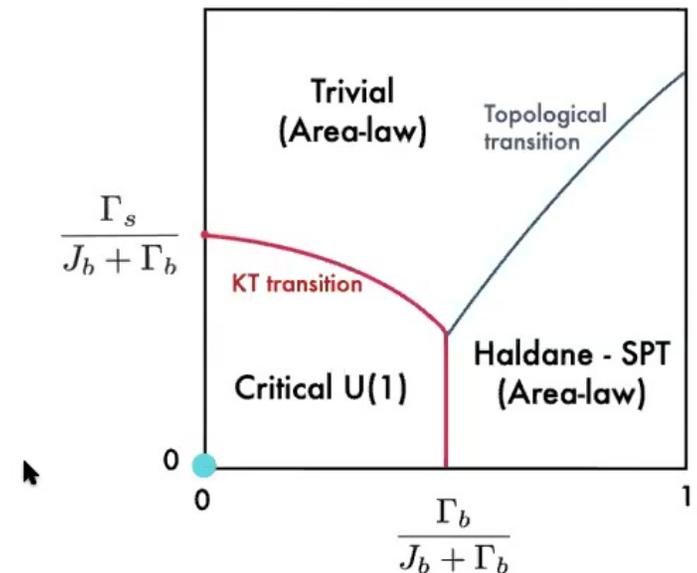
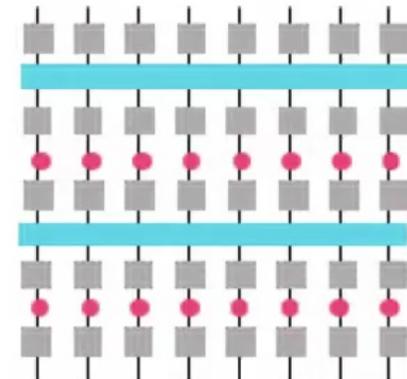


Entanglement Entropy



Summary and conclusions

- Mapping quantum information-dynamics to ground states of an effective Hamiltonian
- Phases classified according to the larger symmetry of the effective Hamiltonian (Broken symmetry states, trivial, SPT)
- Example 1: Spin model with Z_2 symmetry
 Z_2 enlarged to D_4 symmetry !
 How broken symmetries translate to information
- Example 2: Gaussian fermion circuit
 Z_2 fermion parity enlarged to $U(1)$!
 Measurement induced KT transition



Questions and outlook

- What is the phase structure for general replica index n ?
Which phases survive in the replica limit?
- Can the time-reversal-like (transpose) symmetry be spontaneously broken?
Implications for negativity?
- Analogous phase transitions in quantum circuits that show signatures in conventional observables (do not require post-selection of measurements)?
- Relation and application to building fault tolerant circuits?

Summary and conclusions

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