Title: Measurement of quantum fields in curved spacetimes

Speakers: Chris Fewster

Series: Colloquium

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Abstract: A standard account of the measurement chain in quantum mechanics involves a probe (itself a quantum system) coupled temporarily to the system of interest. Once the coupling is removed, the probe is measured and the results are interpreted as the measurement of a system observable. Measurement schemes of this type have been studied extensively in Quantum Measurement Theory, but they are rarely discussed in the context of quantum fields and still less on curved spacetimes.

In this talk I will describe how measurement schemes may be formulated for quantum fields on curved spacetime within the general setting of algebraic QFT. This allows the discussion of the localisation and properties of the system observable induced by a probe measurement, and the way in which a system state can be updated thereafter. The framework is local and fully covariant, allowing the consistent description of measurements made in spacelike separated regions. Furthermore, specific models can be given in which the framework may be exemplified by concrete calculations.

I will also explain how this framework can shed light on an old problem due to Sorkin concerning "impossible measurements" in which measurement apparently conflicts with causality.

The talk is based on work with Rainer Verch [Leipzig], (Comm. Math. Phys. 378, 851â€“889(2020), arXiv:1810.06512; see also arXiv:1904.06944 for a summary) and a recent preprint arXiv:2003.04660 with Henning Bostelmann and Maximilian H. Ruep [York].

Measurement of quantum fields in curved spacetimes

CJ Fewster
University of York

Perimeter Institute Colloquium
October 2020

Comm. Math. Phys. **378** 851–889 (2020) arXiv:1810.06512 - with Rainer Verch
Short summary arXiv:1904.06944
arXiv:2003.04660 with Henning Bostelmann and Maximilian Ruep

A gap in the literature

Measurement theory in quantum mechanics has a long and controversial history. Nonetheless:

- simple rules are taught to students
- measurement chain is analysed in quantum measurement theory.

Much less is said in quantum field theory.

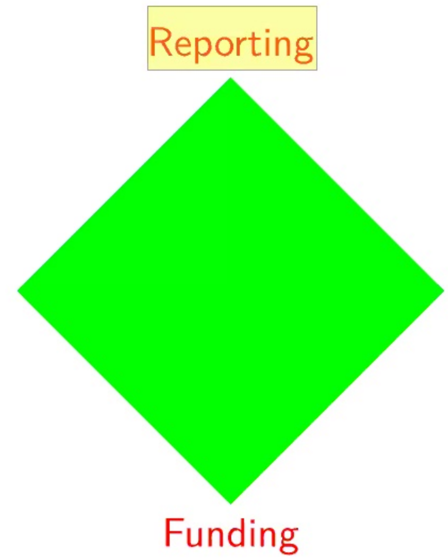
- Lecture courses and texts are silent.
- QMT rarely discussed for QFT; still less in curved spacetimes.
- Algebraic QFT is founded on the idea of local observables, but little discussion of how they are actually measured.

In this talk...

- Analyse the measurement chain in QFT
- Provide a general operational framework for measurement
- Covariant; applies in curved as well as flat spacetime
- Passes consistency tests
- Can be used for calculation

Relativity, quantum theory and measurement – it's complicated

$c < \infty$ measurements occupy bounded spacetime regions



Relativity, quantum theory and measurement – it's complicated

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$\hbar > 0$ ...but are not performed at points

$$\Delta E \Delta t \gtrsim \hbar$$

Relativity, quantum theory and measurement – it's complicated

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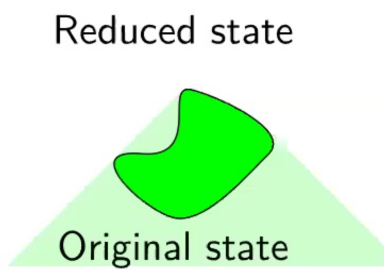
Lorentz invariance	no preferred frame no instantaneous collapse at constant t



Hellwig & Kraus

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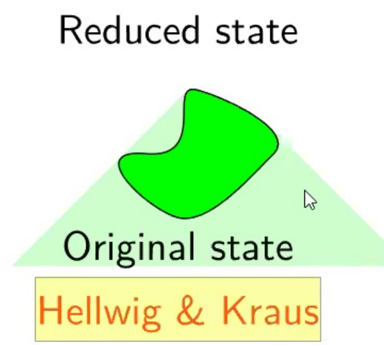

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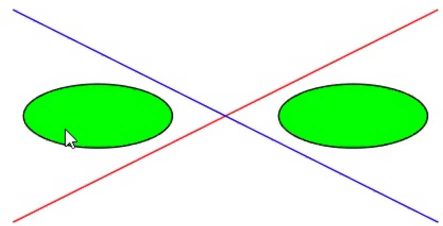

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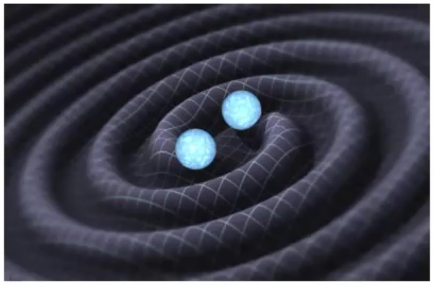
Curved spacetime	lack of symmetry, nontrivial topology...



Ligo

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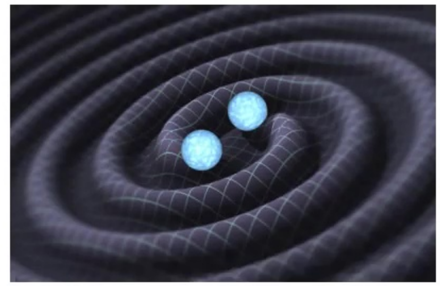
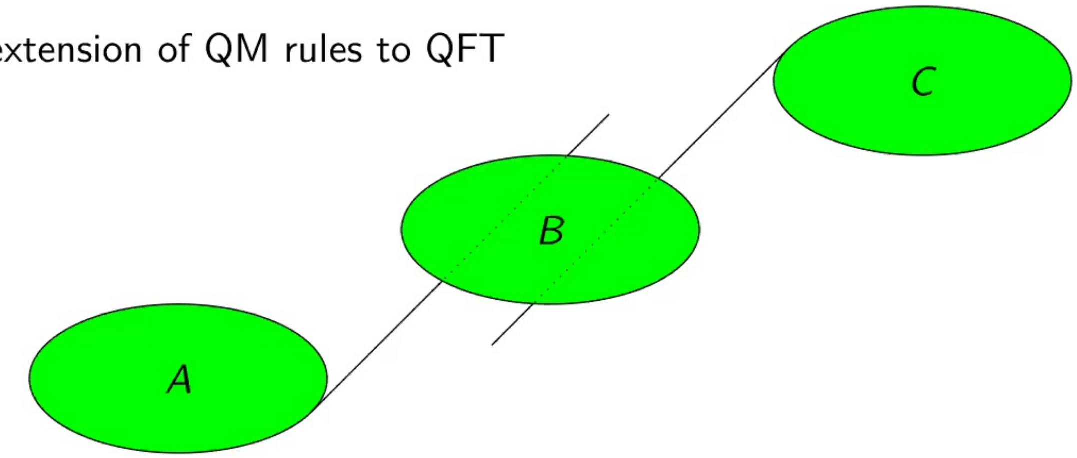


Ligo

Extension of QM measurement rules to QFT is nontrivial and risks pathology

Impossible measurements Sorkin 1993

'By hand' extension of QM rules to QFT

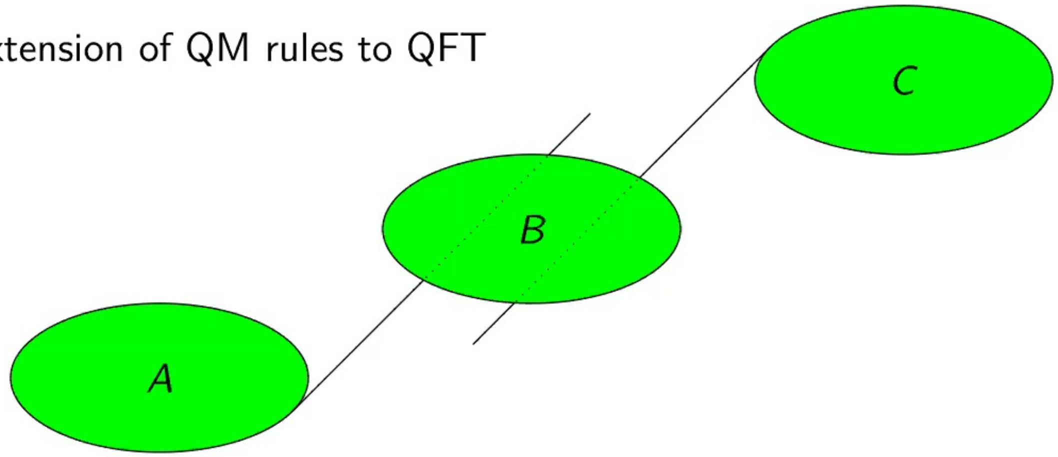


Claim: nonselective measurement of a typical observable B allows C to determine whether A has conducted a measurement – superluminal communication.

Presumably, therefore, B represents an impossible measurement.

Impossible measurements Sorkin 1993

'By hand' extension of QM rules to QFT



"[I]t becomes a priori unclear, for quantum field theory, which observables can be measured consistently with causality and which can't.
This would seem to deprive [QFT] of any definite measurement theory, leaving the issue of what can actually be measured to (at best) a case-by-case analysis"
See e.g., Borsten, Jubb, Kells (2019) for such an analysis.

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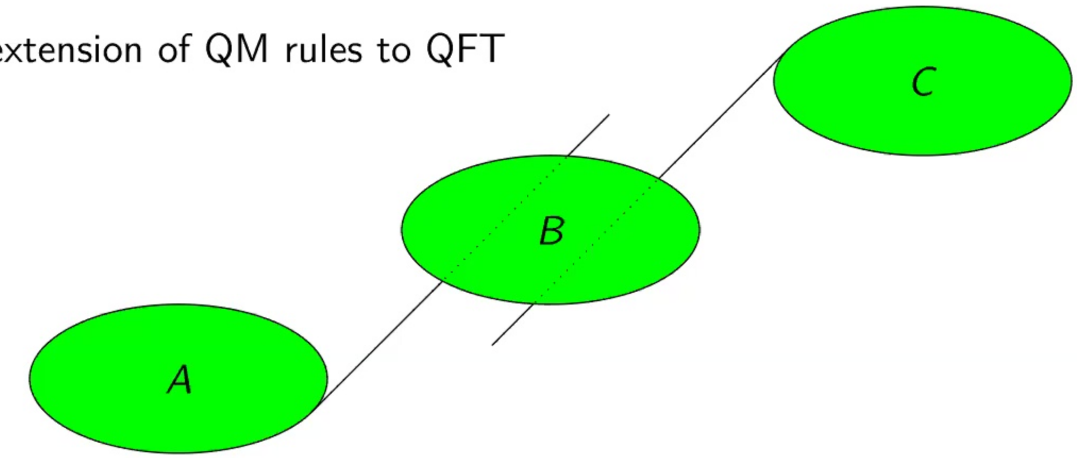

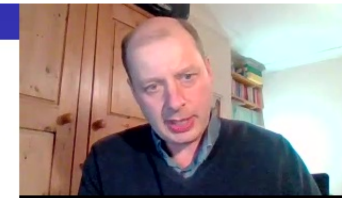

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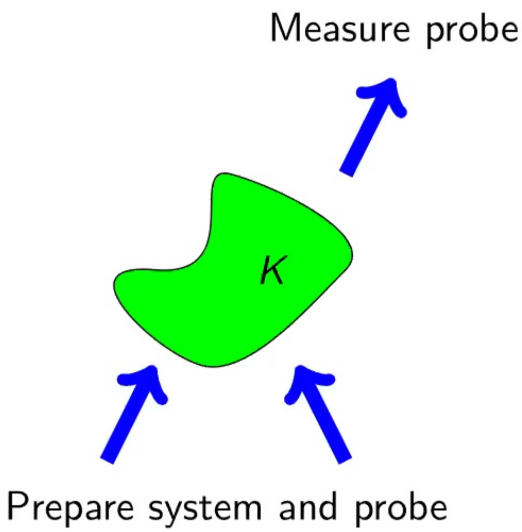
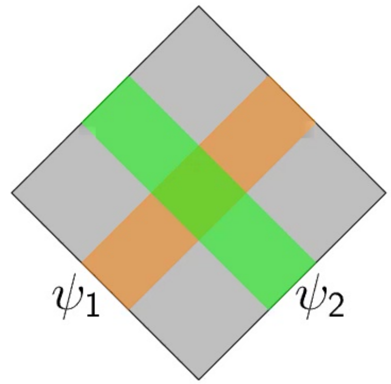
Operational approach CJF & Verch, 2018

Instead of constructing rules for QFT *de novo*, apply a systematic approach by modelling the measurement process.

Operational approach CJF & Verch, 2018

A QFT (system) is coupled to another QFT (probe) in a compact spacetime region K (a proxy for the experimental design). The probe is measured elsewhere.





$\mathcal{L}_{\text{int}} = \psi_1\psi_2\varphi_1\varphi_2$ provides a tunable localised coupling between φ_1 and φ_2.

Operational approach CJF & Verch, 2018

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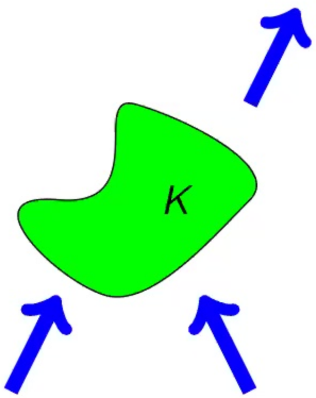
Measure probe



Prepare system and probe

Updated state

Hypothetical local system measurement

Prepare system

Operational approach CJF & Verch, 2018

A QFT (system) is coupled to another QFT (probe) in a compact spacetime region K (a proxy for the experimental design). The probe is measured elsewhere.

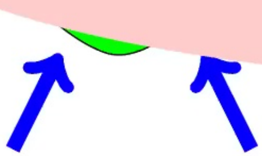
Measure probe

Updated state

Apparent circularity: How do you measure the probe?

Quis metietur ipsos mensores?

Working hypothesis: Someone, somewhere, knows how to measure something.



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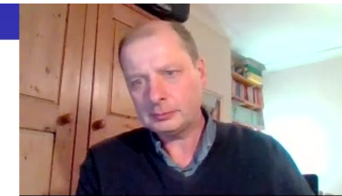

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Algebraic QFT – quickstart (See arXiv:1904.04051 for a pedagogical intro)

Describe a QFT on $\boldsymbol{M}$ in terms of a $*$-algebra $\mathcal{A}(\boldsymbol{M})$ with unit, together with subalgebras $\mathcal{A}(\boldsymbol{M}; N)$ for suitable open regions $N \subset \boldsymbol{M}$. ($\mathcal{A}(\boldsymbol{M}; M) = \mathcal{A}(\boldsymbol{M})$)

$$A + \lambda B, \qquad AB, \qquad A^*, \qquad \mathbf{1}$$

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Typical elements of $\mathcal{A}(\boldsymbol{M}; N)$ include smeared fields

$$\Phi(f) \in \mathcal{A}(\boldsymbol{M}; N) \qquad \text{if } f \equiv 0 \text{ outside } N$$

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Terms and conditions apply

- $N_1 \subset N_2 \implies \mathcal{A}(\boldsymbol{M}; N_1) \subset \mathcal{A}(\boldsymbol{M}; N_2)$ Isotony
- $\mathcal{A}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M})$ if N contains a Cauchy surface of $\boldsymbol{M}$ Timeslice
- ...

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An observable may be localisable in many distinct regions.

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A state is a map $\omega : \mathcal{A}(\boldsymbol{M}) \to \mathbb{C}$, assigning expectation values $\omega(A)$ to $A \in \mathcal{A}(\boldsymbol{M})$.
ω is linear and obeys $\omega(\mathbf{1}) = 1$, $\omega(A^* A) \geq 0$.

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ω is linear and obeys $\omega(\mathbf{1}) = 1$, $\omega(A^*A) \geq 0$.

NB No specific Lagrangian has been assumed.

Outline of the idea

Describe the system and probe by QFTs $\mathcal{A}$, $\mathcal{B}$ on spacetime $\boldsymbol{M}$ (globally hyperbolic). $\mathcal{A}(\boldsymbol{M})$ is the algebra of system observables on $\boldsymbol{M}$. We compare

- the uncoupled combination $\mathcal{U}$ of $\mathcal{A}$ and $\mathcal{B}$

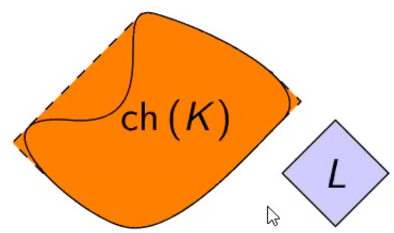
$$\mathcal{U}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M}; N) \otimes \mathcal{B}(\boldsymbol{M}; N)$$

- a coupled combination $\mathcal{C}$ with compact coupling region K.

Minimal assumption on $\mathcal{C}$:

$\mathcal{C}(\boldsymbol{M}; L) \cong \mathcal{U}(\boldsymbol{M}; L) \qquad \forall L$ outside the causal hull $\mathrm{ch}\,(K)$,

and the isomorphisms are compatible with isotony.



$\mathrm{ch}\,(K) = J^{+}(K) \cap J^{-}(K)$

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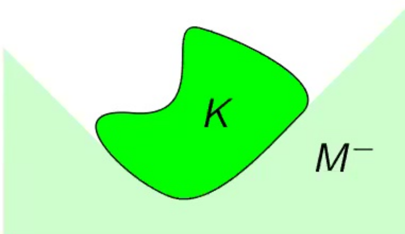
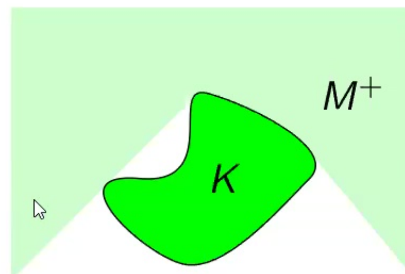
Define geometrical 'in'/'out' regions $M^{-/+}$ on which $\mathcal{U}$ and $\mathcal{C}$ agree.



$$M^{\pm} = M \setminus J^{\mp}(K)$$



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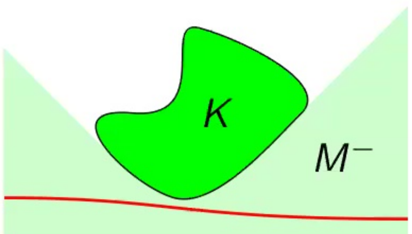
$M^{\pm}$ contain Cauchy surfaces $\implies \exists$ isomorphisms $\tau^{\pm} : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$.



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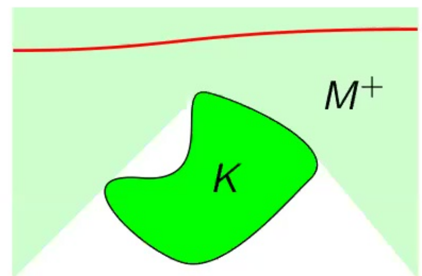

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$M^{\pm}$ contain Cauchy surfaces $\implies \exists$ isomorphisms $\tau^{\pm} : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$.

Combining $\tau^{\pm}$, we define a scattering operator

$$\Theta = (\tau^{-})^{-1} \circ \tau^{+} \in \mathrm{Aut}(\mathcal{U}(\boldsymbol{M}))$$

Locality property: $\Theta \restriction \mathcal{U}(\boldsymbol{M}; N) = \mathrm{id}$, if $N \subset K^{\perp}$.

Measurement scheme: prepare early, measure late

$\tau^{\pm}$ translate fictitious uncoupled language to the physical coupled system.

- 'Prepare system and probe in states ω and σ at early times'

  The uncoupled state $\omega \otimes \sigma$ on $\mathcal{U}(\boldsymbol{M})$ corresponds to the state of $\mathcal{C}(\boldsymbol{M})$,

  $$\underset{\sim}{\omega}_\sigma = ((\tau^-)^{-1})^*(\omega \otimes \sigma) \qquad \underset{\sim}{\omega}_\sigma(X) = (\omega \otimes \sigma)((\tau^-)^{-1}X)$$

- 'Measure a probe observable B at late times'

  Probe observable $\mathbf{1} \otimes B$ in $\mathcal{U}(\boldsymbol{M})$ corresponds to observable

  $$\widetilde{B} := \tau^+(\mathbf{1} \otimes B) \in \mathcal{C}(\boldsymbol{M})$$

- The actual measurement of $\widetilde{B}$ in state $\underset{\sim}{\omega}_\sigma$ corresponds to a fictitious measurement of induced system observable $A \in \mathcal{A}(\boldsymbol{M})$ in state ω so that

  $$\omega(A) = \underset{\sim}{\omega}_\sigma(\widetilde{B}).$$

Measurement scheme: prepare early, measure late

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  The uncoupled state $\omega \otimes \sigma$ on $\mathcal{U}(\boldsymbol{M})$ corresponds to the state of $\mathcal{C}(\boldsymbol{M})$,

Explicitly, $$A = \varepsilon_\sigma(B) \stackrel{\text{def}}{=} \eta_\sigma(\Theta(\mathbf{1} \otimes B))$$

where $\eta_\sigma : \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ linearly extends $P \otimes Q \mapsto \sigma(Q)P$.

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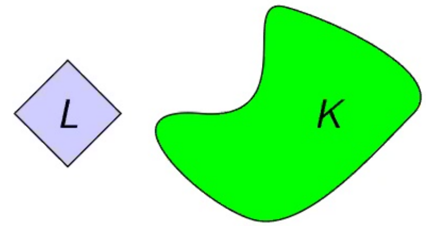
Induced system observables – localisation



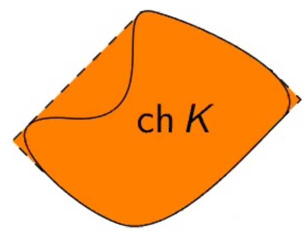
Recall: Θ acts trivially on $\mathcal{U}(\boldsymbol{M}; L)$ if $L \subset K^\perp$.

Theorem (a) If $B \in \mathcal{B}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then $\varepsilon_\sigma(B) = \sigma(B)\mathbf{1}$.

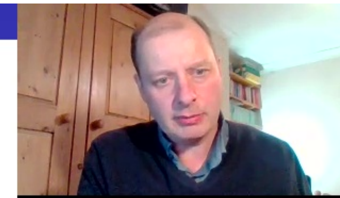
(b) If $\mathcal{A}$ obeys a Haag property, then

$$\varepsilon_\sigma(B) \in \mathcal{A}(\boldsymbol{M}; N) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}),$$

where N is any open connected causally convex set containing K.



NB N must contain $\mathrm{ch}\, K$. The localisation of B is irrelevant.



Induced system observables – fluctuations

True and hypothetical expectation values agree, by construction

$$\underset{\sim}{\omega}_\sigma(\widetilde{B}) = \omega(\varepsilon_\sigma(B)) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}).$$

$\varepsilon_\sigma : \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ is linear, completely positive, and obeys

$$\varepsilon_\sigma(\mathbf{1}) = \mathbf{1}, \qquad \varepsilon_\sigma(B^*) = \varepsilon_\sigma(B)^*, \qquad \varepsilon_\sigma(B)^*\varepsilon_\sigma(B) \leq \varepsilon_\sigma(B^*B).$$

Consequently, the true measurement displays greater variance than the hypothetical one due to detector fluctuations

$$\mathrm{Var}(\widetilde{B}; \underset{\sim}{\omega}_\sigma) \geq \mathrm{Var}(\varepsilon_\sigma(B); \omega).$$

Effects

An effect is an observable s.t. B and $\mathbf{1} - B$ are positive, corresponding to a true/false measurement

$$\mathrm{Prob}(B \mid \omega) = \omega(B), \qquad \mathrm{Prob}(\neg B \mid \omega) = \omega(\mathbf{1} - B).$$

Unsharp unless B is a projection.

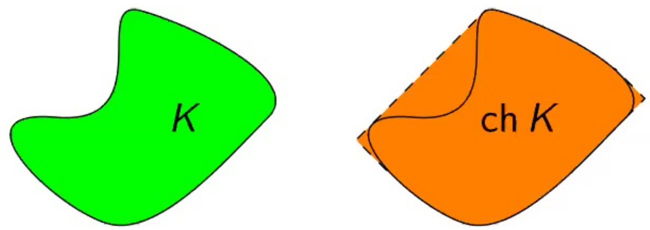
Because ε_σ is completely positive, but not a homomorphism in general:

- probe effects induce system effects
- system effects are typically unsharp, even for sharp probe effects.

Summary so far

To every local probe observable B and probe preparation state σ there is an induced system observable, $\varepsilon_\sigma(B)$ which can be localised in any connected region containing the causal hull of K.

$$\mathrm{ch}(K) = J^+(K) \cap J^-(K)$$



Expectation values of the true and hypothetical measurements match, but the true measurement has greater variance.

Probe observables localisable in $K^\perp$ induce trivial system observables.

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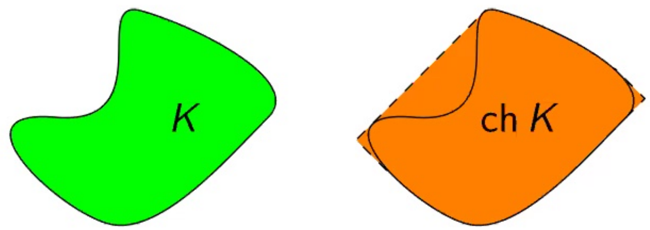
To every local probe observable B and probe preparation state σ there is an induced system observable, $\varepsilon_\sigma(B)$ which can be localised in any connected region containing the causal hull of K.

$$\mathrm{ch}\,(K) = J^+(K) \cap J^-(K)$$



Expectation values of the true and hypothetical measurements match, but the true measurement has greater variance.

Justifies regarding $A \in \mathcal{A}(\boldsymbol{M}; L)$ as 'measureable within' L in the sense that the system and probe are coupled there.

State update rules

How should the instantaneous collapse rule of QM be carried over to QFT in CST?

State update rules

Operational ideology: the purpose of state updates is to facilitate predictions.

State updates – two probes CJF + Verch; CJF + Bostelmann & Ruep

Consider two independent probes coupled to the system, each measuring an effect.

The probes can be regarded as a single 'super-probe' and the effects A, B can be combined as the effect $A\&B$ (success of both) given by $A \otimes B$.

The expectation of B conditioned on successful measurement of A is

$$\mathbb{E}(B|A;\omega) = \frac{\mathbb{E}(A\&B;\omega)}{\mathrm{Prob}(A;\omega)}$$

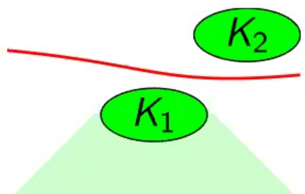
NB The RHS involves outcomes for a single probe; already understood.

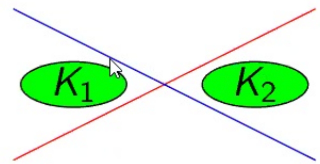
$$\mathbb{E}(B|A;\omega) = \frac{(\omega \otimes \sigma_A \otimes \sigma_B)(\Theta_{AB}(\mathbf{1} \otimes A \otimes B))}{(\omega \otimes \sigma_A)(\Theta_A(\mathbf{1} \otimes A))}$$

where Θ_{AB} is the scattering map for the combined probe, and Θ_A that for A alone.

Causal factorisation and the update rule

Two coupling regions separated by a Cauchy surface are causally orderable.





N regions are causally orderable if they may be labelled $K_1, \ldots, K_N$ so that

$$K_n \text{ lies to the past of a Cauchy surface separating it from } K_{n+1}$$

for all $1 \leq n \leq N-1$.

Any such labelling defines a compatible causal order; write $K_1 \lhd K_2 \lhd \cdots \lhd K_N$.

Causal factorisation and the update rule

Two coupling regions separated by a Cauchy surface are causally orderable.
Causal factorisation: if $K_A \triangleleft K_B$ w.r.t. some compatible causal order,

$$\Theta_{AB} = \hat{\Theta}_A \circ \hat{\Theta}_B = (\Theta_A \otimes \mathrm{id}) \circ (\Theta_B \otimes_2 \mathrm{id})$$

Consequence: if K_A and K_B are spacelike then $\hat{\Theta}_A$ and $\hat{\Theta}_B$ commute.

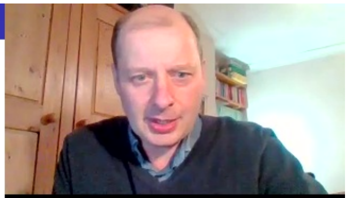
Under this assumption, and for $K_A \triangleleft K_B$,

$$\mathbb{E}(B|A;\omega) = (\omega_A \otimes \sigma_B)(\Theta_B(\mathbf{1} \otimes B)) = \mathbb{E}(B;\omega_A); \qquad \omega_A(C) = \frac{(\omega \otimes \sigma_A)(\Theta_A(C \otimes A))}{(\omega \otimes \sigma_A)(\Theta_A(\mathbf{1} \otimes A))}$$

NB ω_A is independent of B, σ_B and Θ_B.

This justifies the state update rule $\omega \mapsto \omega_A$ given a successful measurement of A.

Causal factorisation and the update rule



Two coupling regions separated by a Cauchy surface are causally orderable.
Causal factori...

It is not necessary to assume that the state actually changes.

The update rule conveniently does the book-keeping needed to compute the conditional expectation, given additional knowledge from the A-measurement.

Co...

Under this assu...

$$\mathbb{E}(B|A;\omega) = (\omega_A \otimes \sigma_B)(\Theta_B(\mathbf{1} \otimes B)) = \mathbb{E}(B;\omega_A); \qquad \omega_A(C) = \frac{(\omega \otimes \sigma_A)(\Theta_A(C \otimes A))}{(\omega \otimes \sigma_A)(\Theta_A(\mathbf{1} \otimes A))}$$

NB ω_A is independent of B, σ_B and Θ_B.

This justifies the state update rule $\omega \mapsto \omega_A$ given a successful measurement of A.

Properties of the update rule

Unspooky action at a distance

Theorem For B localisable in $K_A^\perp$, $\omega_A(B) = \omega(B)$ iff B is uncorrelated with $\varepsilon_\sigma(A)$ in ω.

Consistency For two updates at spacelike separation

$$(\omega_A)_B = (\omega_B)_A$$

Multiple causally orderable probes Assuming causal factorisation (suitably extended)

$$\mathbb{E}(B|A_1\&A_2\&\cdots\&A_N;\omega) = \mathbb{E}(B;((\omega_{A_1})_{A_2})_{\cdots A_N})$$

if effects $A_1, \ldots A_N$ are measured by causally orderable probes with $K_1 \lhd \cdots \lhd K_N \lhd K_B$.
(Valid for any compatible causal ordering.)

Nonselective measurement

If no selection is made on the results of A, the updated state is a convex combination of ω_A and $\omega_{\neg A}$

$$\begin{aligned}
\omega_A^{\text{n.s.}}(C) &= (\omega \otimes \sigma_A)(\Theta_A(C \otimes A)) + (\omega \otimes \sigma_A)(\Theta_A(C \otimes (\mathbf{1} - A))) \\
&= (\omega \otimes \sigma_A)(\Theta_A(C \otimes \mathbf{1}))
\end{aligned}$$

which is independent of A.

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which is independent of A.

The principle of blissful ignorance

Theorem $\omega_A^{\text{n.s.}}(B) = \omega(B)$ for all B localisable in $K_A^{\perp}$.

Multiple nonselective measurements

Consider causally orderable probes with

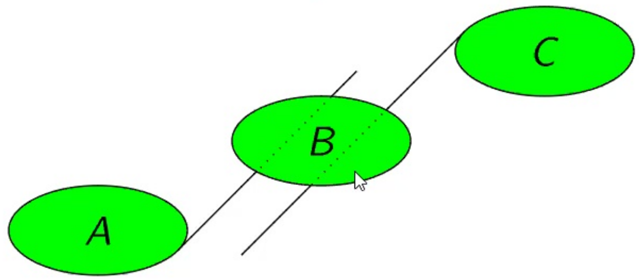
$$K_{A_1} \lhd \cdots \lhd K_{A_M} \lhd {\color{red}K_B} \lhd K_{C_1} \lhd \cdots \lhd K_{C_N}$$

Effects $A_1, \ldots, A_M$, $C_1, \ldots, C_N$ are measured without selection.

$$\mathbb{E}(B;\omega) = \mathbb{E}(B;((\omega_{A_1}^{\text{n.s.}})_{A_2}^{\text{n.s.}})_{\cdots A_N}^{\cdots \text{n.s.}})$$

- Use the updated state for the nonselective measurements in the past of K_B (according to any compatible ordering).
- Nonselective measurements to the future (in some compatible order) may be ignored (just as well).

Impossible measurements? Bostelmann, CJF & Ruep arXiv:2003.04660

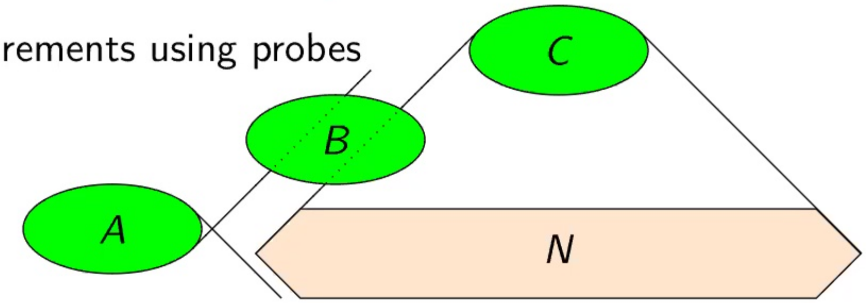


- Alice chooses whether to make a nonselective measurement
- Bob certainly makes a nonselective measurement
- Can Charlie determine whether Alice performed the measurement?

$$\omega^{\text{n.s.}}_{AB}(C) \overset{?}{\neq} \omega^{\text{n.s.}}_{B}(C)$$

Impossible measurements? Bostelmann, CJF & Ruep arXiv:2003.04660

Model A and B measurements using probes



More detailed investigation of scattering map locality properties gives

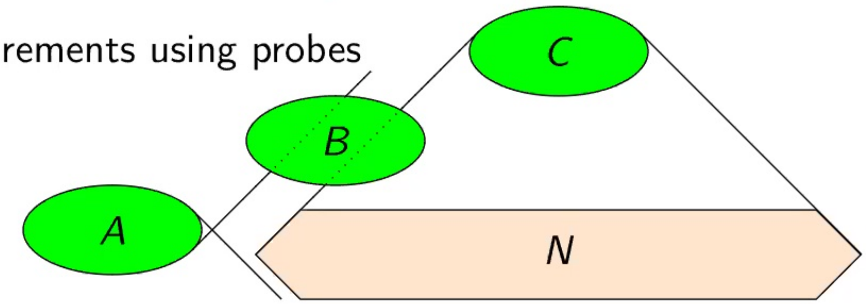
$$\hat{\Theta}_B C \otimes \mathbf{1} \otimes \mathbf{1} \in \mathcal{U}(\boldsymbol{M}; N) \qquad \text{for region } N \subset K_A^{\perp} \cap M_B^{-}$$

Consequently, Charlie cannot determine whether Alice has measured:

$$\omega_{AB}^{\text{n.s.}}(C) = (\omega \otimes \sigma_A \otimes \sigma_B)(\hat{\Theta}_A \hat{\Theta}_B C \otimes \mathbf{1} \otimes \mathbf{1}) = (\omega \otimes \sigma_A \otimes \sigma_B)(\hat{\Theta}_B C \otimes \mathbf{1} \otimes \mathbf{1}) = \omega_B^{\text{n.s.}}(C)$$

Impossible measurements? Bostelmann, CJF & Ruep `arXiv:2003.04660`

Model A and B measurements using probes



The analysis shows that the measurement scheme is free of Sorkin-type pathologies.

Key assumption – the probes and couplings are described by physics respecting locality.

Impossible measurements can only be performed using impossible apparatus.

Summary

- Operational framework of QMT adapted to AQFT
  - covariant, formulated for curved as well as flat spacetimes
  - derived from minimal assumptions
- Probe observables induce local system observables
  - localisable in the causal hull of coupling region
- State update rules
  - derived from required properties rather than posited
  - consistent for any compatible causal ordering of multiple probes
  - principle of blissful ignorance
- Framework is free of impossible measurements
- Induced observables have been computed for a specific model (bonus slides)

A specific probe model

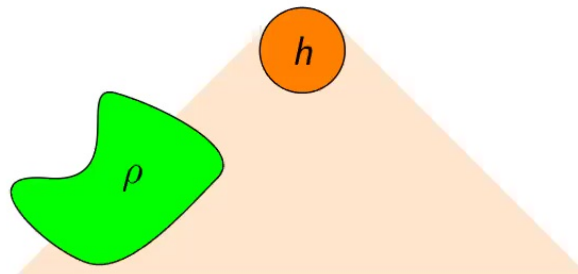
Two free scalar fields: Φ (system) and Ψ (probe) coupled via an interaction term

$$\mathcal{L}_{\mathrm{int}} = -\rho\Phi\Psi, \qquad \rho \in C_0^\infty(M), \qquad K = \operatorname{supp}\rho.$$

Linear equations: standard quantisation applies at least for sufficiently weak coupling. As formal power series in $h \in C_0^\infty(M^+)$,

$$\Theta(\mathbf{1} \otimes e^{i\Psi(h)}) = e^{i\Phi(f^-)} \otimes e^{i\Psi(h^-)}$$

where f^- and $h^- - h$ are supported in $\operatorname{supp}\rho \cap J^-(\operatorname{supp} h)$.



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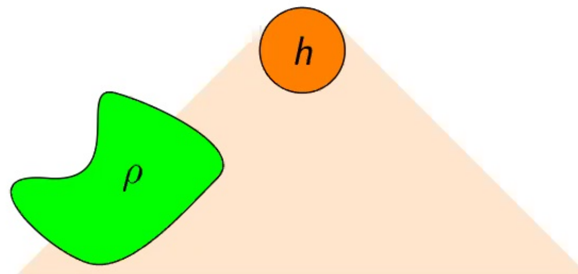


$$\varepsilon_\sigma(e^{i\Psi(h)}) = \sigma\left(e^{i\Psi(h^-)}\right) e^{i\Phi(f^-)} = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

if σ is quasifree with two-point function S.

Examples of induced observables

$$\varepsilon_\sigma(e^{i\Psi(h)}) = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

$$\varepsilon_\sigma(\Psi(h)) = \Phi(f^-)$$

$$\varepsilon_\sigma(\Psi(h)^2) = \Phi(f^-)^2 + \boxed{S(h^-,h^-)\mathbf{1}}$$

Consequently,

$$\mathbb{E}(\widetilde{\Psi(h)}; \underset{\sim}{\omega}_\sigma) = \omega(\Phi(f^-))$$

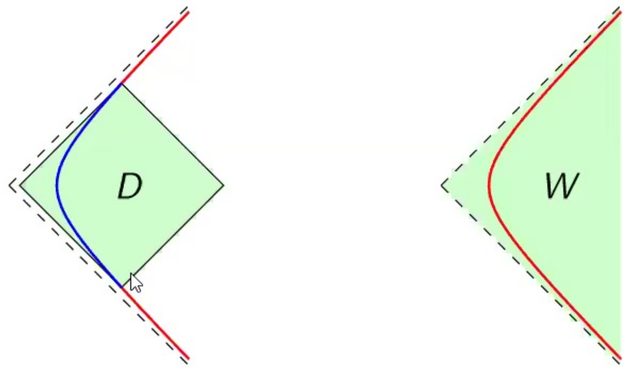
$$\mathrm{Var}(\widetilde{\Psi(h)}; \underset{\sim}{\omega}_\sigma) = \mathrm{Var}(\Phi(f^-);\omega) + S(h^-,h^-)$$

Increased variance in true measurement from detector fluctuations.

Localisation of induced observables

$\varepsilon_\sigma(\Psi(h)^n)$ may be localised in any open causally convex nhd of

$$\mathrm{supp}\, f^- \subset \mathrm{supp}\,\rho \cap J^-(\mathrm{supp}\, h)$$



Localisation region for finite-time coupling is a diamond D.

Localisation region for eternal coupling is a wedge W (can't do better).