

Title: Asymptotic symmetries and celestial CFT

Speakers: Laura Donnay

Series: Quantum Fields and Strings

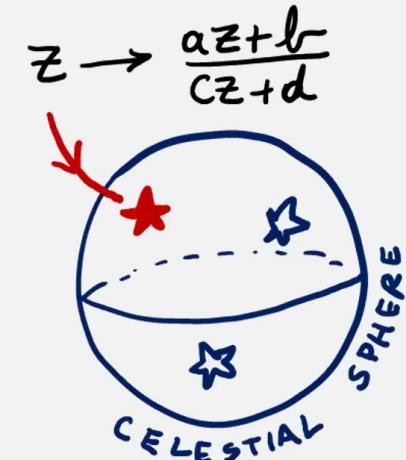
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Abstract: Universal relationships between asymptotic symmetries, QFT soft theorems, and low energy observables have reinvigorated attempts at flat space holography. In this talk, I will review recent advances in the celestial holography proposal, where the 4d S-matrix is reconsidered as a 2d correlator on the celestial sphere at null infinity. In this framework, asymptotic particle states are characterized by the point at which they enter or exit the celestial sphere as well as their SL(2,C) Lorentz quantum numbers: namely their conformal scaling dimension and spin instead of the energy and momentum. I will present a unified treatment of conformally soft Goldstone modes which arise when spin-one or spin-two conformal primary wavefunctions become pure gauge for certain integer values of the conformal dimension.

Asymptotic symmetries *and celestial CFT*

based on 1810.05219 with Andrea Puhm, Andy Strominger
2005.08990 with Andrea Puhm, Sabrina Pasterski



Laura Donnay (Technische Universität Wien)

H2020- Marie Skłodowska-Curie “HoloBH – 746297”

October 27, 2020 – Perimeter Institute, *Quantum Fields and Strings Seminar*



Intro & Motivations

How general is holography?

- **Holographic principle:** beautiful story unfolded over the last 20 years that revealed detailed structure about **quantum gravity** in *Anti-de Sitter* (AdS) spacetimes (cosmological constant $\Lambda < 0$).
- Groundbreaking realization: **AdS/CFT correspondence**
 Quantum gravity in **Anti-de Sitter**
 = **Conformal Field Theory** (CFT) at the *boundary* of spacetime

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- Groundbreaking realization: **AdS/CFT correspondence**
 Quantum gravity in **Anti-de Sitter**
 = **Conformal Field Theory** (CFT) at the *boundary* of spacetime

→ **Holographic formulation** for **flat spacetimes** ($\Lambda = 0$)?

Is there a dual CFT? Where does it live?

What are its properties?

Intro & Motivations

Black holes: entropy counting?

- ✓ for supersymmetric black holes in string theory [Vafa, Strominger '95]
- ✓ extremely spinning black holes (*Kerr/CFT correspondence*)

➤ success resides on the presence of an *AdS* factor in the near-horizon region



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Our understanding of quantum properties of black holes goes hand-in-hand with the **spectacular advances** of the **AdS/CFT correspondence**.

$$S_{BH} = \frac{\mathcal{A}c^3}{4G\hbar} \quad \rightarrow \text{“Primeval holographic relationship”}$$



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But realistic black holes (e.g. Kerr) do not possess an AdS decoupling region.

→ need to develop a **holographic correspondence**
for **asymptotically flat** spacetimes

Outline of the talk

- I. Flat spacetimes and their infinite-dimensional symmetries
- II. Asymptotic symmetries & celestial holography

A surprise in flat spacetime

The BMS symmetries [Bondi-Metzner-van der Burg; Sachs, '62]

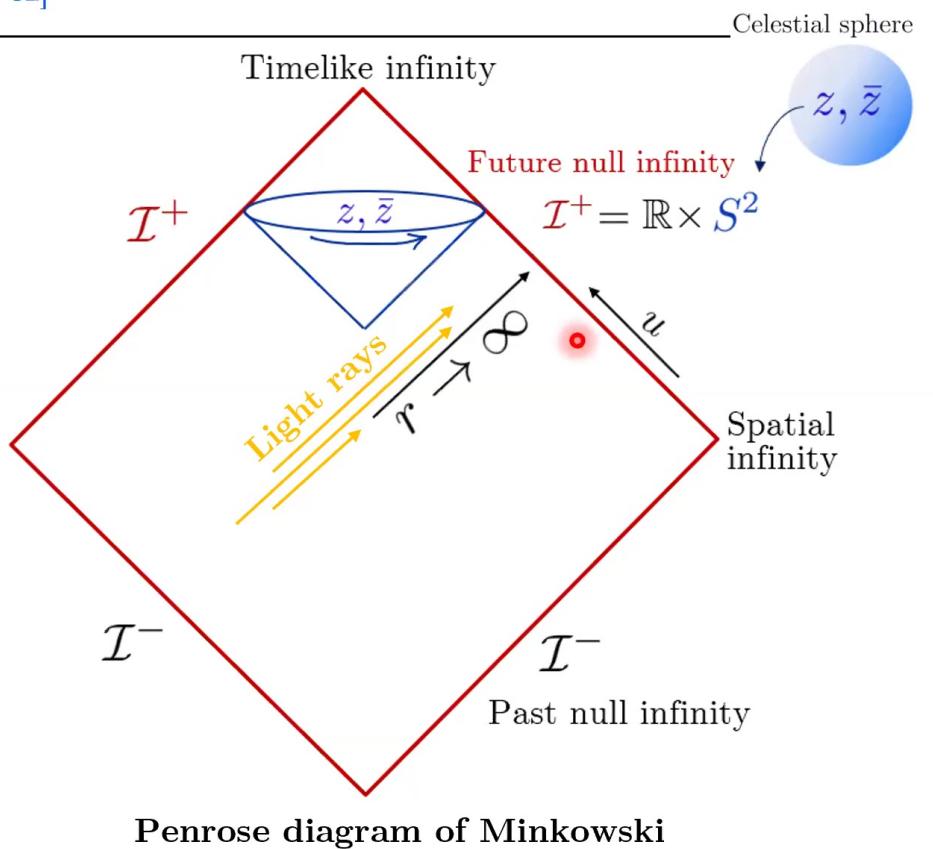
Minkowski metric (flat spacetime) in 4D

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

$$\downarrow u = t - r \text{ : 'retarded' time}$$

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}} dz d\bar{z}$$

$$z = e^{i\phi} \cot \frac{\theta}{2} \quad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$
$$\bar{z} = e^{-i\phi} \cot \frac{\theta}{2}$$



Laura Donnay - Asymptotic symmetries and celestial CFT

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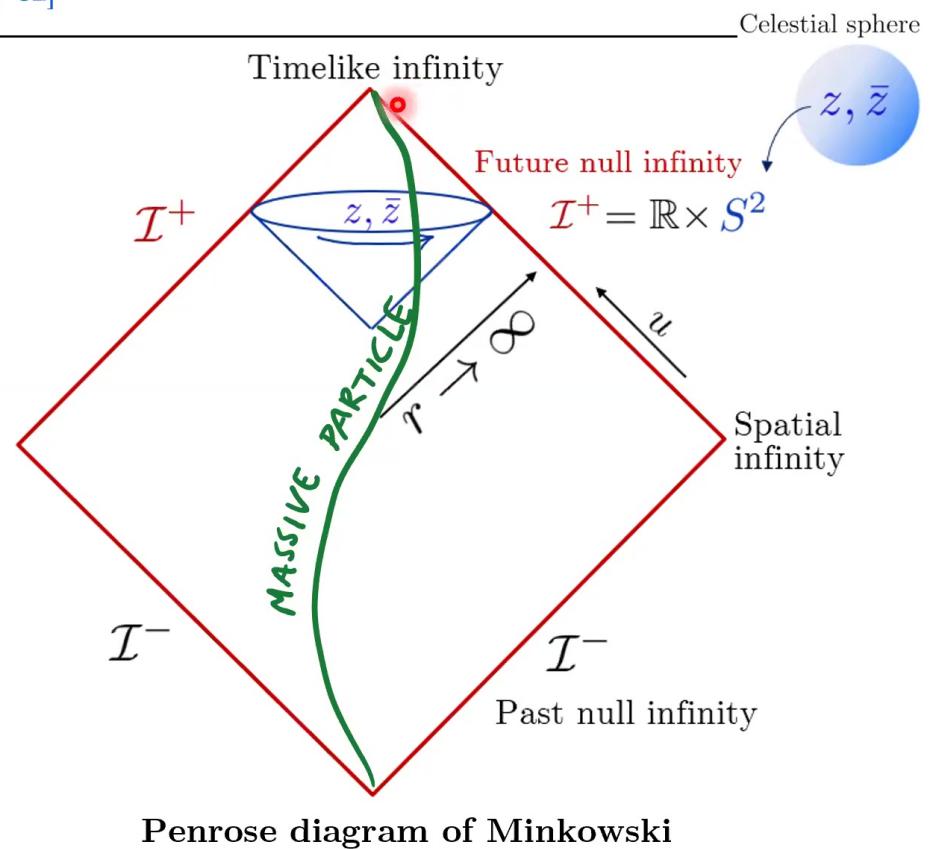
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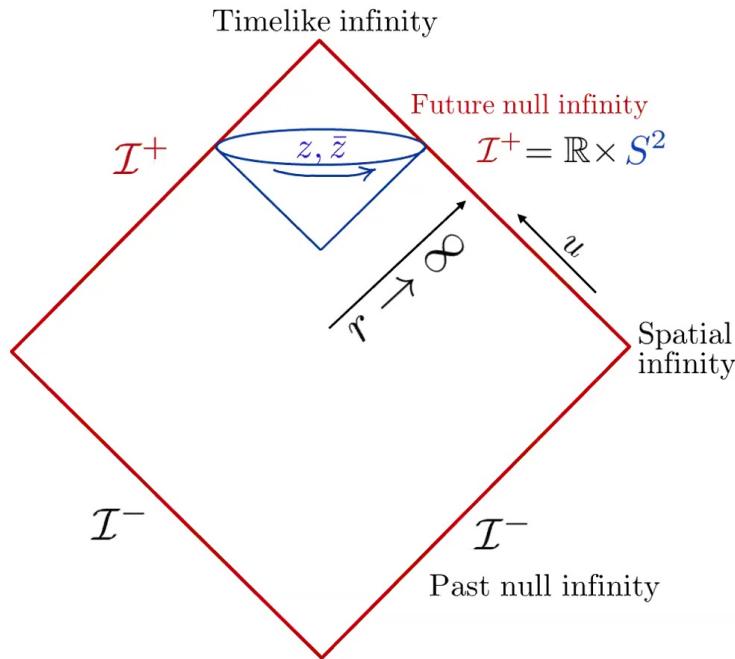


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A surprise in flat spacetime

The BMS symmetries [Bondi-Metzner-van der Burg; Sachs, '62]

Asymptotically flat spacetime (as $r \rightarrow \infty$)

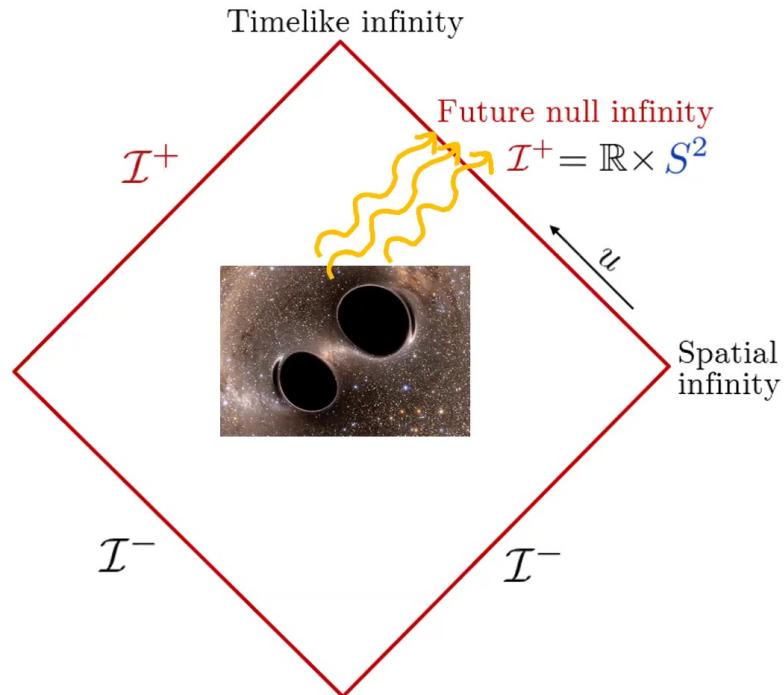


$$\begin{aligned} ds^2 = & -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \circ \\ & + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^zC_{zz}dudz \\ & + \frac{1}{r} \left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz}) \right) dudz + c.c. + \dots \end{aligned}$$

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$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$
$$+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^zC_{zz}dudz$$
$$+ \frac{1}{r} \left(\frac{4}{3}(\mathcal{N}_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz}) \right) dudz + c.c. + \dots$$

$m_B(u, z, \bar{z})$: Bondi mass aspect

$N_z(u, z, \bar{z})$: angular momentum aspect

$C_{zz}(u, z, \bar{z})$: indicates the presence of **gravitational waves**

Bondi news tensor: $N_{zz} \equiv \partial_u C_{zz} \neq 0$

A surprise in flat spacetime

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Asymptotically flat spacetimes

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are preserved by the diffeomorphisms (asymptotic symmetries)

$$\xi_f = f(z, \bar{z})\partial_u + D^zD_zf\partial_r - \frac{1}{r}(D^zf\partial_z + D^{\bar{z}}f\partial_{\bar{z}}) \quad \text{'supertranslations'} \\ u \rightarrow u + f(z, \bar{z})$$



Celestial sphere \mathcal{CS}^2

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$$u \rightarrow u + f(z, \bar{z})$$

Action of supertranslations: $\mathcal{L}_f C_{zz} = f\partial_u C_{zz} - 2D_z^2 f$



Soft supertranslation charge

$$Q_f^{soft} = \frac{1}{16\pi G} \int dud^2z \sqrt{\gamma} (D_z^2 f N^{zz} + D_{\bar{z}}^2 f N^{\bar{z}\bar{z}})$$

Barnich, Troessaert '11
He, Lysov, Mitra, Strominger '15

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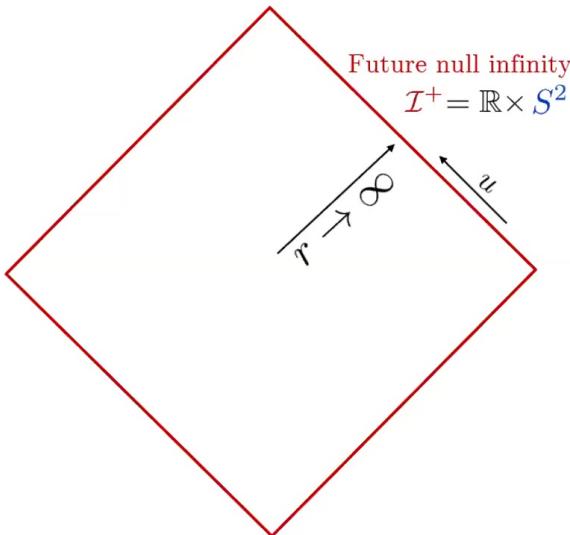
$$\xi_Y = (1 + \frac{u}{2r})Y^z\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^z\partial_{\bar{z}} - \frac{1}{2}(u + r)D_zY^z\partial_r + \frac{u}{2}D_zY^z\partial_u + c.c. \quad \text{‘superrotations’}$$

Barnich, Troessaert '08: allow for local conformal Killing vector on the celestial sphere

$$\begin{array}{ccc} Y^z(z) & \xrightarrow{\hspace{1cm}} & \mathbf{Vir} \times \mathbf{Vir} \\ Y^{\bar{z}}(\bar{z}) & & \end{array} \quad \text{Action of superrotations: } \mathcal{L}_Y C_{zz} = [...] - u D_z^3 Y^z$$

A surprise in flat spacetime

The BMS symmetries [Bondi-Metzner-van der Burg; Sachs, '62][Barnich, Troessaert '08]



Conclusion: The asymptotic symmetry algebra of flat spacetimes is *not* the Poincaré algebra, but rather an **infinite-dimensional extension** of it, known as the **BMS algebra**.

Poincaré = 4 translations \times 6 Lorentz
BMS= *supertranslations* \times *superrotations*

BMS transformations (supertranslations and superrotations) map one geometry into a new **physically inequivalent** one.

They were originally disregarded.



BMS symmetries today: were shown to have **deep** and profound connections with the **infrared structure of gravity**. Implications are still being understood today.

A surprise in flat spacetime

The BMS symmetries [Bondi-Metzner-van der Burg; Sachs, '62]

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Soft theorems from asymptotic symmetries

The **leading soft graviton theorem** [Weinberg '65]

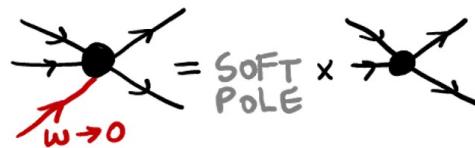
n hard particles (p_k) + external graviton (q)

$$\lim_{\omega \rightarrow 0} \mathcal{A}_{n+1}(q) = S^{(0)} \mathcal{A}_n + \mathcal{O}(q^0)$$

$$S^{(0)} = \sum_{k=1}^n \frac{p_k^\mu p_k^\nu \varepsilon_{\mu\nu}(q)}{p_k \cdot q}$$

$$\mathcal{A}_n = \langle \text{out} | S | \text{in} \rangle$$

+soft particle (energy $\omega \rightarrow 0$)

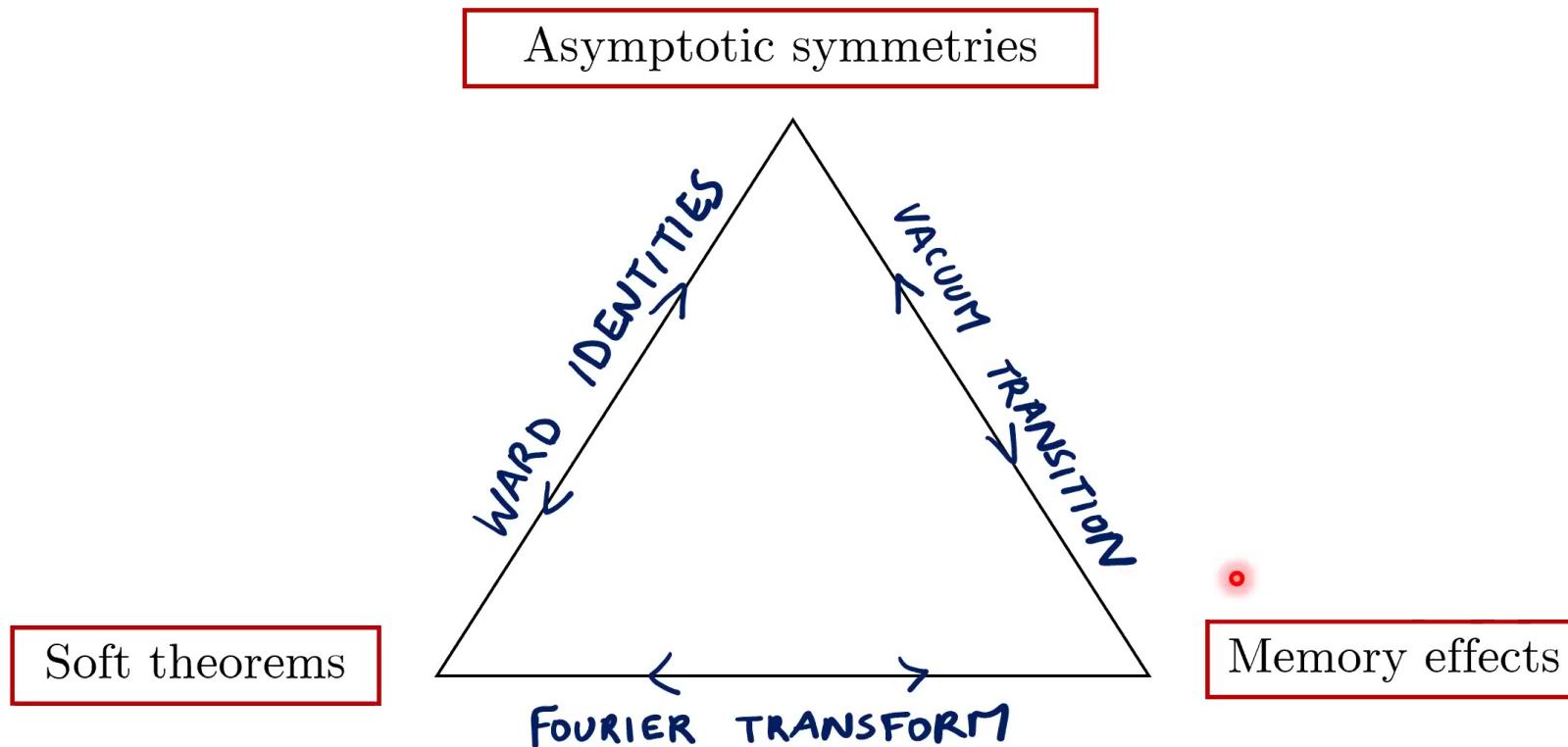


is the **Ward identity** associated to **supertranslation** symmetry [He, Lysov, Mitra, Strominger '15]

$$\langle \text{out} | Q_f^+ \mathcal{S} - \mathcal{S} Q_f^- | \text{in} \rangle = 0$$

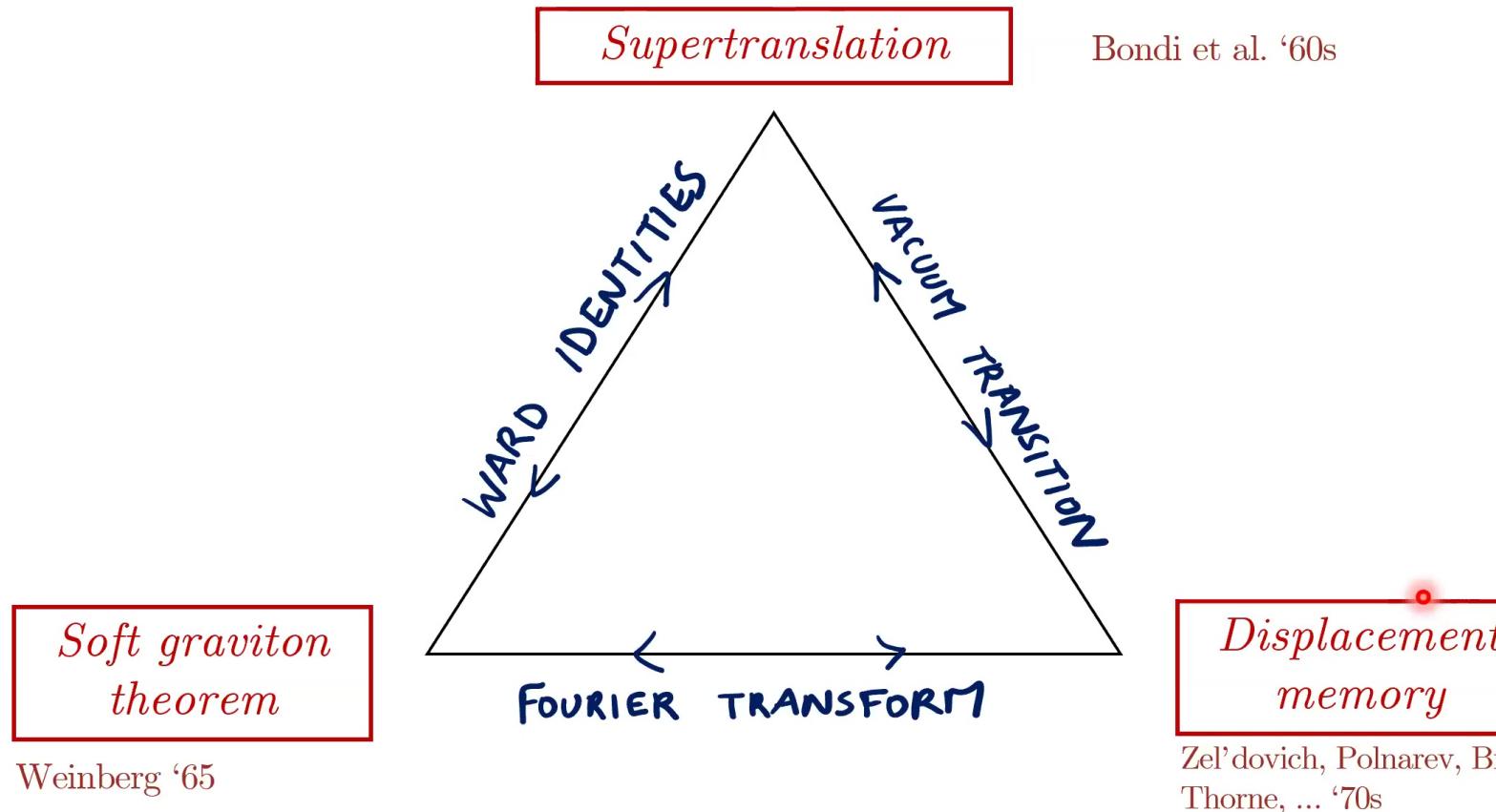
The infrared triangle

[Strominger, 2018]



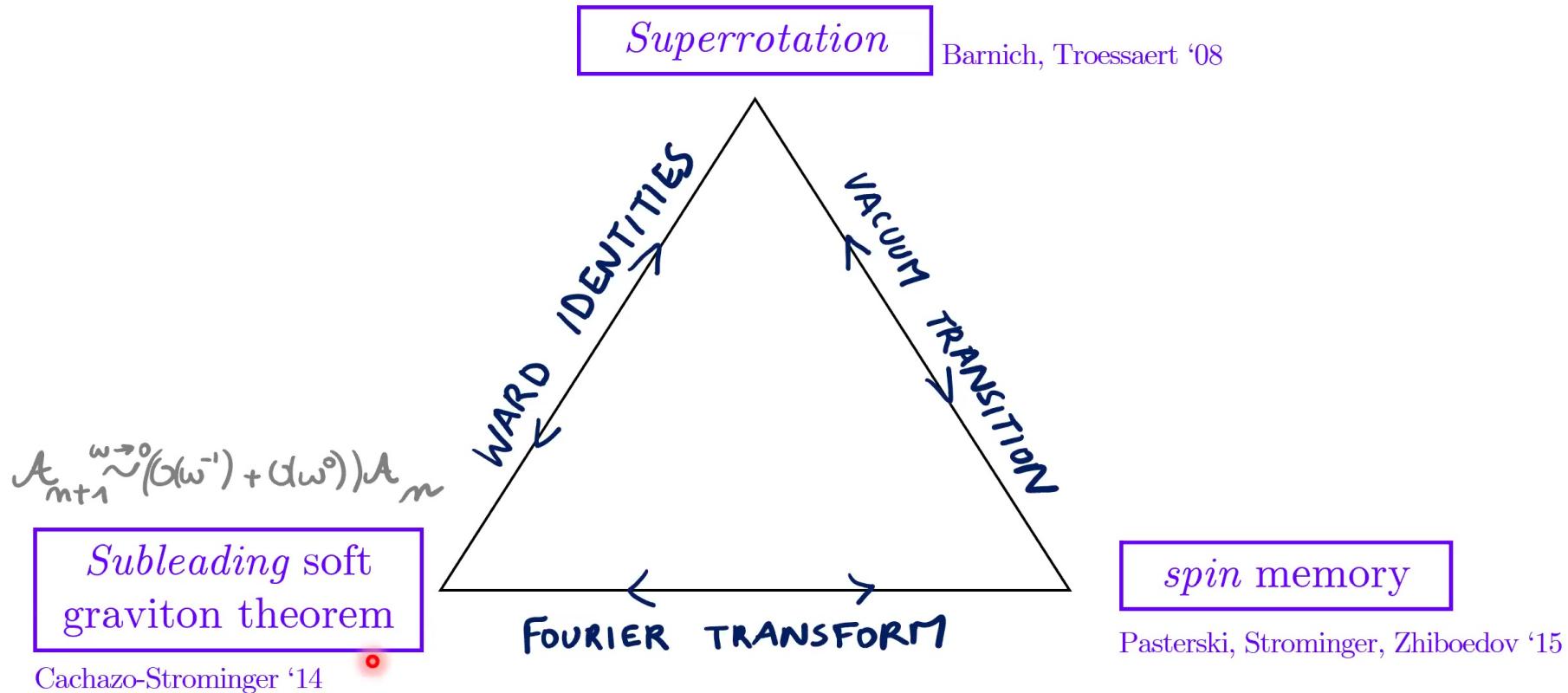
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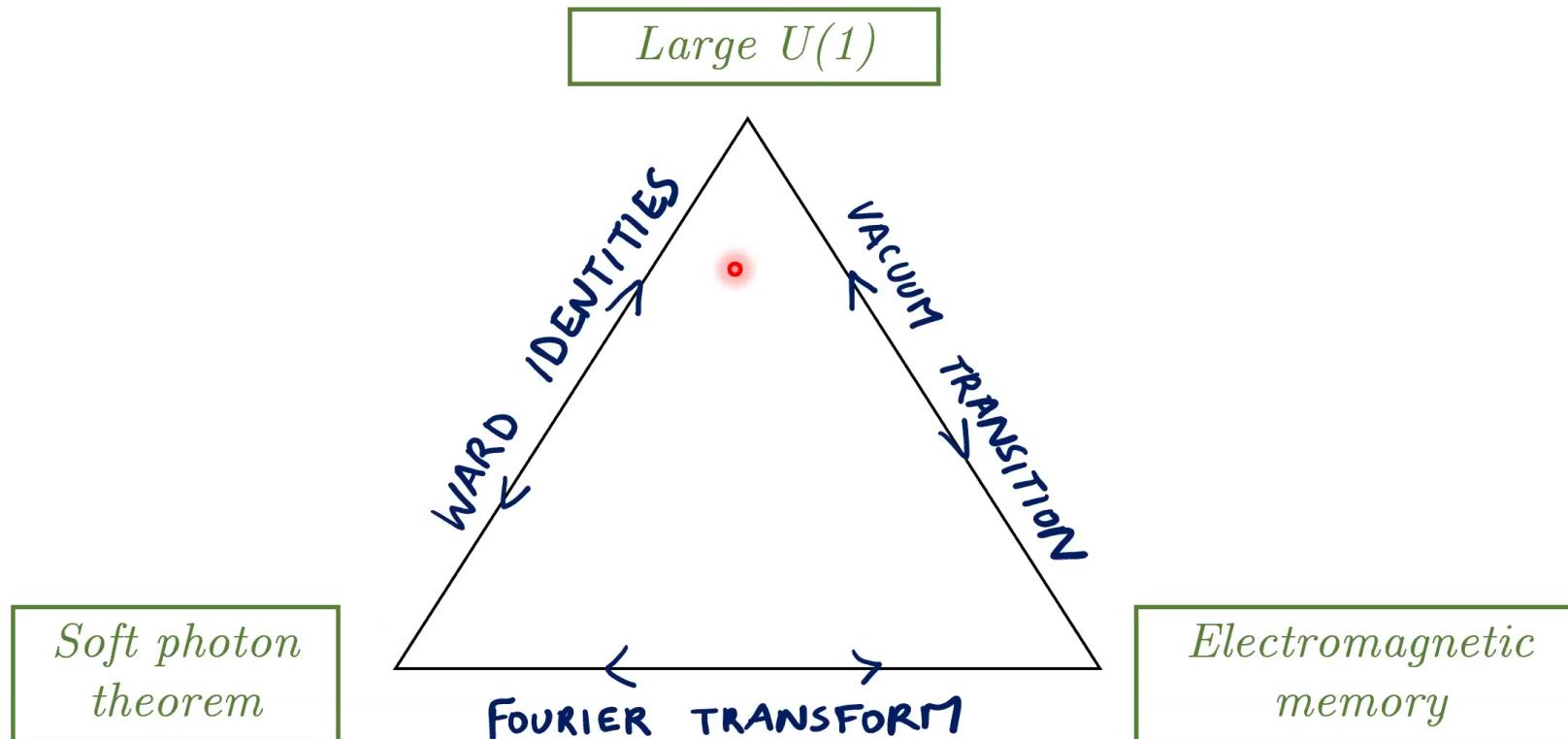
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Laura Donnay - Asymptotic symmetries and celestial CFT

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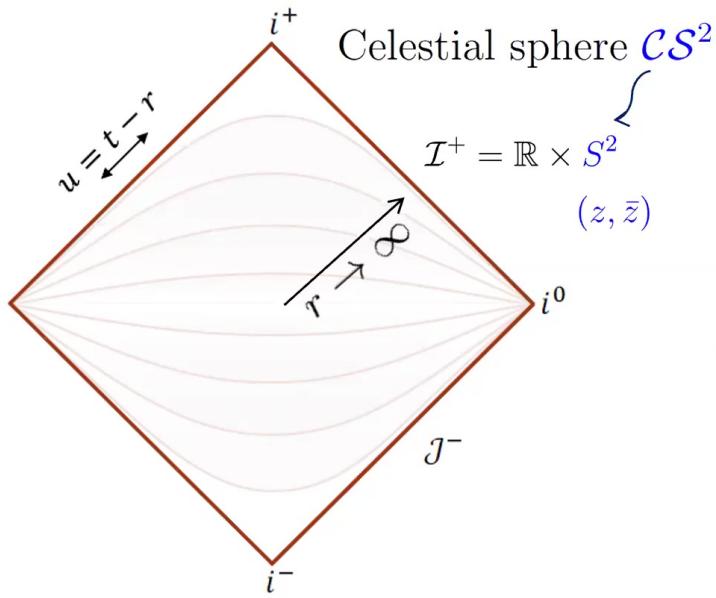
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Towards *flat space* holography...



Symmetry observation

The 4d Lorentz group acts on \mathcal{CS}^2 as the global 2d conformal group:

$$z \rightarrow \frac{az + b}{cz + d}, \quad ad - bc = 1$$

- 4d scattering amplitudes recast as correlators on \mathcal{CS}^2 will share properties of a 2d conformal field theory (CFT)
- Proposition of 4d/2d flat space holography [Strominger et al. '18]

Towards *flat space* holography... ‘celestial holography’

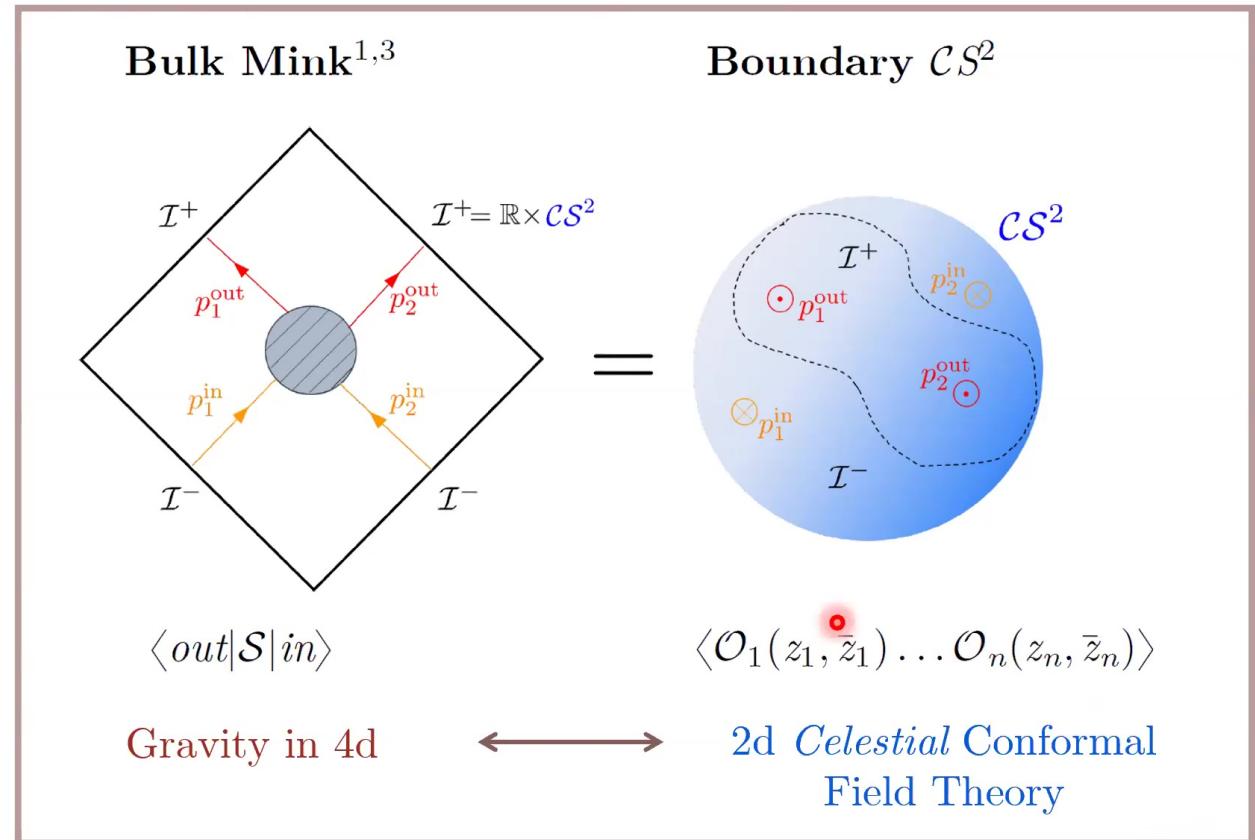
The boundary of flat space is future null infinity

$$\mathcal{I}^+ = \mathbb{R} \times S^2$$

The dual theory lives on the celestial sphere



Celestial sphere \mathcal{CS}^2



A new formulation of asymptotic particles

standard formulation

energy-momentum basis

→ plane waves

$$e^{\pm ip \cdot X} \quad p^\mu = \omega q^\mu(z, \bar{z})$$

4d helicity ℓ

solutions labelled by

$$p^\mu$$

$$\ell$$

translation symmetry manifest
conformal properties obscured

holographic formulation

conformal basis

→ Mellin transform of plane waves

$$\mathcal{O}(\Delta, z, \bar{z}) = \int_0^\infty d\omega \omega^{\Delta-1} e^{\pm ip \cdot X} \quad \textcolor{blue}{z} = \frac{p^1 + ip^2}{p^3 + p^0},$$

2d spin $\textcolor{red}{J}$

$\Delta = h + \bar{h}$: conformal dimension

(z, \bar{z}) : a point on \mathcal{CS}^2

2d spin $\textcolor{red}{J} = h - \bar{h}$

conformal properties manifest
translation symmetry obscured

L.D., Raclariu, Pasterski, Pate,
Puhr, Shao, Strominger,
Stieberger, Taylor, Volovich, ...

Towards *flat space* holography... ‘celestial holography’

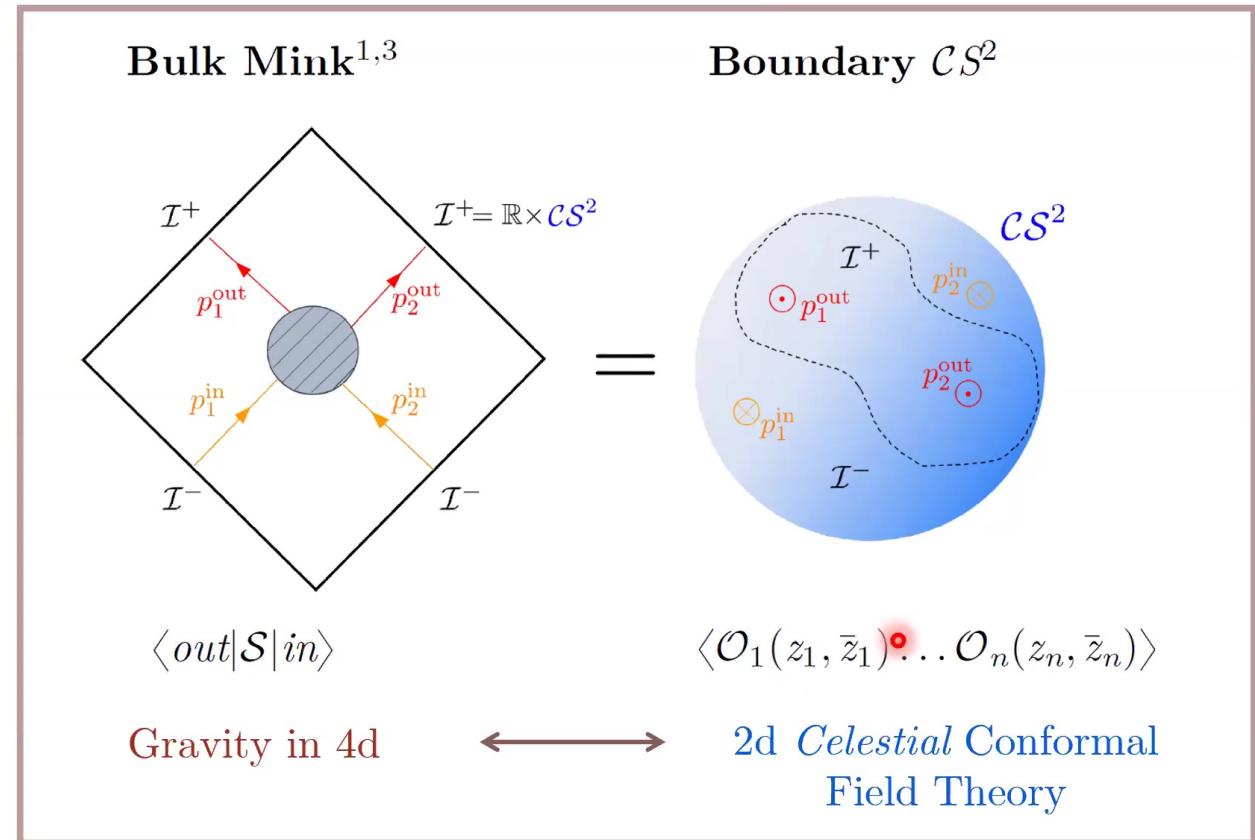
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Celestial sphere \mathcal{CS}^2



Celestial correlators

2d celestial correlators are obtained as Mellin transforms of scattering amplitudes

$$\prod_{k=1}^m \int_0^\infty d\omega_k \omega_k^{\Delta_k - 1} \mathcal{A}(\omega_1, z_1, \bar{z}_1, \dots, \omega_m, z_m, \bar{z}_m) = \langle \mathcal{O}_1(\Delta_1, z_1, \bar{z}_1, \dots, \Delta_m, z_m, \bar{z}_m) \rangle_{CCFT_2}$$

Celestial amplitudes have been computed in a number of examples:

- **scalars:** massive, massless, 1-loop
- tree-level n -point **Yang-Mills**
- **string theory** up to 4-point
- **gravity**

Adamo, Banerjee, Cardona, Guevara, Huang, Law, Mason, Nandan, Pasterski, Pate, Puhm, Raclariu, Sharma, Shao, Schreiber, Strominger, Stieberger, Taylor, Volovich, Zlotnikov,...

Conformally soft particles

holographic formulation

conformal basis

$$\rightarrow \text{Mellin transform of plane waves } \mathcal{O}(\Delta, z, \bar{z}) = \int_0^\infty d\omega \omega^{\Delta-1} e^{\pm ip \cdot X}$$

$\Delta = h + \bar{h}$: conformal dimension

(z, \bar{z}) : a point on \mathcal{CS}^2

An important role (cf. IR triangle) is played by soft particles (a soft particle has energy $\omega \rightarrow 0$).

Celestial primaries

$$X^\mu (\mu = 0, 1, 2, 3) : \text{Cartesian coord. on } \mathbb{R}^{1,3}$$

$$q^\mu(w, \bar{w}) = (1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w})$$

Spin-one: solutions to 4d Maxwell's equations

$$A_{\mu;a}^{\Delta, \pm}(X^\mu; w, \bar{w}) = \frac{\Delta - 1}{\Delta \Gamma(\Delta)(\mp i)^\Delta} V_{\mu;a}^{\Delta, \pm} + \partial_\mu \alpha_a^{\Delta, \pm}$$

•

$\uparrow a=w \text{ or } a=\bar{w}$
 positive/negative helicity

\pm : outgoing
 $-$: incoming

$$\begin{aligned} V_{\mu;a}^{\Delta, \pm} &= \Gamma(\Delta)(\mp i)^\Delta \frac{\partial_a q_\mu}{(-q \cdot X \mp i\varepsilon)^\Delta} \\ &= \partial_a q_\mu \int_0^\infty d\omega \omega^{\Delta-1} e^{\pm i\omega q \cdot X - \varepsilon\omega} \end{aligned}$$

$\underbrace{\quad}_{\text{Mellin transform of the plane wave}}$

$$\alpha_a^{\Delta, \pm} = \frac{\partial_a q \cdot X}{\Delta(-q \cdot X \mp i\varepsilon)^\Delta}$$

\uparrow
 residual gauge parameter

Pasterski, Shao
de Boer, Solodukhin

Celestial primaries

Spin-one: solutions to **4d Maxwell's equations** $\nabla^\rho \nabla_\rho A_{\mu;a}^{\Delta,\pm} = 0$

$$A_{\mu;a}^{\Delta,\pm} = \frac{\Delta - 1}{\Delta \Gamma(\Delta)(\mp i)^\Delta} V_{\mu;a}^{\Delta,\pm} + \partial_\mu \alpha_a^{\Delta,\pm}$$

- Celestial wavefunctions **transform** as $SL(2, \mathbb{C})$ **primaries**

$$A_{\mu;a}^{\Delta,\pm} \left(\Lambda_\nu^\rho X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}} \right) = (cw+d)^{2h} (\bar{c}\bar{w}+\bar{d})^{2\bar{h}} \Lambda_\mu^\sigma A_{\sigma;a}^{\Delta,\pm}(X^\rho; w, \bar{w})$$

of **weights** $(h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$.

- Their conformal **shadows** are also conformal primary wavefunctions

$$\widetilde{A}_{\mu;\bar{a}}^{\Delta,\pm} = (-X_\pm^2)^{1-\Delta} A_{\mu;a}^{2-\Delta,\pm} \equiv \tilde{A}_{\mu;a}^{\tilde{\Delta},\pm}$$

$\tilde{\Delta} = 2 - \Delta$

Pasterski, Shao
de Boer, Solodukhin

Celestial primaries & asymptotic symmetries

Spin-one:

$$A_{\mu;a}^{\Delta,\pm} = \frac{\Delta - 1}{\Delta \Gamma(\Delta)(\mp i)^\Delta} V_{\mu;a}^{\Delta,\pm} + \partial_\mu \alpha_a^{\Delta,\pm} \quad (h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

In the *conformally soft limit* $\Delta \rightarrow 1$, the celestial photon becomes **pure gauge**:
it is the Goldstone mode of **large $U(1)$ symmetry on the celestial sphere.** L.D., Strominger, Puhm

$$A_z^{\Delta=1} = \partial_z \epsilon + \dots \quad (h, \bar{h}) = (1, 0)^\bullet \quad \epsilon = \frac{1}{z - w}$$

Celestial primaries & asymptotic symmetries

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$$A_z^{\Delta=1} = \partial_z \epsilon + \dots \quad (h, \bar{h}) = (1, 0) \quad \epsilon = \frac{1}{z - w}$$

$$J(z)\mathcal{O}_\Delta(w, \bar{w}) \sim \frac{1}{z - w}\mathcal{O}_\Delta(w, \bar{w}) \quad \text{Celestial Kac-Moody current (equiv. to the soft photon theorem)}$$



Celestial primaries & asymptotic symmetries

Spin-one:

$$A_{\mu;a}^{\Delta,\pm} = \frac{\Delta - 1}{\Delta \Gamma(\Delta)(\mp i)^\Delta} V_{\mu;a}^{\Delta,\pm} + \partial_\mu \alpha_a^{\Delta,\pm} \quad (h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

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$$A_z^{\Delta=1} = \partial_z \epsilon + \dots \quad (h, \bar{h}) = (1, 0) \quad \epsilon = \frac{1}{z - w}$$

•

$$J(z) \mathcal{O}_\Delta(w, \bar{w}) \sim \frac{1}{z - w} \mathcal{O}_\Delta(w, \bar{w}) \quad \begin{array}{l} \text{Celestial Kac-Moody current} \\ (\text{equiv. to the soft photon theorem}) \end{array}$$

Computing the inner product at null infinity, we land on the **soft photon charge**

$$i(A, A^{\Delta=1})_{\mathcal{I}^+} = Q_\epsilon^{soft} [\epsilon = \frac{1}{z - w}] \quad Q_\epsilon^{soft} = -\frac{1}{e^2} \int du d^2 z (\partial_z \epsilon F_{u\bar{z}}^{(0)} + \partial_{\bar{z}} \epsilon F_{uz}^{(0)}) .$$

Celestial primaries

Spin-two: solutions to **4d linearized Einstein's equations** $\nabla^\rho \nabla_\rho h_{\mu\nu;a}^{\Delta,\pm} = 0$

$$h_{\mu\nu;a}^{\Delta,\pm} = \frac{\Delta - 1}{\Delta + 1} \frac{\epsilon_{\mu\nu;a}}{(-q \cdot X \mp i\varepsilon)^\Delta} + \nabla_\mu \xi_{\nu;a}^\Delta + \nabla_\nu \xi_{\mu;a}^\Delta \quad (h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

In the *conformally soft limit* $\Delta \rightarrow 1$, the celestial graviton becomes a **pure diffeomorphism**: it is the Goldstone mode of **supertranslation** symmetry on the celestial sphere.

$$h_{z\bullet}^{\Delta=1} = r C_{zz} + \dots \quad C_{zz} = -2 D_z^2 f \quad f = -\frac{(\bar{z} - \bar{w})}{4(z - w)(1 + z\bar{z})}$$

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Interestingly, the descendant of this mode gives rise to an operator which is the **supertranslation current** of the leading soft graviton theorem. Its OPE with an operator is:

• $P(z)\mathcal{O}_\Delta(w, \bar{w}) \sim \frac{1}{z - w}\mathcal{O}_{\Delta+1}(w, \bar{w})$ L.D., Strominger, Puhm

Celestial primaries & asymptotic symmetries

Spin-one:

$$A_{\mu;a}^{\Delta,\pm} = \frac{\Delta - 1}{\Delta \Gamma(\Delta)(\mp i)^\Delta} V_{\mu;a}^{\Delta,\pm} + \partial_\mu \alpha_a^{\Delta,\pm} \quad (h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

In the *conformally soft limit* $\Delta \rightarrow 1$, the celestial photon becomes **pure gauge**:
it is the Goldstone mode of **large $U(1)$ symmetry on the celestial sphere**. L.D., Strominger, Puhm

$$A_z^{\Delta=1} = \partial_z \epsilon + \dots \quad (h, \bar{h}) = (1, 0) \quad \epsilon = \frac{1}{z-w}$$

$$J(z)\mathcal{O}_\Delta(w, \bar{w}) \sim \frac{1}{z-w}\mathcal{O}_\Delta(w, \bar{w}) \quad \begin{array}{l} \text{Celestial Kac-Moody current} \\ (\text{equiv. to the soft photon theorem}) \end{array}$$

Computing the inner product at null infinity, we land on the **soft photon charge**

$$i(A, A^{\Delta=1})_{\mathcal{I}^+} = Q_\epsilon^{soft}[\epsilon = \frac{1}{z-w}] \quad Q_\epsilon^{soft} = -\frac{1}{e^2} \int du d^2 z (\partial_z \epsilon F_{u\bar{z}}^{(0)} + \partial_{\bar{z}} \epsilon F_{uz}^{(0)}) .$$

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Superrotations: Virasoro or $\text{Diff}(S^2)$?

Barnich, Troessaert '08

Kapec, Lysov, Pasterski, Strominger '14

Campiglia, Laddha '14

Virasoro Ward ID

$$\langle \text{out} | [Q_{Y(z)}, \mathcal{S}] | \text{in} \rangle = 0$$

not bijective

$\text{Diff}(S^2)$ Ward ID

$$\langle \text{out} | [Q_{Y(z, \bar{z})}, \mathcal{S}] | \text{in} \rangle = 0$$

enlarged phase space required

Subleading Soft Graviton Theorem

$$\lim_{\omega \rightarrow 0} (1 + \omega \partial_\omega) \langle \text{out} | \mathfrak{a}(q) \mathcal{S} | \text{in} \rangle = S^{(1)} \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

Cachazo, Strominger '14

Celestial gravitons: $\tilde{\Delta} = 2$ and $\Delta_{\bullet} = 0$

L.D., Pasterski, Puhr ‘20

These **2 modes** reduce to **pure diffeomorphisms** (‘Goldstone modes’):

Shadow primary with $\tilde{\Delta} = 2$

$$\tilde{h}_{\mu\nu;a}^{\tilde{\Delta}=2} = \nabla_\mu \xi_{\nu;a} + \nabla_\nu \xi_{\mu;a} \rightarrow \tilde{C}_{zz;ww}^2 = -uD_z^3 Y_{ww}^z \text{ with } Y_{ww}^z = \frac{1}{6(z-w)}$$

$$\square \xi = 0$$

Primary with $\Delta = 0$

$$h_{\mu\nu;a}^{\Delta=0} = \nabla_\mu \zeta_{\nu;a} + \nabla_\nu \zeta_{\mu;a} \rightarrow C_{zz;\bar{w}\bar{w}}^0 = -uD_z^3 Y_{\bar{w}\bar{w}}^z \text{ with } Y_{\bar{w}\bar{w}}^z = -\frac{(z-w)^2}{2(\bar{z}-\bar{w})}$$

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Virasoro (superrotations): the CKV condition is violated at **isolated points**

$$\mathcal{L}_Y \gamma_{AB} \underset{\bullet}{\sim} \delta^{(2)}(z-w)$$

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$$\square \zeta = 0$$

Diff(S²): modifies the **celestial sphere metric**



$$\mathcal{L}_Y \gamma_{AB} \neq 0$$

Superrotations and $\text{Diff}(\text{S}^2)$ soft charges

LD, Pasterski, Puhm

Iyer-Wald symplectic structure

$$\Omega[\delta g, h'; g] = \int_{\mathcal{I}^+} \omega[\delta g, h'; g] \quad h' = h^{\Delta=0} \text{ or } h' = \tilde{h}^{\tilde{\Delta}=2}$$

Because the Bondi **fall-offs** are **violated**, a **renormalization** procedure is required

[Compère, Fiorucci, Ruzziconi '18]

$$\Omega^{ren}[\delta g, h'; g] \rightarrow \delta Q_Y^{soft}$$

We find

$$Q_Y^{soft} = \frac{1}{16\pi G} \int du d^2z \sqrt{\gamma} u D_z^3 Y^z N^{zz}$$

Kapec, Lysov, Pasterski, Strominger
Compère, Fiorucci, Ruzziconi
Distler, Flauger, Horn
Campiglia, Laddha

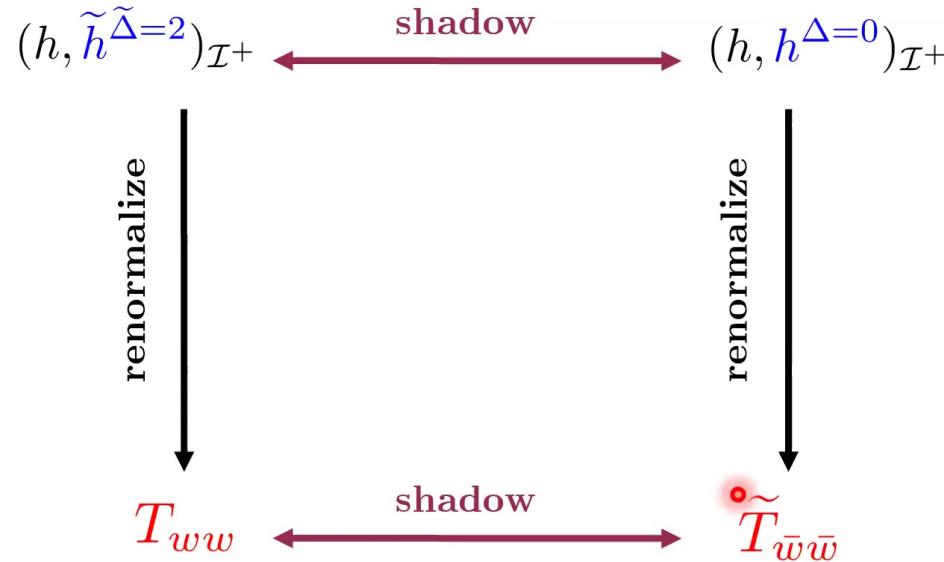
$$h' = \tilde{h}^{\tilde{\Delta}=2} \rightarrow Q_Y^{soft} = -\frac{1}{16\pi G} \int du d^2z \sqrt{\gamma} \frac{1}{(z-w)^4} u N^{zz} = T_{ww} \quad (2,0) \text{ stress tensor}$$

cf. Kapec, Mitra, Raclaru, Strominger

$$h' = h^{\Delta=0} \rightarrow Q_Y^{soft} = \int d^2w' \frac{(w-w')^2}{(\bar{w}-\bar{w}')^2} T_{w'w'} = \tilde{T}_{\bar{w}\bar{w}} \quad (-1,1) \text{ shadow stress tensor}$$

*I have dropped numerical factors

Summary



Virasoro superrotations and **Diff(S²)** are on **equal footing** via the **shadow transform**.

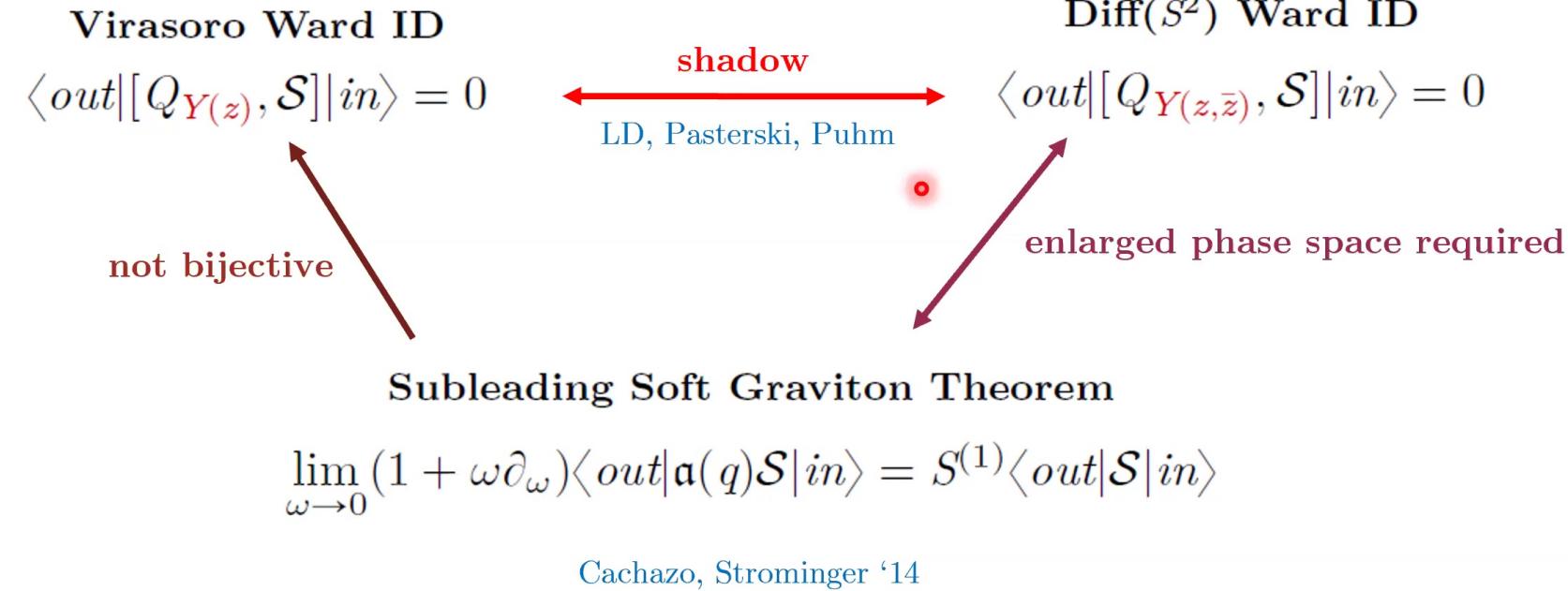
LD, Pasterski, Puhm

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Celestial primaries

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$$A_{\mu;a}^{\Delta,\pm} = \frac{\Delta - 1}{\Delta \Gamma(\Delta)(\mp i)^\Delta} V_{\mu;a}^{\Delta,\pm} + \partial_\mu \alpha_a^{\Delta,\pm}$$

- Celestial wavefunctions **transform** as $SL(2, \mathbb{C})$ **primaries**

$$A_{\mu;a}^{\Delta,\pm} \left(\Lambda_\nu^\rho X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}} \right) = (cw+d)^{2h} (\bar{c}\bar{w}+\bar{d})^{2\bar{h}} \Lambda_\mu^\sigma A_{\sigma;a}^{\Delta,\pm}(X^\rho; w, \bar{w})$$

of **weights** $(h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$.

- Their conformal **shadows** are also conformal primary wavefunctions

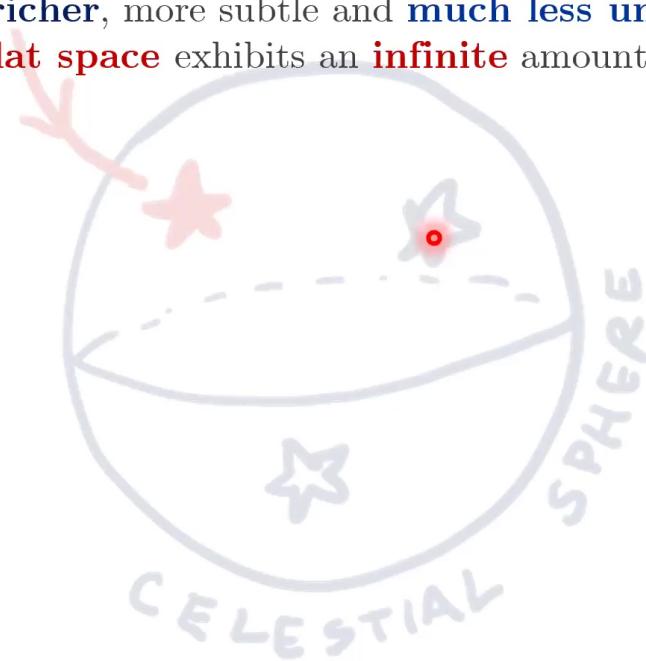
$$\widetilde{A}_{\mu;\bar{a}}^{\Delta,\pm} = (-X_\pm^2)^{1-\Delta} A_{\mu;a}^{2-\Delta,\pm} \equiv \widetilde{A}_{\mu;a}^{\tilde{\Delta},\pm}$$

$\tilde{\Delta} = 2 - \Delta$

Pasterski, Shao
de Boer, Solodukhin

Conclusions

- Physics in the deep infrared is **much richer**, more subtle and **much less understood** than we previously thought. The boundary of **flat space** exhibits an **infinite** amount of **symmetries**.



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- Physics in the deep infrared is **much richer**, more subtle and **much less understood** than we previously thought. The boundary of **flat space** exhibits an **infinite** amount of **symmetries**.
- We have a unified treatment of **conformally soft** Goldstone modes of asymptotic symmetries for celestial holography.
- Properties of celestial CFTs have just started to be unraveled...

They **share** some properties with usual CFTs, but also differ from the latter in ways that are not yet understood. They are subject to **multiple infinities** of **asymptotic symmetry constraints** beyond the ones coming from conformal invariance. *These constraints have no analogs in conventional CFTs.* (e.g. supertranslation symmetry)

Further explorations are keys towards a **holographic formulation of flat spacetimes**.

e.g.: subleading soft photon, sub-subleading soft graviton theorem,... [L.D., Pasterski, Puhm, to appear]

Thank you very much!