Title: Beyond BCS: An Exact Model for Superconductivity and Mottness

Speakers: Philip Phillips

Series: Quantum Matter

Date: October 27, 2020 - 3:30 PM

URL: http://pirsa.org/20100065

Abstract: High-temperature superconductivity in the cuprates remains an unsolved problem because the cuprates start off their lives as Mott insulators in which no organizing principle such a Fermi surface can be invoked to treat the electron interactions. Consequently, it would be advantageous to solve even a toy model that exhibits both Mottness and superconductivity. In 1992 Hatsugai and Khomoto wrote down a momentum-space model for a Mott insulator which is safe to say was largely overlooked, their paper garnering just 21 citations (6 due to our group). I will show exactly[1] that this model when appended with a weak pairing interaction exhibits not only the analogue of Cooper's instability but also a superconducting ground state, thereby demonstrating that a model for a doped Mott insulator can exhibit superconductivity. The properties of the superconducting state differ drastically from that of the standard BCS theory. The elementary excitations of this superconductor are not linear combinations of particle and hole states but rather are superpositions of doublons and holons, composite excitations signaling that the superconducting ground state of the doped Mott insulator inherits the non-Fermi liquid character of the normal state.

Additional unexpected features of this model are that it exhibits a superconductivity-induced transfer of spectral weight from high to low energies and a suppression of the superfluid density as seen in the cuprates.

[1] https://www.nature.com/articles/s41567-020-0988-4.

Superconductivity and Mottness: Exact Results

arXiv: 1912.01008
https://www.nature.com/articles/s41567-020-0988-4
with N&V by J. Zaanen

Luke Yeo

Edwin Huang

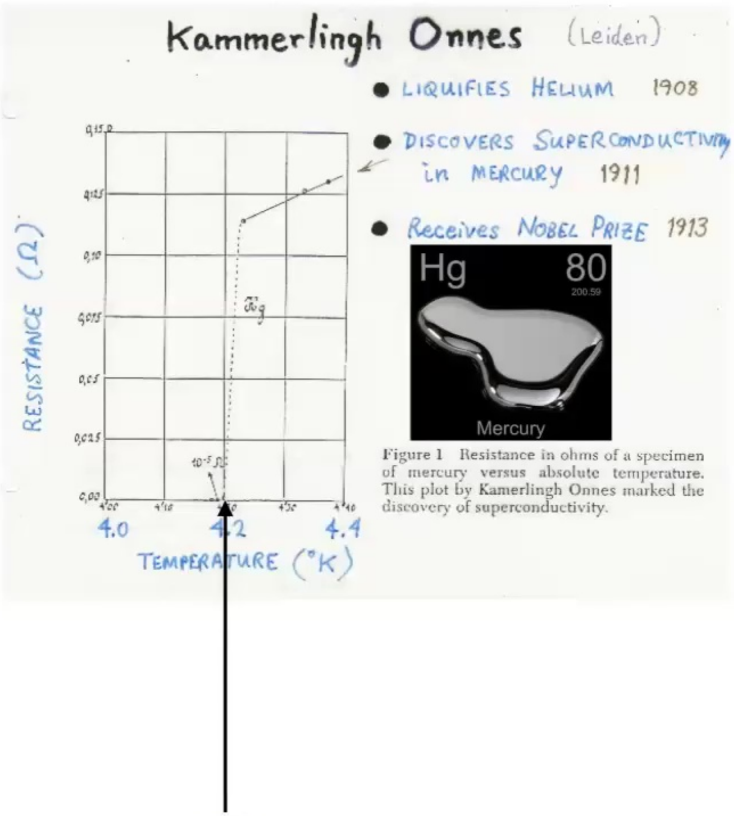
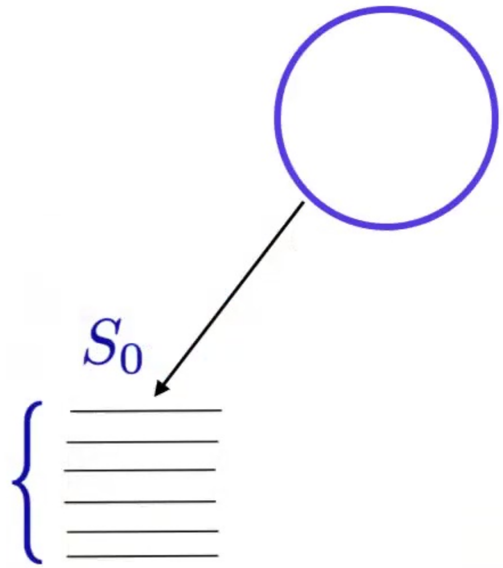


Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

T_c

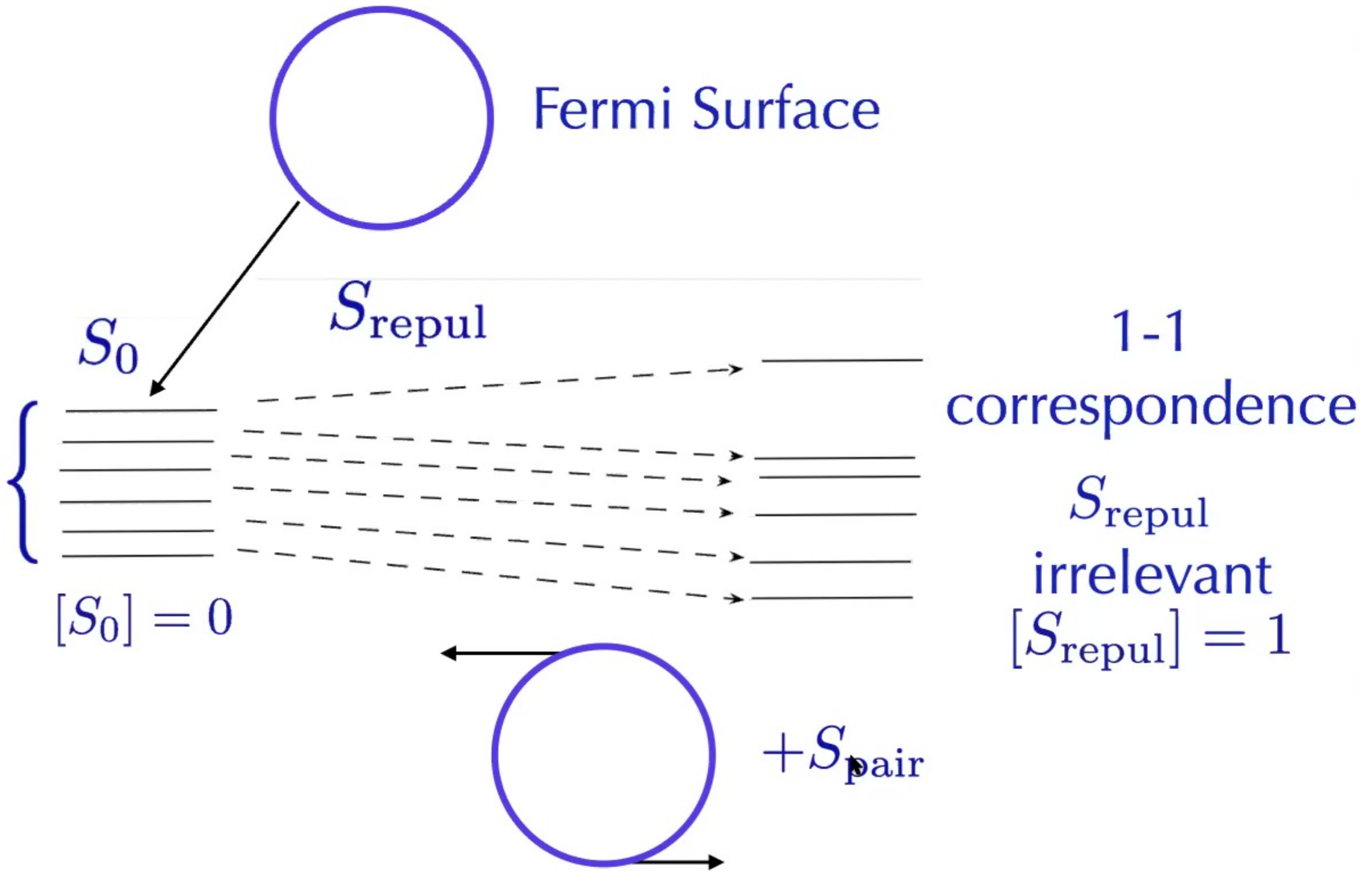
FL $\rightarrow$ BCS



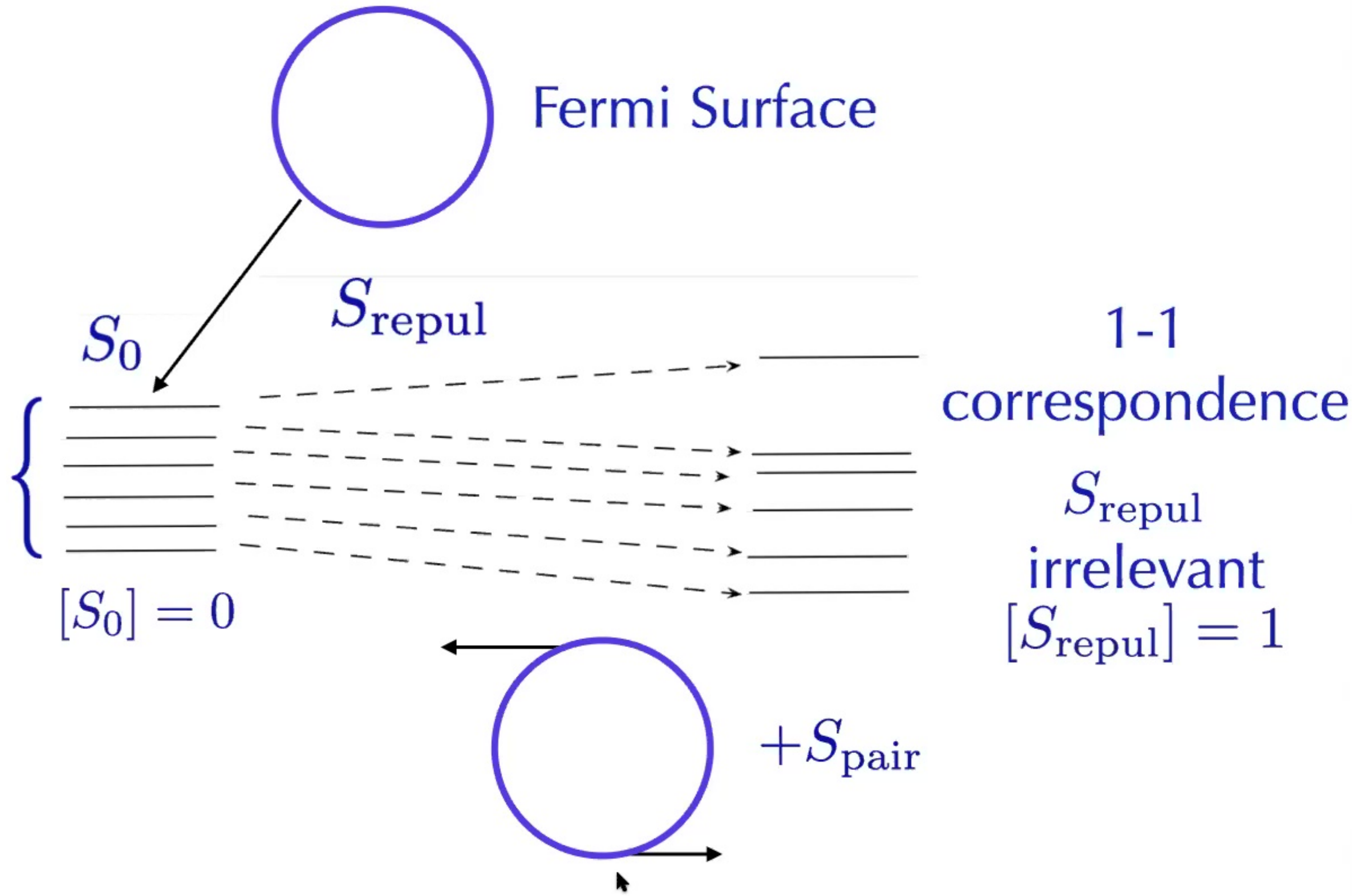
$[S_0] = 0$

$[S_{\mathrm{repul}}] = 1$

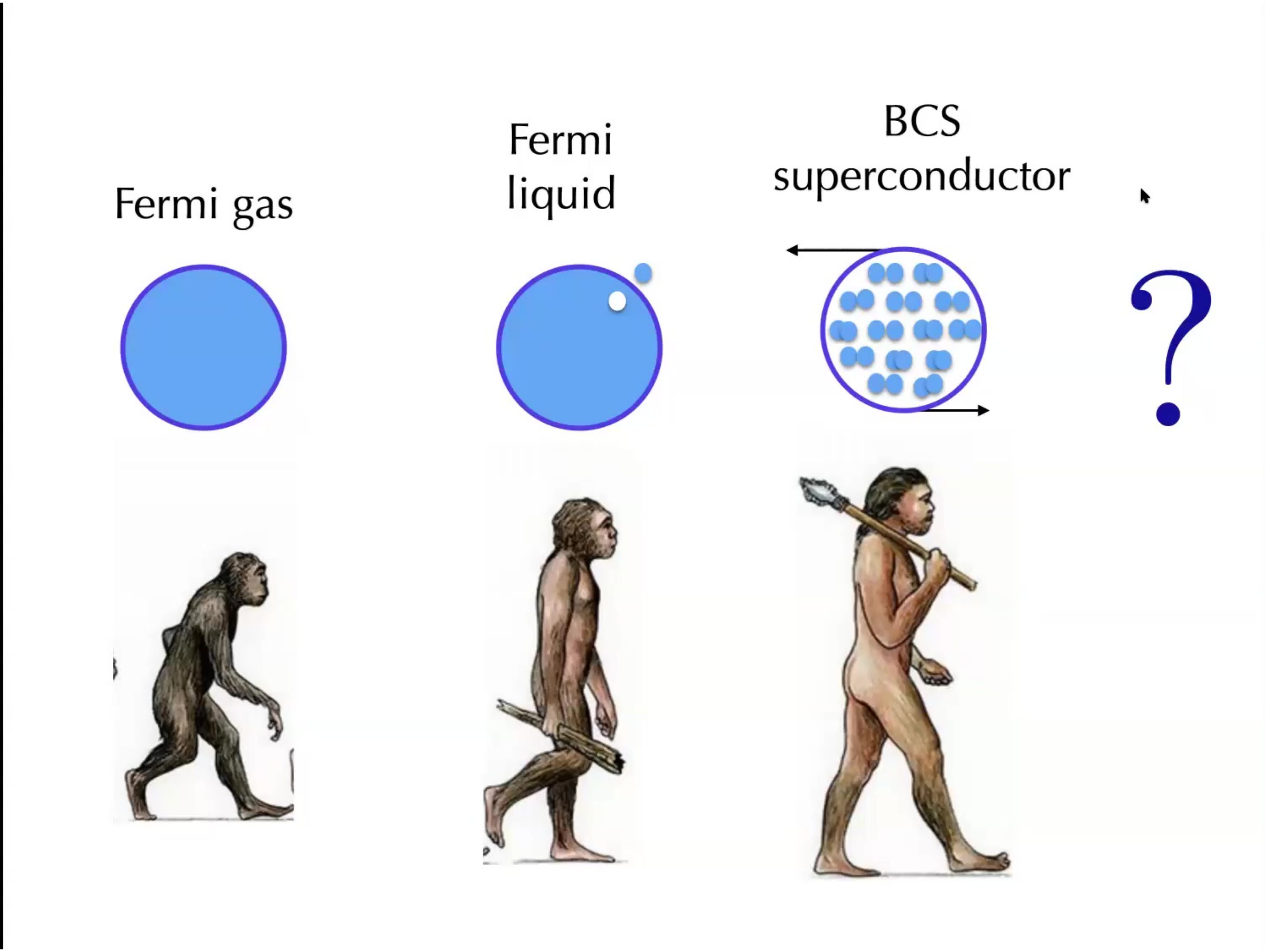
$\mathrm{FL} \rightarrow \mathrm{BCS}$



$\mathrm{FL} \to \mathrm{BCS}$

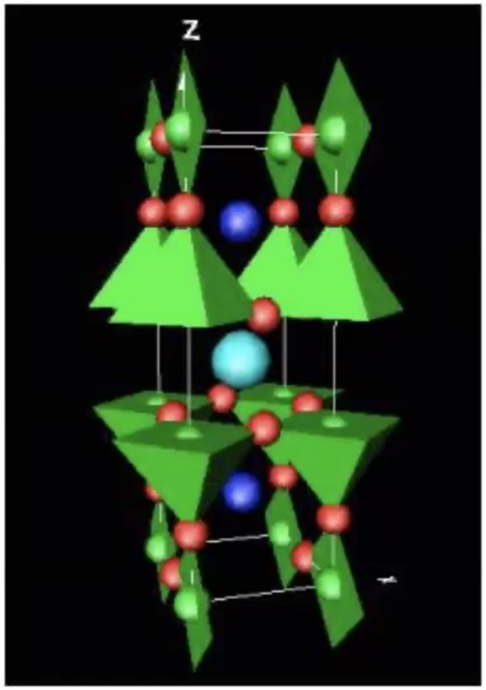


superconductivity $\frac{2\Delta}{T_c} = 3.5$

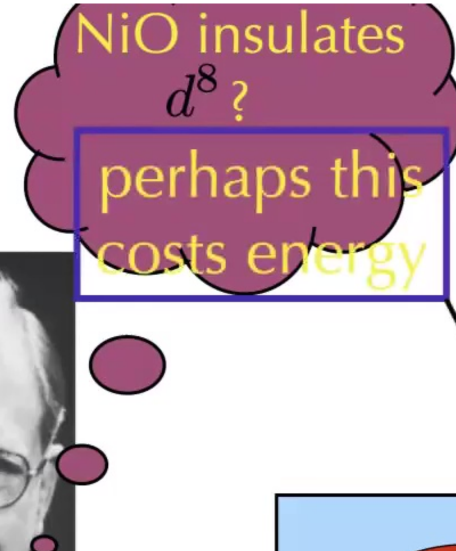

Fermi gas
Fermi liquid
BCS superconductor
?

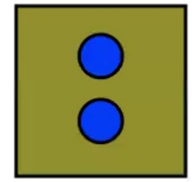
Is there physics beyond BCS?

fixed point beyond FL?

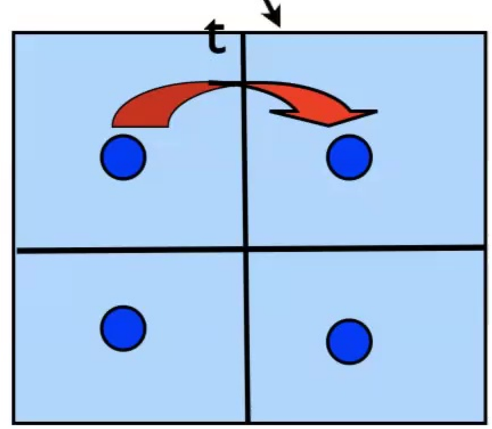


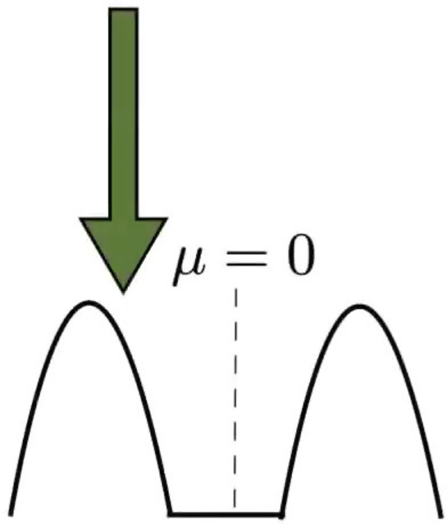
$Y\,Ba_2Cu_3O_7$
Cuprate Superconductors

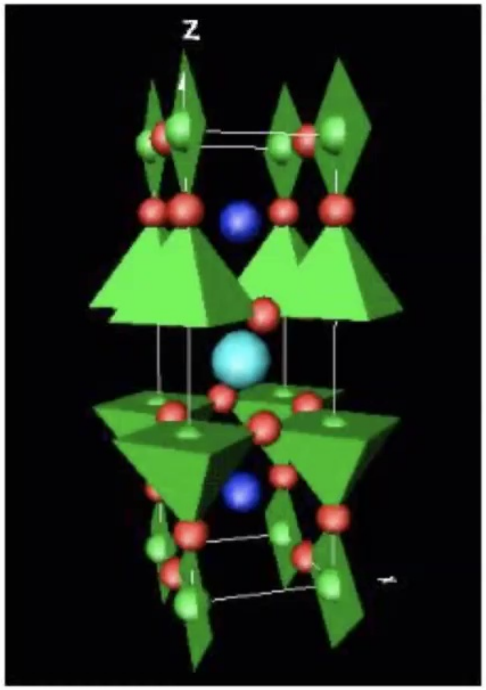




$$U \gg t$$







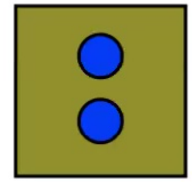
$Y\,Ba_2Cu_3O_7$

Cuprate Superconductors

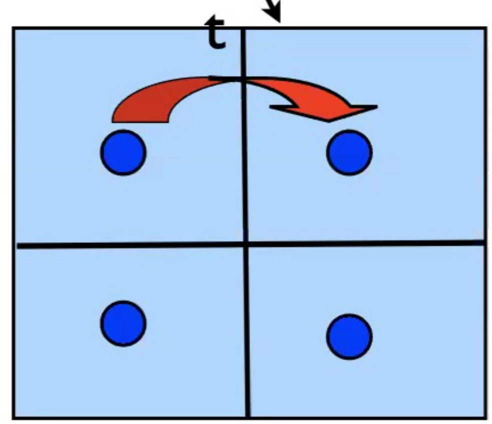
NiO insulates

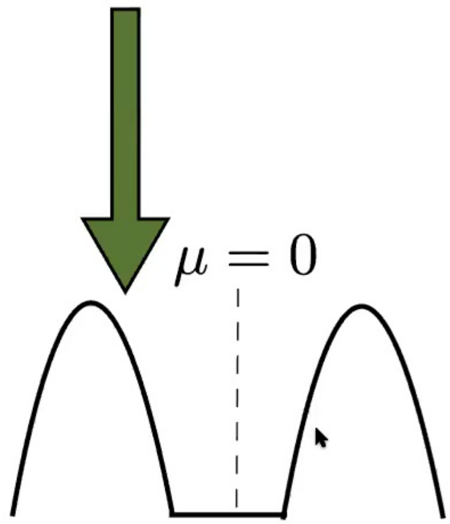
d^8 ?

perhaps this costs energy



$U \gg t$





no change in size of Brillouin zone

solve the Hubbard Model!!

Cooper instability??

solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

R. B. Laughlin
Departrment of Physics
Stanford University
Stanford, California 94305

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

A Critique of "A Critique of Two Metals"

Philip W. Anderson and G. Baskaran

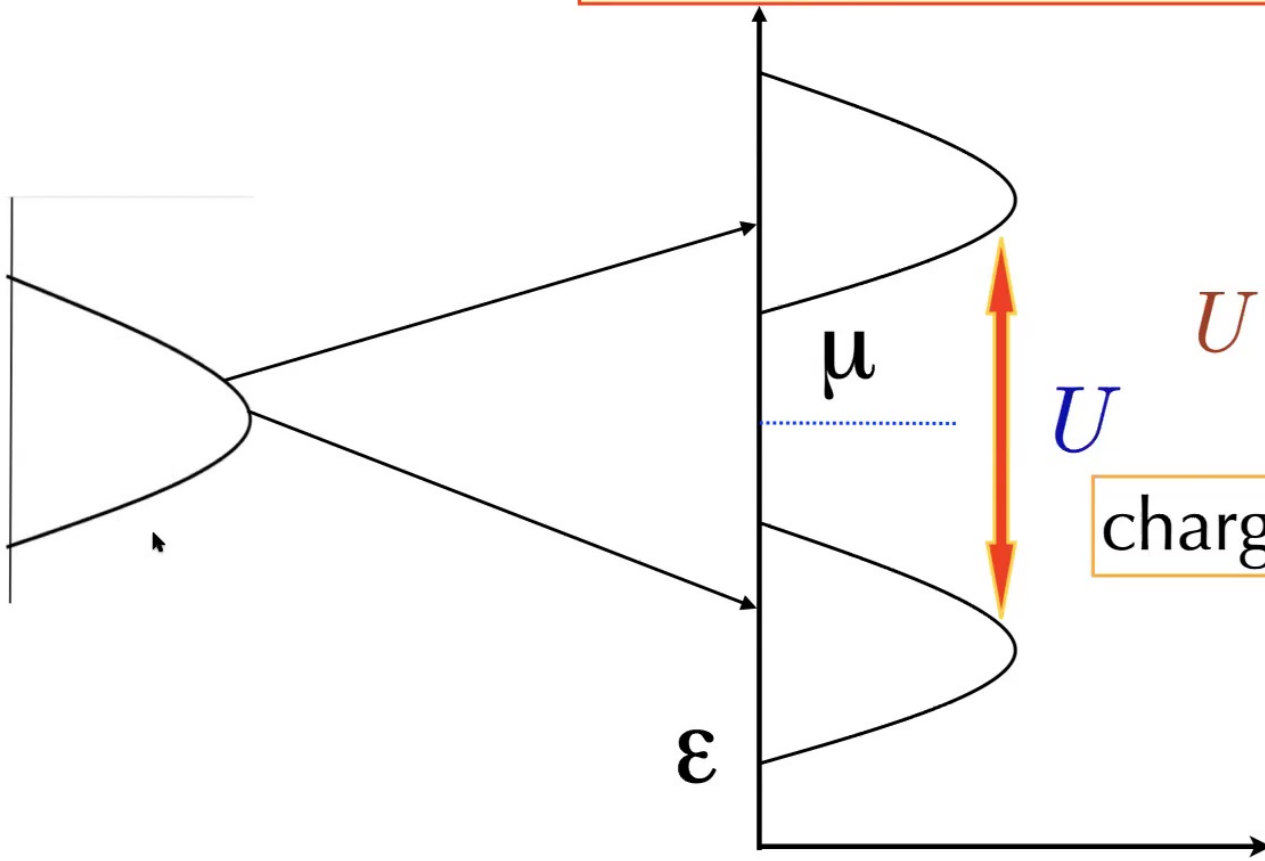
Joseph Henry Laboratories of Physics

Princeton University, Princeton, NJ 08544

The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott

Laughlin's objection:

gap with no symmetry breaking not demonstrated!!

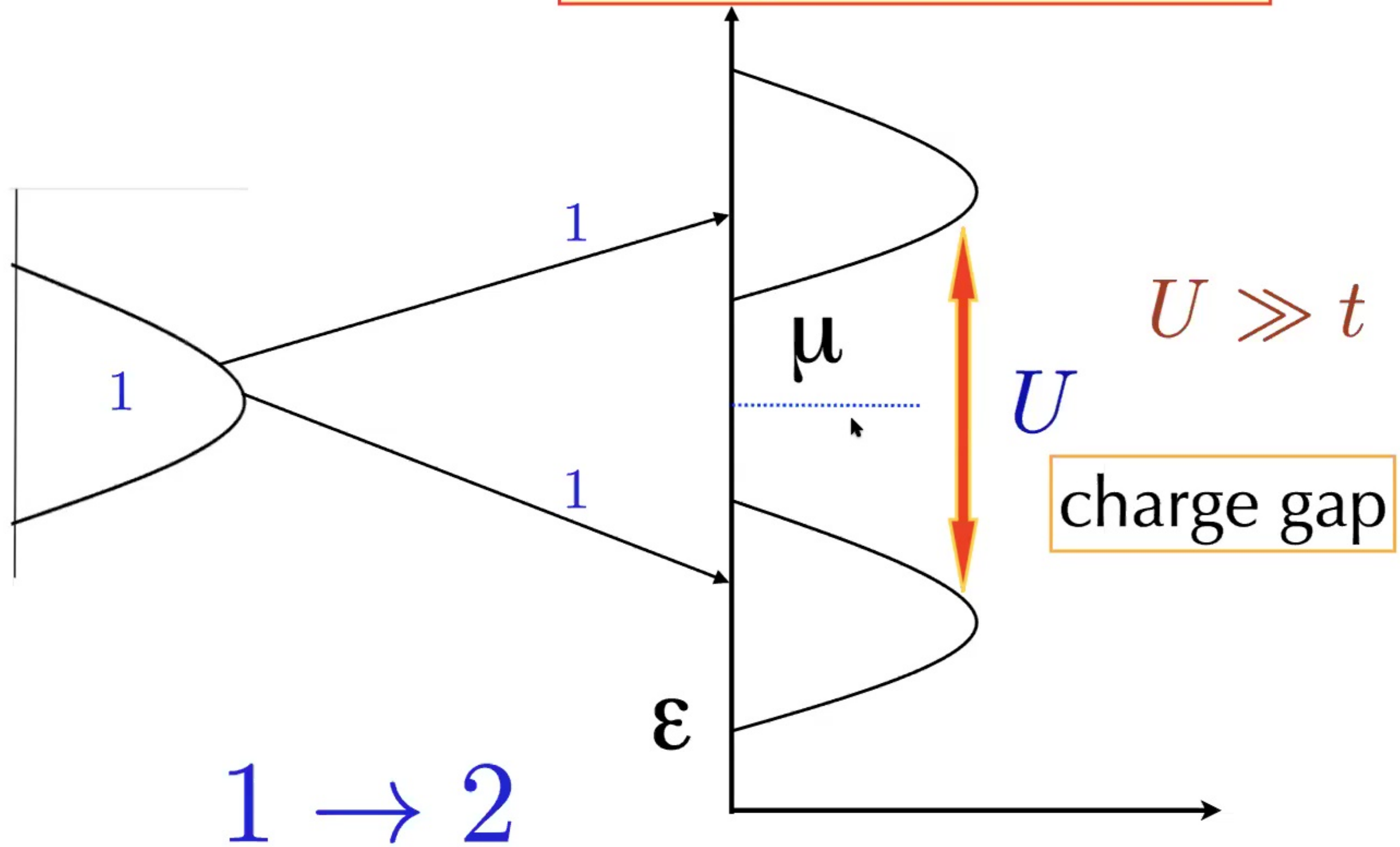


$U \gg t$

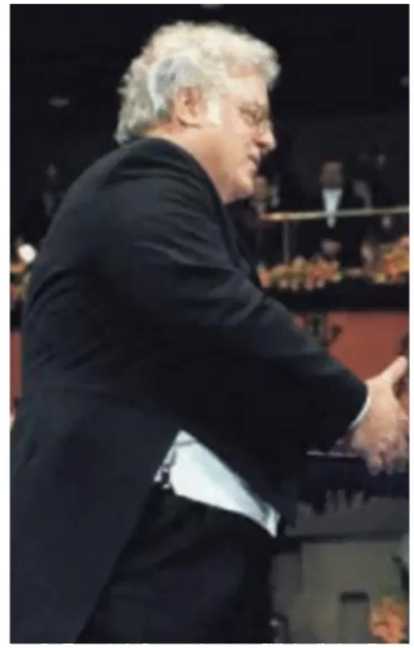
charge gap

Laughlin's objection:

gap with no symmetry breaking not demonstrated!!



$-$



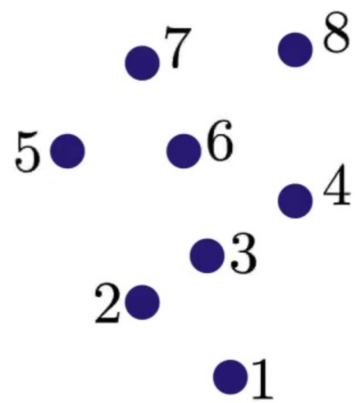
$=$ **Mottness**

zeros

$= 0$ $\neq 0$

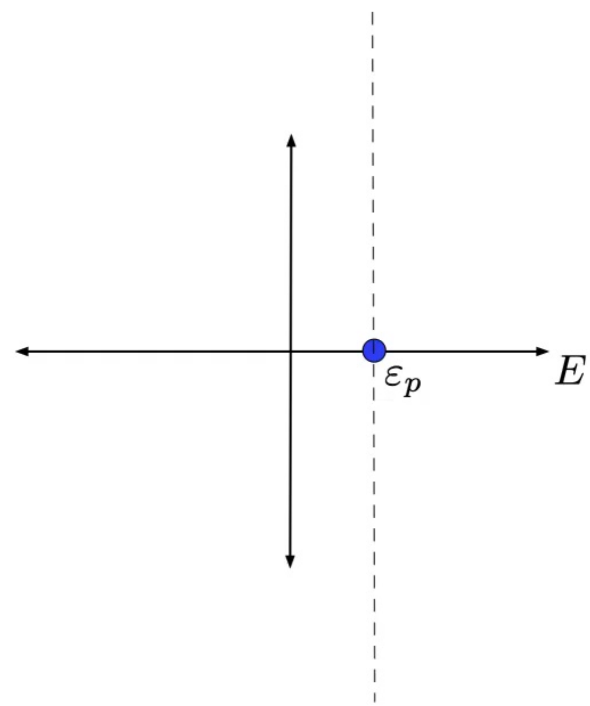
$$\mathrm{DetReG}(\omega = 0, \mathrm{p})$$

counting particles



Luttinger counting theorem

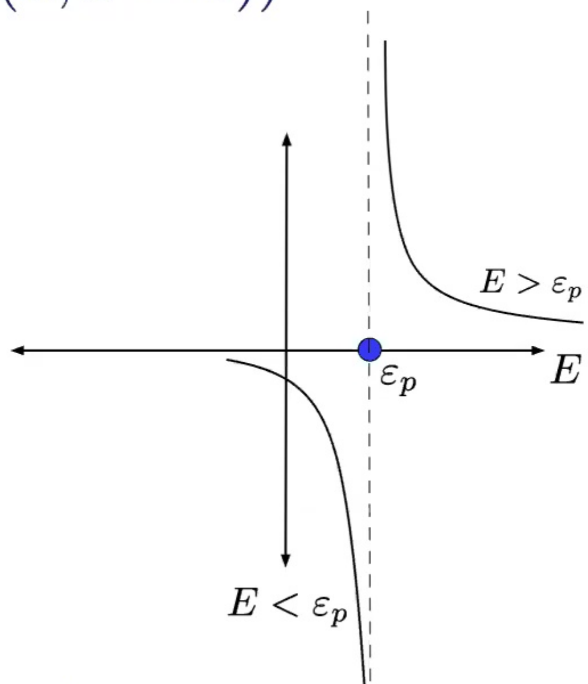
$$G(E) = \frac{1}{E - \varepsilon_p}$$



Luttinger counting theorem

$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$



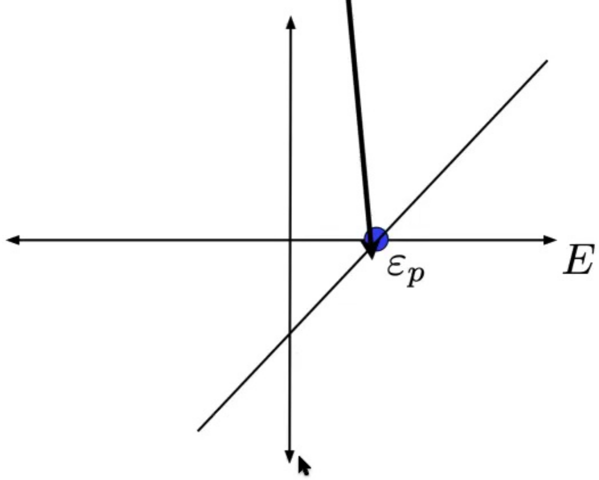
counting poles (qp)

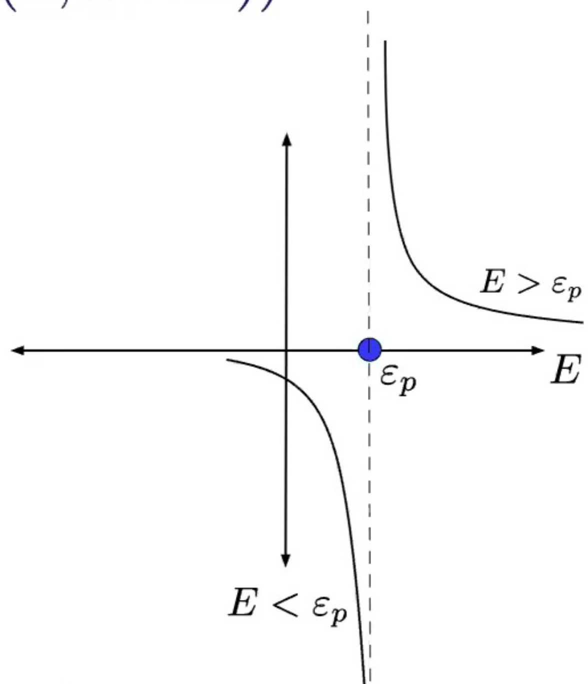
Luttinger counting theorem

$$G(E) = \frac{1}{E - \varepsilon_p}$$

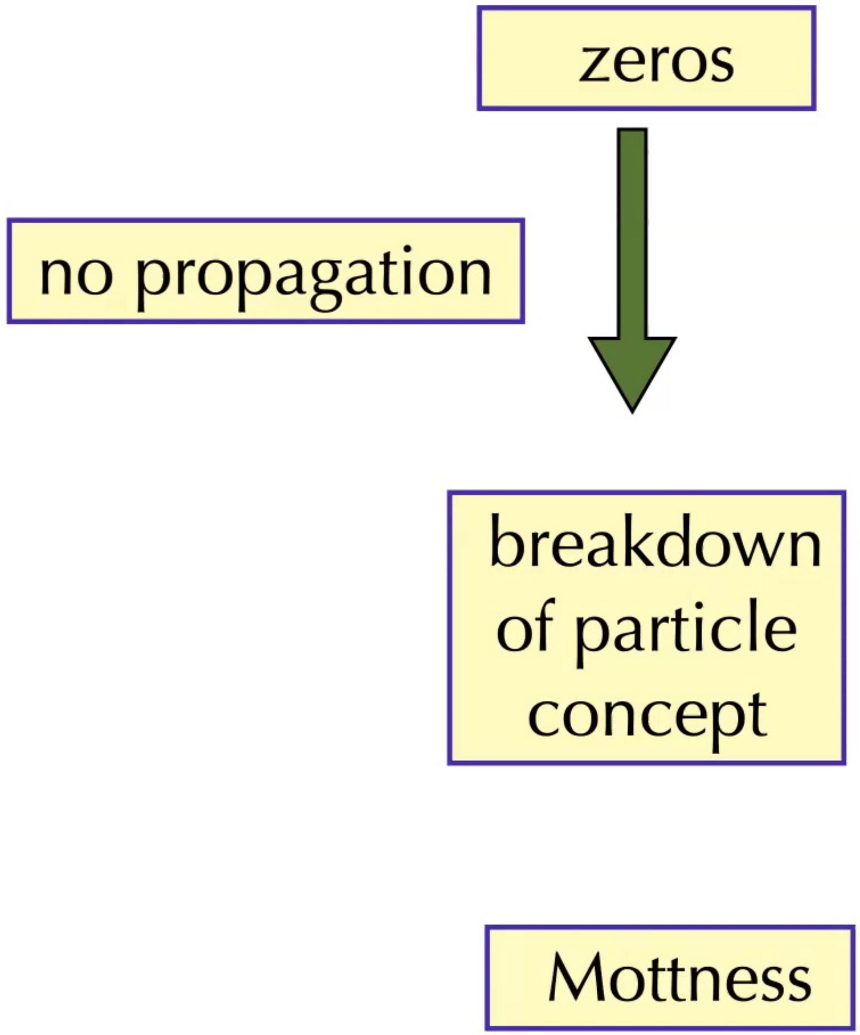
$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

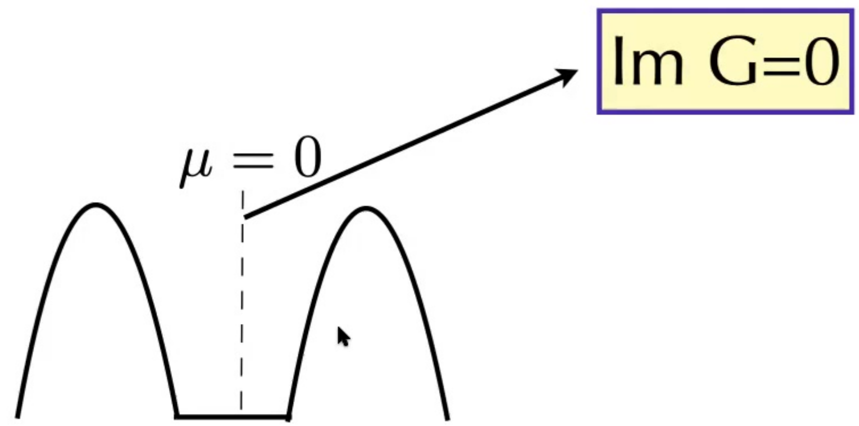




counting poles (qp)


zeros
no propagation
breakdown of particle concept
Mottness

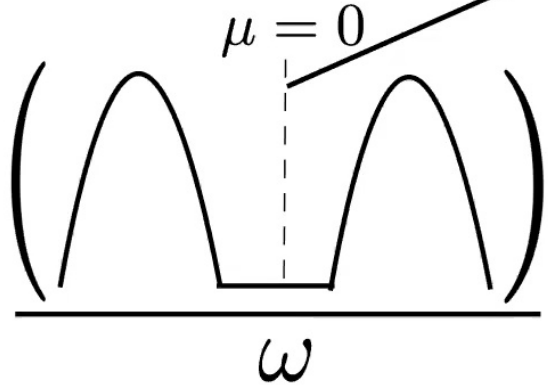
How do zeros obtain?



How do zeros obtain?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty}$$



$$d\omega$$

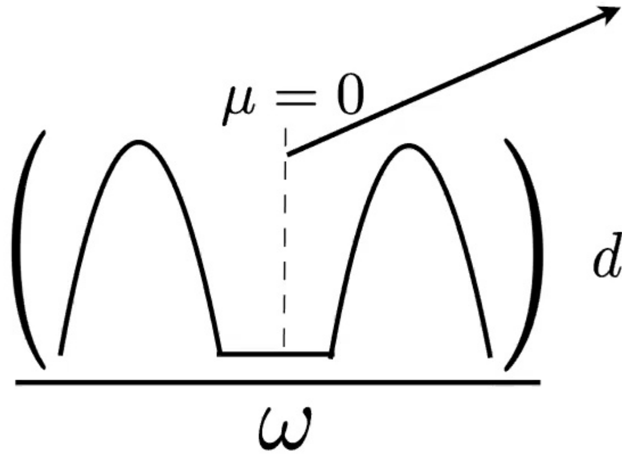
Kramers -Kronig

= below gap+above gap = 0

How do zeros obtain?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty}$$



$d\omega$

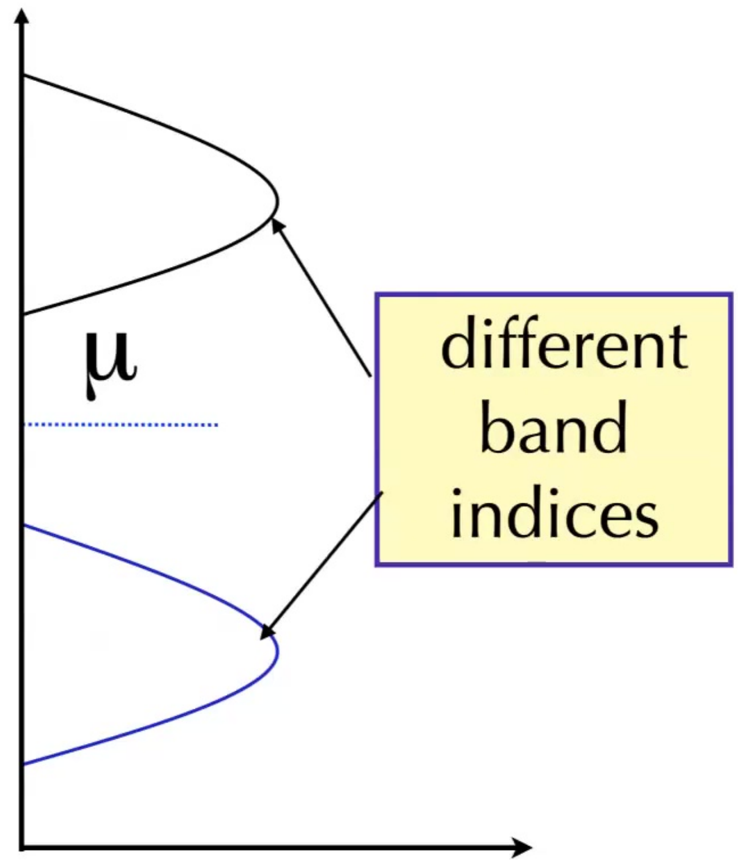
Kramers -Kronig

= below gap+above gap = 0

$\mathrm{Det}G(\mathbf{k},\omega=\mathbf{0})=\mathbf{0}$ (single band)

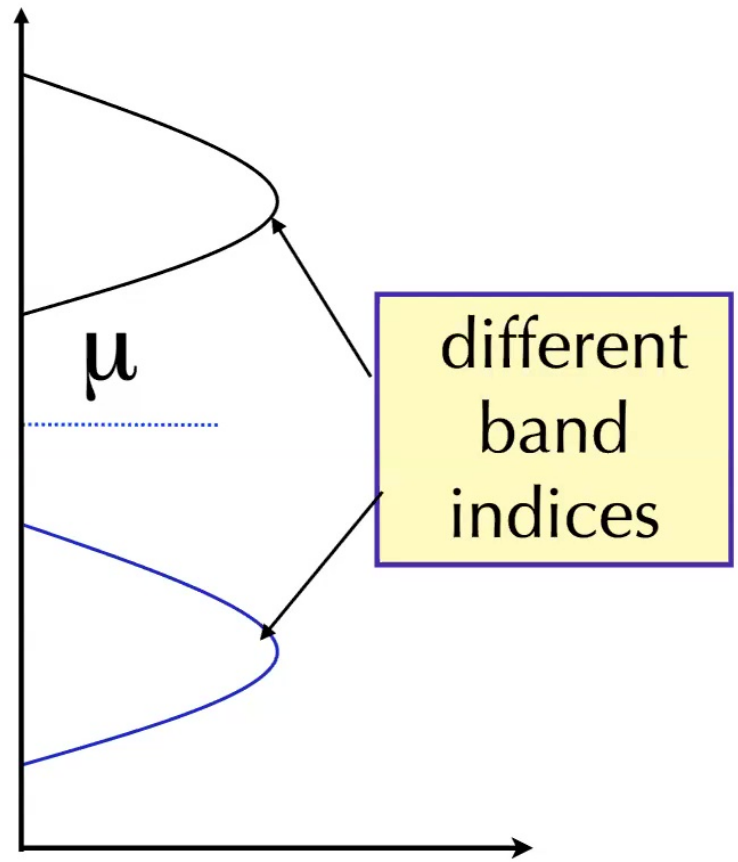
Symmetry Breaking

$$G_k(\omega) = \begin{pmatrix} \dfrac{1}{\omega - E_k^+} & 0 \\ 0 & \dfrac{1}{\omega - E_k^-} \end{pmatrix}$$



Symmetry Breaking

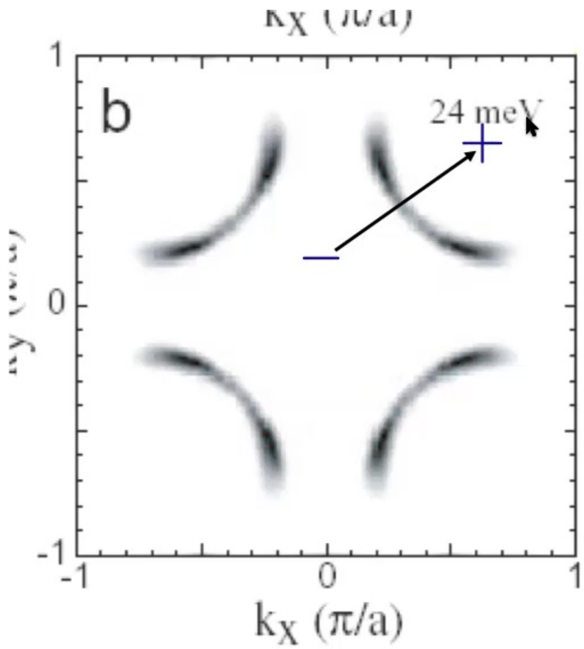
$$G_k(\omega) = \begin{pmatrix} \frac{1}{\omega - E_k^+} & 0 \\ 0 & \frac{1}{\omega - E_k^-} \end{pmatrix}$$



$$\mathrm{Det} G \neq 0$$

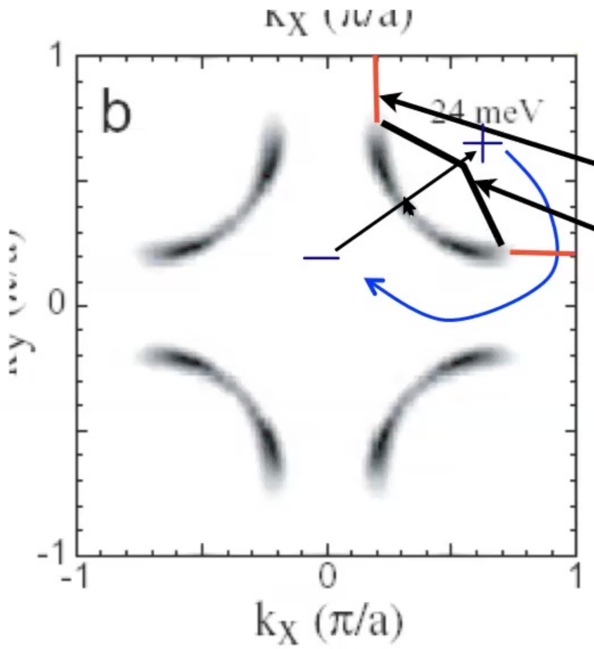
no Mottness

Fermi Arcs



Fermi Arcs

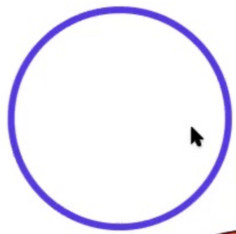
Re G
Changes
Sign across
An arc



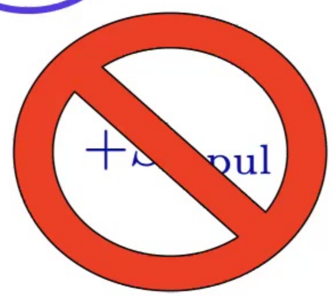
Must cross
A zero line
(DetG=0)!!!

Fermi arcs necessarily imply zeros exist.

Where's Mottness??

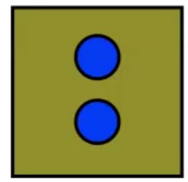


Fermi Surface

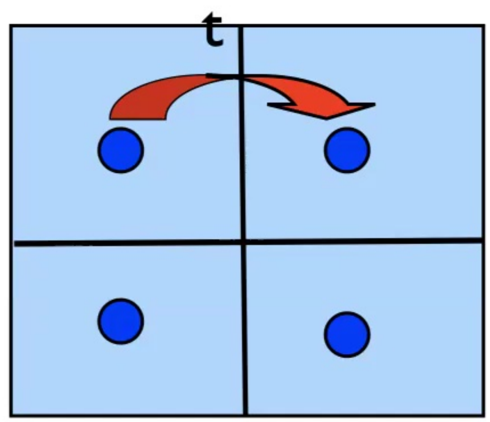


irrelevant

Hubbard Model



$$U \gg t$$



Mott insulator in momentum space???

Hatsugai-Khomoto Model (1992)

$$H_{\mathrm{HK}} = -t\sum_{\langle j,l\rangle,\sigma}\left(c^\dagger_{j\sigma}c_{l\sigma}+h.c.\right)-\mu\sum_{j\sigma}c^\dagger_{j\sigma}c_{j\sigma}$$

$$+\ \frac{U}{L^d}\sum_{j_1..j_4}\delta_{j_1+j_3,j_2+j_4}\,c^\dagger_{j_1\uparrow}c_{j_2\uparrow}c^\dagger_{j_3\downarrow}c_{j_4\downarrow}$$

Hatsugai-Khomoto Model (1992)

$$H_{\mathrm{HK}} = -t \sum_{\langle j,l\rangle,\sigma} \left(c^\dagger_{j\sigma} c_{l\sigma} + h.c. \right) - \mu \sum_{j\sigma} c^\dagger_{j\sigma} c_{j\sigma}$$

$$+ \frac{U}{L^d} \sum_{j_1 \cdot\cdot j_4} \delta_{j_1+j_3, j_2+j_4} c^\dagger_{j_1\uparrow} c_{j_2\uparrow} c^\dagger_{j_3\downarrow} c_{j_4\downarrow},$$

$$c_{k\sigma} = \sum_j e^{ikj} c_{j\sigma}$$

$$H_{\mathrm{HK}} = \sum_k H_k = \sum_k \left(\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow} \right).$$

$$\xi_k = \epsilon_k - \mu$$

$$\tilde{S}_{\text{int}} = U \int dt d\ell d^{d-1}\vec{k}\psi^\dagger_\uparrow(\vec{k})\psi_\uparrow(\vec{k})\psi^\dagger_\downarrow(\vec{k})\psi_\downarrow(\vec{k}), \quad [\tilde{S}_{\text{int}}] = -2$$

$$[n_{k\uparrow}n_{k'\downarrow}] = -1$$

FL+ $U n_{k\uparrow}n_{k'\downarrow} \neq$ F.L.

Hubbard band operators

$$c^\dagger_{k\sigma}$$

Hubbard band operators

$$c^\dagger_{k\sigma}$$

$$\zeta_{k\sigma} = c^\dagger_{k\sigma}(1 - n_{k\bar{\sigma}})$$

$+$

$$\eta_{k\sigma} = c^\dagger_{k\sigma} n_{k\bar{\sigma}}$$

$$\langle n_{k\sigma} \rangle = \frac{1}{2}$$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\omega + i0^+ - (\xi_k + U/2)}}$$

$$\Sigma(k,\omega)$$

$$\omega = \xi_k + U/2$$

$$\Re\Sigma = \Im\Sigma = \infty$$

Hubbard band operators

$c^\dagger_{k\sigma}$

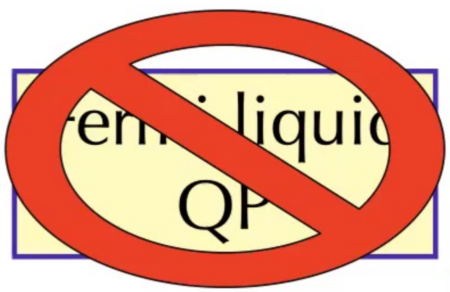
$\zeta_{k\sigma} = c^\dagger_{k\sigma}(1 - n_{k\bar{\sigma}})$ $+$ $\eta_{k\sigma} = c^\dagger_{k\sigma} n_{k\bar{\sigma}}$

$\langle n_{k\sigma} \rangle = \frac{1}{2}$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\omega + i0^+ - (\xi_k + U/2)}}$$

$\Sigma(k,\omega)$

$\omega = \xi_k + U/2$

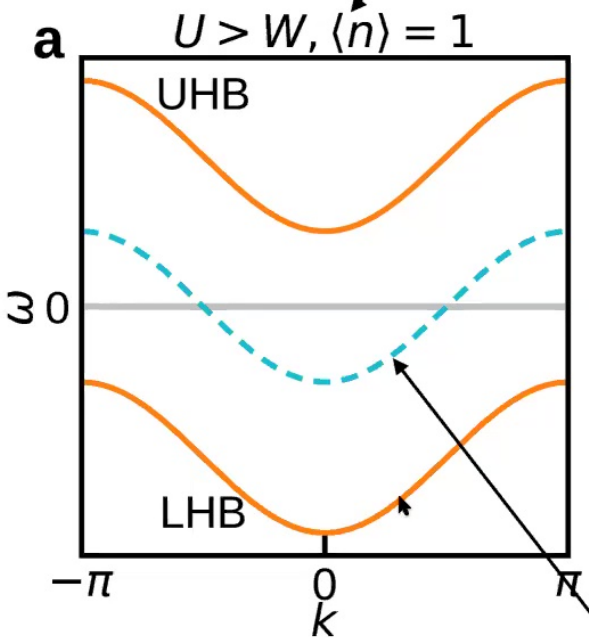


zeros

$\Re\Sigma = \Im\Sigma = \infty$

Mott transition: composite excitations

$$\Delta E = U - 4dt = U - W$$

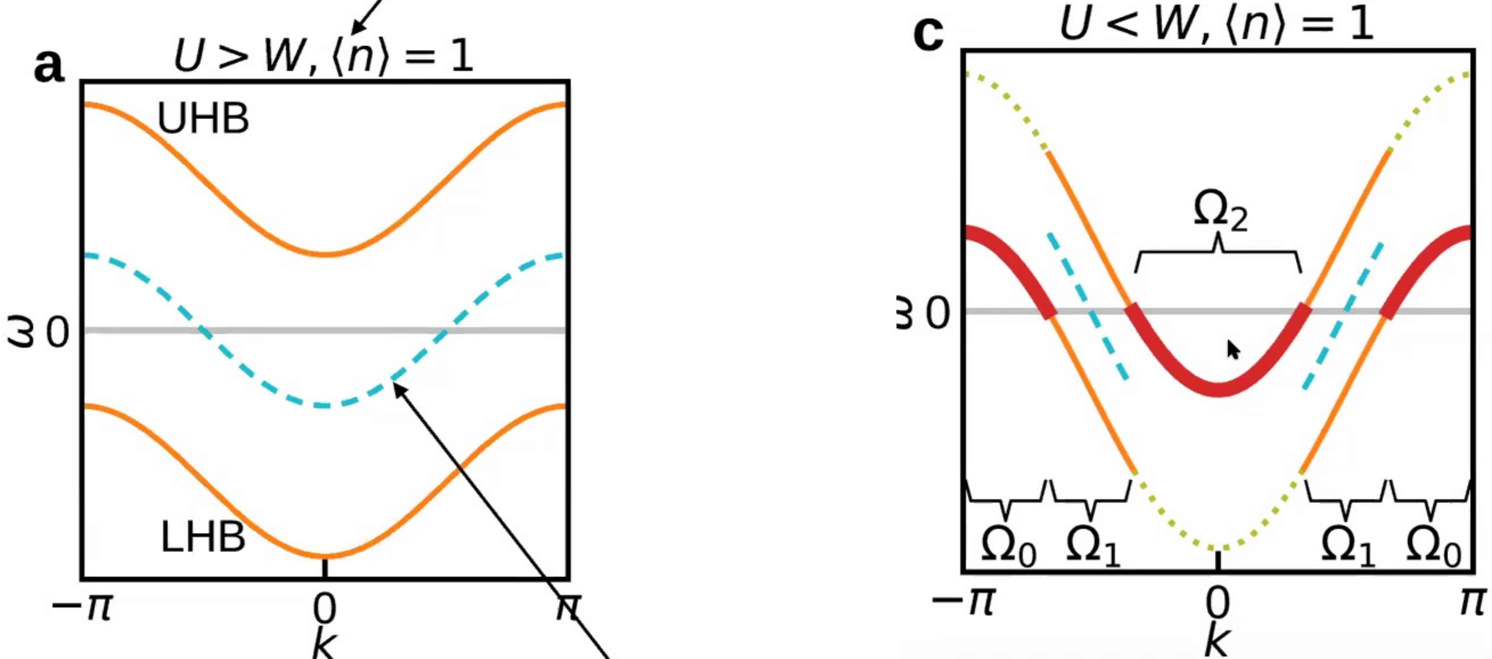


insulator

Luttinger Surface

Mott transition: composite excitations

$$\Delta E = U - 4dt = U - W$$

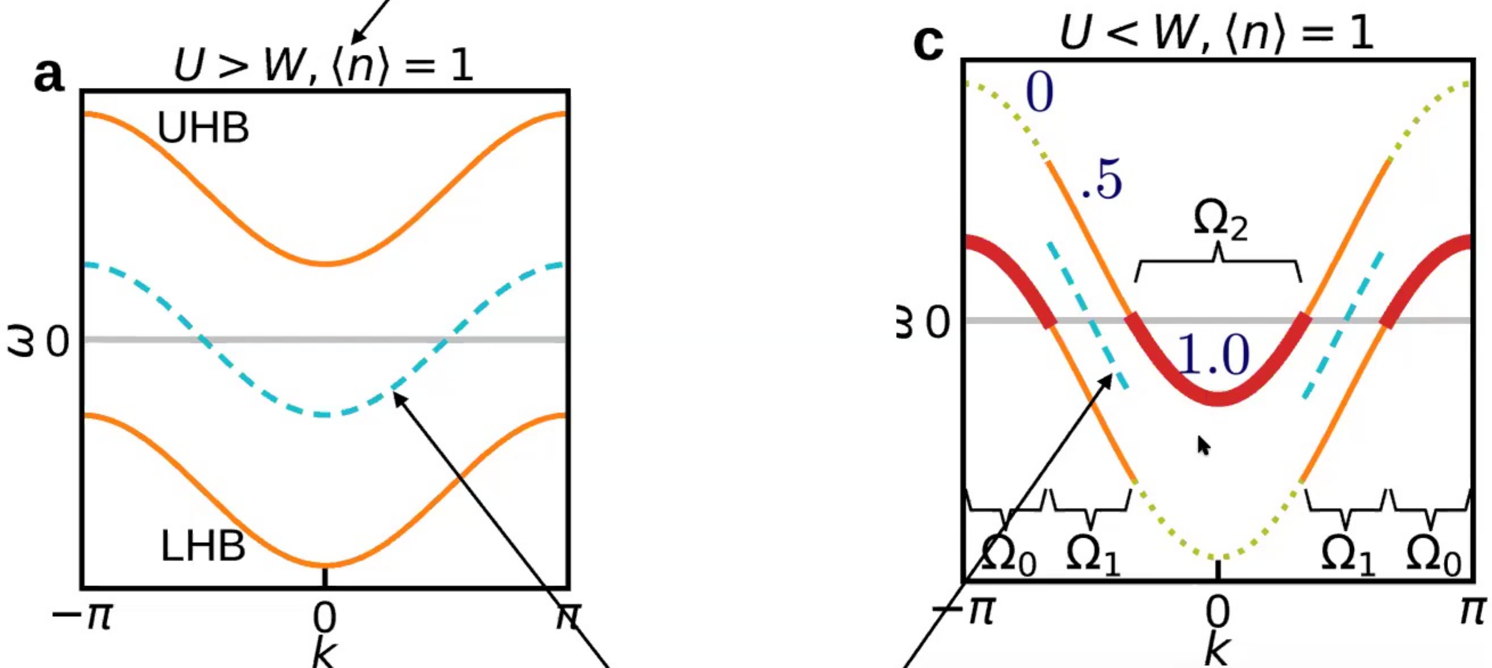


insulator

Luttinger Surface

Mott transition: composite excitations

$$\Delta E = U - 4dt = U - W$$



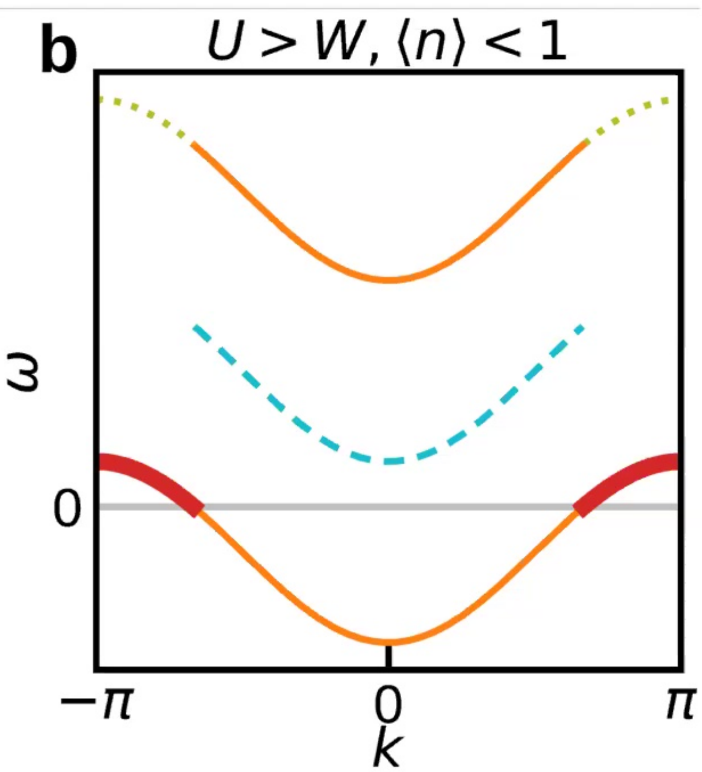
insulator

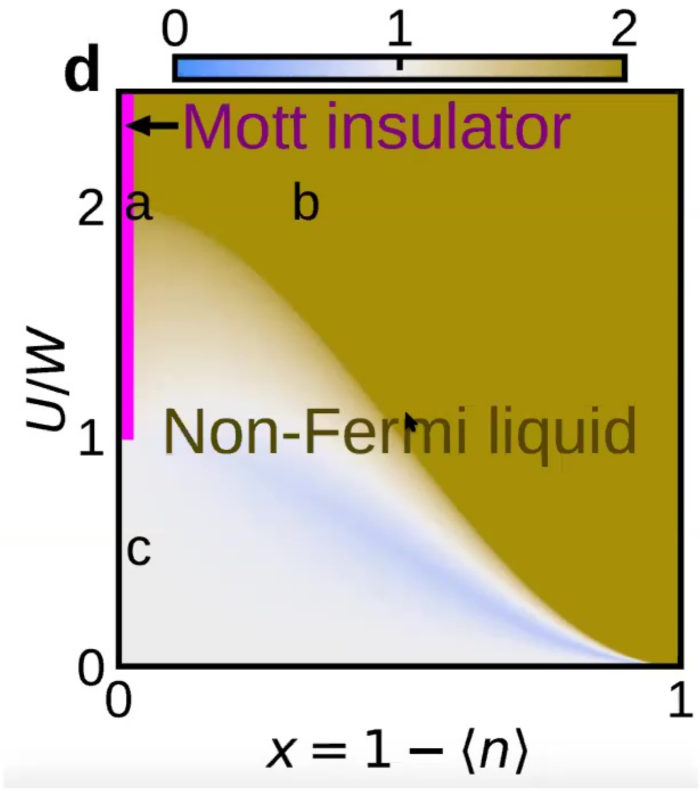
Luttinger Surface

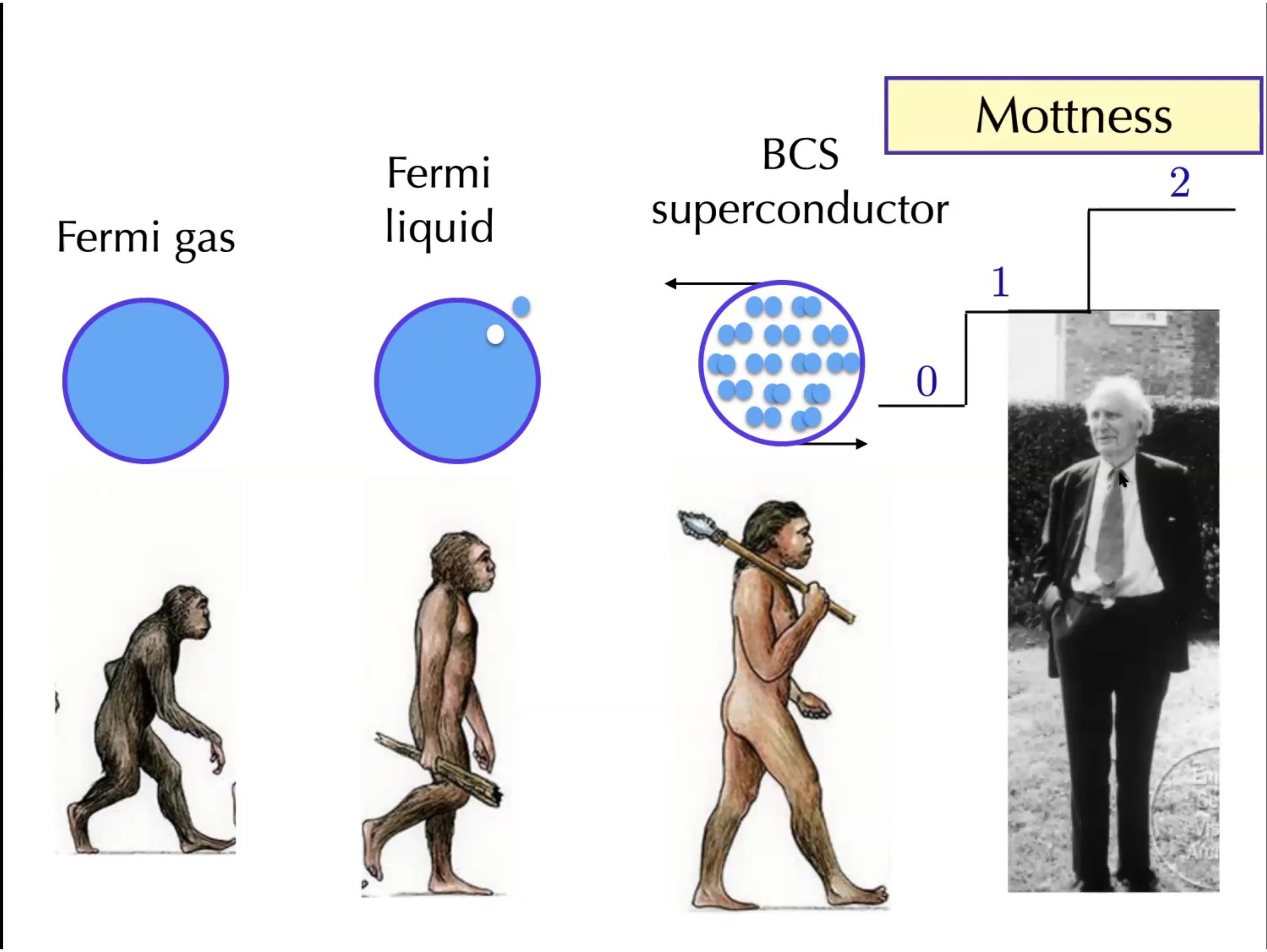
metal

metal is degenerate: SU(2) invariance

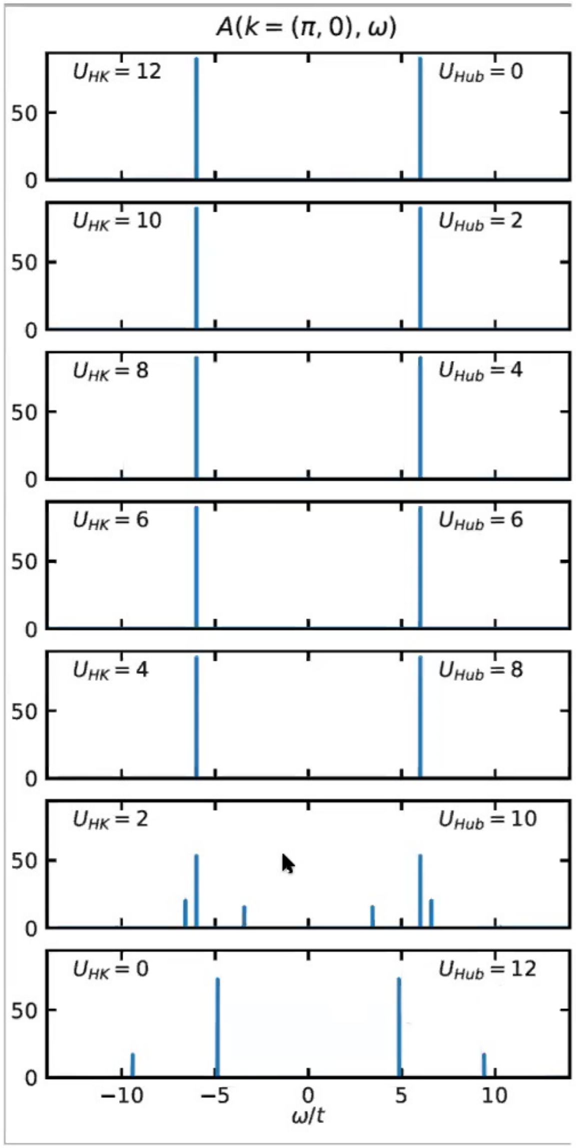
$$|\Psi_G;\{\sigma_k\}\rangle = \prod_{k\in\Omega_1} c^\dagger_{k\sigma_k} \prod_{k\in\Omega_2} c^\dagger_{k\uparrow}c^\dagger_{k\downarrow}|0\rangle,$$






Mottness
Fermi gas
Fermi liquid
BCS superconductor
0
1
2

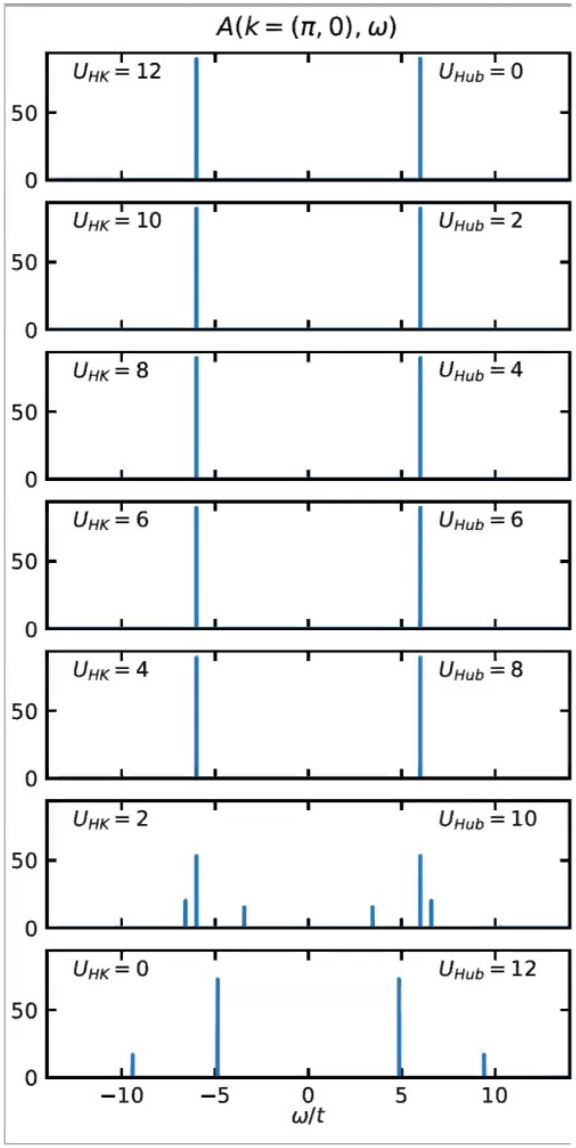
$n = 1.0$



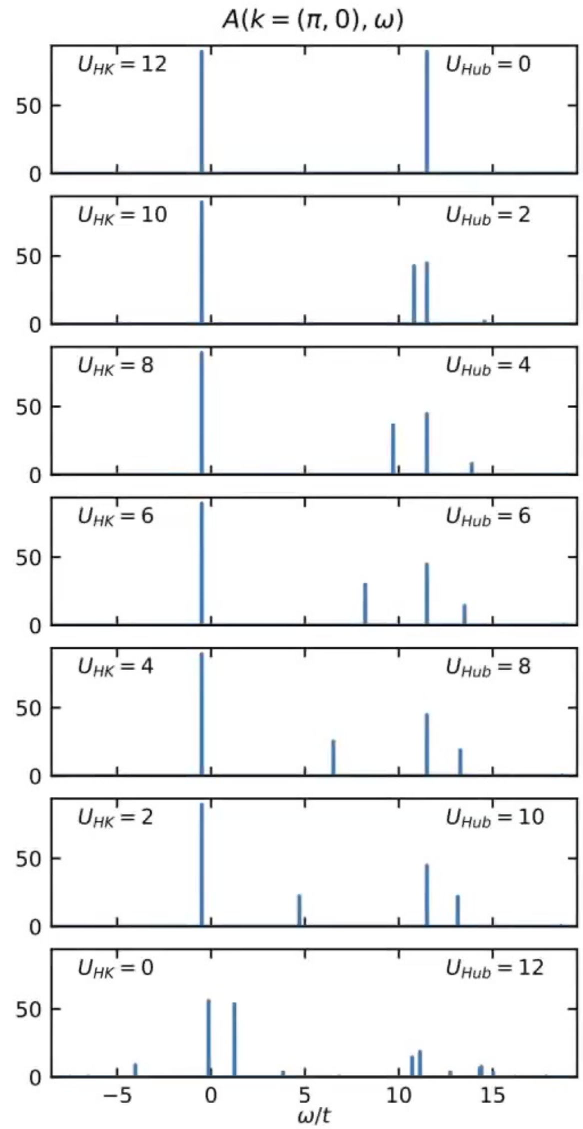
A(k = (π, 0), ω)
U_HK = 12
U_Hub = 0
U_HK = 10
U_Hub = 2
U_HK = 8
U_Hub = 4
U_HK = 6
U_Hub = 6
U_HK = 4
U_Hub = 8
U_HK = 2
U_Hub = 10
U_HK = 0
U_Hub = 12
50
0
−10
−5
0
5
10
ω/t

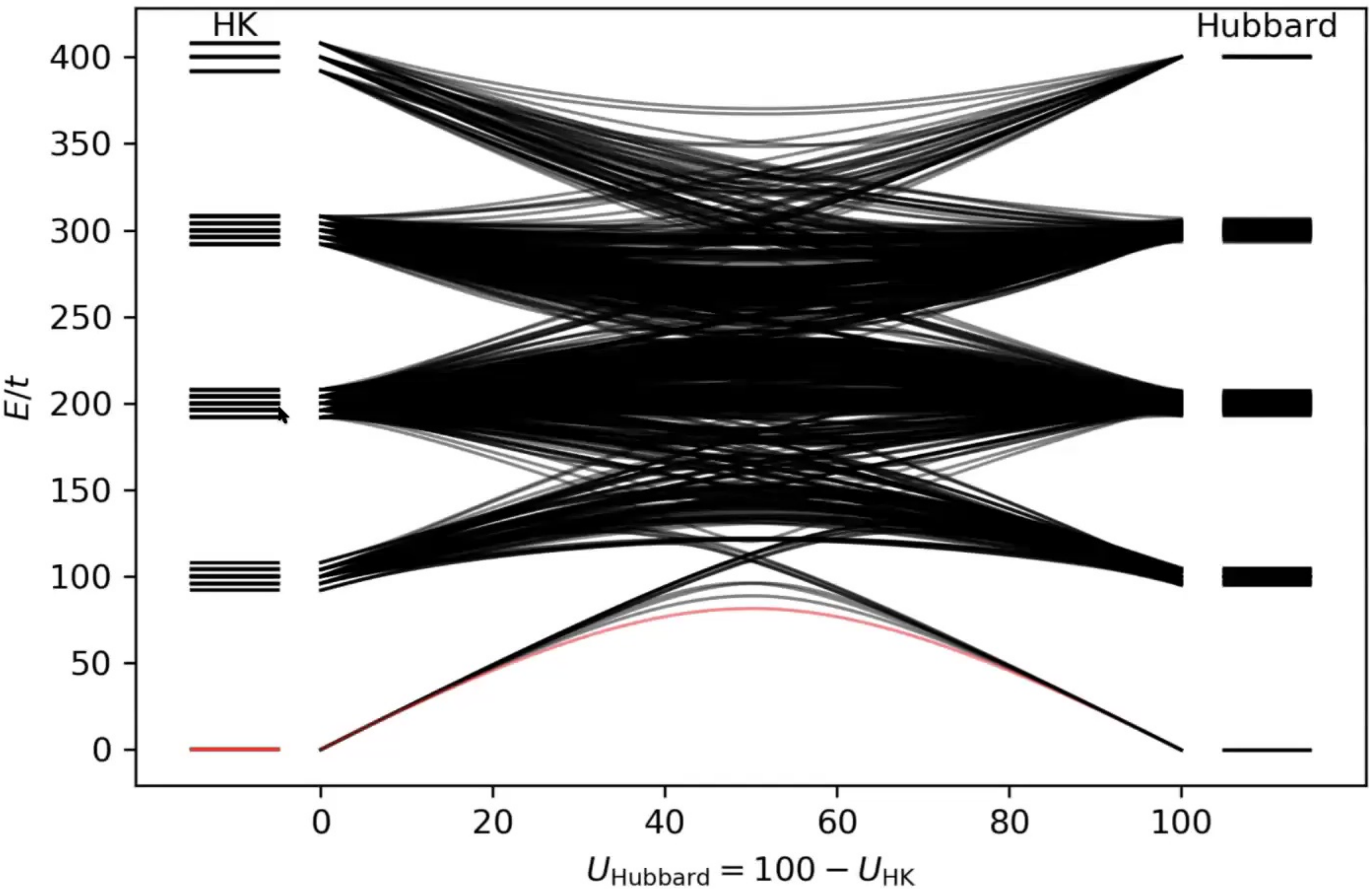
$n = 1.0$

$H_{\mathrm{HK}} \approx H_{\mathrm{Hubb}}$

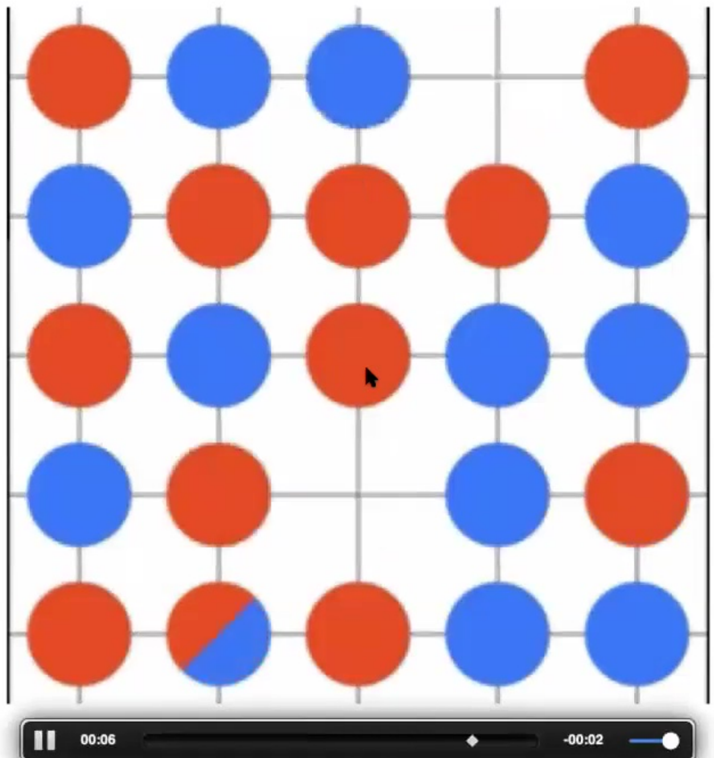


$n = 0.875$




HK
Hubbard
400
350
300
250
200
150
100
50
0
E/t
0
20
40
60
80
100
$U_{\text{Hubbard}} = 100 - U_{\text{HK}}$

what the HK model leaves out



Superdconductivity?

C. Setty
2019

ILS+ Pair
fluctuations=
SYK

Is this true in
general?

Cooper Instability

$$H = H_{\mathrm{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

$$\langle n_{k\sigma}\rangle = 0$$

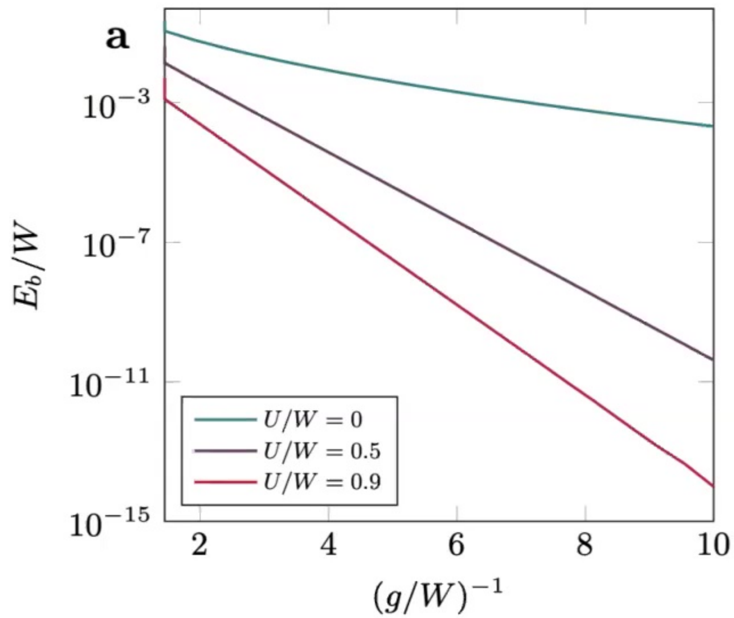
$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \lesseqgtr 0$$

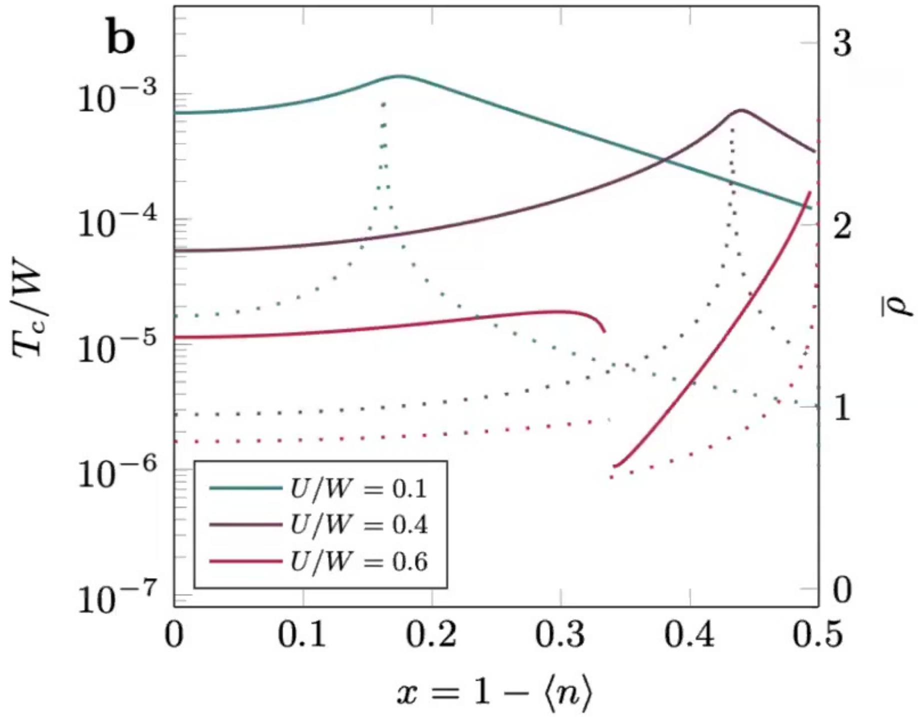
$$1 = -\frac{g}{L^d}\sum_{k\in\Omega_0}\frac{\langle 1 - n_{k\uparrow} - n_{-k\downarrow}\rangle}{E - 2\xi_k - U\langle n_{k\downarrow} + n_{-k\uparrow}\rangle}$$

$$1 = -g\int_{\mu}^{W/2} d\epsilon \frac{\rho(\epsilon)}{E - 2\epsilon + 2\mu},$$

Cooper Instability

$$E_b = -E \sim W(1 - (U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$





$W > U$

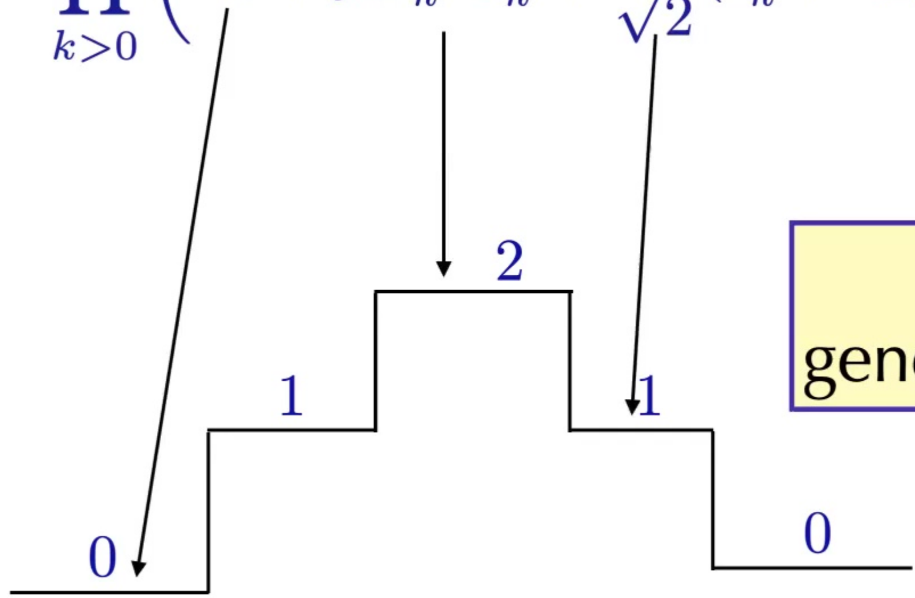
$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational wave function

$$|\psi_{\rm BCS}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle$$

$$|\psi_{\rm BCS}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b_{-k}^\dagger + u_k v_k (b_k^\dagger + b_{-k}^\dagger))|0\rangle$$

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK generalization

three variational parameters

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$1 = \frac{g}{W}\sinh^{-1}\left(\frac{W-U}{2\Delta}\right) + \frac{g}{W}\sinh^{-1}\left(\frac{U}{2\Delta}\right)$$

$$\Big\downarrow \quad \Delta \ll U, W$$

$$\Delta = (W-U)^{1/2}U^{1/2}e^{-\frac{W}{2g}}$$

gap/T_c ratio

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}} .$$

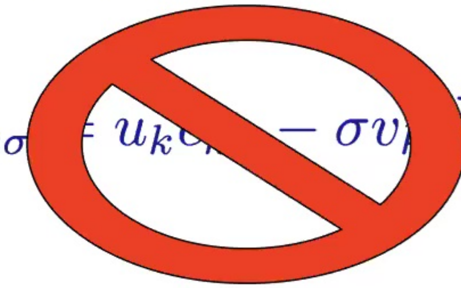
$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\rm BCS}\rangle = 0$$

Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$



$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\bar{\sigma}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\bar{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}}$$

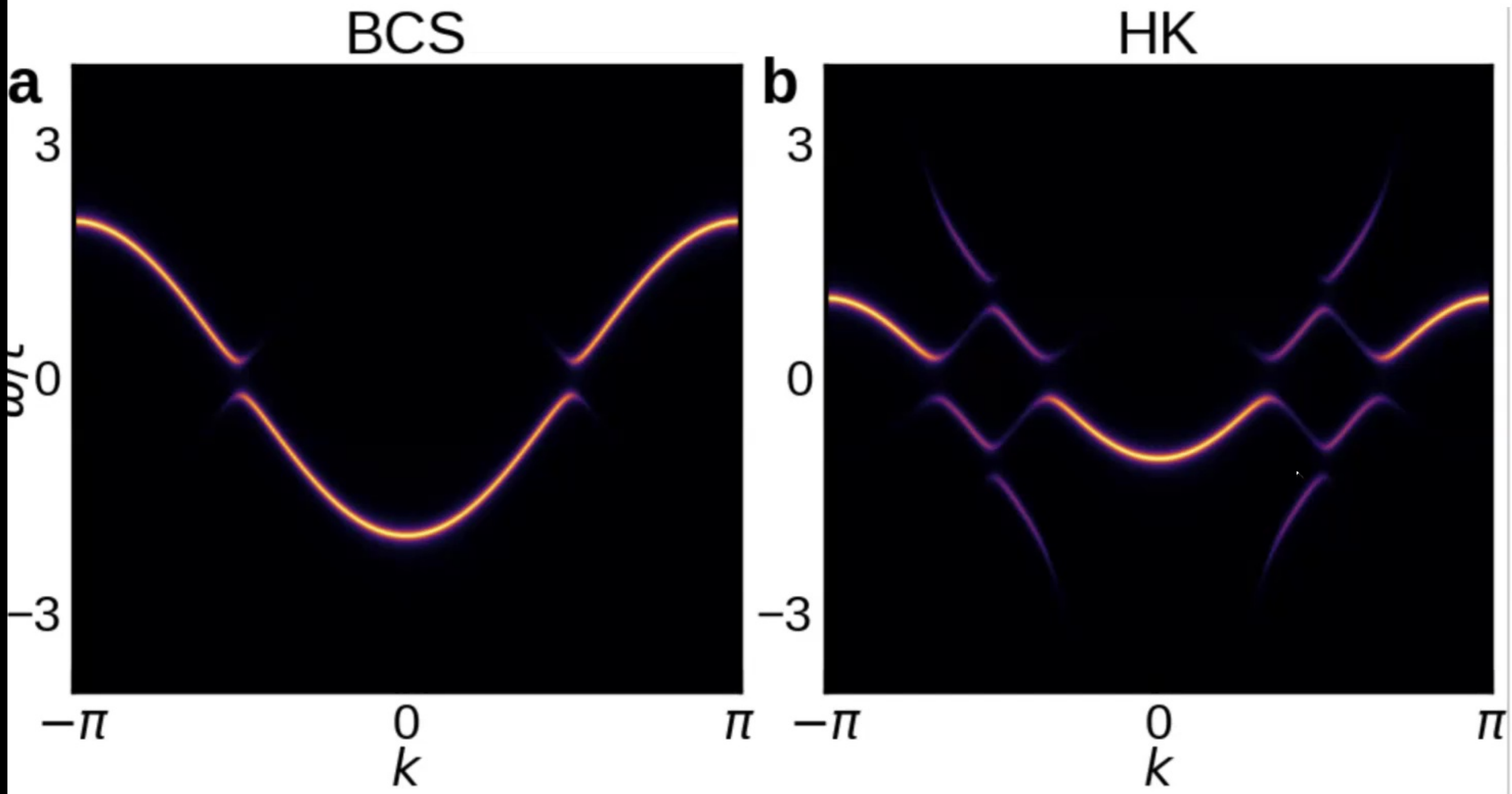
Excitation spectrum

$$\gamma_{k\sigma}^{u/l}|\psi\rangle = 0$$

$$\langle\psi|\gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l})^{\dagger}|\psi\rangle = \langle\psi|H|\psi\rangle + E_k^{u/l}$$

$$E_k^{u/l} = \sqrt{{\xi_k^{u/l}}^2 + \Delta^2}$$

superconductivity affects both bands!


BCS
HK
a
b
3
0
−3
−π
0
π
k

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

can we explain the color change?

REPORT

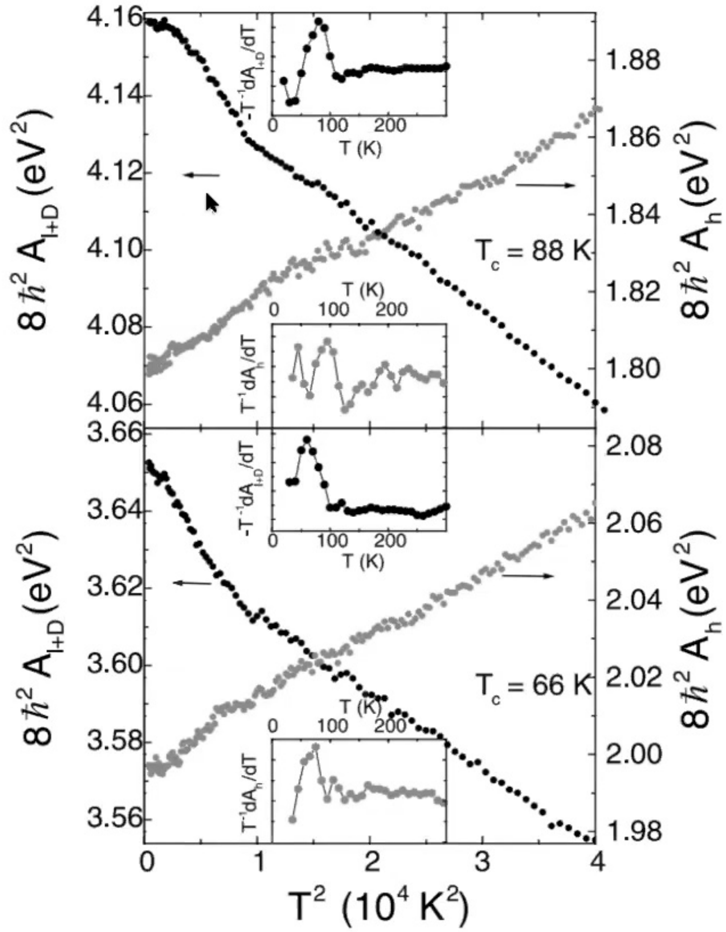
Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

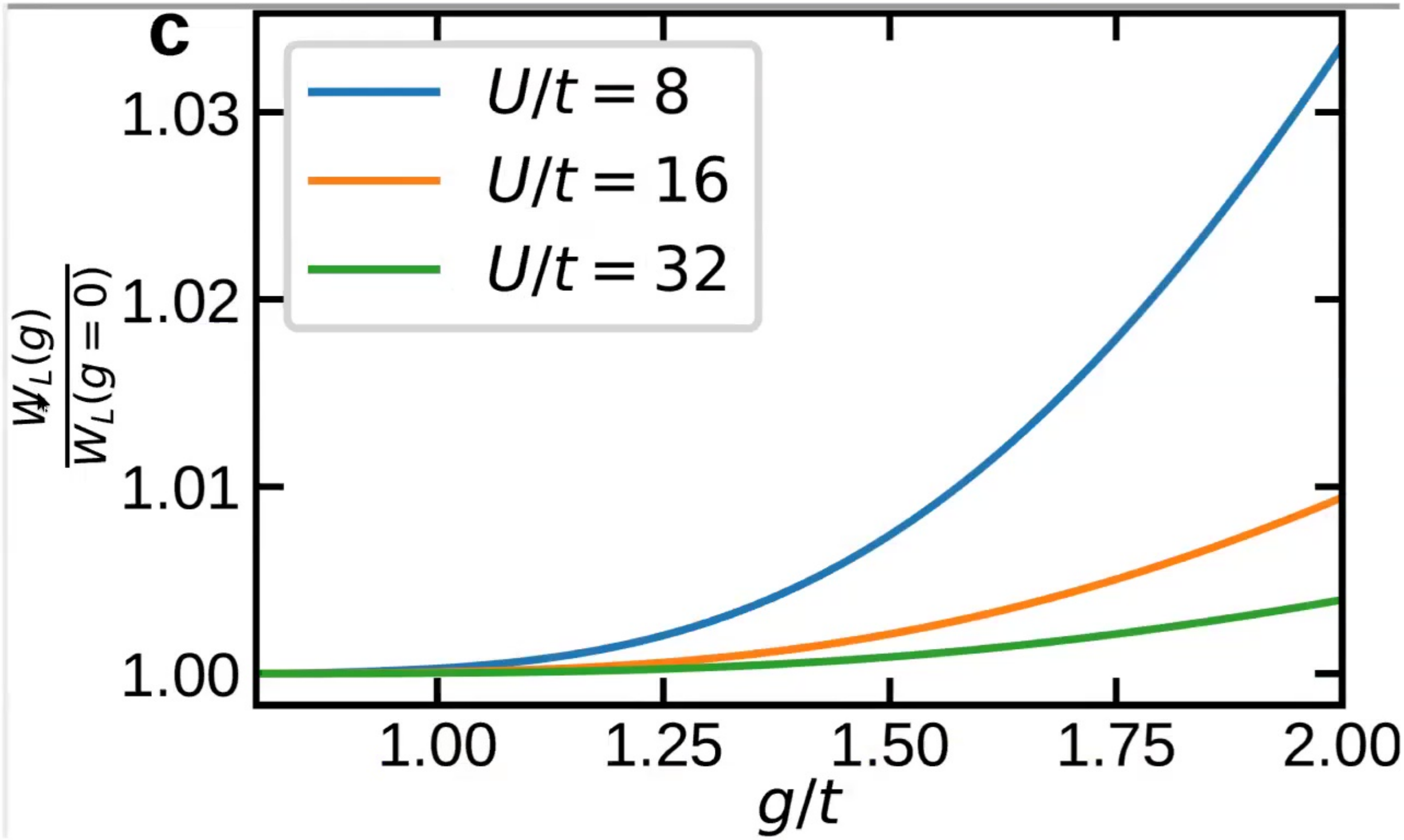


condensation energy

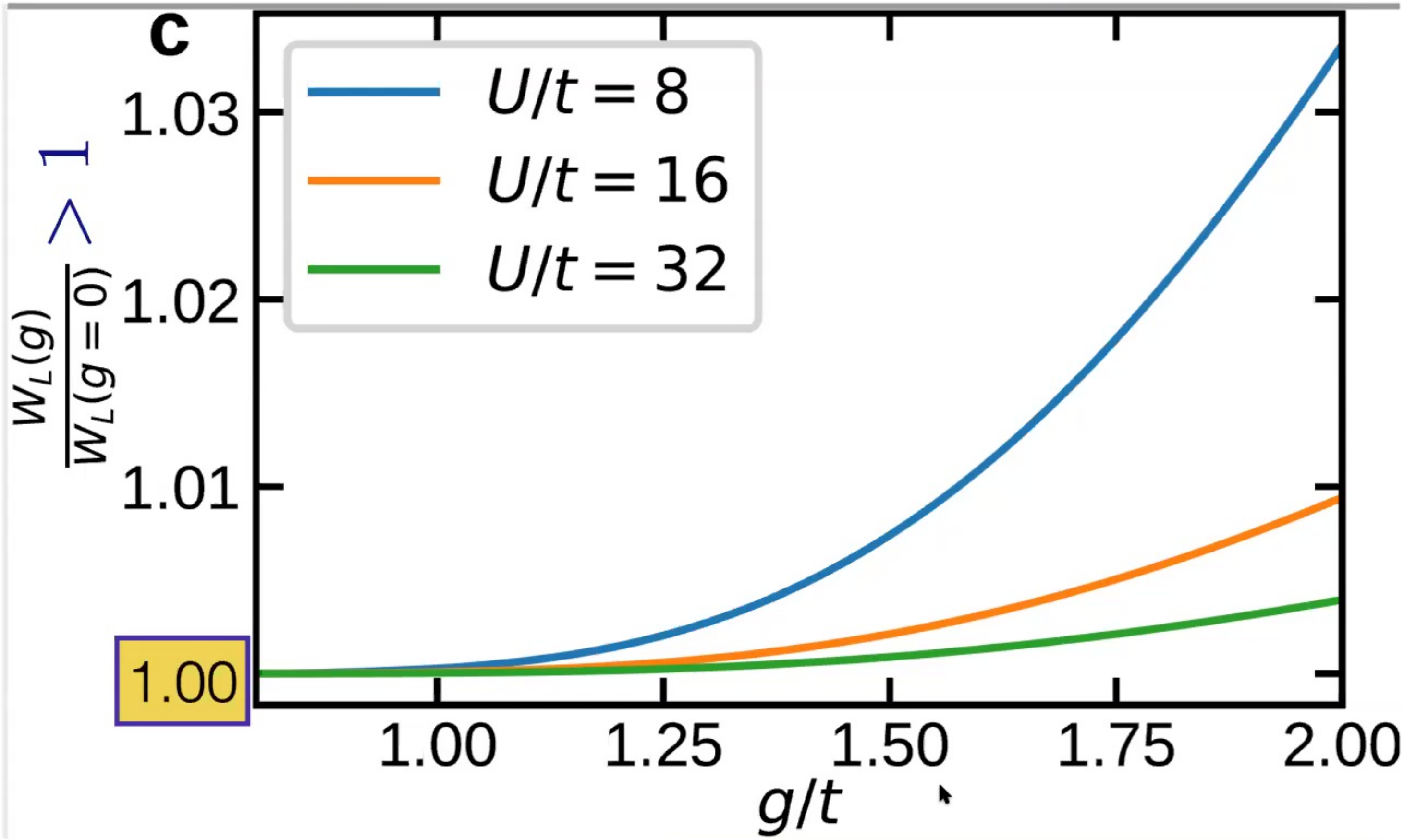
Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing

condensation energy:HK model



condensation energy:HK model



$\Delta W_L \propto O(2-3\%)$ as in experiments why?

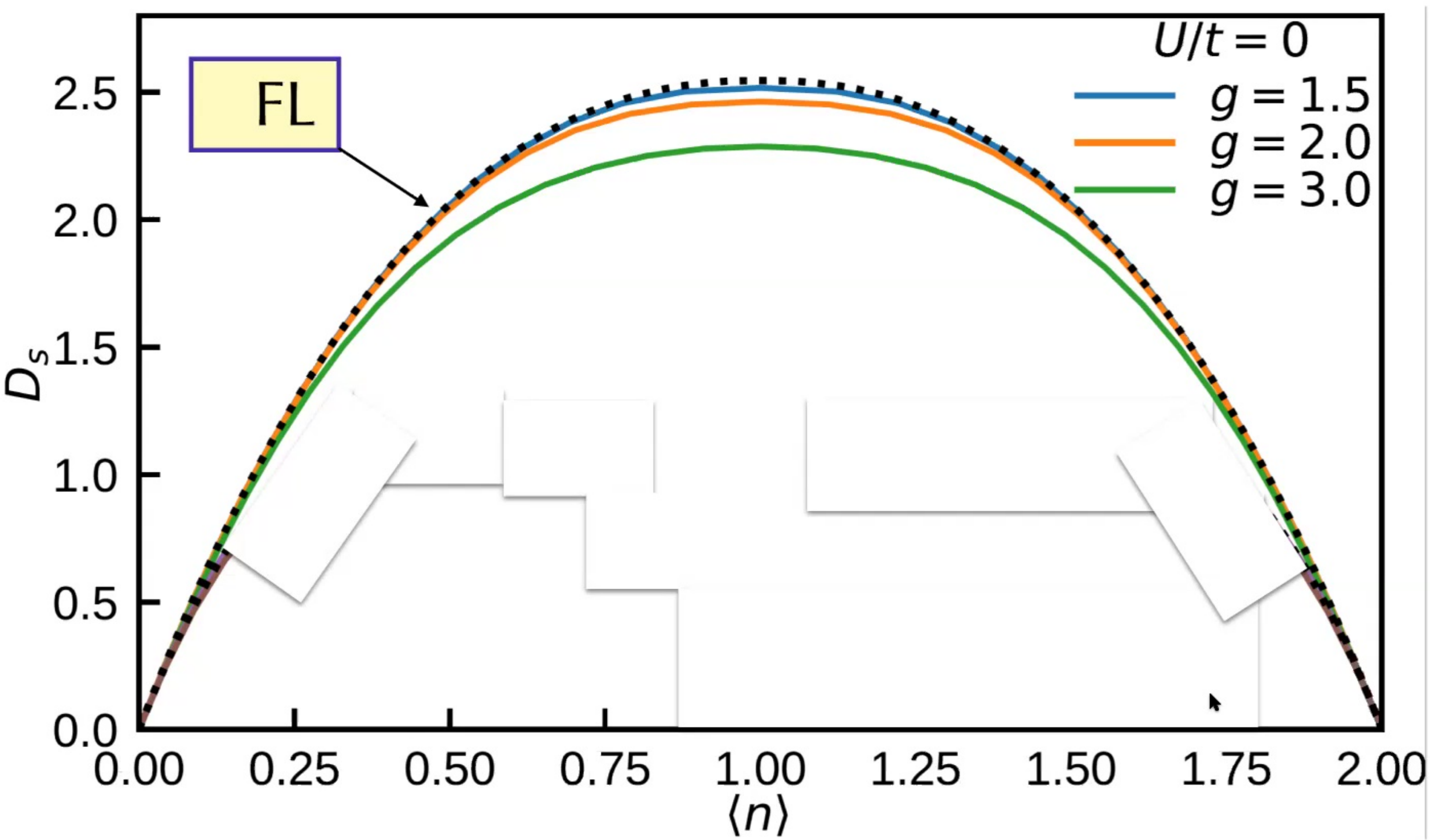
$$H = H_{\mathrm{HK}} + H_p$$

$$[H_{\mathrm{HK}}, H_p] \neq 0$$

↓

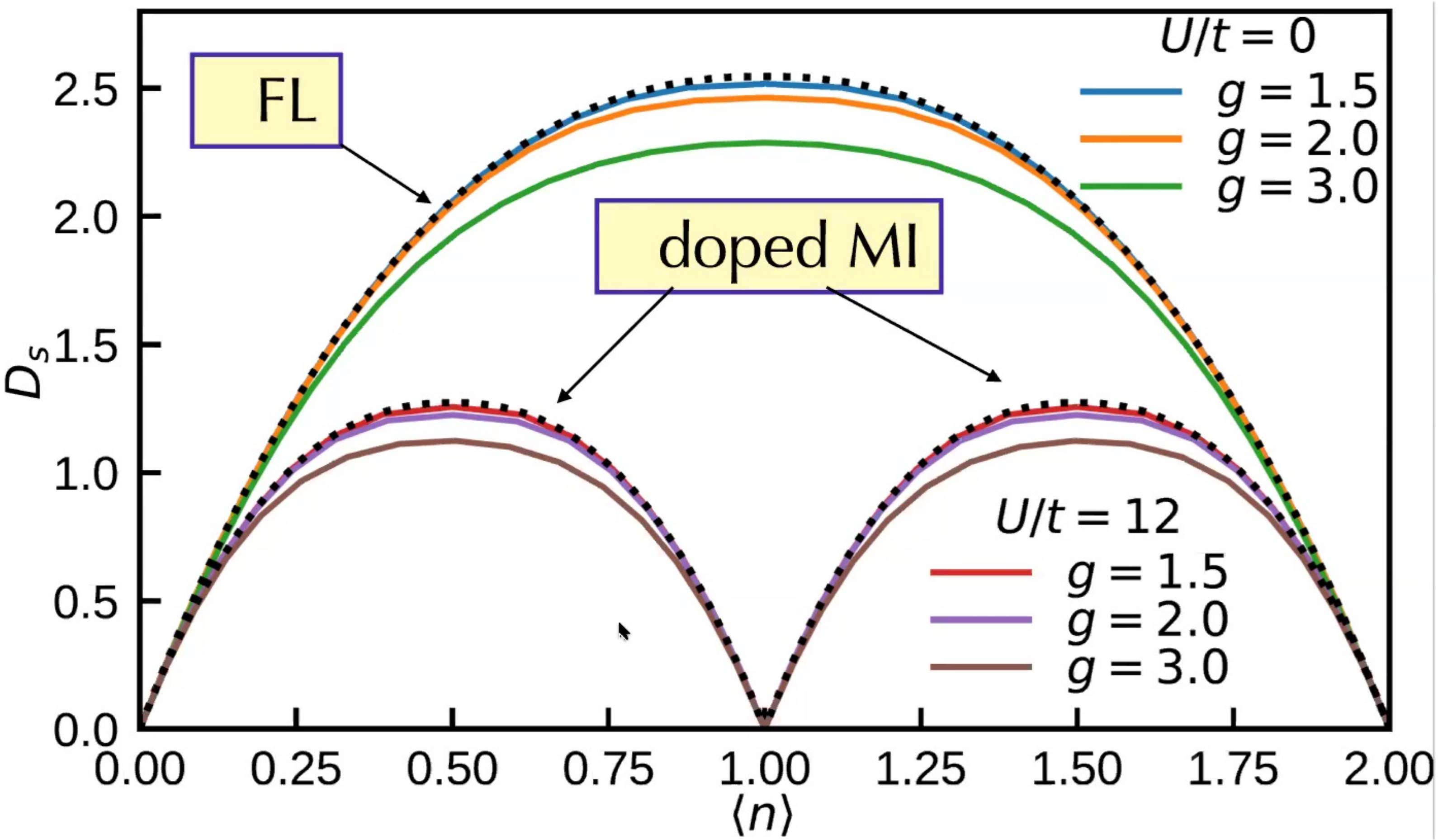
dynamical
spectral weight
transfer

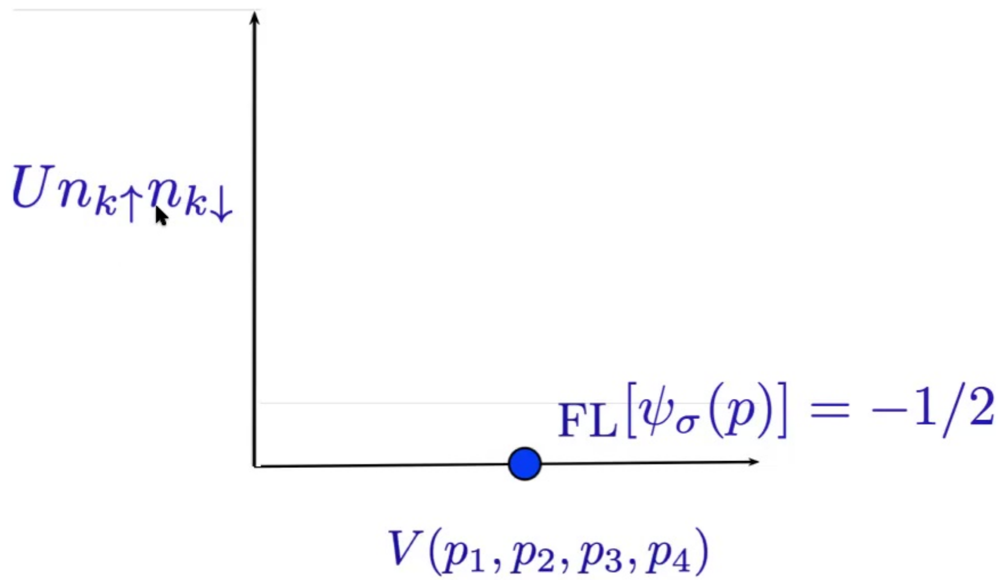
Superfluid Density

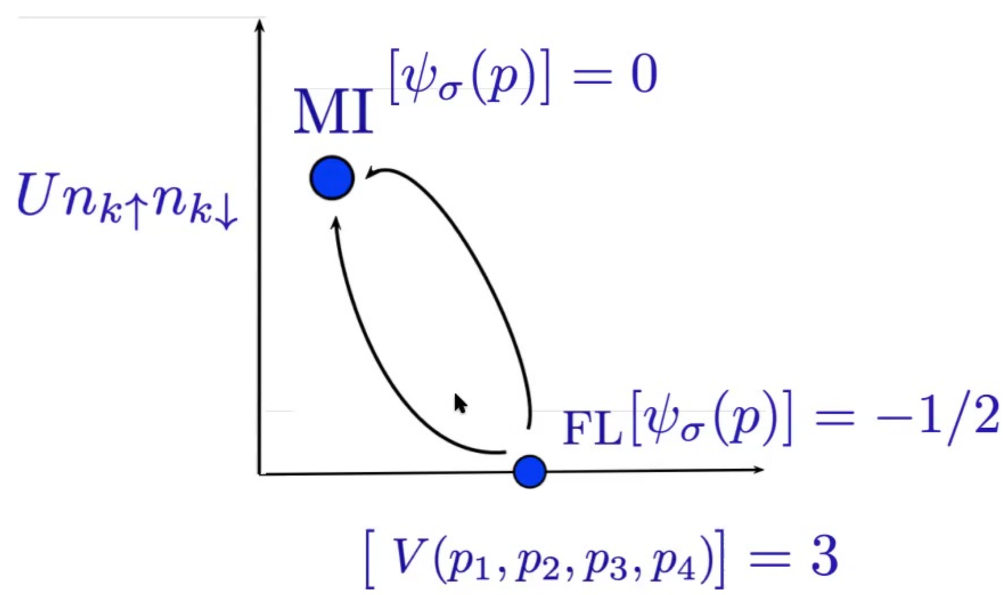


Superfluid Density

Mottness-induced suppression




$Un_{k\uparrow}n_{k\downarrow}$
$\mathrm{FL}[\psi_\sigma(p)] = -1/2$
$V(p_1, p_2, p_3, p_4)$

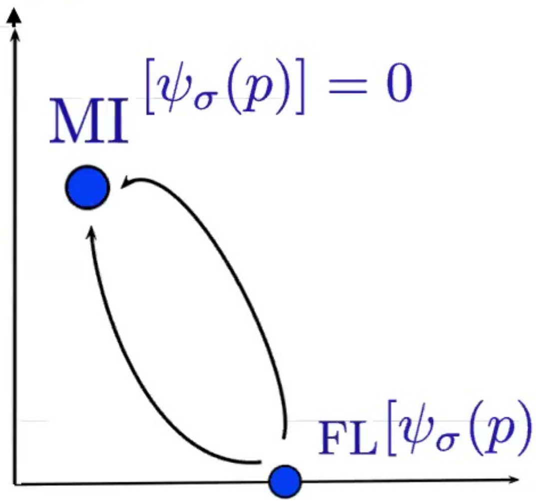

MI
$[\psi_\sigma(p)] = 0$
$U n_{k\uparrow} n_{k\downarrow}$
FL $[\psi_\sigma(p)] = -1/2$
$[\, V(p_1, p_2, p_3, p_4)] = 3$

Mott gap

$[H_t, H_V] = 0$

Mottness in momentum space

$Un_{k\uparrow}n_{k\downarrow}$



$[\psi_\sigma(p)] = 0$

FL $[\psi_\sigma(p)] = -1/2$

$$\gamma^l_{k\sigma} \propto \sqrt{2}x_k\zeta^\dagger_{k\sigma} - \sigma z_k\zeta_{-k\bar{\sigma}}$$

$[\, V(p_1, p_2, p_3, p_4)] = 3$

$$\gamma^u_{k\sigma} \propto z_k\eta^\dagger_{k\sigma} - \sigma\sqrt{2}y_k\eta_{-k\bar{\sigma}}$$

$$\frac{\Delta}{T_c} \propto \infty$$

PYHons

$[H_{\rm HK}, H_p] \neq 0$

non-BCS superconductivity

$$\frac{W_L(g)}{W_L(g=0)} > 1$$

Hatsugai-Khomoto Model (1992)

$$H_{\mathrm{HK}} = -t \sum_{\langle j,l \rangle,\sigma} \left(c^{\dagger}_{j\sigma} c_{l\sigma} + h.c. \right) - \mu \sum_{j\sigma} c^{\dagger}_{j\sigma} c_{j\sigma}$$

$$+ \frac{U}{L^d} \sum_{j_1 .. j_4} \delta_{j_1 + j_3, j_2 + j_4} c^{\dagger}_{j_1\uparrow} c_{j_2\uparrow} c^{\dagger}_{j_3\downarrow} c_{j_4\downarrow},$$