Title: A note on dual gravitational charges

Speakers: Roberto Olivieri

Series: Quantum Gravity

Date: October 29, 2020 - 2:30 PM

URL: http://pirsa.org/20100061

Abstract: Dual gravitational charges (DGCs) have been originally computed in the first-order formalism by means of covariant phase space methods using tetrad variables. I show i) why DGCs do not arise using the metric variables and ii) how they can be set to zero by exploiting the freedom to add exact 3-forms to the symplectic potential.

Without exploiting that freedom, DGCs can be understood as Hamiltonian charges associated to the Kosmann variation. I then discuss the implications of this observation for asymptotic symmetries and comment about subleading contributions thereof.

Finally, I also show that DGCs can be equally derived by means of cohomological methods. In this case, DGCs depends on the order of the Lagrangian: they exist only in the first-order formalism.

A note on dual gravitational charges

Roberto Oliveri

Institute of Physics, Czech Academy of Sciences
CEICO - Central European Institute for Cosmology and Fundamental Physics

Talk at *Perimeter Institute* - October 29th, 2020

based on arXiv:1912.01016 [gr-qc], arXiv:2010.01111 [hep-th] with Simone Speziale
see also arXiv:2007:07144 [hep-th] and arXiv:2007.01267 [hep-th] by H. Godazgar, M. Godazgar, M. Perry

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 1 (4 di 43) 217% Strumenti Compila e firma Commento

Introduction

- Dual supertranslation charges introduced "dualising" supertranslation charges at future null infinity

  [Godazgar et al. - 1812.01641], [Kol&Porrati - 1907.00990]

$$\not\delta Q_\xi = \frac{1}{8\pi G}\int_{S_\infty} \star\mathbf{k}_\xi, \qquad \text{(electric)}$$

$$\not\delta \widetilde{Q}_\xi = \frac{1}{8\pi G}\int_{S_\infty} \mathbf{k}_\xi, \qquad \text{(magnetic)}$$

  where $\mathbf{k}_\xi$ is the Barnich-Brandt charge density.

- Hamiltonian derivation of the dual supertranslation charge $\not\delta\widetilde{Q}_\xi$ provided in

  [Godazgar at al. - 2007.01257 & 2007.07144]

  More precisely, they computed $\not\delta\widetilde{Q}_\xi$
  - in the first-order formalism (Palatini-Holst action)
  - using tetrad variables
  - by means of covariant phase-space methods (à la Iyer-Wald)

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 2 (5 di 43) 217% Strumenti Compila e firma Commento

Motivation

In this talk, I will show

+ why dual charges do not arise using metric variables
+ how they can be set to zero by exploiting the freedom to add exact 3-forms to the pre-symplectic potential
+ how they can be understood from Hamiltonian charges associated to the Kosmann variation
+ how they can be computed by means of cohomological methods (à la Barnich-Brandt)

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 3 (6 di 43) 217% Strumenti Compila e firma Commento

Outline

1. Covariant phase-space methods - basics
2. "Dressing" the pre-symplectic potential
3. Dual charges from Hamiltonian charges associated to the Kosmann variation
4. Dual charges from cohomological methods
5. Conclusions

3

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 4 (10 di 43) 217% Strumenti Compila e firma Commento

Lagrangian and pre-symplectic potential

We adopt the first-order formalism: independent variables (e,ω) and (g,Γ)

$$L^{(e,\gamma)} = \frac{1}{2}\epsilon_{IJKL}\, \Sigma^{IJ} \wedge F^{KL}(\omega) + \frac{1}{\gamma}\, \Sigma_{IJ} \wedge F^{IJ}(\omega), \quad \Sigma^{IJ} = e^I \wedge e^J$$

$$L^{(g,\gamma)} = \left(g^{\mu\nu} R_{\mu\nu}(\Gamma) - \frac{1}{2\gamma}\epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}(\Gamma)\right)\epsilon.$$

Taking arbitary variations of the Lagrangian $\delta L[\phi] = E[\phi]\delta\phi + d\theta[\phi,\delta\phi]$, where

$$\theta^{(e,\gamma)\mu}(\delta) = 2e_I^{[\mu} e_J^{\nu]}\delta\omega_\nu^{IJ} - \frac{1}{\gamma}\epsilon^{\mu\nu\rho\sigma} e_{\nu I} e_{\rho J}\delta\omega_\sigma^{IJ},$$

$$\theta^{(g,\gamma)\mu}(\delta) = \left(2g^{\rho[\mu} g^{\nu]\sigma} - \frac{1}{\gamma}\epsilon^{\mu\nu\rho\sigma}\right) g_{\rho\lambda}\delta\Gamma^\lambda_{\nu\sigma}.$$

They differ by an exact 3-form, i.e., $\theta^{(g,\gamma)}(\delta) = \theta^{(e,\gamma)}(\delta) + d\alpha(\delta)$

[De Paoli&Speziale - 1804.09685]

$$\alpha(\delta) = \star(e_I \wedge \delta e^I) + \frac{1}{\gamma} e_I \wedge \delta e^I.$$

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 5 (11 di 43) 217% Strumenti Compila e firma Commento

Lagrangian and pre-symplectic potential

We adopt the first-order formalism: independent variables (e, ω) and (g, Γ)

$$L^{(e,\gamma)} = \frac{1}{2}\epsilon_{IJKL}\,\Sigma^{IJ} \wedge F^{KL}(\omega) + \frac{1}{\gamma}\,\Sigma_{IJ} \wedge F^{IJ}(\omega), \quad \Sigma^{IJ} = e^I \wedge e^J$$

$$L^{(g,\gamma)} = \left(g^{\mu\nu}R_{\mu\nu}(\Gamma) - \frac{1}{2\gamma}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}(\Gamma)\right)\epsilon.$$

Taking arbitary variations of the Lagrangian $\delta L[\phi] = E[\phi]\delta\phi + d\theta[\phi, \delta\phi]$, where

$$\theta^{(e,\gamma)\mu}(\delta) = 2e_I^{[\mu}e_J^{\nu]}\delta\omega_\nu^{IJ} - \frac{1}{\gamma}\epsilon^{\mu\nu\rho\sigma}e_{\nu I}e_{\rho J}\delta\omega_\sigma^{IJ},$$

$$\theta^{(g,\gamma)\mu}(\delta) = \left(2g^{\rho[\mu}g^{\nu]\sigma} - \frac{1}{\gamma}\epsilon^{\mu\nu\rho\sigma}\right)g_{\rho\lambda}\delta\Gamma^\lambda_{\nu\sigma}.$$

They differ by an exact 3-form, i.e., $\theta^{(g,\gamma)}(\delta) = \theta^{(e,\gamma)}(\delta) + d\alpha(\delta)$

[De Paoli&Speziale - 1804.09685], [RO&Speziale - 1912.01016]

$$\alpha(\delta) = \star(e_I \wedge \delta e^I) + \frac{1}{\gamma}e_I \wedge \delta e^I.$$

Notice that $\Sigma_{IJ} \wedge F^{IJ} = T^I \wedge T_I - d(e_I \wedge T^I)$ vanishes for $T^I = 0$,
but the corresponding θ-contribution $\Sigma_{IJ} \wedge \delta\omega^{IJ} = -d(e_I \wedge \delta e^I)$ does not.

5

Hamiltonian charges: tetrad & metric variables

Symplectic 2-form: $\Omega(\delta_1,\delta_2)=\delta_1\Theta(\delta_2)-\delta_2\Theta(\delta_1), \quad \Theta(\delta)=\int_\Sigma \theta(\delta)$

Metric Hamiltonian charges $\not\delta H^{(g,\gamma)}_\xi=\Omega^{(g,\gamma)}(\delta,\delta_\xi)$

(without torsion, recall $1/\gamma\ \epsilon^{\mu\nu\rho\sigma}g_{\rho\lambda}\delta\Gamma^\lambda_{\nu\sigma}=0$)

$$\not\delta H^{(g,\gamma)}_\xi=\int_S \delta\kappa_\xi - i_\xi\theta^g(\delta)=\not\delta H^g_\xi, \qquad \text{(Iyer\&Wald charge)}$$

where $\kappa_\xi=-\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho\xi^\sigma dx^\mu\wedge dx^\nu$ is the Komar 2-form.

Tetrad Hamiltonian charges $\not\delta H^{(e,\gamma)}_{(\lambda,\xi)}=\Omega^{(e,\gamma)}(\delta,\delta_\lambda+\delta_\xi)$

$$\not\delta H^{(e,\gamma)}_{(\lambda,\xi)}=\int_S \delta q^{(e,\gamma)}_\lambda+\delta q^{(e,\gamma)}_\xi - i_\xi\theta^{(e,\gamma)}(\delta)=\not\delta H^e_\xi+\delta H^e_\lambda+\frac{1}{\gamma}\left(\not\delta\widetilde{H}^e_\xi+\delta\widetilde{H}^e_\lambda\right)$$

with $q^{(e,\gamma)}_\xi=P_{IJKL}i_\xi\omega^{IJ}\Sigma^{KL}$, $q^{(e,\gamma)}_\lambda=P_{IJKL}\lambda^{IJ}\Sigma^{KL}$, $P_{IJKL}=\frac{1}{2}\epsilon_{IJKL}+\frac{1}{\gamma}\eta_{I[K}\eta_{L]J}$

Hamiltonian charges: tetrad & metric variables

Symplectic 2-form: $\Omega(\delta_1, \delta_2) = \delta_1 \Theta(\delta_2) - \delta_2 \Theta(\delta_1), \quad \Theta(\delta) = \int_\Sigma \theta(\delta)$

Metric Hamiltonian charges $\not\delta H_\xi^{(g,\gamma)} = \Omega^{(g,\gamma)}(\delta, \delta_\xi)$

(without torsion, recall $1/\gamma \; \epsilon^{\mu\nu\rho\sigma} g_{\rho\lambda} \delta \Gamma^\lambda_{\nu\sigma} = 0$)

$$\not\delta H_\xi^{(g,\gamma)} = \int_S \delta \kappa_\xi - i_\xi \theta^g(\delta) = \not\delta H_\xi^g, \qquad \text{(Iyer\&Wald charge)}$$

where $\kappa_\xi = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \nabla^\rho \xi^\sigma dx^\mu \wedge dx^\nu$ is the Komar 2-form.

Tetrad Hamiltonian charges $\not\delta H_{(\lambda,\xi)}^{(e,\gamma)} = \Omega^{(e,\gamma)}(\delta, \delta_\lambda + \delta_\xi)$

$$\not\delta H_{(\lambda,\xi)}^{(e,\gamma)} = \int_S \delta q_\lambda^{(e,\gamma)} + \delta q_\xi^{(e,\gamma)} - i_\xi \theta^{(e,\gamma)}(\delta) = \not\delta H_\xi^e + \delta H_\lambda^e + \frac{1}{\gamma}\left(\not\delta \widetilde{H}_\xi^e + \delta \widetilde{H}_\lambda^e \right)$$

with $q_\xi^{(e,\gamma)} = P_{IJKL} i_\xi \omega^{IJ} \Sigma^{KL}$, $q_\lambda^{(e,\gamma)} = P_{IJKL} \lambda^{IJ} \Sigma^{KL}$, $P_{IJKL} = \frac{1}{2}\epsilon_{IJKL} + \frac{1}{\gamma} \eta_{I[K}\eta_{L]J}$

In particular,

. $\not\delta H_\xi^e = \int_S \delta q_\xi^e - i_\xi \theta^e(\delta) = \int_S \frac{1}{2} \epsilon_{IJKL} \left(i_\xi \omega^{IJ} \delta \Sigma^{KL} - i_\xi \Sigma^{IJ} \wedge \delta \omega^{KL} \right) \neq \not\delta H_\xi^g$

.. $\not\delta \widetilde{H}_\xi^e = \int_S 2 \mathcal{L}_\xi e_I \wedge \delta e^I$ — see also [Corichi et al., - 1312.7828]

... $\delta H_\lambda^e + \frac{1}{\gamma} \delta \widetilde{H}_\lambda^e = - \int_S P_{IJKL} \lambda^{IJ} \delta \Sigma^{KL}$

This difference in the charges from different Lagrangians is also studied in [Freidel&Geiller&Pranzetti - 2006.12527 & 2007.03563]

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 8 (19 di 43) 217% Strumenti Compila e firma Commento

What is next?

We have three question to address

1. We can use the ambiguity of the pre-symplectic potential
   (recall, $\delta L[\phi] = E[\phi]\delta\phi + d\theta[\phi, \delta\phi]$)
   $$\theta(\delta) \to \theta(\delta) + d\alpha(\delta)$$
   $$\Omega(\delta_1, \delta_2) \to \Omega(\delta_1, \delta_2) + \Omega^{\alpha}(\delta_1, \delta_2)$$
   Can α set to zero dual and internal Lorentz charges simultaneously?
   Alternative: add a boundary Lagrangian $\to$ boundary dof $\to$ edge modes
   [Freidel et al - 2006.12527 & 2007.03563], [Wieland - 1905.06357]
2. If we stay with $\theta(\delta)$, can we at least recover matching charges in the case of isometries? Recall that $\oint H^e_\xi \neq \oint H^g_\xi$.
   We introduce a notion of isometry for tetrads, $\mathcal{K}_\xi e^I = 0$ if ξ is Killing.
   Recall that $\mathcal{L}_\xi e^I = \lambda_\xi{}^I{}_J e^J \neq 0$ if ξ is Killing, in contrast with $\mathcal{L}_\xi g_{\mu\nu} = 0$.
   What is $\Omega(\delta, \mathcal{K}_\xi)$?
3. We can compute the surface charges by means of cohomological methods. Ambiguity-free methods.

   Do Barnich-Brandt charges “see” the dual charges?

8

"Dressing" the pre-symplectic potential

The variation of the Lagrangian defines $\theta(\delta)$ up to an exact form!

One is allowed to choose a new representative for $\theta^{(e,\gamma)}(\delta)$, say,

$$\theta'^{(e,\gamma)}(\delta) = \theta^{(e,\gamma)}(\delta) + d\alpha(\delta) = \theta^{(g,\gamma)}(\delta)$$

to set to zero dual and internal Lorentz charges and recover the metric charges.

Schetch of the proof. From

$$\alpha(\delta) = \star(e_I \wedge \delta e^I) + \frac{1}{\gamma} e_I \wedge \delta e^I = \alpha_\star(\delta) + \frac{1}{\gamma}\alpha_\gamma(\delta)$$

compute

$$\Omega^\alpha(\delta, \underline{\delta_\lambda} + \delta_\xi) = \int_S \delta(\kappa_\xi - q^e_\xi) - i_\xi d\alpha_\star(\delta) - \underline{\delta H^e_\lambda} - \frac{1}{\gamma}\left(\not{\delta}\widetilde{H}^e_\xi + \underline{\delta\widetilde{H}^e_\lambda}\right)$$

and add it to $\not{\delta} H^{(e,\gamma)}_{(\lambda,\xi)} = \not{\delta} H^e_\xi + \delta H^e_\lambda + \frac{1}{\gamma}\left(\not{\delta}\widetilde{H}^e_\xi + \delta\widetilde{H}^e_\lambda\right)$ to get

$$\not{\delta} H'^e_\xi = \not{\delta} H^e_\xi + \int_S \delta(\kappa_\xi - q^e_\xi) - i_\xi d\alpha_\star(\delta) = \int_S \delta\kappa_\xi - i_\xi \theta^g(\delta) = \not{\delta} H^g_\xi$$

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 9 (24 di 43) 217% Strumenti Compila e firma Commento

Dual charges from Hamiltonian charges associated to the Kosmann variation

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 10 (27 di 43) 217% Strumenti Compila e firma Commento

The Kosmann derivative and dual charges

The notion of isometry for tetrads is given by the Kosmann derivative $\mathcal{K}_\xi$ [Jacobson&Mohd - 1507.01054], [Prabhu - 1511.00388]

$$\mathcal{K}_\xi := \mathcal{L}_\xi + \delta_{\bar\lambda}, \qquad \bar\lambda^{IJ} := D^{[I}\xi^{J]} + i_\xi \omega^{IJ}$$

It obeys $\mathcal{K}_\xi e^I = 0$ if ξ is Killing.

Thanks to this definition, the Noether charges are recovered exactly for any ξ. For Hamiltonian charges, they are recovered for Killing but not in general.

- Noether charges: same result with tetrad and metric variables for any ξ

  In fact, $\theta^{(e,\gamma)}(\delta_{\bar\lambda}) = d\alpha(\mathcal{L}_\xi)$ and $\theta^{(e,\gamma)}(\delta) + d\alpha(\delta) = \theta^{(g,\gamma)}(\delta)$ imply

  $$j^{(e,\gamma)}_\xi(\mathcal{K}_\xi) = \theta^{(e,\gamma)}(\mathcal{L}_\xi) + \theta^{(e,\gamma)}(\delta_{\bar\lambda}) - i_\xi L^{(e,\gamma)} = \theta^{(g,\gamma)}(\mathcal{L}_\xi) - i_\xi L^{(g,\gamma)} = j^{(g,\gamma)}(\mathcal{L}_\xi)$$

- Hamiltonian charges:

$$\Omega^{(e,\gamma)}(\delta, \mathcal{K}_\xi) = \Omega^{(e,\gamma)}(\delta, \mathcal{L}_\xi) + \Omega^{(e,\gamma)}(\delta, \delta_{\bar\lambda}) = \not\delta H^g_\xi + \int_S \mathcal{A}_\xi(\delta)$$

$$\mathcal{A}_\xi(\delta) := \left[\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(e^I_\alpha \mathcal{L}_\xi g^{\alpha\rho} + \nabla_\alpha \xi^\alpha e^\rho_I) - \frac{1}{\gamma}\mathcal{L}_\xi g_{\mu\sigma} e^I_\nu\right] \delta e^\sigma_I \, dx^\mu \wedge dx^\nu$$

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 11 (30 di 43) 217% Strumenti Compila e firma Commento

Isometries and asymptotic symmetries at spatial infinity

Hamiltonian charges associated to the Kosmann variation

[Recall $\Omega^{(e,\gamma)}(\delta, \mathcal{L}_\xi) = \not{\delta} H_\xi^{(e,\gamma)} = \not{\delta} H_\xi^e + \frac{1}{\gamma}\not{\delta}\widetilde{H}_\xi^e$]

$$\Omega^{(e,\gamma)}(\delta, \mathcal{K}_\xi) = \Omega^{(e,\gamma)}(\delta, \mathcal{L}_\xi) + \Omega^{(e,\gamma)}(\delta, \delta_{\bar{\lambda}}) = \not{\delta} H_\xi^g + \int_S \mathcal{A}_\xi(\delta)$$

$$\mathcal{A}_\xi(\delta) := \left[\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(e_\alpha^I \mathcal{L}_\xi g^{\alpha\rho} + \nabla_\alpha \xi^\alpha e_I^\rho) - \frac{1}{\gamma}\mathcal{L}_\xi g_{\mu\sigma} e_\nu^I\right] \delta e_I^\sigma dx^\mu \wedge dx^\nu$$

- Isometries. Trivial, $\mathcal{A}_\xi(\delta) = 0$. We recover the metric charges: $\not{\delta} H_\xi^e = \not{\delta} H_\xi^g$
  Or, mutual cancellation between dual and internal Lorentz charges:

$$\not{\delta}\widetilde{H}_\xi^e = \int_S 2\mathcal{L}_\xi e^I \wedge \delta e_I = -\int_S 2\bar{\lambda}^{IJ} e_J \wedge \delta e_I = -\Omega^{(e,\gamma)}(\delta, \delta_{\bar{\lambda}})$$

- Asymptotic symmetries at spatial infinity. Less trivial, $\mathcal{A}_\xi(\delta) \to 0$ at i^0.
  Poincaré charges recovered! [Ashtekar et al. - 0802.2527]
  Or, no contribution from $\alpha(\delta)$ [De Paoli&Speziale - 1804.09685] in the limit.

11

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 12 (33 di 43) 217% Strumenti Compila e firma Commento

Asymptotic symmetries at future null infinity

Hamiltonian charges associated to the Kosmann variation

[Recall $\Omega^{(e,\gamma)}(\delta, \mathcal{L}_\xi) = \not\delta H_\xi^{(e,\gamma)} = \not\delta H_\xi^e + \frac{1}{\gamma}\not\delta \widetilde{H}_\xi^e$]

$$\Omega^{(e,\gamma)}(\delta, \mathcal{K}_\xi) = \Omega^{(e,\gamma)}(\delta, \mathcal{L}_\xi) + \Omega^{(e,\gamma)}(\delta, \delta_{\bar{\lambda}}) = \not\delta H_\xi^g + \int_S \mathcal{A}_\xi(\delta)$$

$$\mathcal{A}_\xi(\delta) := \left[\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(e_\alpha^I \mathcal{L}_\xi g^{\alpha\rho} + \nabla_\alpha \xi^\alpha e_I^\rho) - \frac{1}{\gamma}\mathcal{L}_\xi g_{\mu\sigma} e_\nu^I\right] \delta e_I^\sigma \, dx^\mu \wedge dx^\nu$$

Using Bondi gauge and fall-off conditions* for the tetrads to future null infinity:

$$\int_{S^2_\infty} \mathcal{A}_\xi(\delta) = \frac{1}{2\gamma}\int_{S_\infty} \delta g_{BD}\left(\nabla_A \xi^D + \nabla^D \xi_A\right) dx^A \wedge dx^B$$

The RHS is exactly the result of [Godazgar et al. - 2007.07144]

In summary, the Kosmann variation recovers the standard BMS charges, but gives rise to the dual BMS charges!

$$\Omega^{(e,\gamma)}(\delta, \mathcal{K}_\xi) = (\text{standard BMS charges}) + (\text{dual BMS charges})$$

Question*: extended BMS fall-off conditions: dual BMS still present?

12

dual_charges_PItalk.pdf - Adobe Reader

File Modifica Vista Finestra ?

Apri 12 (34 di 43) 217% Strumenti Compila e firma Commento

Dual charges from cohomological methods

Dual charges from the Barnich-Brandt charges

Cohomological methods are ambiguity-free: focus on EoM!

Recall that $\theta(\delta) = \mathcal{I}^{(4)} L$ [Anderson] [Barnich&Brandt - hep-th/0111246]

For tetrad gravity [Barnich et al. - 1611.01777], [Frodden&Hidalgo - 1703.10120]

$$\delta_{(\lambda,\xi)} e^I \wedge E^{(e)}_I + \delta_{(\lambda,\xi)} \omega^{IJ} \wedge E^{(\omega)}_{IJ} = N(\lambda,\xi) + dS(\lambda,\xi)$$

where $N(\lambda,\xi)$ and $S(\lambda,\xi)$ are, resp., the Noether identities and current

$$N(\lambda,\xi) = 2P_{IJKL}[(\lambda^{IJ} - i_\xi \omega^{IJ}) e^K \wedge (d_\omega T^L - F^{LM} \wedge e_M) + i_\xi e^I e^J \wedge d_\omega F^{KL}]$$

$$S(\lambda,\xi) = 2P_{IJKL}\left[i_\xi e^I e^J \wedge F^{KL} + (i_\xi \omega^{IJ} - \lambda^{IJ}) e^K \wedge T^L)\right]$$

We need an operator to map 3- to 2-forms. It is the homotopy operator

(see e.g. the review [Compère - 1801.07064])

$$\mathcal{I}^{(3)}_\delta = \left[\frac{1}{2}\delta\phi \frac{\delta}{\delta\partial_\mu\phi} - \frac{1}{3}\delta\phi\partial_\nu \frac{\delta}{\delta\partial_\mu\partial_\nu\phi} + \frac{2}{3}\partial_\nu\delta\phi\frac{\delta}{\delta\partial_\mu\partial_\nu\phi} + \dots\right] i_{\partial_\mu}$$

Then, the BB charges read as

$$\not\delta Q^{BB}_{(\lambda,\xi)} = \int_S \mathcal{I}^{(3)}_\delta S(\lambda,\xi) = \int_S P_{IJKL}\left[(i_\xi\omega^{IJ} - \lambda^{IJ})\delta\Sigma^{KL} - i_\xi \Sigma^{IJ} \wedge \delta\omega^{KL}\right]$$

BB charges reproduce dual gravitational charges in the first-order formalism!

Conclusions

Dual gravitational charges are only present in the tetrad first-order formalism:

- arise from an exact 3-form originated from the Holst sector.
- are fully exposed to the ambiguity of the pre-symplectic potential.
- can be understood from Hamiltonian charges with the Kosmann variation
- can be computed by means of cohomological methods. No ambiguity!

Suprisingly, they survive* at future null infinity!

– standard BMS charges: Bondi mass ($l = 0$) & supermomenta ($l \geq 1$)

– dual BMS charges: conserved Taub-NUT ($l = 0$) & ($l \geq 1$) harmonics

Caveat*: use of standard BMS fall-off conditions for the tetrad to null infinity

Open questions:

1. Do they survive with different fall-off conditions, e.g., extended BMS group?
2. Fluxes (and memory) from dual BMS charges?

(for BMS [Compère, RO, Seraj - 1912.03164])

$$\frac{d}{du}(dual\ BMS) = (radiative\ flux) + (memory)?$$

14