Title: Asymptotic charges in gravity

Speakers: Mahdi Godazgar

Series: Quantum Gravity

Date: October 22, 2020 - 2:30 PM

URL: http://pirsa.org/20100058

Abstract: I will discuss the recent Hamiltonian derivation of dual BMS charges at null infinity using the first order formalism. More generally, I will discuss how this idea can be used to classify asymptotic charges in gravity.

Asymptotic charges in gravity

with Hadi Godazgar, Malcolm Perry and Chris Pope
(see also ArXiv:2006.12527 and ArXiv:2010.01111)

with Hadi Godazgar, Malcolm Perry and Chris Pope
(see also ArXiv:2006.12527 and ArXiv:2010.01111)

Introduction / motivation

Asymptotically flat spacetimes

$\exists$ Bondi coords (u, r, x^I) $\{\theta, \phi\}$ s.t. the metric

$$ds^2 = -Fe^{2\beta}du^2 - 2e^{2\beta}du\,dr + r^2 h_{IJ}(dx^I - C^I du)(dx^J - C^J du)$$

large r :

$$F = 1 + \frac{F_0}{r} + \dots, \quad \beta = \frac{\beta_0}{r^2} + \dots, \quad \det(h) = \det(\gamma) = \sin^2\theta$$

$$C^I = \frac{C_0^I}{r^2} + \dots, \quad h_{IJ} = \gamma_{IJ} + \frac{C_{IJ}}{r} + \dots$$

Asymptotically flat spacetimes

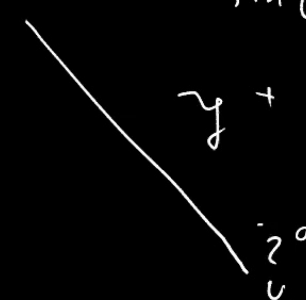
$\exists$ Bondi coords (u, r, x^I) $\leftarrow \{\theta, \phi\}$ s.t. the metric

$$ds^2 = -Fe^{2\beta}du^2 - 2e^{2\beta}du\,dr + r^2 h_{IJ}(dx^I - C^I du)(dx^J - C^J du)$$

large r:

$$F = 1 + \frac{F_0}{r} + \dots, \quad \beta = \frac{\beta_0}{r^2} + \dots, \quad \det(h) = \det(\gamma) = \sin^2\theta$$

$$C^I = \frac{C_0^I}{r^2} + \dots, \quad h_{IJ} = \gamma_{IJ} + \frac{C_{IJ}}{r} + \dots$$



"isolated system observed @ ∞" $\leftarrow$ "Minkowski"

Asymptotically flat spacetimes

$\exists$ Bondi coords (u, r, x^I) s.t. the metric, $x^I \leftarrow \{\theta, \phi\}$

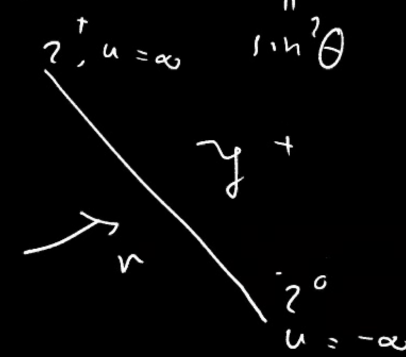
$$ds^2 = -Fe^{2\beta}du^2 - 2e^{2\beta}du\,dr + r^2 h_{IJ}(dx^I - C^I du)(dx^J - C^J du)$$

large r:

$$F = 1 + \frac{F_0}{r} + \dots, \quad \beta = \frac{\beta_0}{r^2} + \dots$$

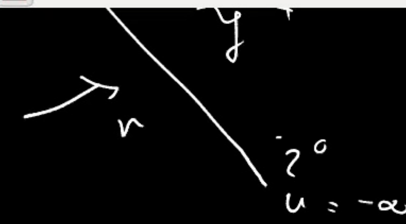
$$C^I = \frac{C_0^I}{r^2} + \dots, \quad h_{IJ} = \gamma_{IJ} + \frac{C_{IJ}}{r} + \dots, \quad \det(h) = \det(\gamma) = \sin^2\theta$$

"isolated system observed @ ∞" $\leftarrow$ "Minkowski"



Symmetry group @ large r $\equiv$ ASG

"isolated system observed @ ∞" $\longleftarrow$ "Minkowski"



Symmetry group @ large r $\equiv$ ASG

$$\text{BMS} = \underbrace{SL(2,\mathbb{C})}_{\text{global conf on } S^2} \ltimes \underbrace{S.T.}_{S(x^I)}$$

Noether: symmetry $\longleftrightarrow$ charges global conf on S^2

$S(x^I)$

Noether : Symmetry $\longleftrightarrow$ charges

What are the charges ?

Why ?

What are the charges?

Why?

* Ashtekar: Asymptotic quantisation

* Strominger: Soft graviton scattering (Weinberg soft thm)

* Hawking, Perry, Strominger: applying these ideas to black holes

* Understand Newman - Penrose (1968) & their relation to BMS group.

→ necessity of having dual BMS charges

generalisations of NUT charge.

((Cf. standard BMS charges are gen. of Bondi 4-mtm)

- Covariant phase space formalism
- the Theory (Palatini – Holst)
- Charges
- Conclusions

- Covariant phase space formalism
- The theory (Palatini – Holst)
- Charges
- Conclusions

- Conclusions

Covariant phase space (Peierls, $\overbrace{\text{Bergmann-Schiller}}^{\text{'53}}$, ..., Wald et al)

Theory: fields $\{\phi\}$, Lagrangian

— Conclusions

'53

Covariant phase space (Peierls, Bergmann–Schiller, ..., Wald et al)

Theory: fields $\{\phi\}$, Lagrangian L ← top-form

$$\delta L = \underset{\text{e.o.m.}}{E(\phi)}\,\delta\phi + d\Theta(\phi, \delta\phi)$$

Theory : fields $\{\phi\}$, Lagrangian L ← top-form

$$\delta L = E(\phi)\,\delta\phi + d\Theta(\phi, \delta\phi)$$

e.o.m.

Presymplectic potential

Presymplectic form

$$\omega(\phi, \delta_1\phi, \delta_2\phi) = \delta_1\Theta(\phi, \delta_2\phi) - \delta_2\Theta(\phi, \delta_1\phi)$$

Hamiltonian flow (local coords $A, B, \ldots$)

$$(dH_T)_A = \underline{\omega_{AB}} \, T^B$$

Hamiltonian (conjugate)

Hamiltonian vector

Hamiltonian (conjugate)

Hamiltonian vector

Equivalently

$$\delta H_{\xi} = \int_{\Sigma} \omega(\phi, \delta\phi, \delta_{\xi}\phi)$$

Diffeos: $\delta_{\xi} \to \mathcal{L}_{\xi}$

$$\omega(\phi, \delta\phi, \mathcal{L}_{\xi}\phi) \simeq d\left(\delta Q_{\xi} - i_{\xi}\Theta(\phi, \delta\phi)\right)$$

$$\omega(\phi, \delta\phi, \mathcal{L}_\xi \phi) \approx d\left(\delta Q_\xi - \iota_\xi \Theta(\phi, \delta\phi) \right)$$

Noether charge

$$J_\xi = dQ_\xi = \Theta(\phi, \mathcal{L}_\xi \phi) - \iota_\xi L$$

$$\left(\text{Cartan} : \mathcal{L}_\xi = d\iota_\xi + d\iota_\xi \right)$$

$$\delta H_\xi = \int_{\partial\Sigma} \left(\delta Q_\xi \right.$$

Page 3 of 12 Layer Layer 1

305%

$$\omega(\phi, \delta\phi, \mathcal{L}_\xi \phi) \approx d\left(\delta Q_\xi - \imath_\xi \Theta(\phi, \delta\phi) \right)$$

Nœther charge

$$J_\xi = dQ_\xi = \Theta(\phi, \mathcal{L}_\xi \phi) - \imath_\xi L$$

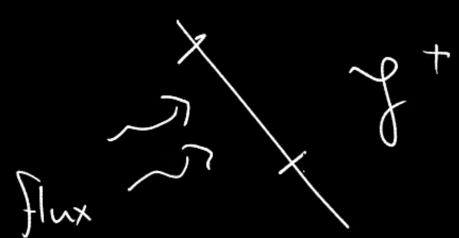
$$\left(\text{Cartan} : \mathcal{L}_\xi = d\imath_\xi + \imath_\xi d \right)$$

$$\delta H_\xi = \int_{\partial\Sigma}$$

$$\delta H_\xi = \int_{\partial\Sigma} \left(\delta Q_\xi - i_\xi \Theta(\phi, \delta\phi) \right)$$

Integrability:

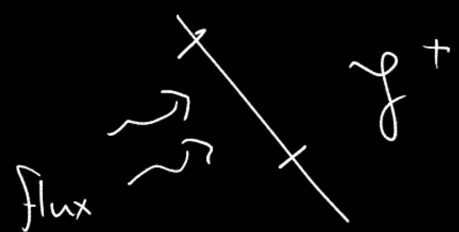
$$(\delta_1\delta_2 - \delta_2\delta_1) H_\xi = -\int_{\partial\Sigma} i_\xi\, \omega(\phi, \delta_1\phi, \delta_2\phi) \neq 0$$



$$\delta H_\xi = \int_{\partial\Sigma} \left(\delta Q_\xi - \iota_\xi \Theta(\phi, \delta\phi) \right)$$

Integrability:

$$(\delta_1\delta_2 - \delta_2\delta_1) H_\xi = - \int_{\partial\Sigma} \iota_\xi\, \omega(\phi, \delta_1\phi, \delta_2\phi) \neq 0$$



$$\delta H_\xi \longrightarrow \not{\delta} H_\xi = \delta \mathcal{H}_\xi + N_\xi$$

BMS charges

$$\oint H^{IW}_{\xi} = \frac{1}{32\pi G}\int_{\partial\Sigma} \varepsilon_{\mu\nu\rho\sigma}\Big\{ 3\, g^{\eta[\tau}\xi^{\sigma}\nabla^{\rho]}\delta g_{\eta\tau} - g^{\tau\sigma}\delta g_{\tau\eta}\nabla^{\eta}\xi^{\rho} - \delta\big(\log\sqrt{-g}\big)\nabla^{\rho}\xi^{\sigma}\Big\}\, dx^{\mu}\wedge dx^{\nu}$$

Bondi form

Leading order

$$\longrightarrow \frac{1}{\quad}$$

Leading order

$$\longrightarrow \frac{1}{16\pi G}\int_{S^2} d\Omega \left\{ \underbrace{4s\,\delta F_0}_{} + \frac{1}{2} s\, \underbrace{\partial_u C_{IJ}\, \delta C^{IJ}}_{N_\xi} \right\}$$

Bondi mass aspect

$s=1$ "global"

$\longrightarrow m$

Bondi news

What about NUT charges?

What about NCT changes?

1^{st} order + Holst term

$$S_{PH} = \frac{1}{16\pi G}\int_M P_{abcd}\, R^{ab}(\omega) \wedge e^c \wedge e^d$$

$$S_{PH} = \frac{1}{16\pi G}\int_M P_{abcd}\, R^{ab}(\omega) \wedge e^c \wedge e^d$$

$$P_{abcd} = \underbrace{\tfrac{1}{2}\varepsilon_{abcd}}_{\substack{\text{Palatini} \\ \text{"electric"}}} + \frac{1}{\gamma}\underbrace{\eta_{a[c}\eta_{d]b}}_{\substack{\text{Holst} \\ \text{"magnetic"}}}$$

γ — arbitrary param.

$$S_{PH} = \frac{1}{16\pi G}\int_M P_{abcd}\, \mathcal{R}^{ab}(\omega) \wedge e^c \wedge e^d$$

$$\underbrace{\frac{1}{2}\varepsilon_{abcd}}_{\substack{\text{Palatini} \\ \text{"electric"}}} + i\,\gamma\, \underbrace{\eta_{a[c}\eta_{d]b}}_{\substack{\text{Holst} \\ \text{"magnetic"}}}$$

arbitrary param. (Immirzi)

e^a : $g = e^a \otimes e^b \eta_{ab}$

ASG : BMS

$$S_{PH} = \frac{1}{16\pi G}\int_M P_{abcd}\,\mathcal{R}^{ab}(\omega)\wedge e^c\wedge e^d$$

$$P_{abcd} = \underbrace{\frac{1}{2}\varepsilon_{abcd}}_{\substack{\text{Palatini} \\ \text{"electric"}}} + \frac{1}{\gamma}\underbrace{\eta_{a[c}\eta_{d]b}}_{\substack{\text{Holst} \\ \text{"magnetic"}}}$$

γ — arbitrary param. (Immirzi)

e^a : $g = e^a \otimes e^b \eta_{ab}$

ASG : BMS + Lorentz transfs

Presymplectic potential:

$$\Theta_E = \frac{1}{32\pi G} \varepsilon_{abcd}\, \delta\omega^{ab} \wedge e^c \wedge e^d$$

$$\Theta_M = \frac{i}{16\pi G}$$

Page 5 of 12 Layer Layer 1

305%

Presymplectic potential:

$$\Theta_E = \frac{1}{32\pi G} \varepsilon_{abcd}\, \delta\omega^{ab} \wedge e^c \wedge e^d$$

$$\Theta_M = \frac{i}{16\pi G} \delta\omega_{ab} \wedge e^a \wedge e^b$$

Noether charge:

$$Q_E = \frac{1}{32\pi G} \varepsilon_{abcd}\, \imath_\xi \omega^{ab}$$

$$Q_E = \frac{1}{32\pi G}\,\varepsilon_{abcd}\,\iota_\xi\,\omega^{ab}\,e^c\wedge e^d$$

$$Q_M = \frac{i}{16\pi G}\,\iota_\xi\,\omega^{ab}\,e_a\wedge e_b$$

$$\rightsquigarrow \quad \not{\delta} H^E_\xi = \frac{1}{16\pi G}\,\varepsilon_{abcd}\int_{\partial\Sigma}\iota_\xi e^c\,\delta\omega^{ab}\wedge e^d = \not{\delta} H_\xi^{IW}$$

$$Q_E = \frac{1}{32\pi G}\,\varepsilon_{abcd}\,\iota_\xi \omega^{ab}\, e^c \wedge e^d$$

$$Q_M = \frac{i}{16\pi G}\,\iota_\xi \omega^{ab}\, e_a \wedge e_b$$

$$\rightsquigarrow \quad \not\delta H_\xi^E = \frac{1}{16\pi G}\,\varepsilon_{abcd}\int_{\partial\Sigma} \iota_\xi e^c\, \delta\omega^{ab} \wedge e^d = \not\delta H_\xi^{IW}$$

$$\not\delta H_\xi^M = -\frac{1}{8\pi G}\int_{\partial\Sigma} \delta e^a \wedge \mathcal{L}_\xi e_a = \not\delta \tilde{H}_\xi$$

Page 5 of 12 Layer Layer 1

305%

$$\delta H^{M}_{\xi} = -\frac{1}{8\pi G}\int_{\partial\Sigma} \delta e^{a}\wedge \mathcal{L}_{\xi} e_{a} = \delta \tilde{H}_{\xi}$$

Leading order

$$\mathcal{H}^{M}_{\xi} = -\frac{1}{8\pi G}\int_{S^2} s \, dC_0^{\,b} \qquad C_0{}^{b}{}_{I} = \gamma_{IJ}\, C_0^{\,J}$$

$s=1$ NUT charge

$$\rightsquigarrow \quad \not{\delta} H^{E}_{\xi} = \frac{1}{16\pi G}\,\epsilon_{abcd}\int_{\partial\Sigma} \iota_{\xi} e^{c}\,\delta\omega^{ab}\wedge e^{d} = \not{\delta} H^{1W}_{\xi}$$

$$\not{\delta} H^{M}_{\xi} = -\frac{1}{8\pi G}\int_{\partial\Sigma} \delta e^{a}\wedge \mathcal{L}_{\xi} e_{a} = \not{\delta}\tilde{H}_{\xi}$$

Leading order

$$\mathcal{H}^{M}_{\xi} = -\frac{1}{8\pi G}\int_{S^2} s\, dC_0^{\,b} \qquad C_0{}^{b}{}_{I} = \gamma_{IJ}\, C_0^{\,J}$$

$s=1$ NUT charge

$$\delta H_\xi = \frac{1}{16\pi G}\epsilon_{abcd}\int_{\partial\Sigma} \eta_\xi e^c\, \delta\omega^{ab}\wedge e^d = \delta H_\xi^{\omega}$$

$$\delta H_\xi^M = -\frac{1}{8\pi G}\int_{\partial\Sigma} \delta e^a \wedge \mathcal{L}_\xi e_a = \delta \tilde{H}_\xi$$

Leading order

$$\mathcal{H}_\xi^M = -\frac{1}{8\pi G}\int_{S^2} s\, dC_0^{\,b} \qquad C_0{}^b{}_I = \gamma_{IJ}\, C_0^{\,J}$$

$s=1$ NUT charge

$$N_\xi^M \sim \int_{S^2} \frac{s}{2}\,\partial_u C_{IJ}\, \delta \tilde{C}_{IJ}$$

$$\tilde{C}_{IJ} = \epsilon_{IK}\, C^K{}_J$$

Remarks

* General approach to charges: Lagrangian matters!
(Gauß-Bonnet or Pontryagin)

* Can be done also in Barnich-Brandt (Oliveri-Speziale)

* λ: $\lambda = -1 \longrightarrow$ Ashtekar variables
$\longrightarrow$ reproduce NP charge

Remarks

* General approach to charges: Lagrangian matters!
  (Gauß-Bonnet or Pontryagin)
* Can be done also in Barnich-Brandt (Oliveri-Speziale)
* λ : $\underline{\lambda = -1}$ $\longrightarrow$ Ashtekar variables
  $\longrightarrow$ reproduce NP charge
  $\longrightarrow$ consistent phase space