Title: Signals of a Quantum Universe

Speakers: Daniel Green

Series: Colloquium

Date: October 14, 2020 - 2:00 PM

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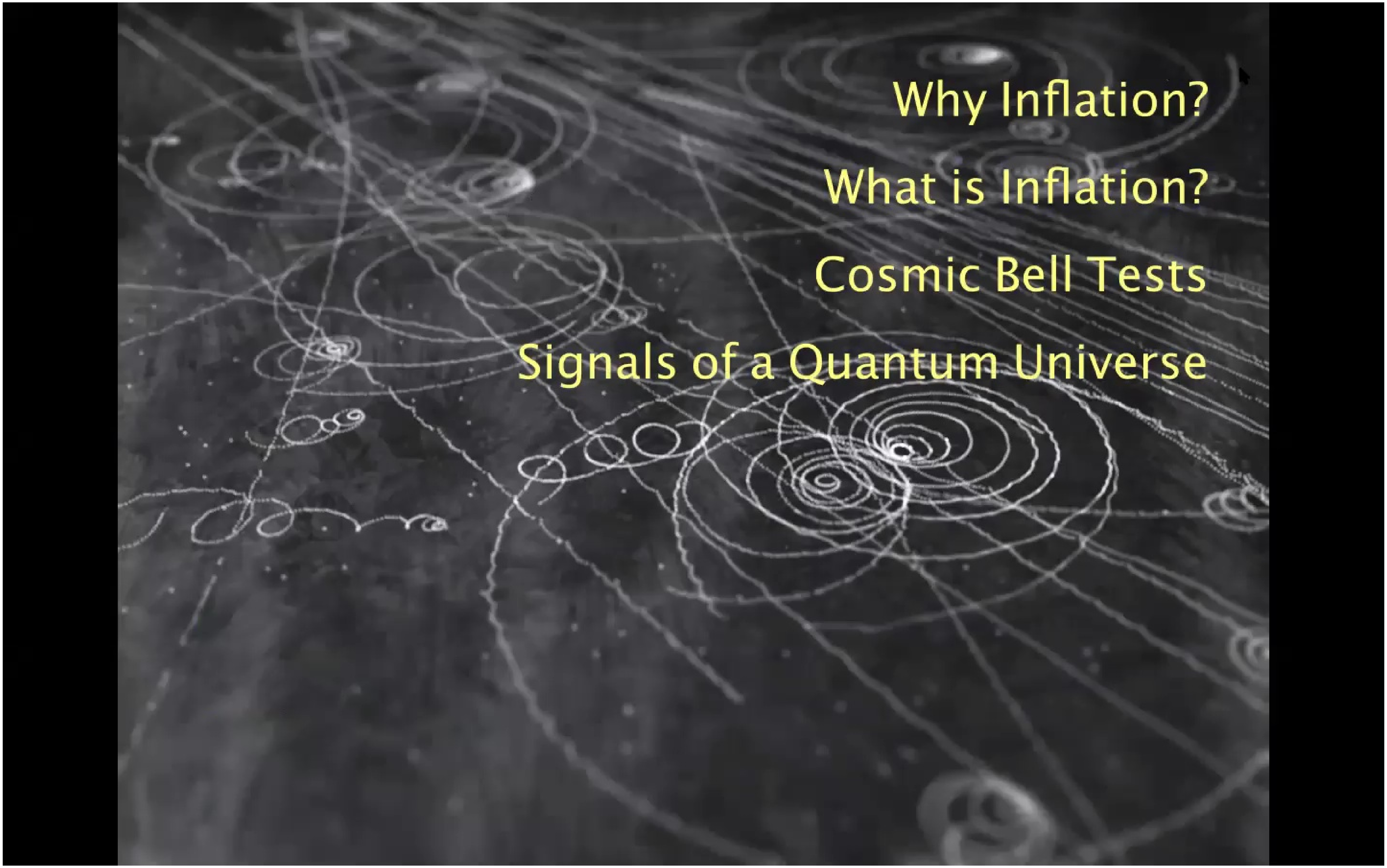
Abstract: The idea that structure in the Universe was created from quantum mechanical vacuum fluctuations during inflation is very compelling, but unproven. Finding a test of this proposal has been challenging because the universe we observe is effectively classical. I will explain how quantum fluctuations can give rise to the density fluctuations we observe and will show that we can test this hypothesis using the statistical properties of maps of the universe.


Why Inflation?
What is Inflation?
Cosmic Bell Tests
Signals of a Quantum Universe

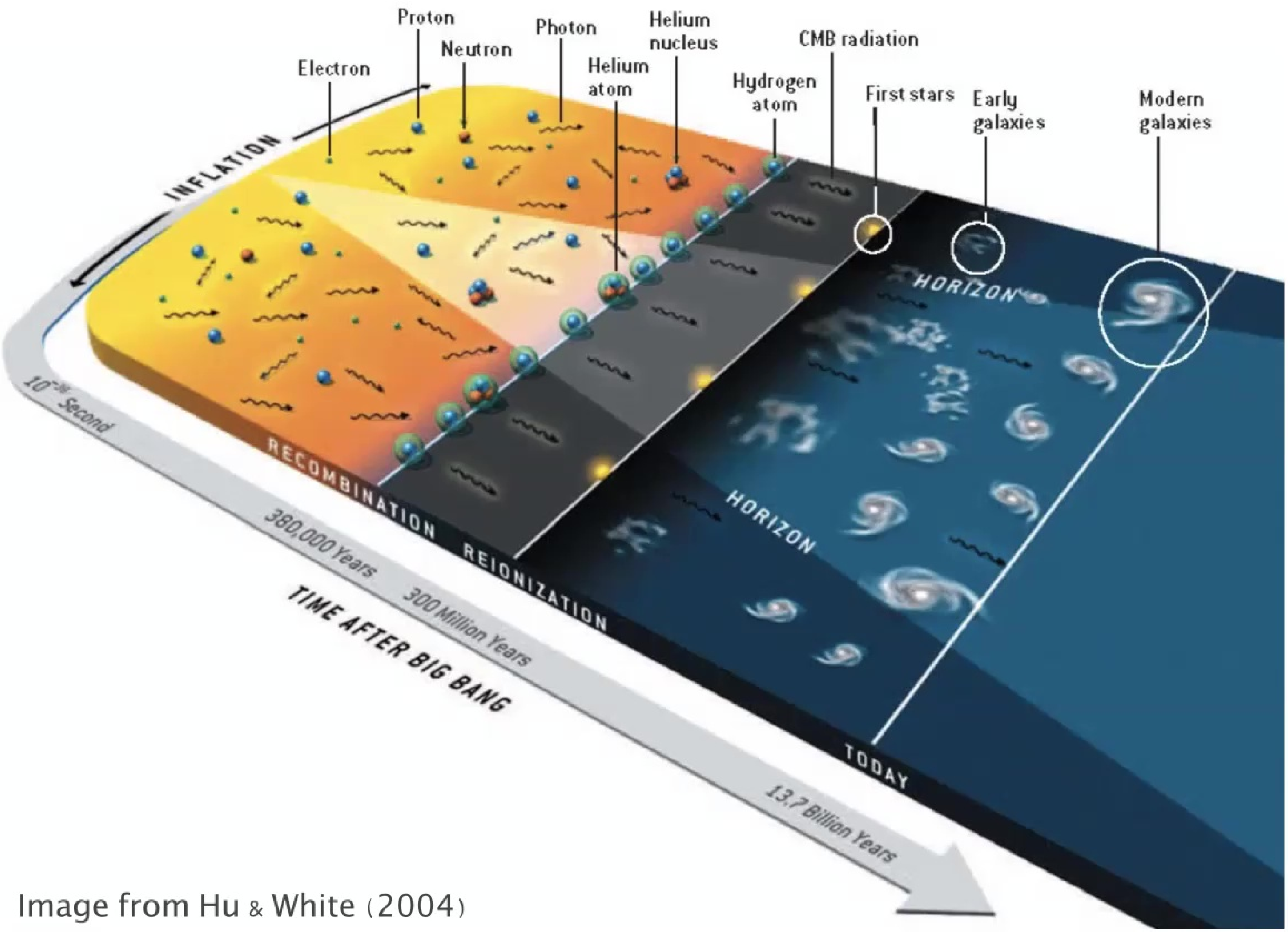
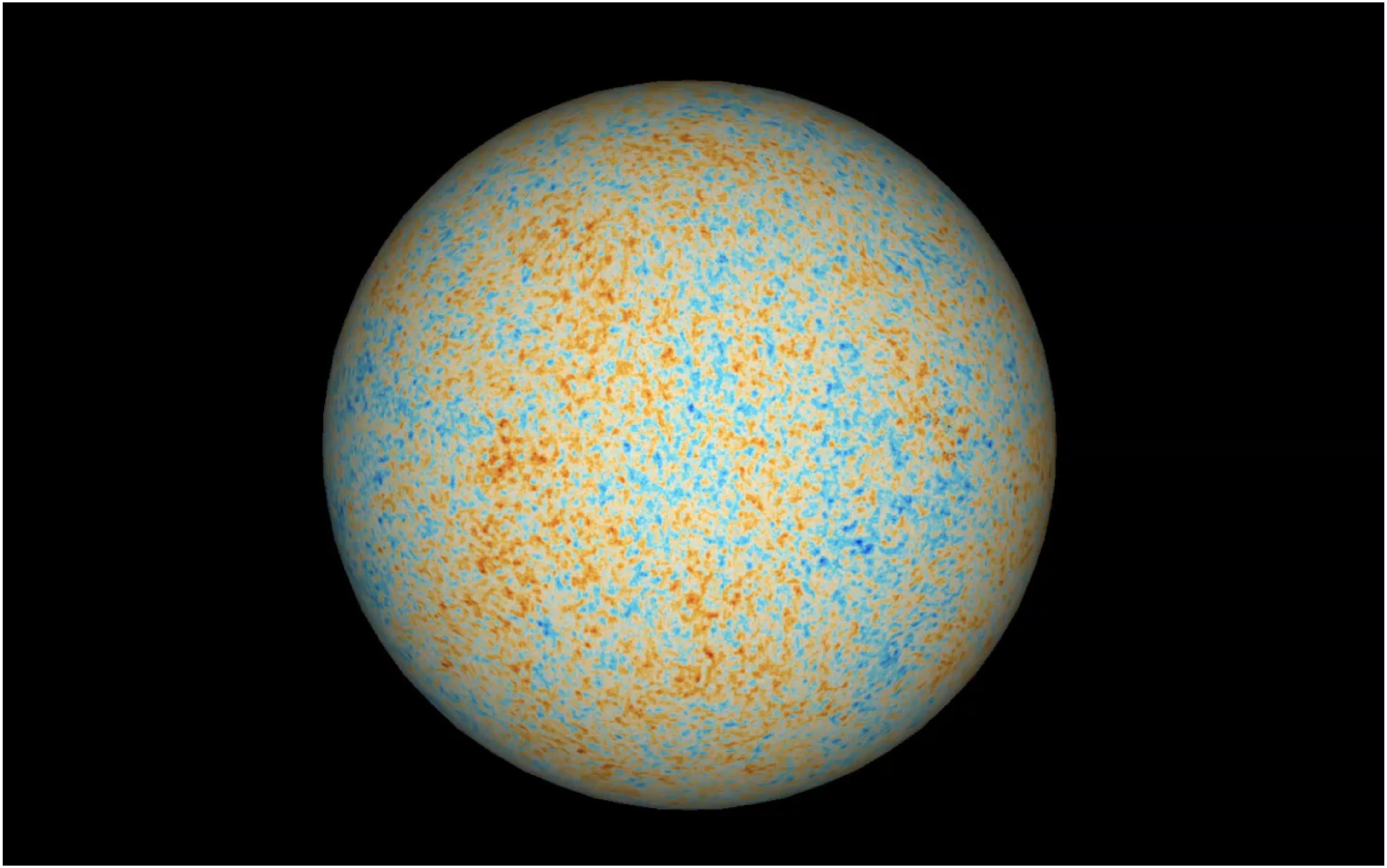
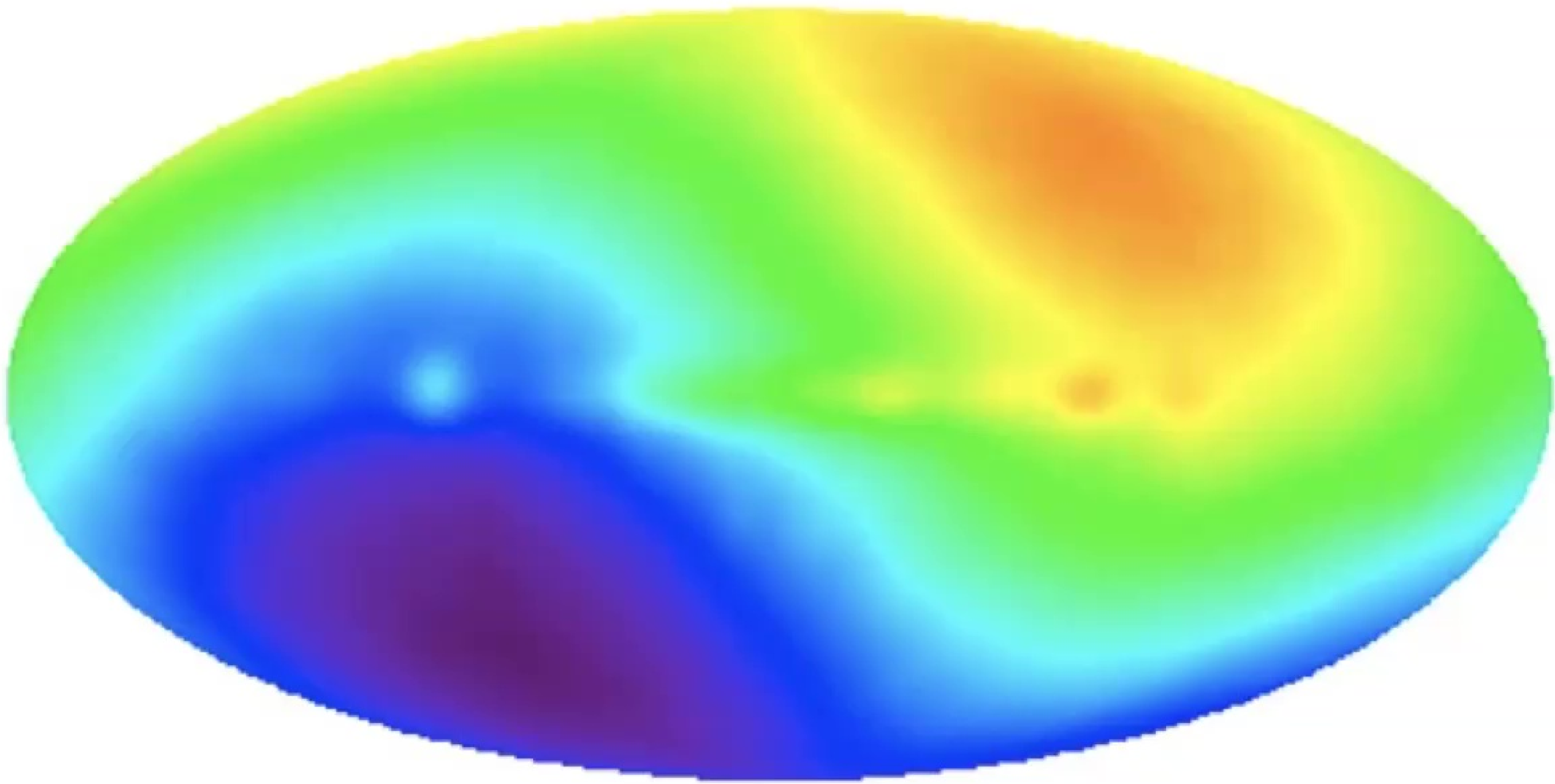


Image from Hu & White (2004)



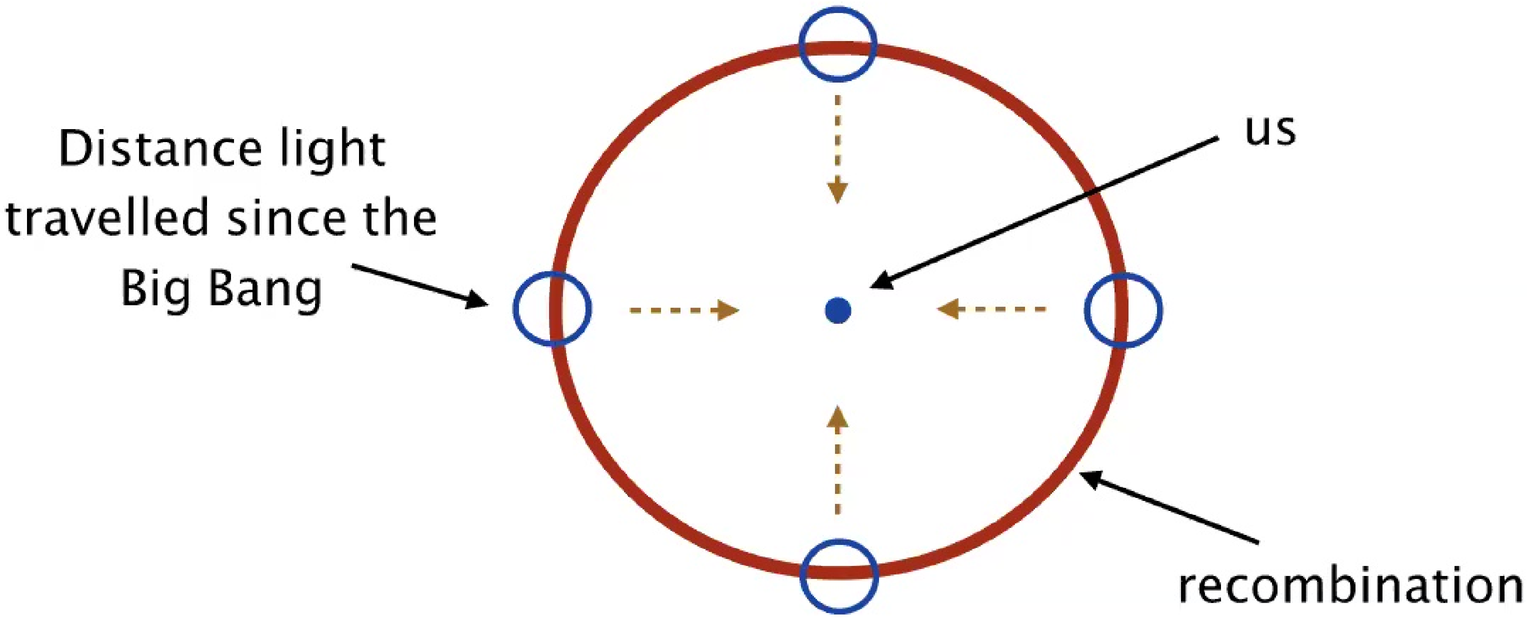


1969

$\Delta T \approx 3\,\mathrm{mK}$

Why inflation? (I)

Uniformity demands explanation



Hot big bang doesn't give enough time to equilibrate

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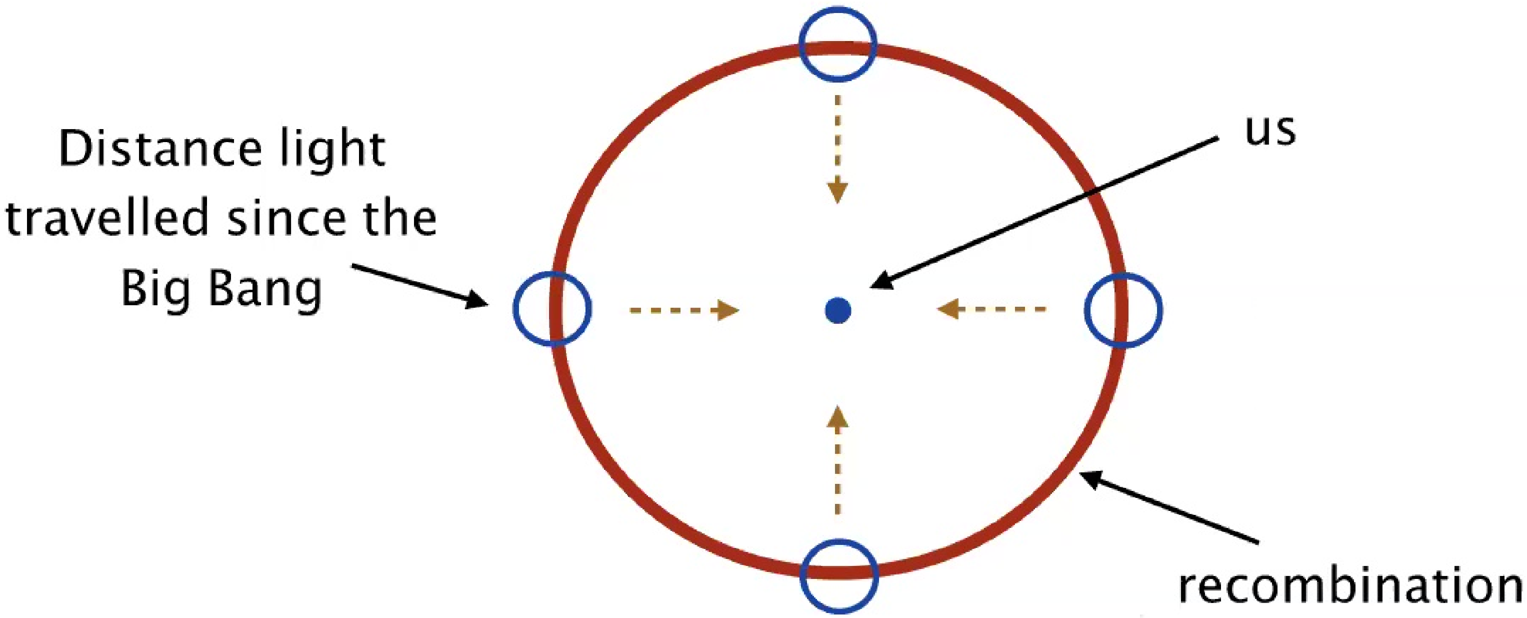
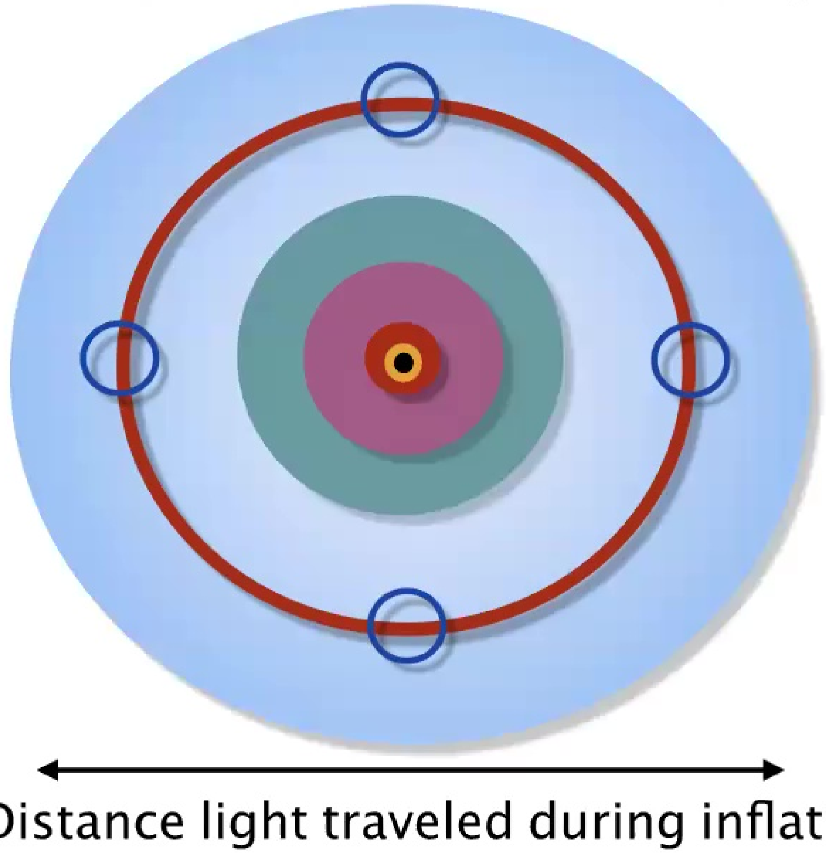


Hot big bang doesn't give enough time to equilibrate

Inflation

Solution: (1) add time in the past (add negative time)

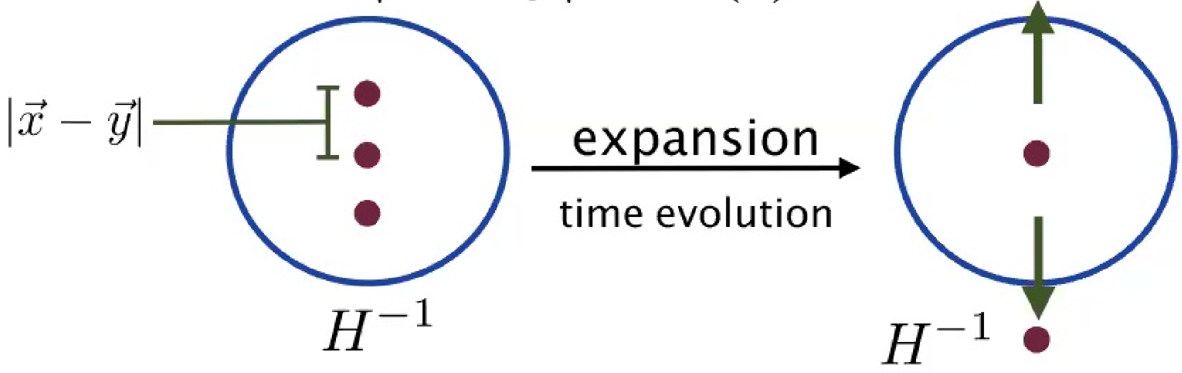


Guth (1980)

Inflation

(2) Take advantage of exponential expansion

$$|\vec{x}-\vec{y}| \propto a(t) = e^{Ht}$$



Causal contact set by Hubble parameter $H^{-1} = \left(\frac{\dot{a}}{a}\right)^{-1}$

Guth (1980)

Inflation

Problem: inflation can't last forever

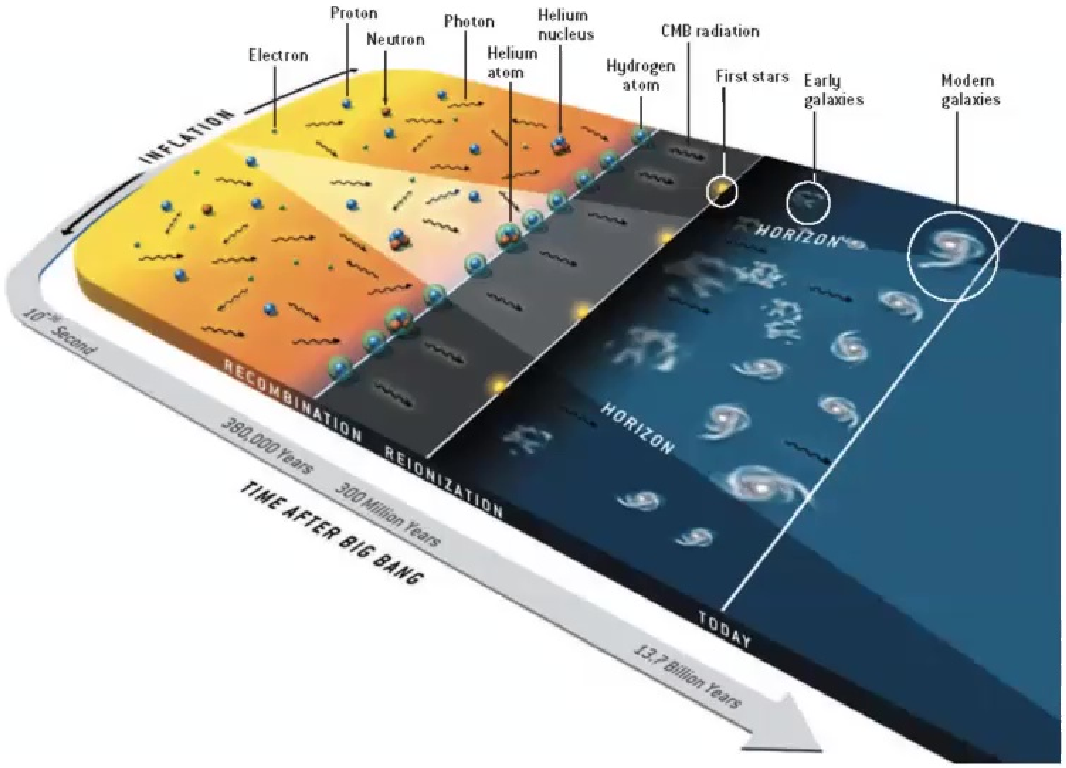
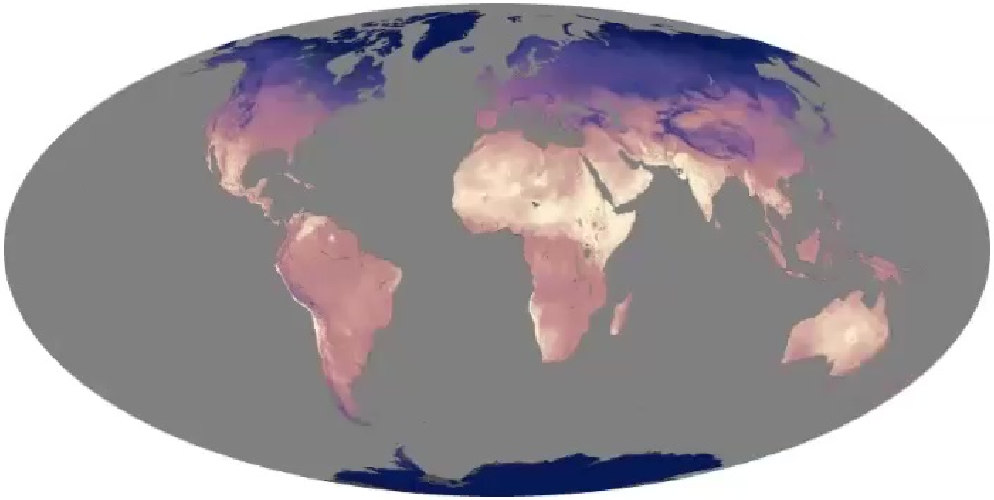


Image from Hu & White (2004)

Inflation

End inflation with a scalar field

Scalar field is just a number for each point in space



Think of $T(\vec{x}, t)$

Each point evolves in time

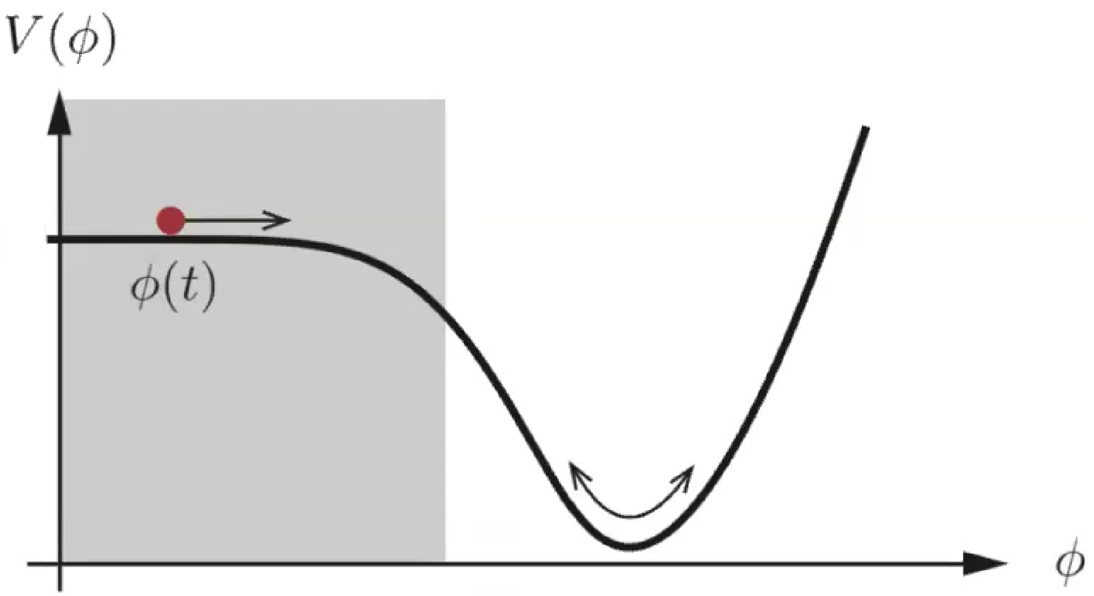
Linde; Albrecht & Steinhardt (1982)

Slow-Roll Inflation

Slow-roll inflation $\phi(\vec{x}, t) = \phi(t)$

$$H^2 \propto \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\approx V_0 = \text{const.}$$



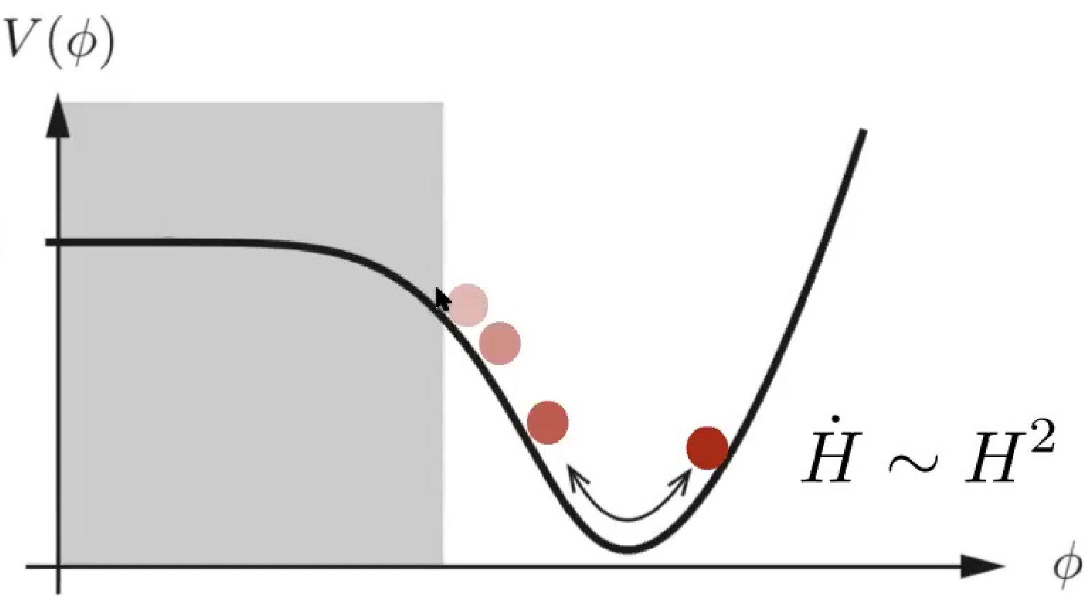
From Baumann & McAllister

Linde; Albrecht & Steinhardt (1982)

Slow-Roll Inflation

Inflation ends: potential converted to kinetic energy

$$H^2 \propto \frac{1}{2}\dot{\phi}^2 + V(\phi)$$



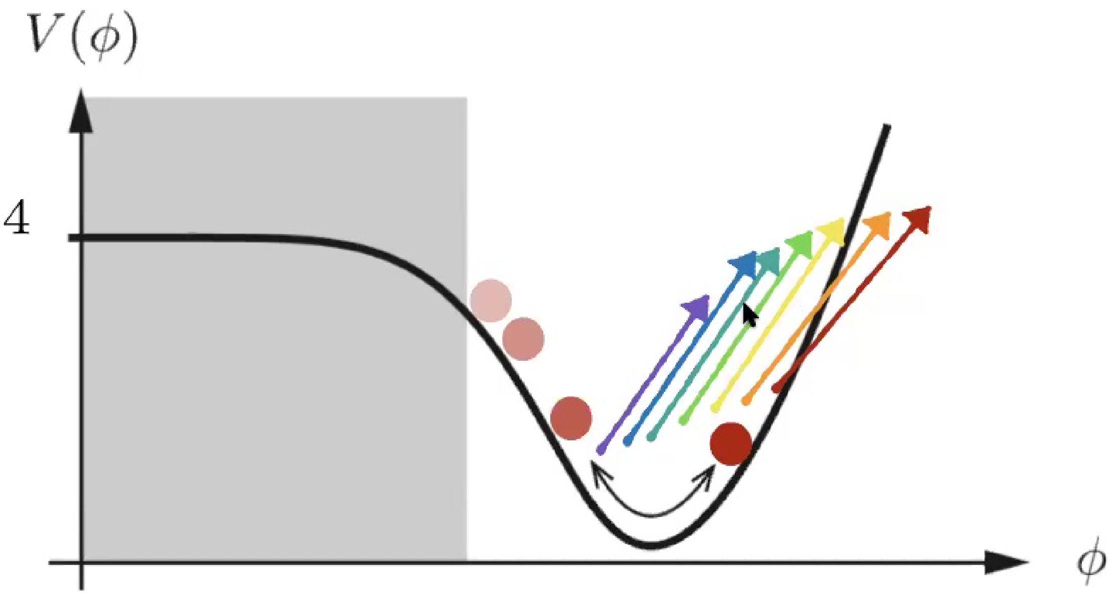
Linde; Albrecht & Steinhardt (1982)

From Baumann & McAllister

Slow-Roll Inflation

Reheating: kinetic energy converted to radiation

$$H^2 \propto T^4 \propto a^{-4}$$



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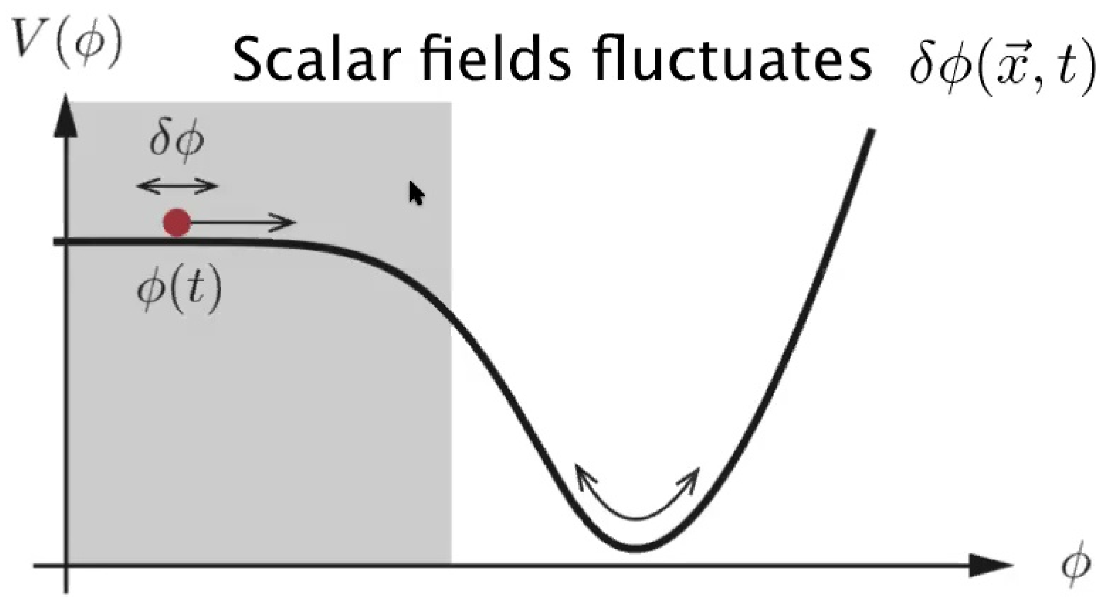
Origin of Structure

Inflation sources long wavelength initial conditions



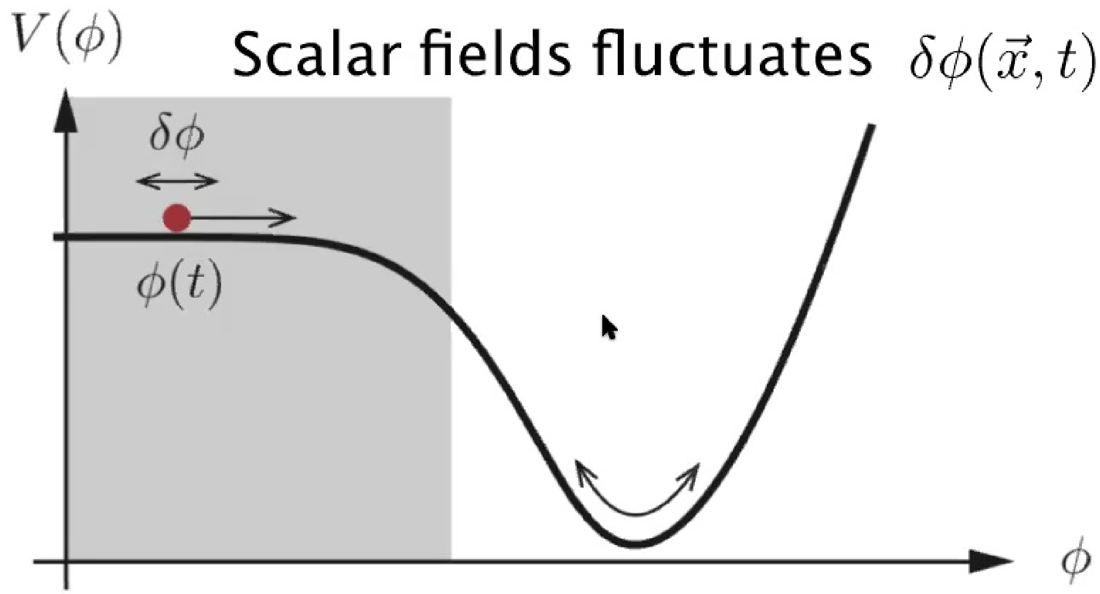
Length of inflation varies in space $a(t) \rightarrow a(t, \vec{x})$

Mukhanov & Chibisov; Hawking; Starobinsky; Guth & So-Young Pi; Bardeen, Steinhardt & Turner (1982)

Origin of Structure

Inflation sources long wavelength initial conditions

Scalar fields fluctuates $\delta\phi(\vec{x}, t)$



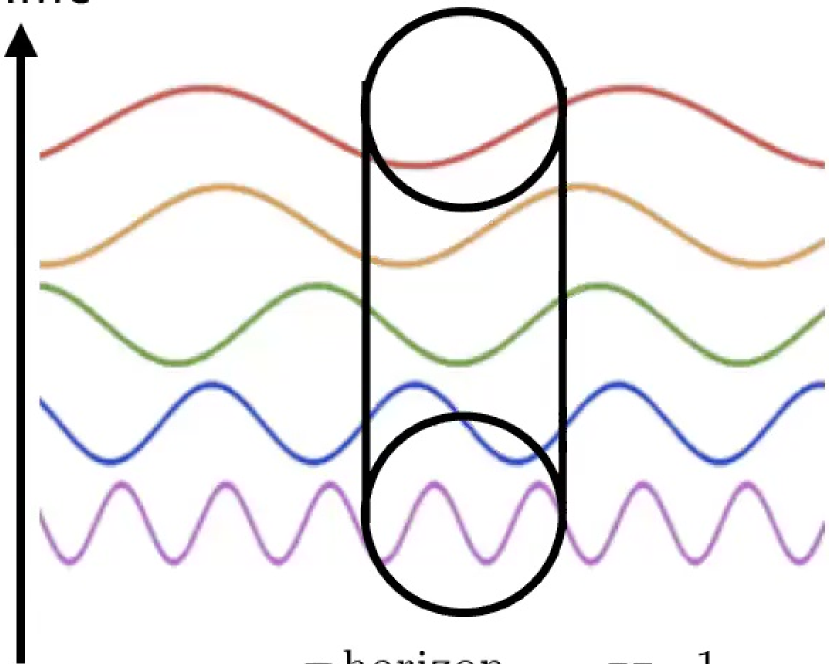
Length of inflation varies in space $a(t) \to a(t, \vec{x})$

Mukhanov & Chibisov; Hawking; Starobinsky; Guth & So-Young Pi; Bardeen, Steinhardt & Turner (1982)

Origin of Structure

Expansion sources classical from quantum

Time

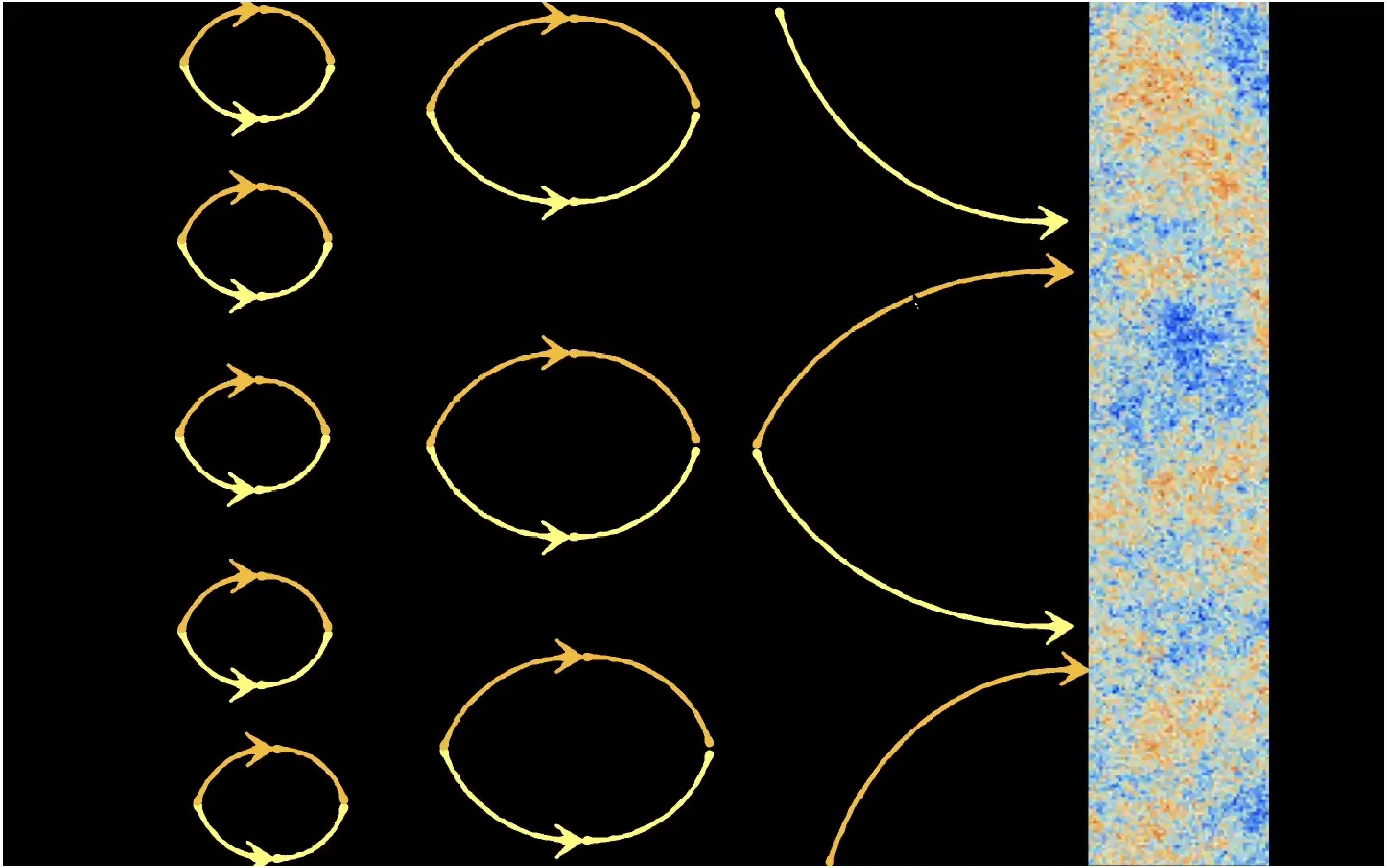


Classical initial density fluctuations

Quantum Vacuum

$$R^{\text{horizon}} = H^{-1}$$

Mukhanov & Chibisov; Hawking; Starobinsky; Guth & So-Young Pi; Bardeen, Steinhardt & Turner (1982)



Inflationary Predictions

"Predictions" of slow-roll inflation (circa ~1985)

On large scales:

- Homogeneous & Isotropic
- Universe is spatially flat

Density fluctuations are:

- Adiabatic (i.e. uniform in all energy densities)
- Nearly scale-invariant
- Gaussian

Similar prediction for gravitational waves

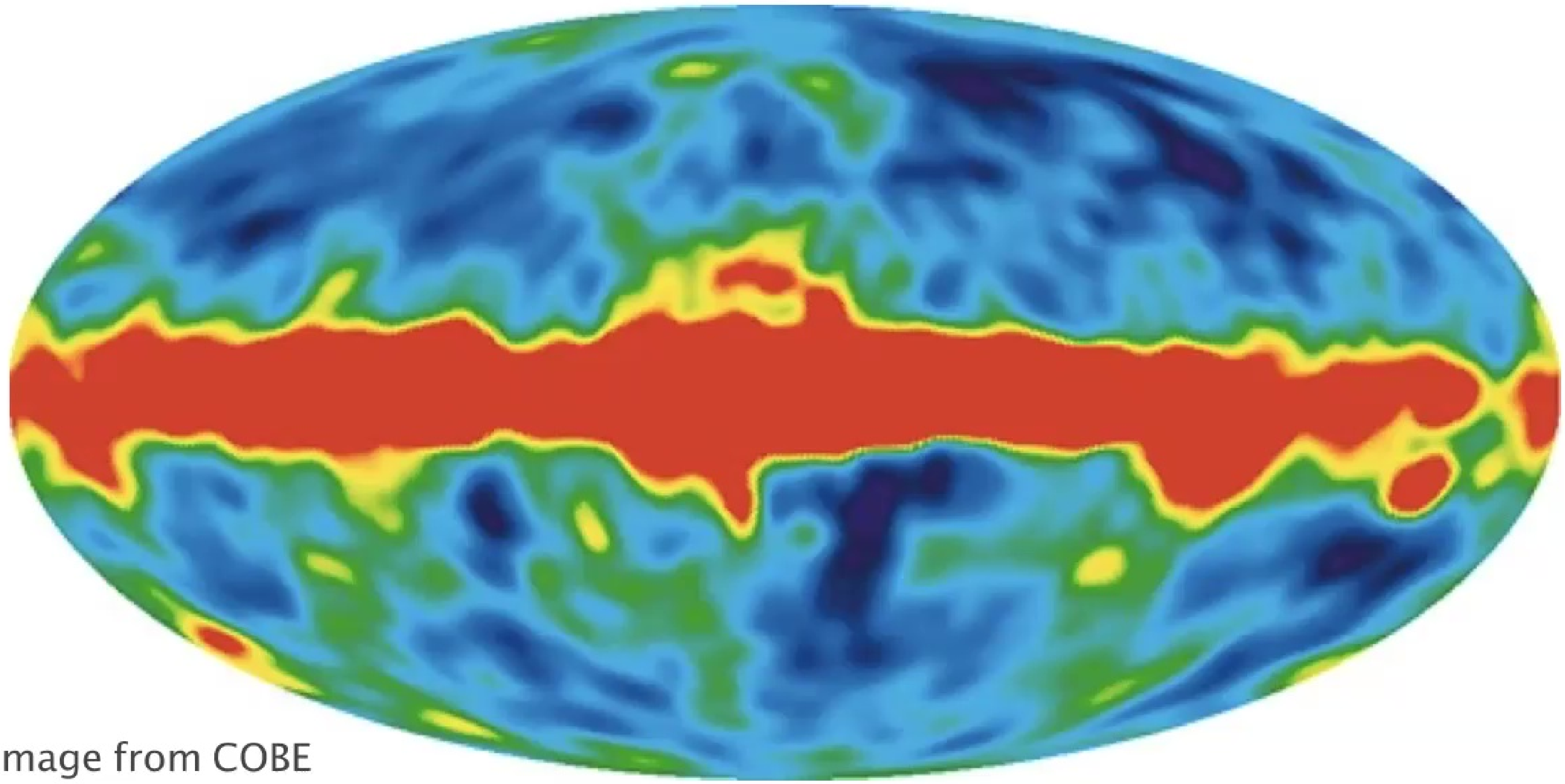
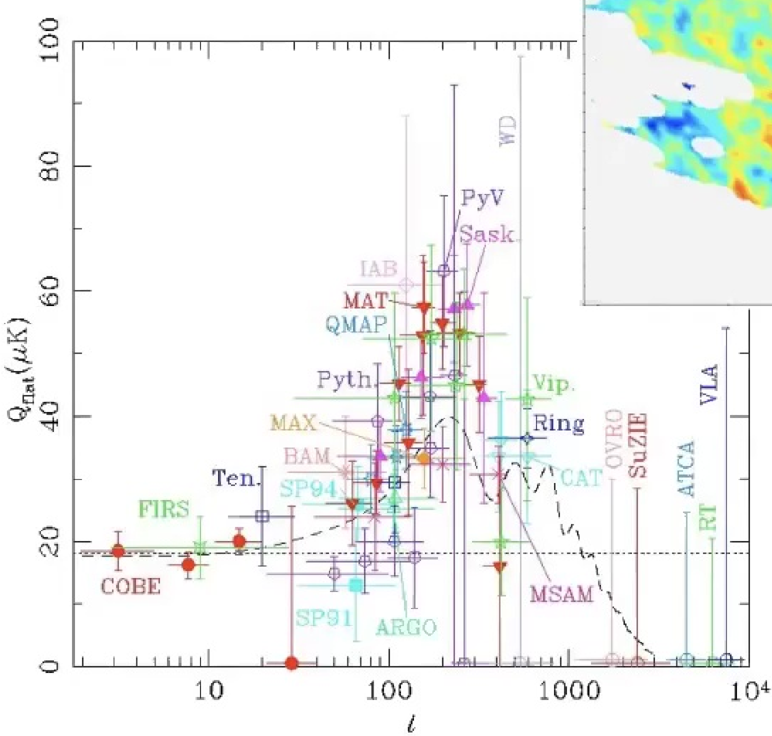


Image from COBE

1992

$$\Delta T = \mathcal{O}(10)\,\mu\mathrm{K}$$



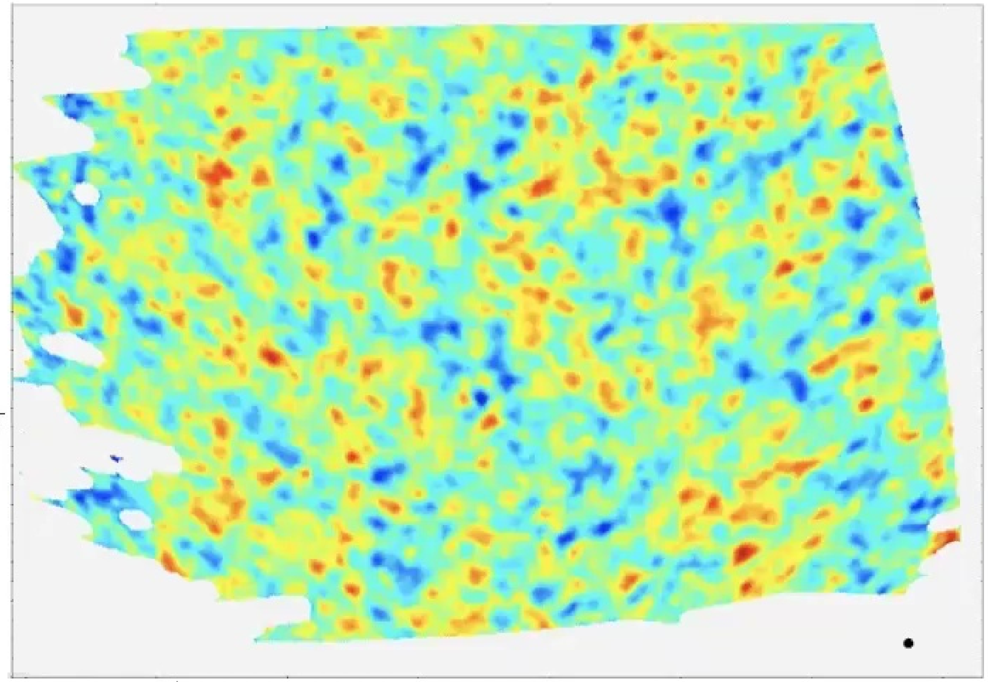
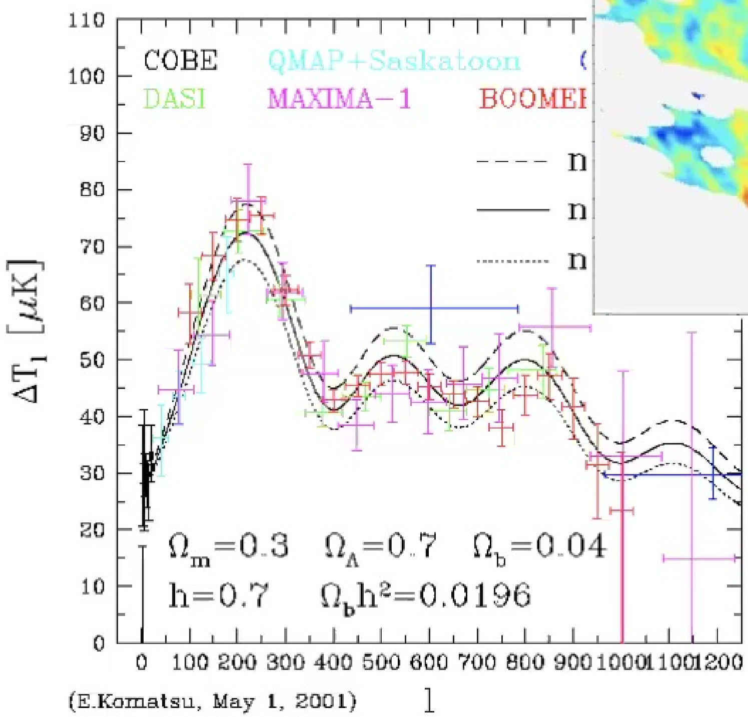
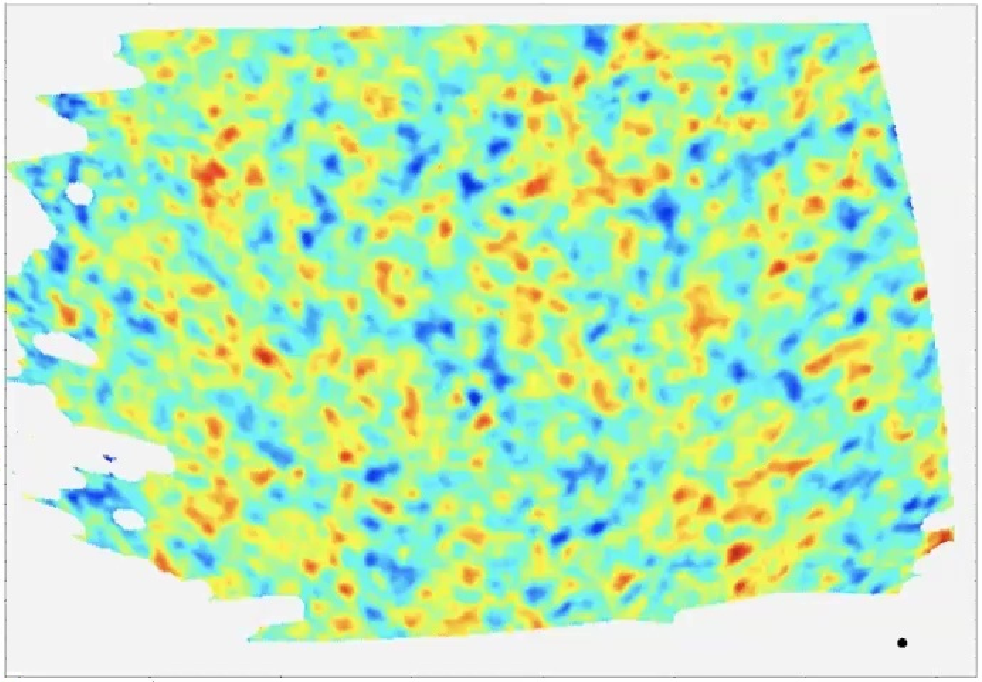


Image from Boomerang

1992-2001





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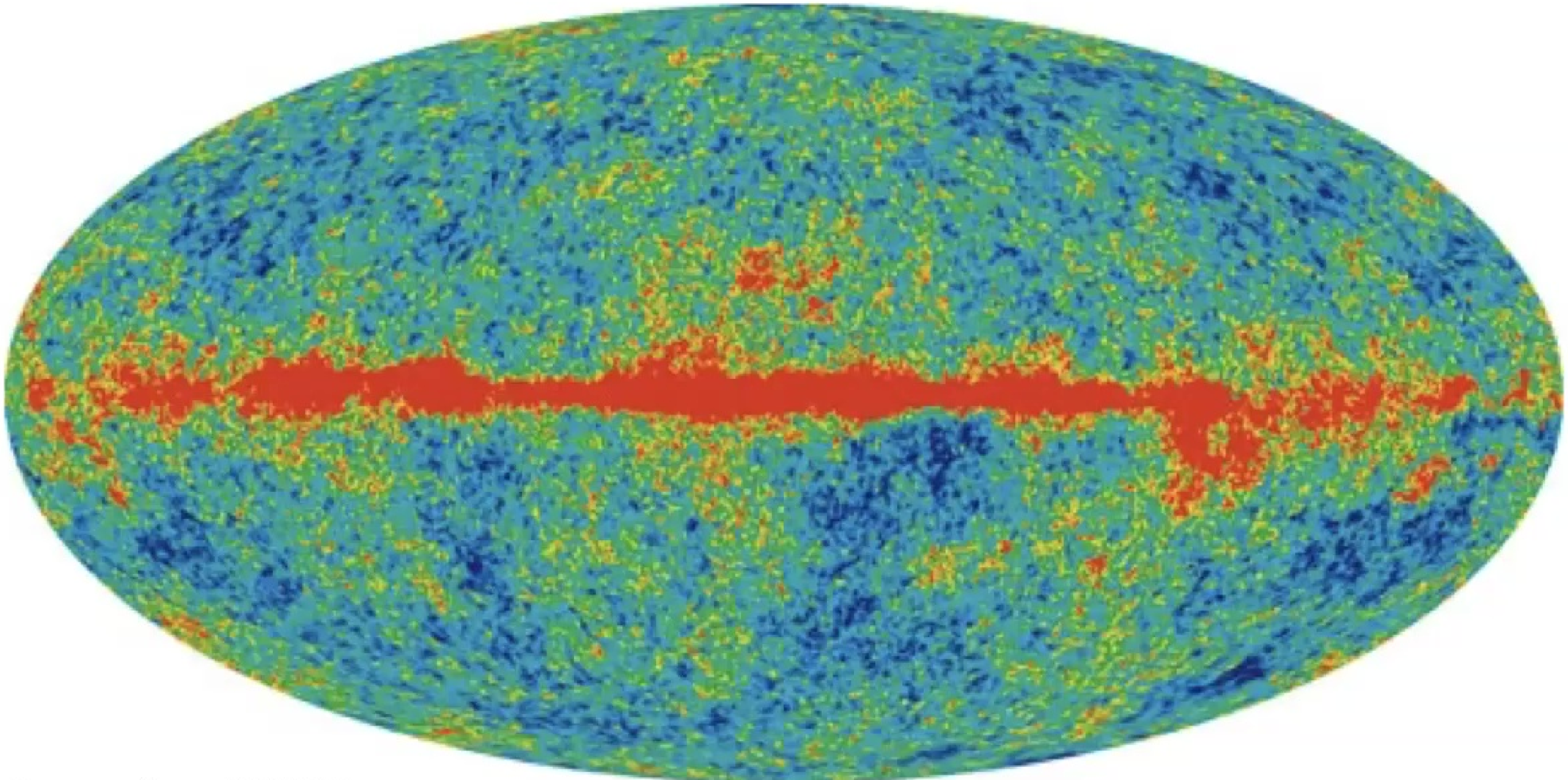
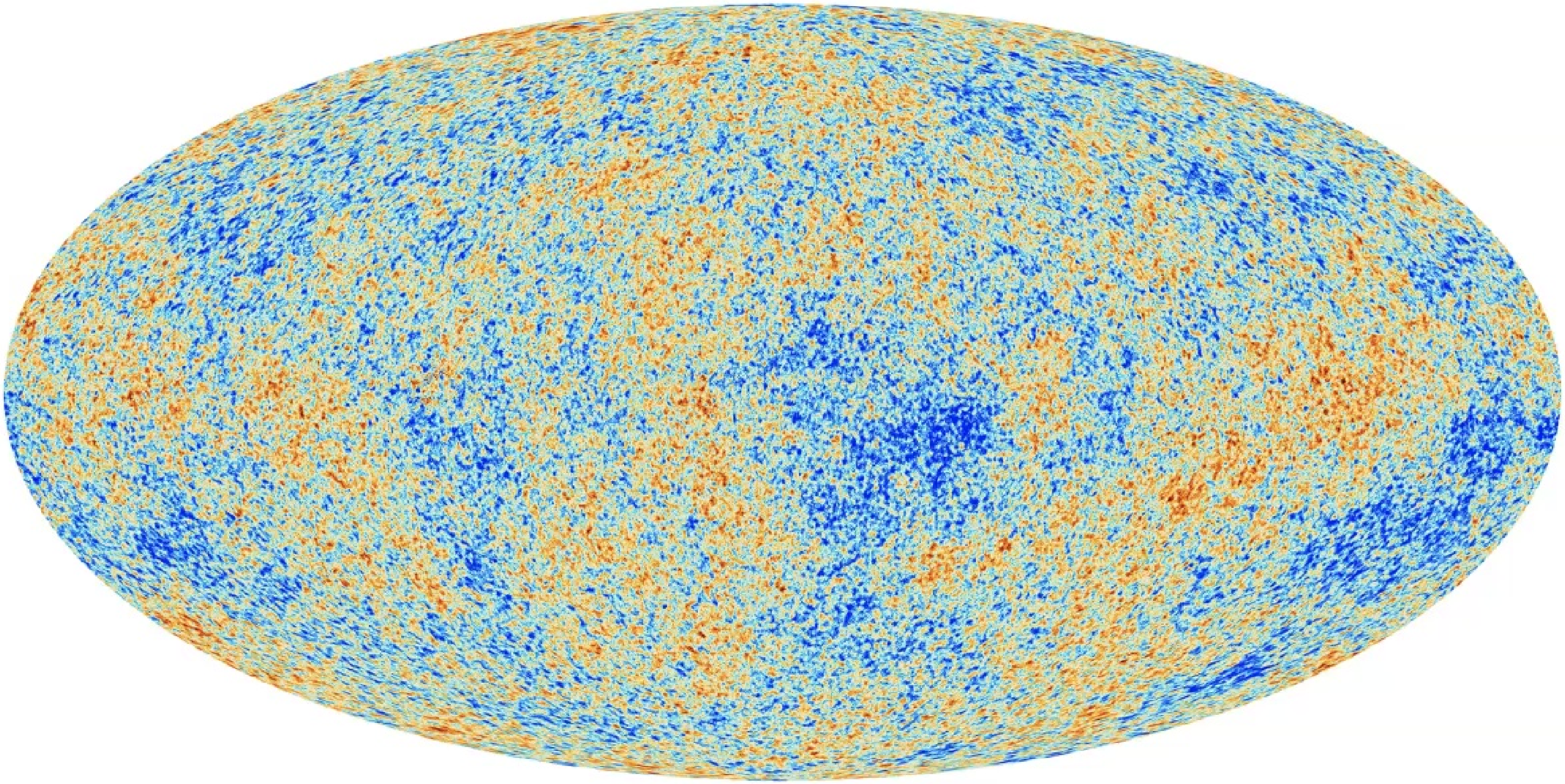


Image from WMAP

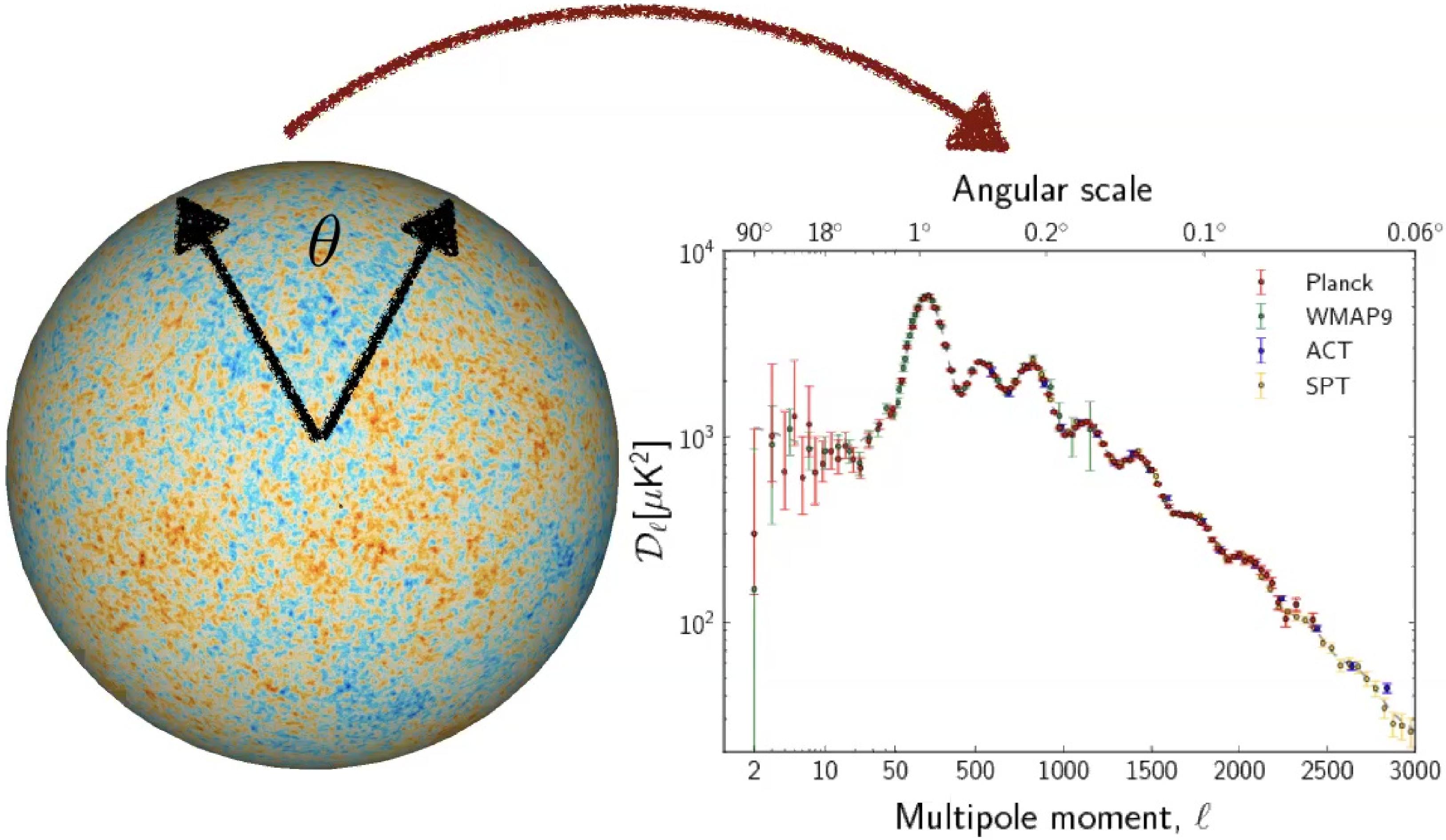
2003–2012



2013-2018

Image from Planck

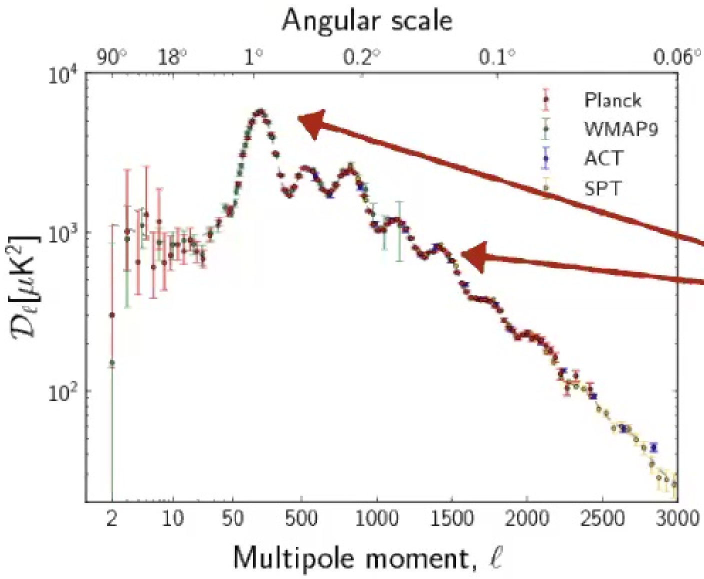
Why inflation (II)



Courtesy of thecmb.org

Why inflation (II)

This is snapshot of a sound waves



Waves all add in phase

$$\frac{\delta T}{T} \approx A_{\vec{k}} \cos(c_s k\tau) + \cancel{B_{\vec{k}} \sin(c_s k\tau)}$$

$$d\tau = a^{-1} dt$$

Why inflation (II)

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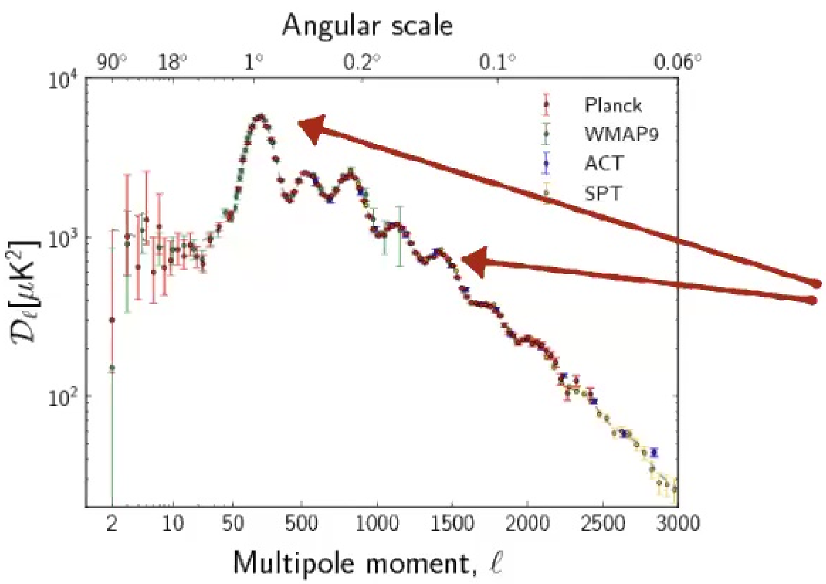


Waves all add in phase

$$\frac{\delta T}{T} \approx A_{\vec{k}} \cos(c_s k\tau) + B_{\vec{k}} \sin(c_s k\tau)$$

$$d\tau = a^{-1} dt$$

Why inflation (II)

Explained if fluctuations created long before

Long wavelength sound waves don't oscillate

$$\frac{\delta T}{T} \approx A_{\vec{k}} + b_{\vec{k}} a^{-3}$$

$$B_{\vec{k}} \sim b_{\vec{k}} a^{-3} \to 0$$

Second mode is redshifted away [remember for later]

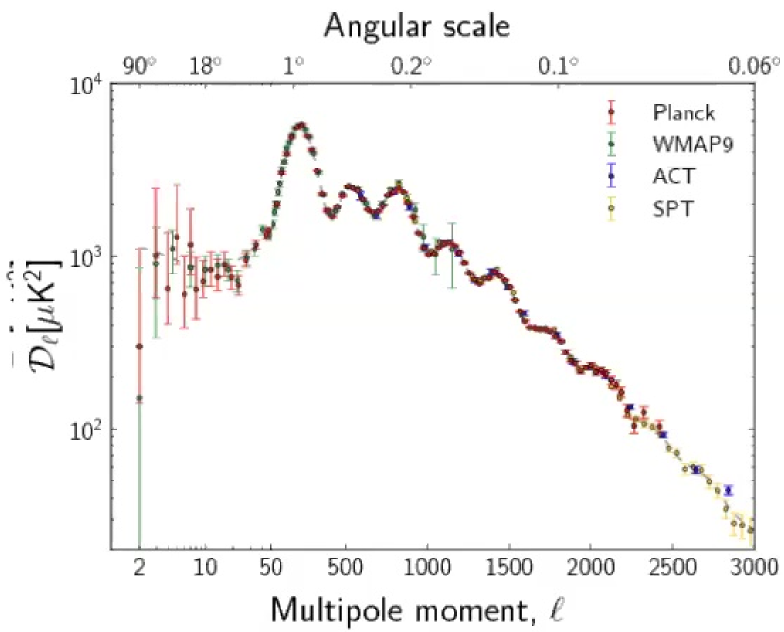
$$\frac{\delta T}{T} \sim A_{\vec{k}} \cos(c_s k \tau)$$

"Inflation": statistical initial conditions

Sound waves at recombination:

Inflationary Predictions

Data (today):

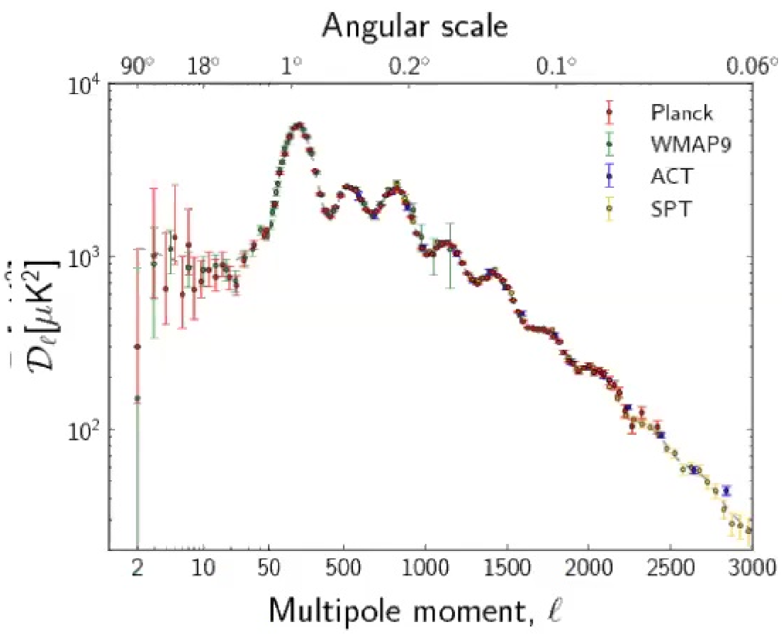


	Planck		*Planck*+lensing		*Planck*+WP	
Parameter	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6964	0.693 ± 0.019	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3036	0.307 ± 0.019	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8285	0.823 ± 0.018	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	68.14	67.9 ± 1.5	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.19^{+0.12}_{-0.14}$	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14094	0.1414 ± 0.0029	0.14305	0.1426 ± 0.0025
$\Omega_m h^3$	0.09597	0.09590 ± 0.00059	0.09603	0.09593 ± 0.00058	0.09591	0.09589 ± 0.00057
Y_P	0.247710	0.24771 ± 0.00014	0.247785	0.24775 ± 0.00014	0.247695	0.24770 ± 0.00012
Age/Gyr	13.819	13.813 ± 0.058	13.784	13.796 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.01	1090.16 ± 0.65	1090.48	1090.43 ± 0.54
r_*	144.58	144.75 ± 0.66	145.02	144.96 ± 0.66	144.58	144.71 ± 0.60
$100\theta_*$	1.04139	1.04148 ± 0.00066	1.04164	1.04156 ± 0.00066	1.04136	1.04147 ± 0.00062
z_{drag}	1059.32	1059.29 ± 0.65	1059.59	1059.43 ± 0.64	1059.25	1059.25 ± 0.58
r_{drag}	147.34	147.53 ± 0.64	147.74	147.70 ± 0.63	147.36	147.49 ± 0.59
k_D	0.14026	0.14007 ± 0.00064	0.13998	0.13996 ± 0.00062	0.14022	0.14009 ± 0.00063
$100\theta_D$	0.161332	0.16137 ± 0.00037	0.161196	0.16129 ± 0.00036	0.161375	0.16140 ± 0.00034
z_{eq}	3402	3386 ± 69	3352	3362 ± 69	3403	3391 ± 60
$100\theta_{eq}$	0.8128	0.816 ± 0.013	0.8224	0.821 ± 0.013	0.8125	0.815 ± 0.011
$r_{drag}/D_V(0.57)$	0.07130	0.0716 ± 0.0011	0.07207	0.0719 ± 0.0011	0.07126	0.07147 ± 0.00091

Incredible agreement with slow-roll inflation

Inflationary Predictions

Data (today):



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Incredible agreement with slow-roll inflation

Inflationary Predictions??

Does this confirm slow-roll inflation?

A lot has happened since 1982
(Guth's paper cited 8000 times)

Many seemingly different mechanisms give the same or similar predictions

What is inflation and how do we test the framework?

What is inflation?

A definition:

1. A period of quasi-dS expansion

$$\frac{\dot{H}}{H^2} \ll 1$$

2. A physical clock

Needed to define the end of inflation   Cheung et al.

In slow roll, the clock is defined by $\phi(t)$

What is inflation?

A definition:

1. A period of quasi-dS expansion

$$\frac{\dot{H}}{H^2} \ll 1$$

This was Guth's original insight

Allowed for structure to have a causal origin

What is inflation?

A definition:

2. A physical clock

Essentially Linde, Albrecht & Steinhardt's insight

"Time" is not well-defined without a clock

Inflation must end everywhere at the same "time"

Different regions synched their clocks in the past

What is inflation?

A definition:

2. A physical clock

Essentially Linde, Albrecht & Steinhardt's insight

"Time" is not well-defined without a clock

Inflation must end everywhere at the same "time"

Different regions synched their clocks in the past

What is inflation?

No clock is perfect (uncertainty principle)

The amount of inflation will vary from place to place:

$$\zeta(x) \sim \frac{\delta a(x)}{a} \sim \frac{\dot{a}\delta t(x)}{a} \equiv H\delta t$$

RMS fluctuations of the clock $\sqrt{\langle(\delta t)^2\rangle} \sim \frac{H}{f_\pi^2}$

Time between "ticks" defines an energy scale f_π

For slow-roll inflation $\delta t \sim \frac{\delta\phi}{\dot{\phi}}$ and $f_\pi^2 = \dot{\phi}$

What is inflation?

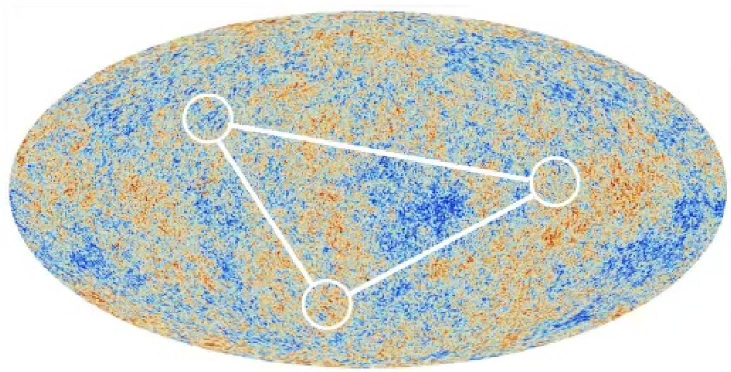
These are the adiabatic fluctuations

Determine CMB temperature fluctuations

$$\frac{\delta T(\mathbf{n})}{T} \sim \int d^3k F(\mathbf{k}\cdot\mathbf{n}, k)\zeta_{\mathbf{k}}$$

Think of

$$\langle \zeta(x_1)..\zeta(x_n)\rangle \rightarrow$$



Quantum Origin of Structure

Raises the question: what is the clock?

Real world clocks are affected by thermal noise too

Can we tell the difference?

Quantum explanation captures the imagination:

The largest objects in the universe are explained by the unusual phenomena of the smallest scales

But the universe need not follow out preference

Quantum Origin of Structure

Didn't Bell solve this problem for us?

Bell gives a definitive way to tell a state IS quantum

Unfortunately, the universe we observe is classical

We need a way to tell:

WAS the universe in a (pure) quantum state?

Review of Bell's Inequalities

Really just a reflections of two kinds of probability

See e.g. Hardy (2001)

Quantum: given $\mathcal{A}_{if} \in \mathbb{C}$ for each path

$$P_f = \left| \sum_i \mathcal{A}_{if} \right|^2 \qquad \sum_f P_f = 1$$

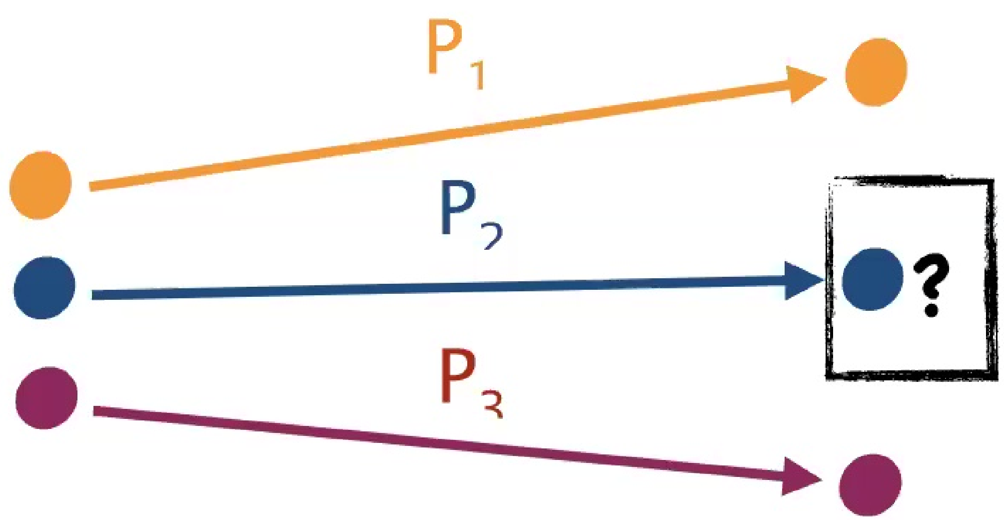
Classical: given $p_{if} \in [0,1]$ for each path

$$P_f = \sum_i p_{if} \qquad \sum_f P_f = 1$$

Review of Bell's Inequalities

Classical Probability = lack of knowledge

time

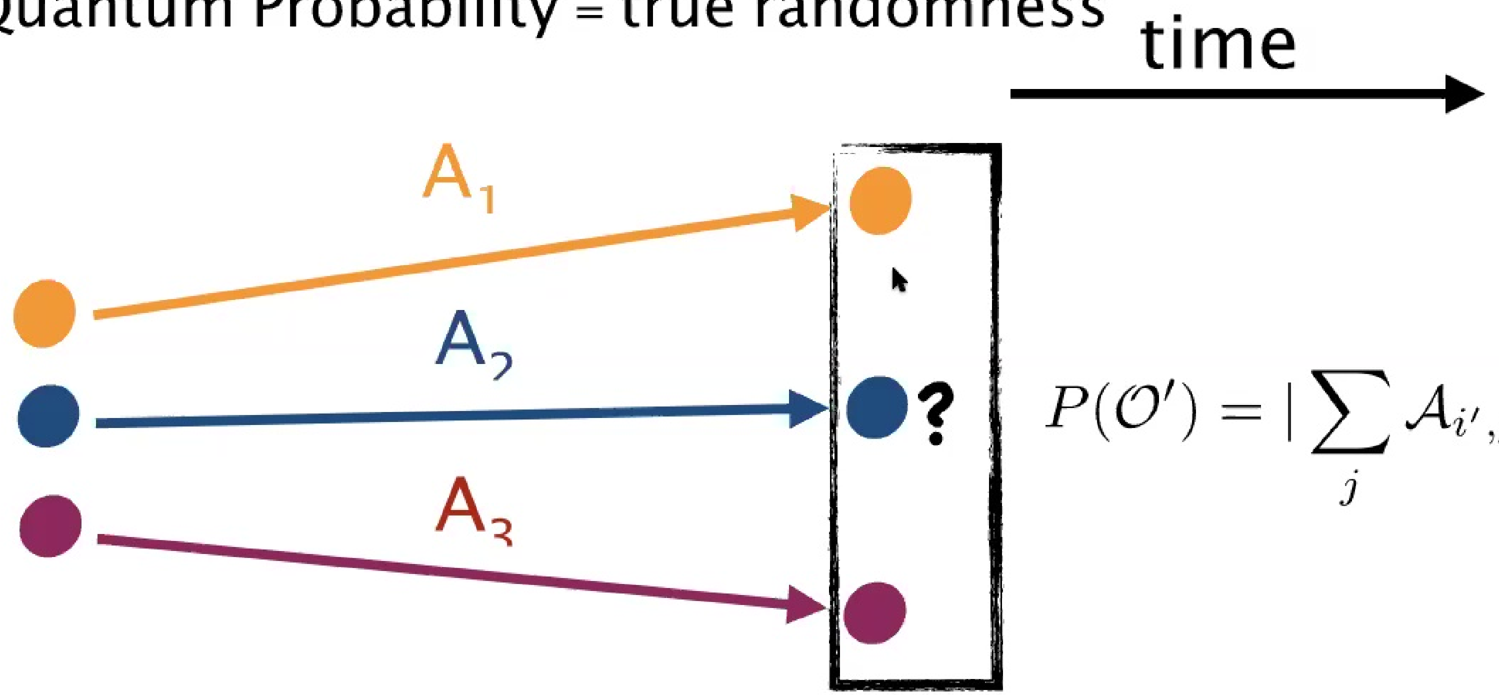


$$P(\mathcal{O}) = P_i$$

The universe picks one trajectory

Review of Bell's Inequalities

Quantum Probability = true randomness

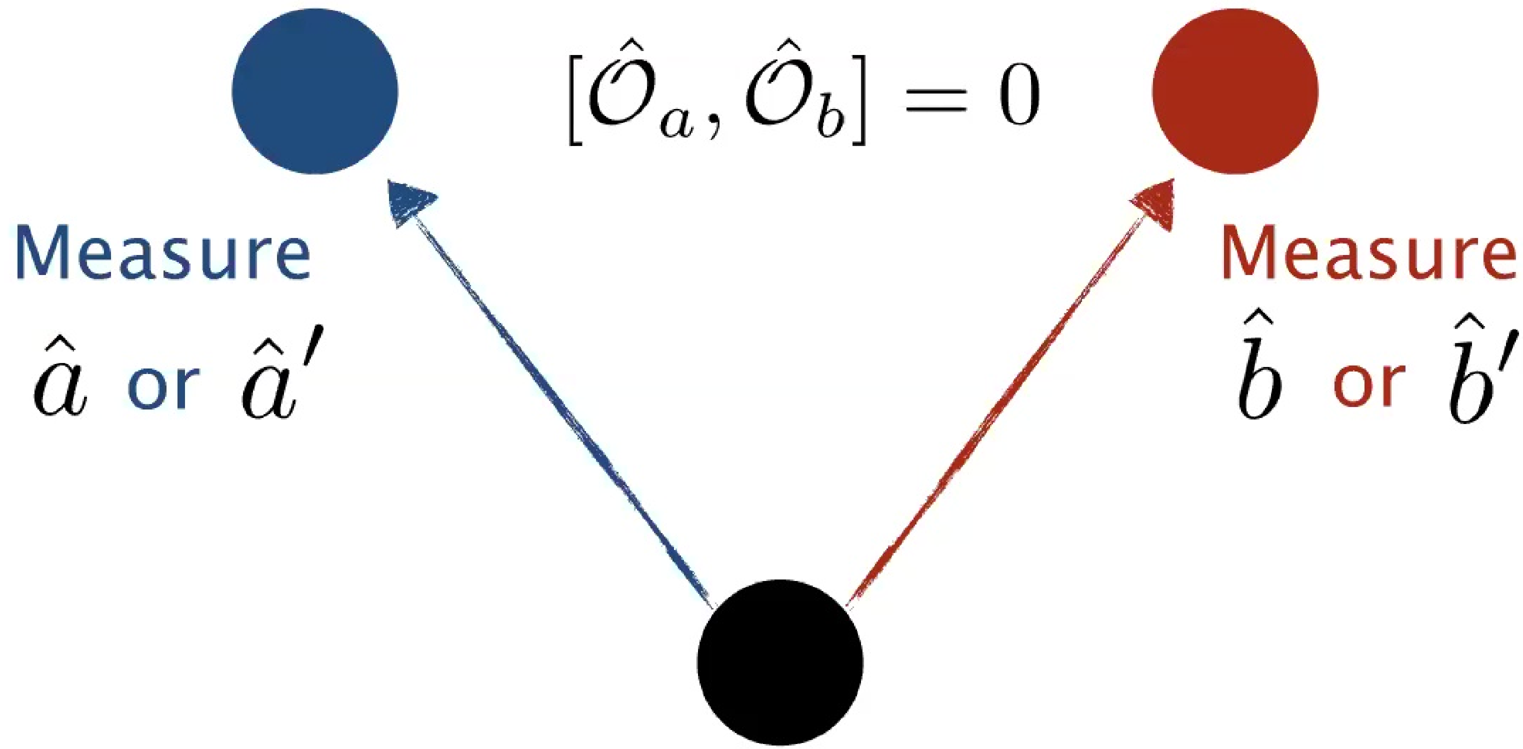


$$P(\mathcal{O}') = |\sum_j \mathcal{A}_{i',j}|^2$$

But not if we measure in basis $[\mathcal{O}', \mathcal{O}] \neq 0$

Review of Bell's Inequalities

$$[\hat{\mathcal{O}}_a, \hat{\mathcal{O}}_b] = 0$$



$$\hat{a}^2 = \hat{a}'^2 = \hat{b}^2 = \hat{b}'^2 = 1$$

Review of Bell's Inequalities

Key idea: I can choose what I measure locally

$\hat{a}|\psi_a\rangle$ $\hat{a}'|\psi_a\rangle$

[pure or mixed state]

Assume Q variables don't commute $[\hat{a}, \hat{a}'] \neq 0$

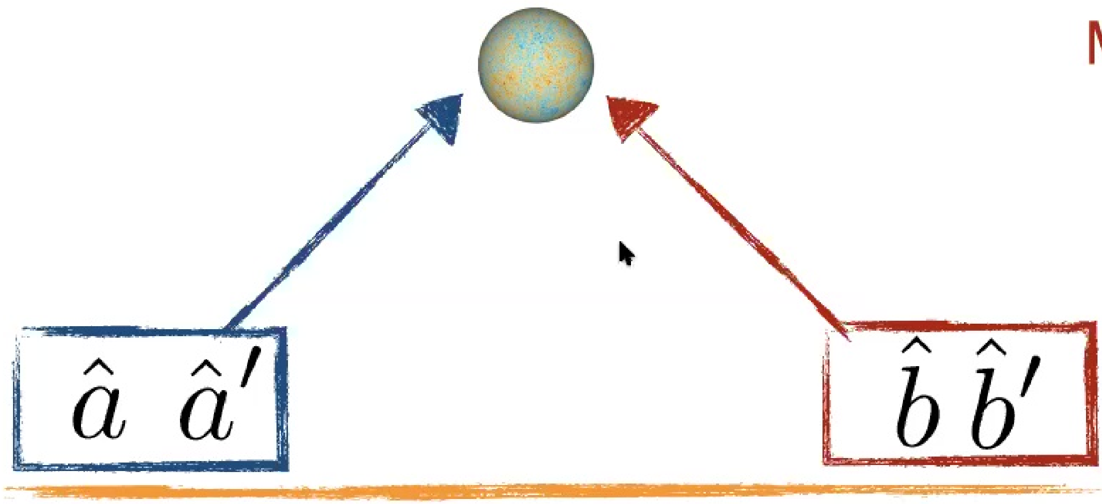
But both will only return ± 1

Q vs C distinguished by $\hat{C} = \hat{a}\hat{b} + \hat{a}'\hat{b} + \hat{a}'\hat{b}' - \hat{a}\hat{b}'$

Challenge for Cosmology

Try repeating a Bell-type measurement in cosmology

Many authors



Reheating

Inflation

Suppressed by 10^{-115} Martin & Vennin (2017), de Putter & Doré (2019)

Challenge for Cosmology

The key is measuring a nonzero commutator

Problem: given two real solutions for a mode (wave)

$$\zeta(\vec{k},t) = A(\vec{k},t) + iB(\vec{k},t)$$

The commutator is $[\hat{\zeta}, \dot{\hat{\zeta}}] = i(\dot{A}B - \dot{B}A)$

Now remember that $B \propto a^{-3} \to 0$

Inflation works because this commutator vanishes

Challenge for Cosmology

The vanishing of the commutator is general

$$[\zeta(\vec{x},t),\dot{\zeta}(\vec{x}',t)] \propto \frac{1}{\sqrt{-g}}\delta(\vec{x}-\vec{x}') = \frac{1}{a^3}\delta(\vec{x}-\vec{x}')$$

e.g. Grishchuk & Sidorov

Suppressed by the volume of the universe

$$\langle\hat{\zeta}(\vec{x},t)\ldots\rangle = \int d\zeta\,(\zeta(\vec{x},t)\ldots)\,|\Psi[\zeta]|^2$$

This is completely equivalent to classical statistics

$$|\Psi[\zeta]|^2 \to P[\zeta]$$


Signals of a
Quantum Universe
DG & Porto
arXiv:2001.09149 [Phys. Rev. Lett. 124, 25]

Bell Test from Time Evolution

Our only hope is to use statistics. Since

$$\langle \hat{\zeta}(\vec{x},t)\ldots\rangle = \int d\zeta\,(\zeta(\vec{x},t)\ldots)\,|\Psi[\zeta]|^2$$

We must ask whether

$$|\Psi[\zeta]|^2 \to P[\zeta] \neq P_{\text{classical}}[\zeta]$$

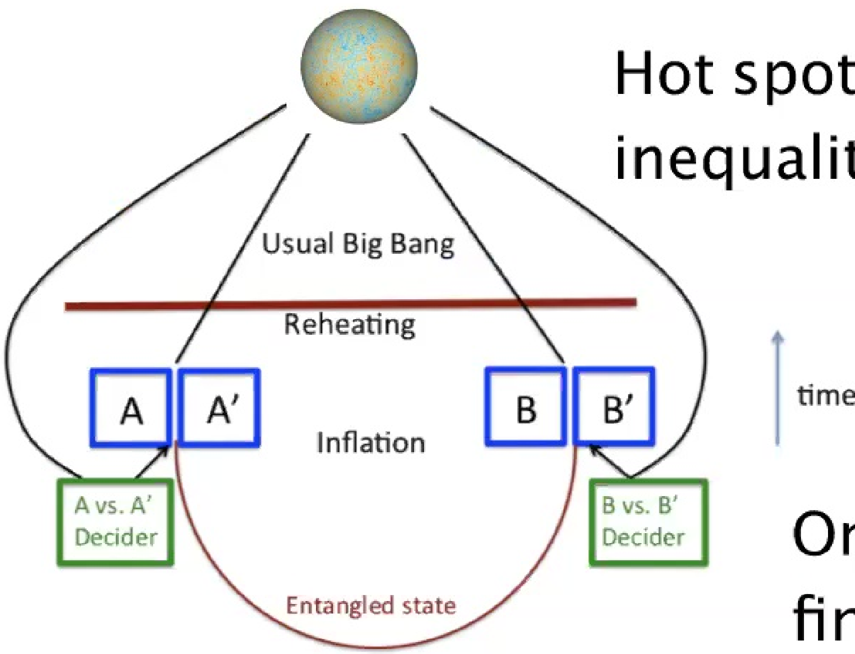
I.e. Quantum dynamics leads to unique distributions

Need to understand of how we generate $P[\zeta]$

Proof of Principle

Model with a Bell measurement during inflation

Maldacena (2015)



Hot spots in CMB stores Bell inequality violation

Only works in this specific finely-tuned model

We want something more generic

Key Idea

Classical fluctuations are always real/physical

Classical statistics just represents our uncertainty

Classical modes are sines and cosines

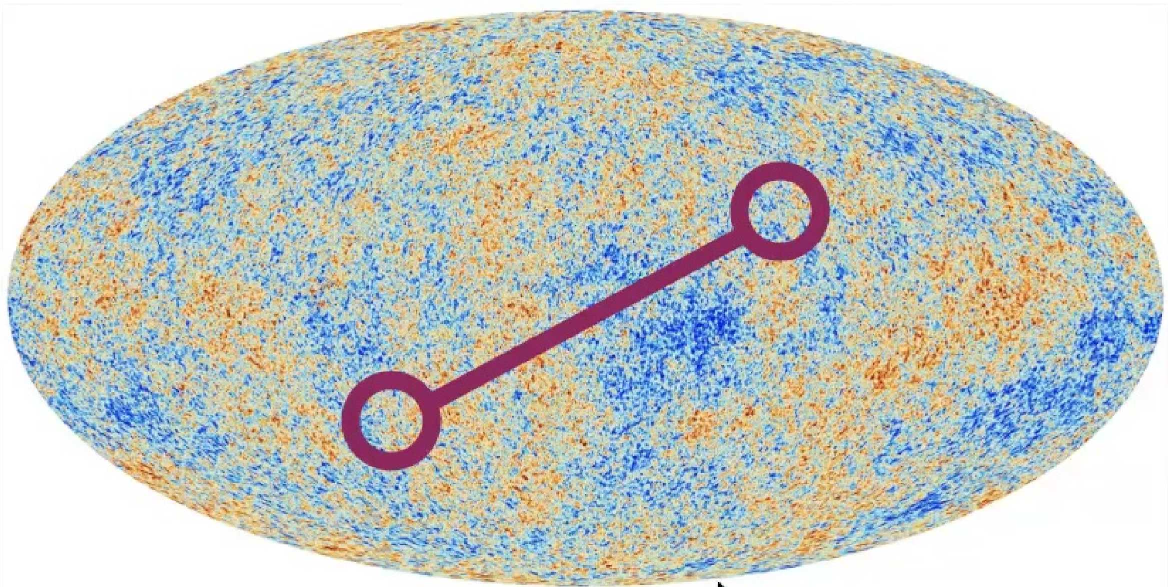
Quantum (vacuum) fluctuations become physical

Quantum modes are positive / negative frequencies

Direct reflect of quantum vs classical statistics

Gaussian Fluctuations

Fixed by two-point statistics



$$\langle \delta T(\vec{x}) \delta T(\vec{x}') \rangle = f(|\vec{x} - \vec{x}'|) \leftrightarrow \langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = \tilde{f}(k)(2\pi)^3 \delta(\vec{k} + \vec{k}')$$

Gaussian Fluctuations

Gaussian modes described as

$$\hat{\zeta}(\mathbf{x},\tau) = \int \frac{d^3k}{(2\pi)^3} \Delta_\zeta e^{i\mathbf{k}\cdot\mathbf{x}} \Big[\underbrace{a^\dagger_{\mathbf{k}} \zeta(\vec{k},\tau)}_{\approx e^{ik\tau}} + \underbrace{a_{-\mathbf{k}} \zeta^*(\vec{k},\tau)}_{\approx e^{-ik\tau}} \Big]$$

positive frequency — negative frequency

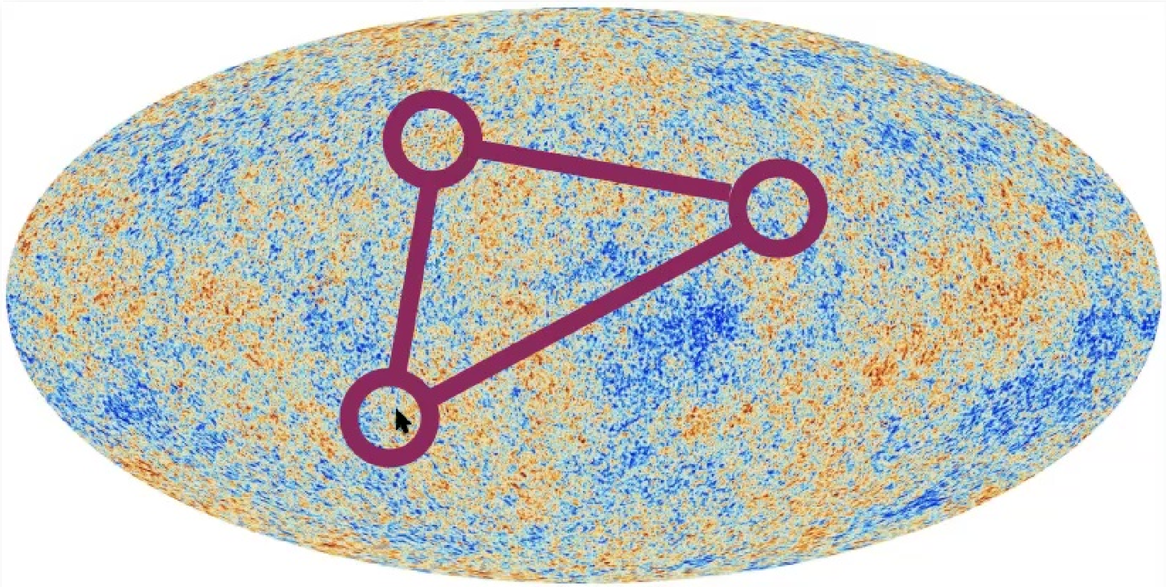
Quantum: $\langle 0 | a_{\mathbf{k}'} a^\dagger_{\mathbf{k}} | 0 \rangle = \delta(\mathbf{k}-\mathbf{k}')\,, \quad \langle 0 | a^\dagger_{\mathbf{k}} a_{\mathbf{k}'} | 0 \rangle = 0$

Classical: $\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}'} \rangle_c = \frac{1}{2}\delta(\mathbf{k}-\mathbf{k}') = \langle a_{\mathbf{k}'} a^\dagger_{\mathbf{k}} \rangle_c$

Gaussian correlators are identical

Non-Gaussian Fluctuations

What about non-Gaussian correlators? E.g. 3-point



$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = B(k_1, k_2, k_3)(2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Non-Gaussian Fluctuations

We will assume NG arises from nonlinear evolution

I.e. no non-local correlations in the initial state

We do not want to assume special interactions

E.g. not allowed to arrange a Bell measurement

Key difference between cosmology and computing

Quantum NG

Let us work out a simple example

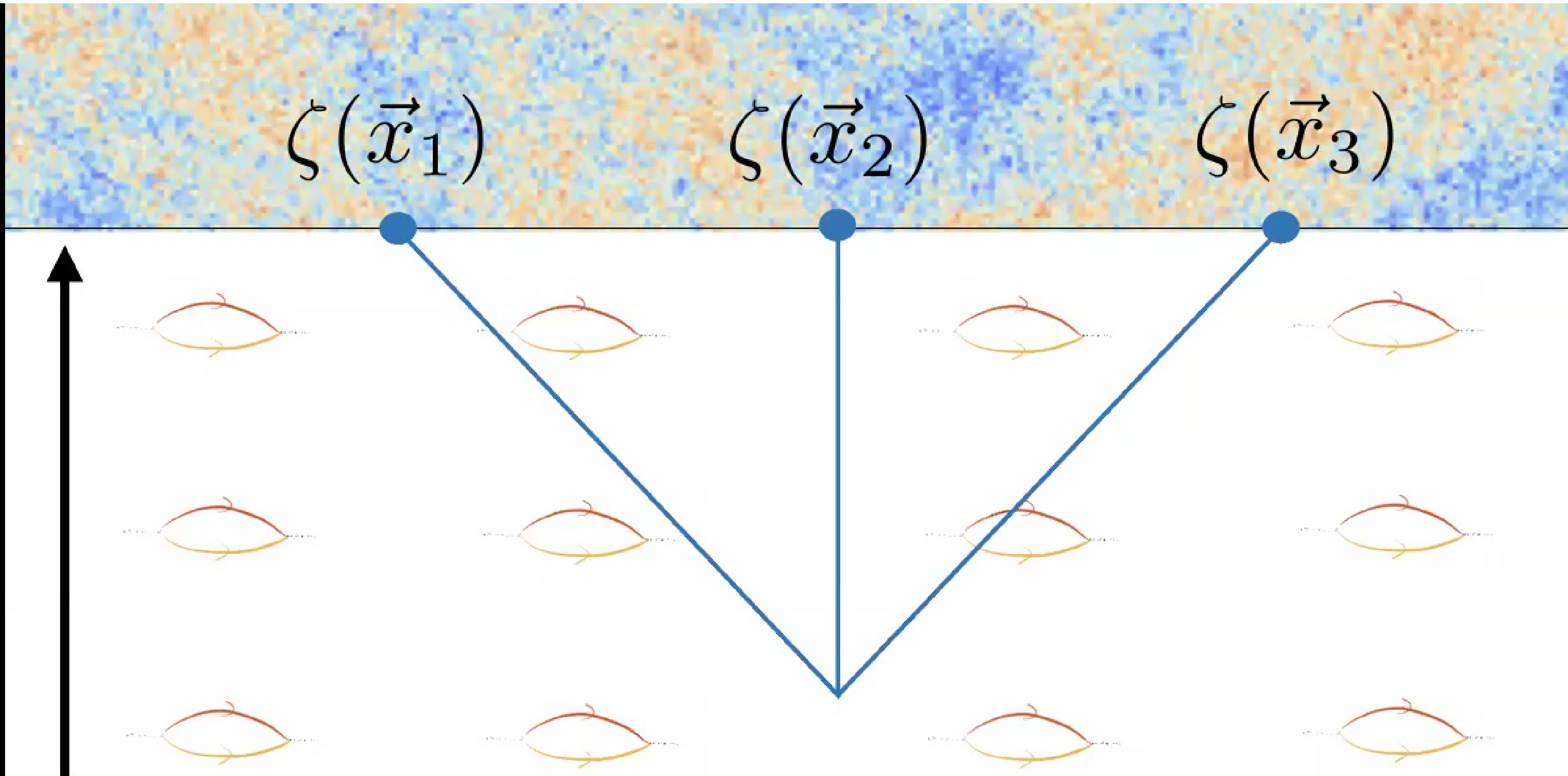
$$H_{\rm int} = -\frac{\lambda}{3!}\int d^3x\,\dot{\zeta}(\vec{x})^3$$

Standard perturbation theory

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle'_q = i\int d\tau'\,\langle[H_{\rm int}(\tau'),\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}(0)]\rangle$$

$$= \frac{4\lambda H^{-1}\Delta_\zeta^6}{\boxed{(k_1+k_2+k_3)^3}\,k_1k_2k_3}$$

"total energy pole"



Time

Creation of three particles from vacuum

$$\delta(E_1 + E_2 + E_3) \rightarrow \frac{1}{(E_1+E_2+E_3)^n}$$

Scattering Uncertainty Principle

Quantum NG

Residue is the related to the S-matrix

Maldacena & Pimentel; Raju

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle_q' = \frac{4\lambda H^{-1} \Delta_\zeta^6}{\boxed{(k_1 + k_2 + k_3)^3} k_1 k_2 k_3}$$

"total energy pole"

$$\propto \frac{\mathcal{A}_{\text{on-shell}}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{(k_1 + k_2 + k_3)^3 \, k_1^2 k_2^2 k_3^2}$$

The pole itself is just signaling energy conservation

Classical NG

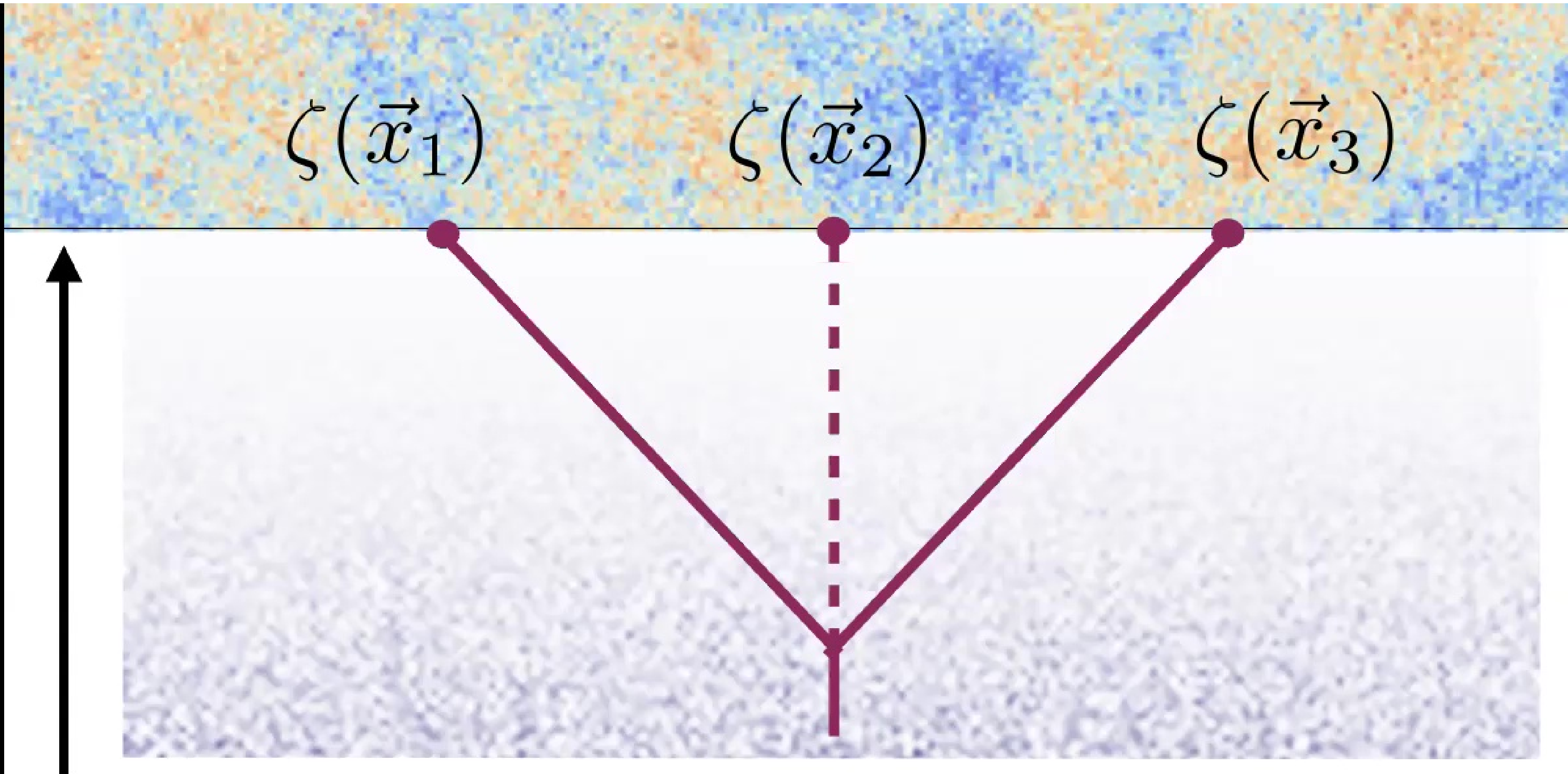
Classical analogue: solve EoM perturbatively

$$\zeta_{\mathbf{k}}^{(2)}(\tau) = \lambda \int \frac{d\tau' d^3p}{(2\pi)^3} (-H\tau')^{-1} \left(\partial_{\tau'} G_{\mathbf{k}}(\tau,\tau')\right) \partial_{\tau'}\zeta_{\mathbf{p}}^{(1)}(\tau')\, \partial_{\tau'}\zeta_{\mathbf{k}-\mathbf{p}}^{(1)}(\tau')$$

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle'_c = \frac{\lambda H^{-1}\Delta_\zeta^6}{3k_1k_2k_3}\left[\frac{3}{(k_1+k_2+k_3)^3} + \frac{1}{(k_1+k_2-k_3)^3} + \frac{1}{(k_1-k_2+k_3)^3} + \frac{1}{(k_2-k_1+k_3)^3}\right]$$

"total energy pole"

On-shell pole or "Folded Shape"



Time

Creation of three particles from vacuum

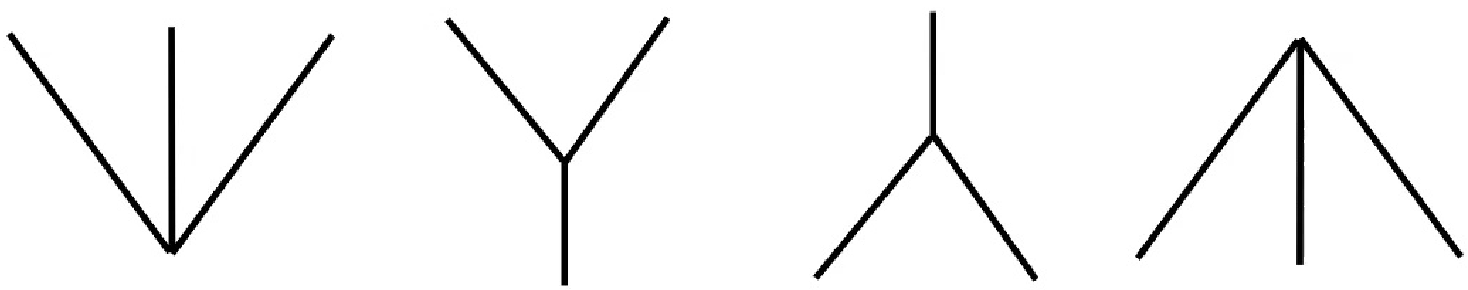
$$\delta(E_1 - E_2 - E_3) \rightarrow \frac{1}{(E_1 - E_2 - E_3)^n}$$

Scattering On-Shell Decay

Classical NG

A result of crossing symmetry of the S-matrix

Relates all processes of these processes



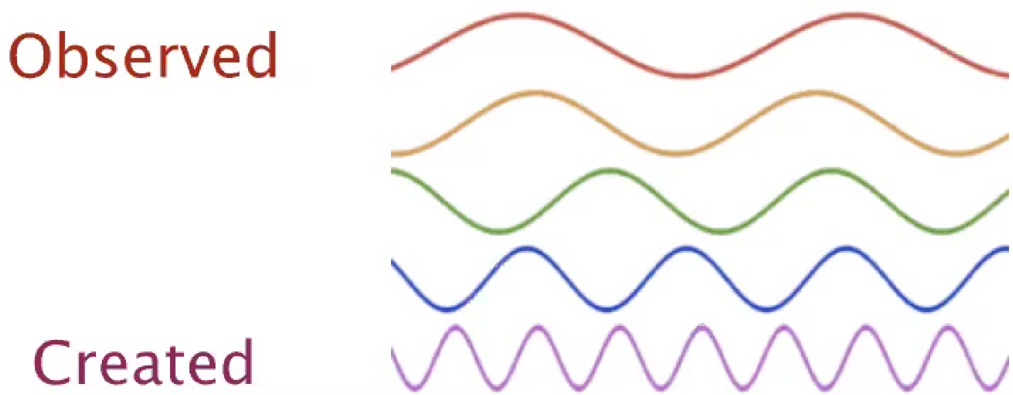
$0 \to 3$ $\quad$ $1 \to 2$ $\quad$ $2 \to 1$ $\quad$ $3 \to 0$

Only differ by number of particles in the initial state

Classical NG

Classical fluctuations = particles in the initial state

Observed fluctuations exist in the far past

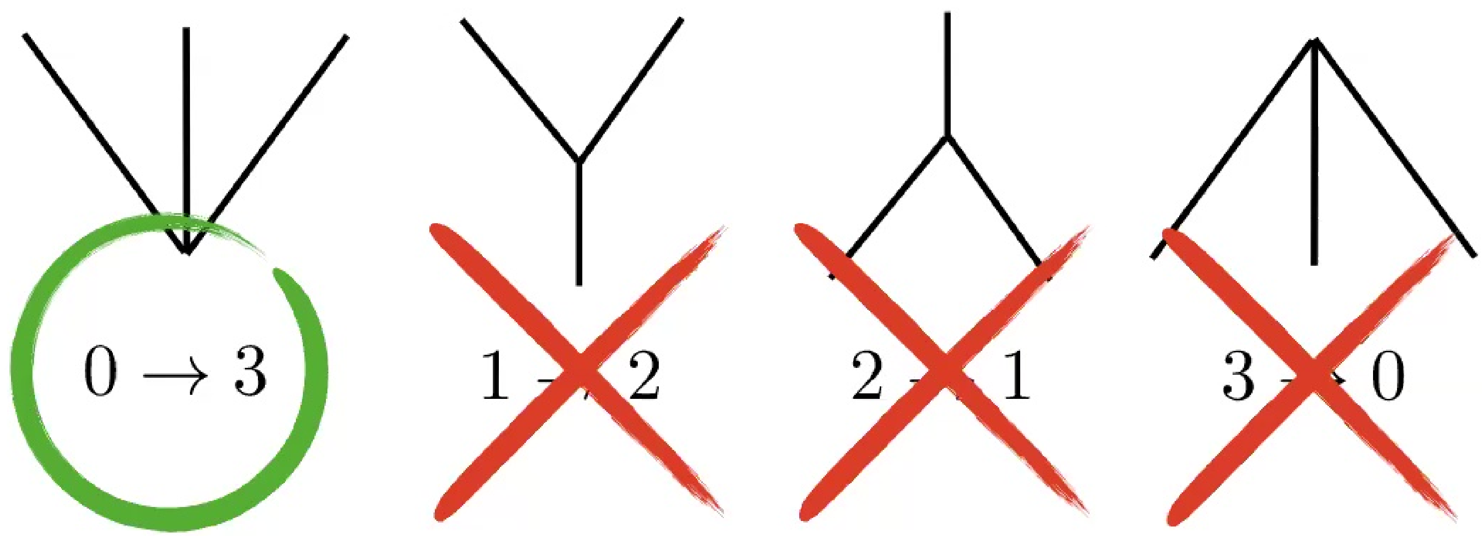


Classical probability is just lack of knowledge

Quantum vs Classical

No classical analogue of vacuum fluctuations

Quantum vacuum has no initial particles

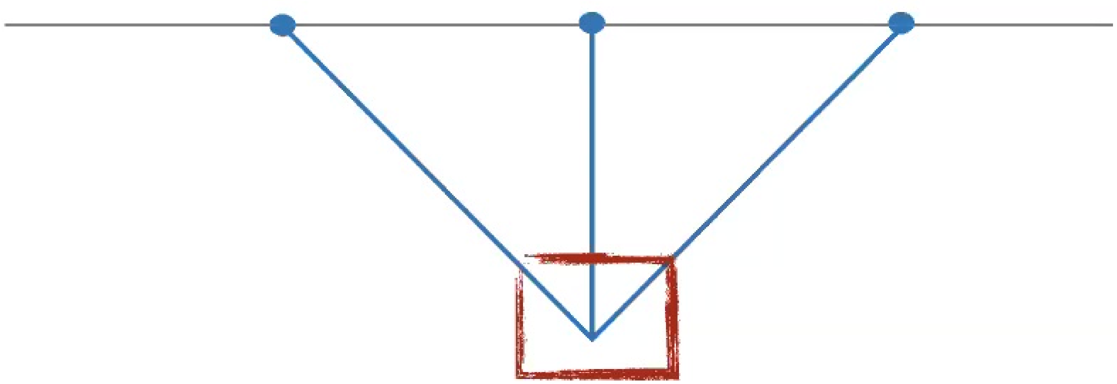


Quantum vs Classical

Difference in poles is direct result of commutator

$$\langle\zeta(\mathbf{x}_1,\tau)\,\zeta(\mathbf{x}_2,\tau)\,\zeta(\mathbf{x}_3,\tau)\rangle_q - \langle\zeta(\mathbf{x}_1,\tau)\,\zeta(\mathbf{x}_2,\tau)\,\zeta(\mathbf{x}_3,\tau)\rangle_c =$$

$$+\frac{i\lambda}{4}\int_{-\infty}^{\tau} d\tau' d^3x' a^4(\tau')\left[\zeta(\mathbf{x}_1,\tau),\dot{\zeta}(\mathbf{x}',\tau')\right]\left[\zeta(\mathbf{x}_2,\tau),\dot{\zeta}(\mathbf{x}',\tau')\right]\left[\zeta(\mathbf{x}_3,\tau),\dot{\zeta}(\mathbf{x}',\tau')\right]$$



Commutator non-zero at the intersection of past light-cones

Hidden Variables

We can evade Bell's inequality if we give up locality

Is there a similar result in cosmological signature?

Idea: Classical theory prefers positive frequencies

Use a complex scalar

$$\hat{\phi}_{\mathbf{k}}(\tau) = \frac{\Delta_\phi}{k^{3/2}} \left[a^\dagger_{\mathbf{k}} \varphi(\vec{k},\tau) + b_{-\mathbf{k}} \varphi^*(\vec{k},\tau) \right]$$

Only excite the positive frequencies

$$\left\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}'} \right\rangle_c = \left\langle a_{\mathbf{k}'} a^\dagger_{\mathbf{k}} \right\rangle_c = \delta\left(\mathbf{k} - \mathbf{k}'\right), \quad \left\langle b_{\mathbf{k}} b^\dagger_{\mathbf{k}'} \right\rangle_c = 0$$

Hidden Variables

Now we will simply invent a "Green's function"

Propagation mixes positive and negative frequencies

$$\dot{G}_{\mathbf{k}}(\tau \to 0, \tau') = \frac{1}{k}\sin(k\tau')$$

Remove the negative frequency by hand

$$\dot{G}_{\mathbf{k}} \to G^{\mathrm{eff}}_{\mathbf{k}}(\tau \to 0, \tau') = \frac{1}{k}e^{-ik\tau'}$$

Hidden Variables

Using this to compute the bispectrum gives

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' = i\lambda_\phi H^{-1} \frac{\Delta_\phi^6}{k_1 k_2 k_3} \int_{-\infty}^{0} d\tau' \tau'^2 e^{-i(k_1+k_2+k_3)\tau'}$$

$$= \frac{2\lambda_\phi H^{-1} \Delta_\phi^6}{(k_1 + k_2 + k_3)^3 \, k_1 k_2 k_3}$$

Describes 1 particle decaying to 2 anti-particles

Only a total energy pole

Hidden Variables

This theory is non-local

$$G_{\mathbf{k}}^{\text{eff}}\left(\tau \to 0, \tau' \to 0\right) = \frac{1}{k}$$

Non-zero at space-like separations

$$G^{\text{eff}}\left(\vec{x}, \tau \to 0, \tau' \to 0\right) \propto \frac{1}{x^2}$$

In a local theory, must vanish outside the light-cone

$$G_{\mathbf{k}}^{\text{eff}} \sim \frac{1}{k}\sin(k\tau) \to G^{\text{eff}}(\vec{x}) \propto \delta(\vec{x})$$

Hidden Variables

In this local theory

$$\phi_{\mathbf{k}}(\tau \to 0) = \frac{i}{3}\lambda_\phi \int \frac{d^3p}{(2\pi)^3} \int d\tau' \frac{1}{k} \sin(k\tau') \dot{\phi}^*_{\mathbf{p}}(\tau')\, \dot{\phi}^*_{\mathbf{k}-\mathbf{p}}(\tau')$$

and

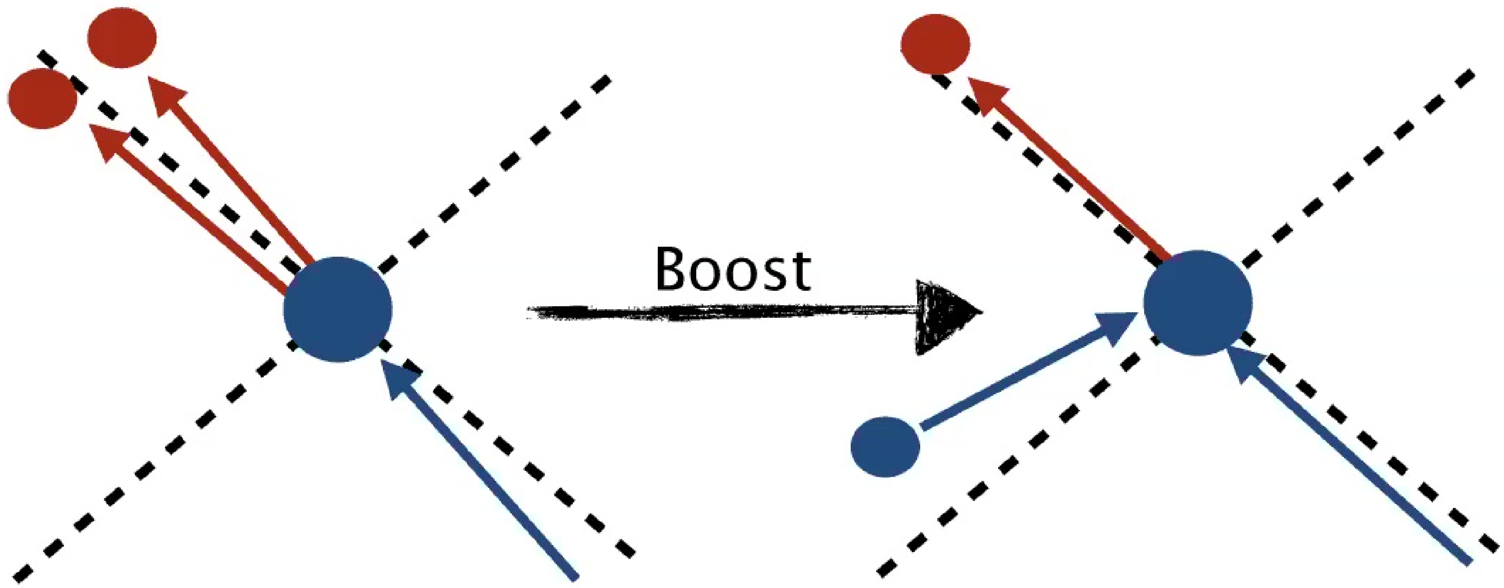
$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' \supset \frac{\lambda_\phi H^{-1} \Delta_\phi^6}{(k_1 - k_2 + k_3)^3\, k_1 k_2 k_3}$$

Include 2 particles annihilating to 1 anti-particle

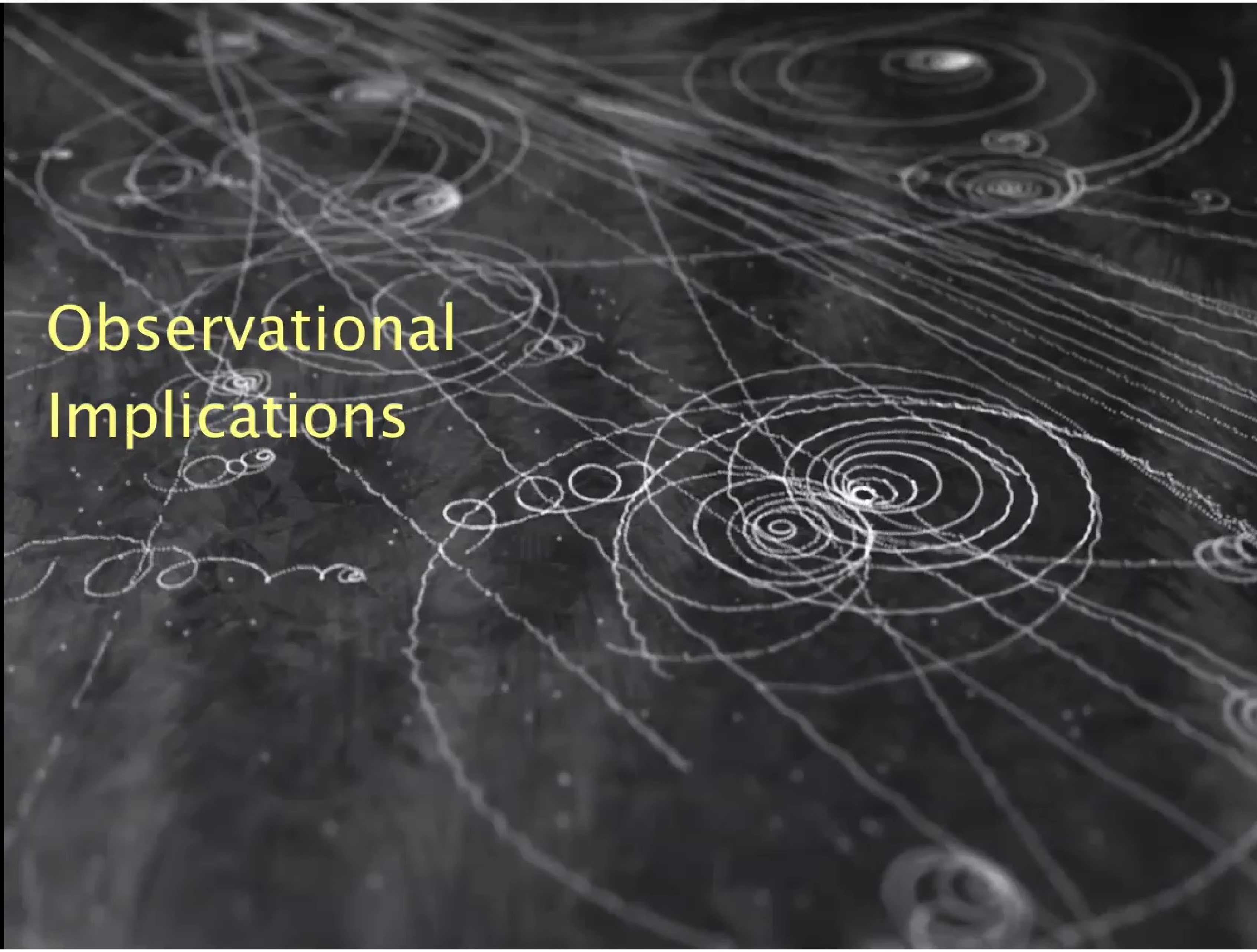
Non-trivial relation between crossing and causality

Classical NG

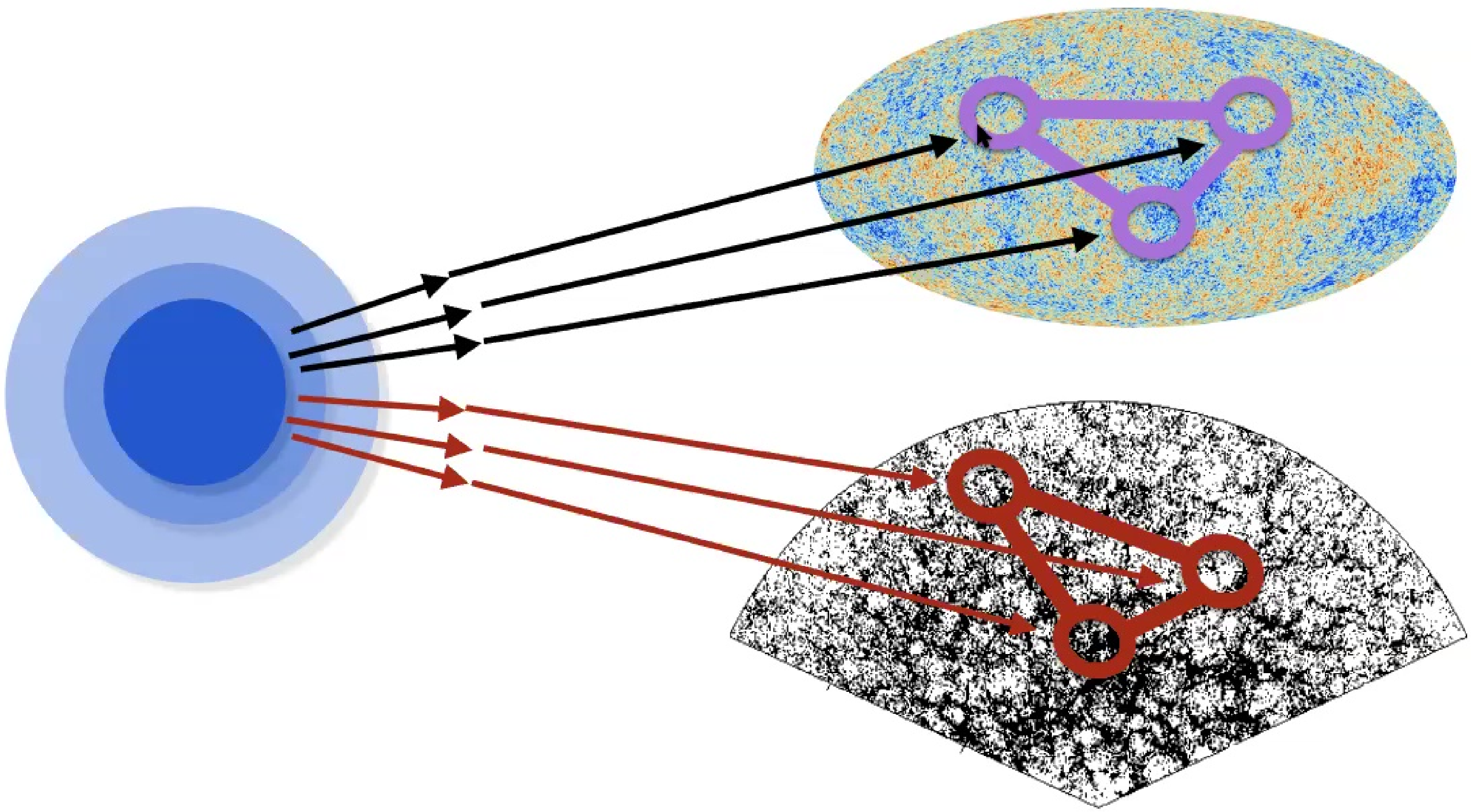
This is the same as the need for antiparticles



Lorentz invariance requires they come together


Observational
Implications

Primordial Non-Gaussianity

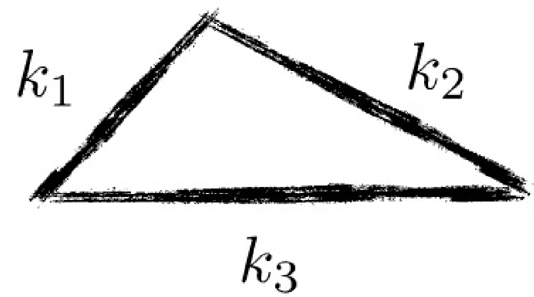


Primordial Non-Gaussianity

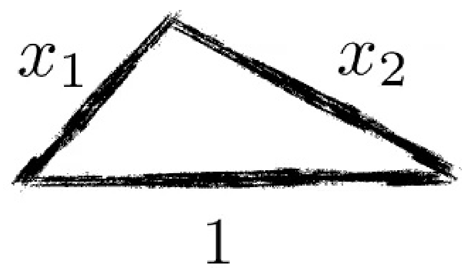
On general grounds, bispectra take the form

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = B(k_1, k_2, k_3)(2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Momentum conservation:



Scale invariance:



$$\times \frac{1}{k_3^6}$$

Primordial Non-Gaussianity

Defined by amplitude and “shape”

$$B(k_1, k_2, k_3) = f_{\mathrm{NL}} \frac{18}{5} \frac{\Delta_\zeta^4}{k_3^6 x_1^2 x_2^2} S(x_1, x_2)$$

The shapes live in a basis of orthogonal functions

$$\int dx_1 dx_2 S_1(x_1, x_2) S_2(x_1, x_2) = S_1 \cdot S_2 = \cos_{12}$$

Cosine is how easily they are distinguish (in 3pt)

Primordial Non-Gaussianity

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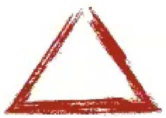
Cosine is how easily they are distinguish (in 3pt)

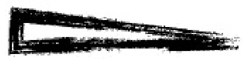
Current Limits

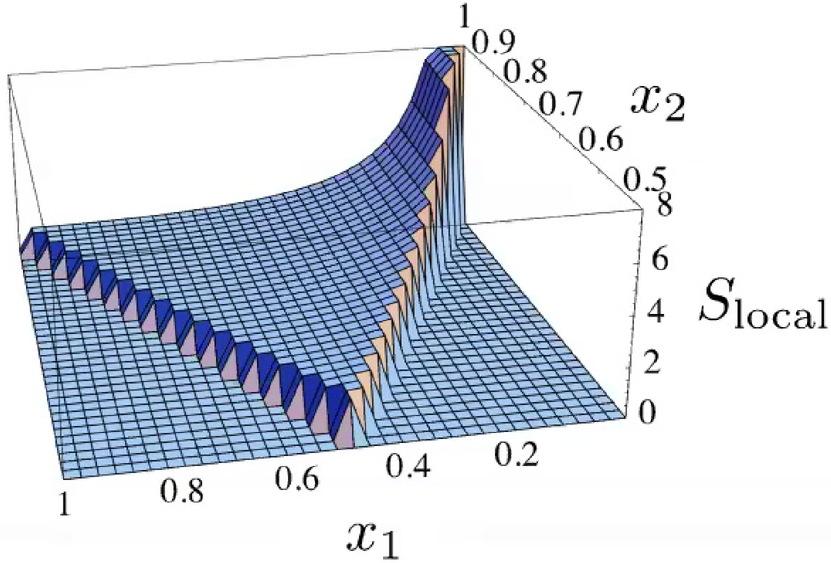
The "Local Shape"

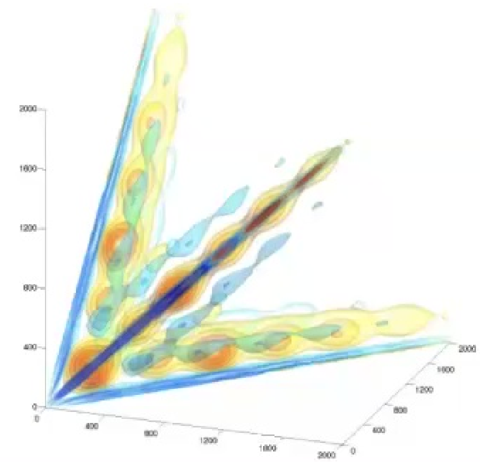
$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$$

Planck 2018









Courtesy of Fergusson & Shellard

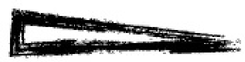
Babich et al. (2004)

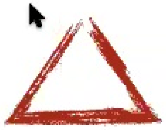
Current Limits

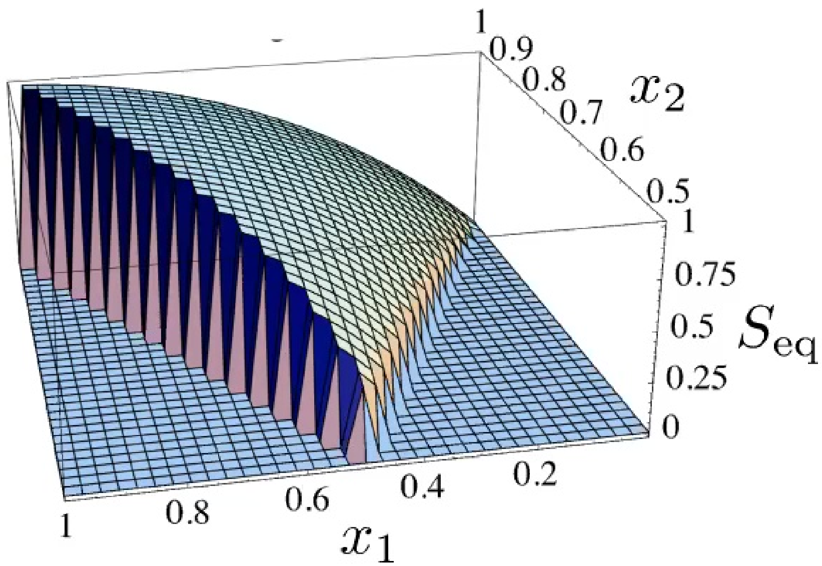
The "Equilateral Shape"

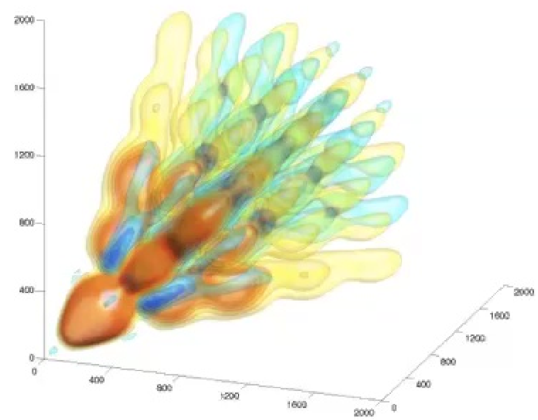
$$f_{\mathrm{NL}}^{\mathrm{equil}} = -26 \pm 47$$

Planck 2018









Courtesy of Fergusson & Shellard

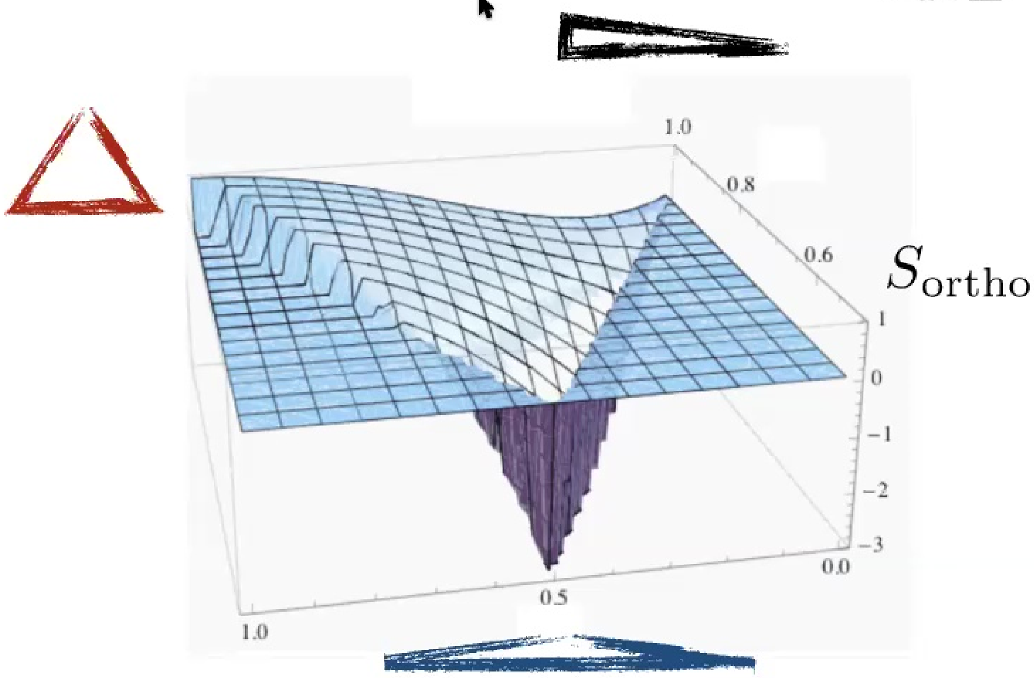
Babich et al. (2004)

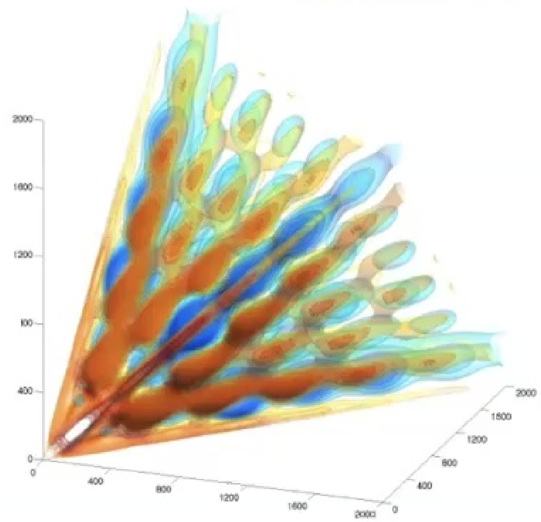
Current Limits

The "Orthogonal Shape"

$$f_{\rm NL}^{\rm ortho} = -38 \pm 24$$

Planck 2018





Courtesy of Fergusson & Shellard

Smith et al. (2009)

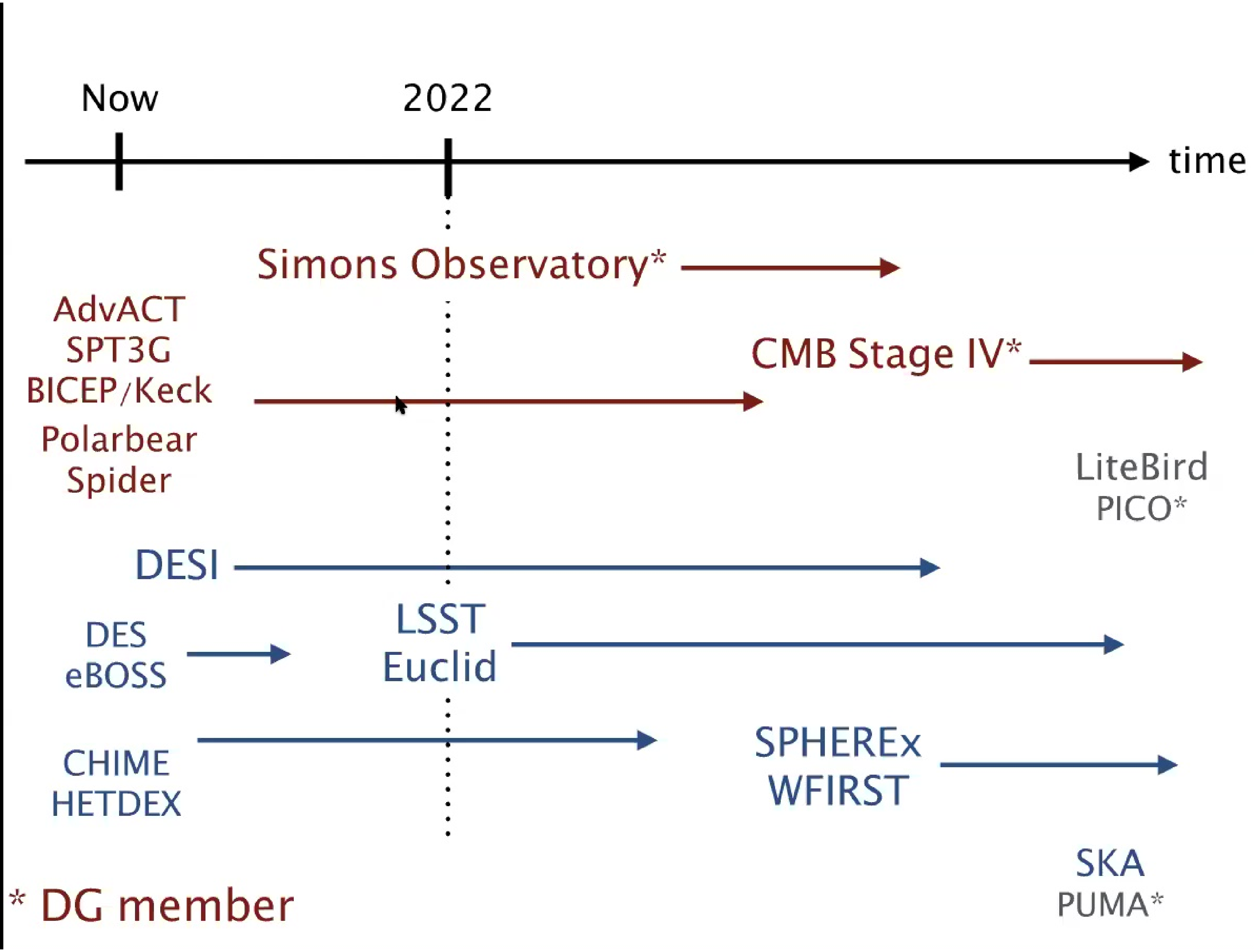
Observational Implications

Quantum NG = Equilateral NG

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'_q = \frac{4\lambda H^{-1} \Delta_\zeta^6}{(k_1 + k_2 + k_3)^3 \, k_1 k_2 k_3}$$

Classical NG = Pole in folded limit

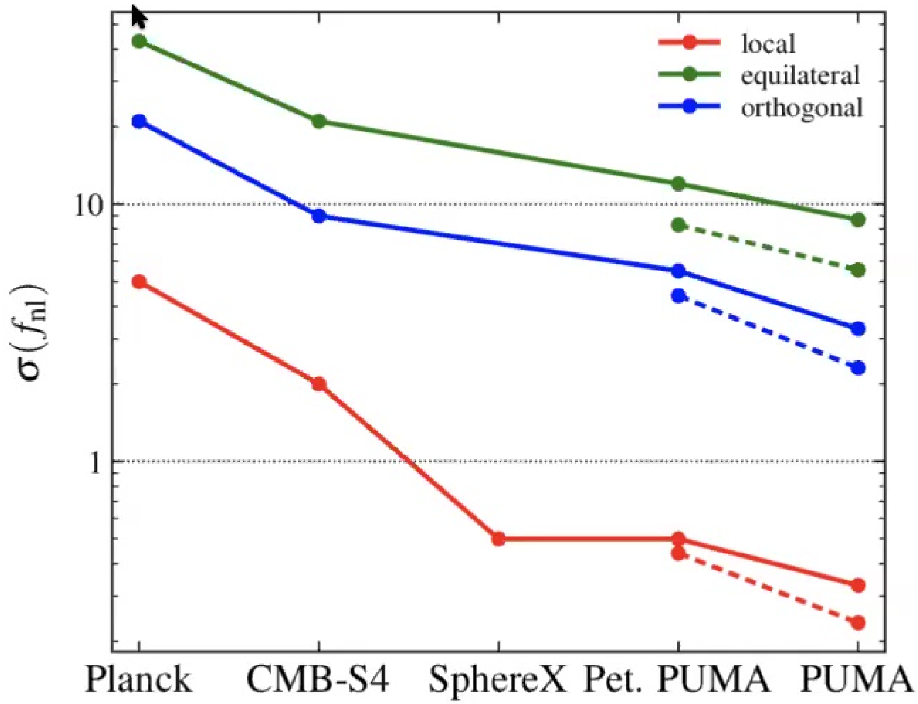
$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_1} \zeta_{\vec{k}_1} \rangle'_c \supset \frac{\lambda H^{-1} \Delta_\zeta^6}{(k_1 - k_2 + k_3)^3 k_1 k_2 k_3}$$



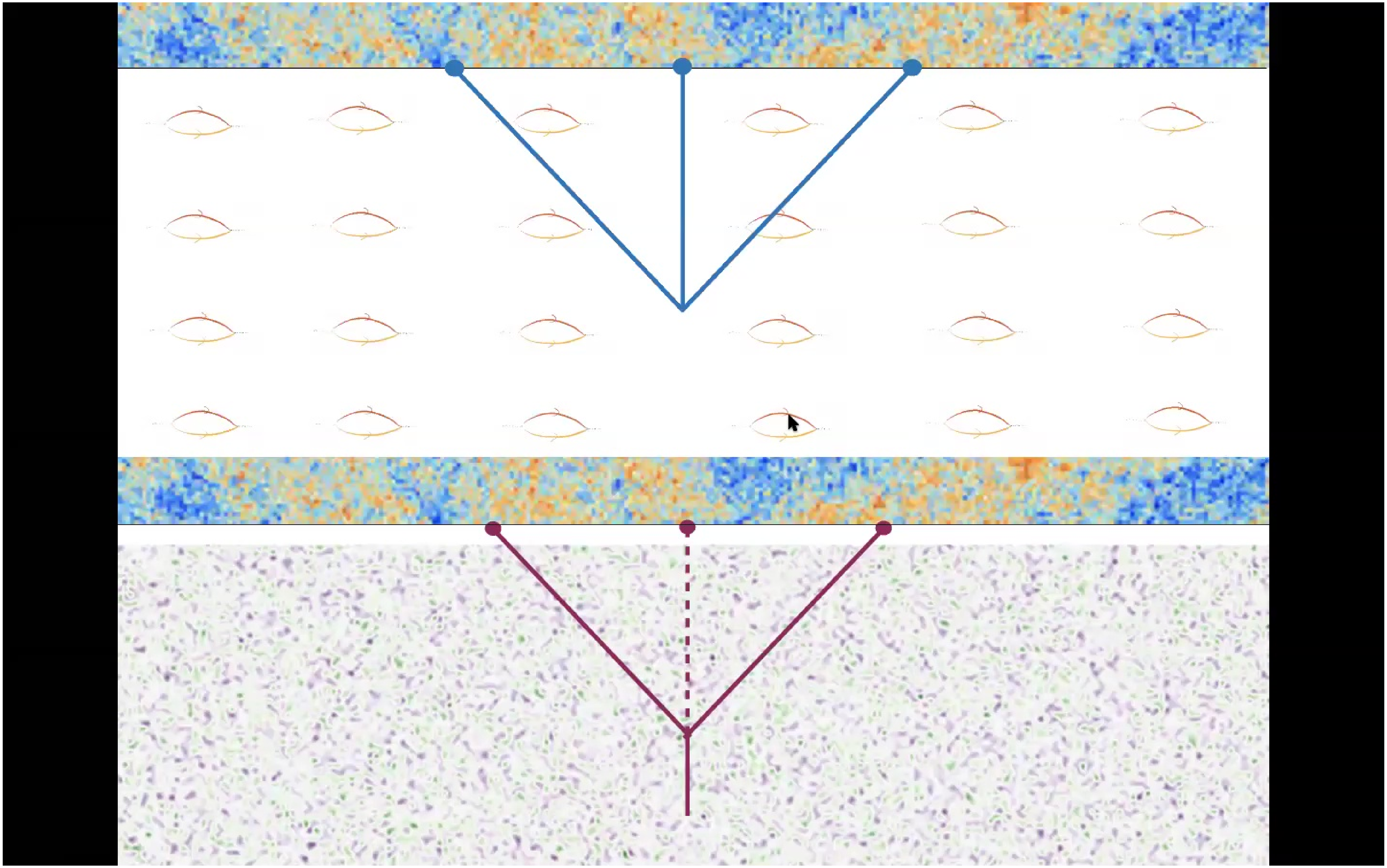
Now
2022
time
Simons Observatory*
AdvACT
SPT3G
BICEP/Keck
Polarbear
Spider
CMB Stage IV*
LiteBird
PICO*
DESI
DES
eBOSS
LSST
Euclid
CHIME
HETDEX
SPHEREx
WFIRST
SKA
PUMA*
* DG member

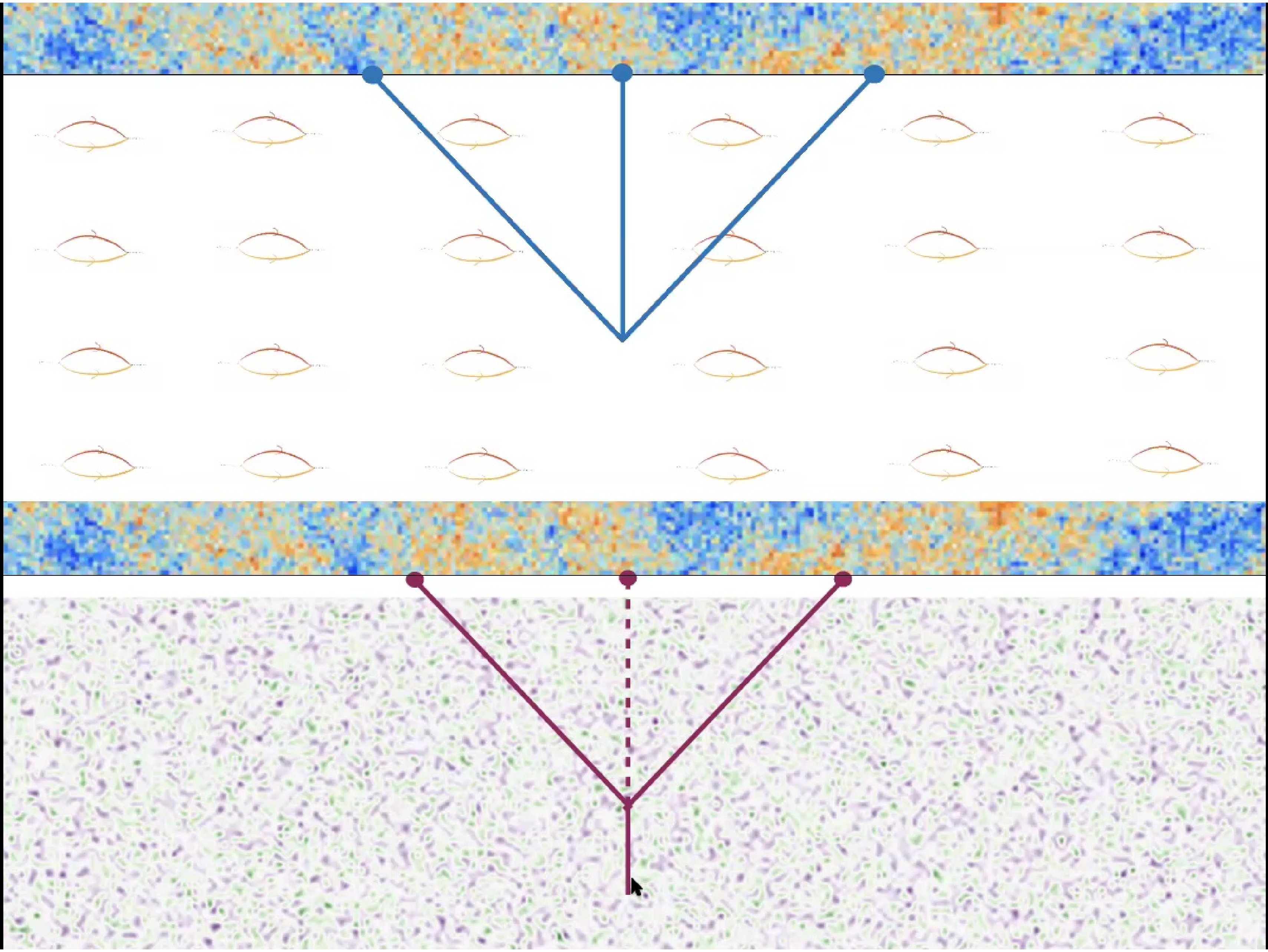
Future Limits

Forecasted improvements on current limits:



PUMA collaboration (2019)





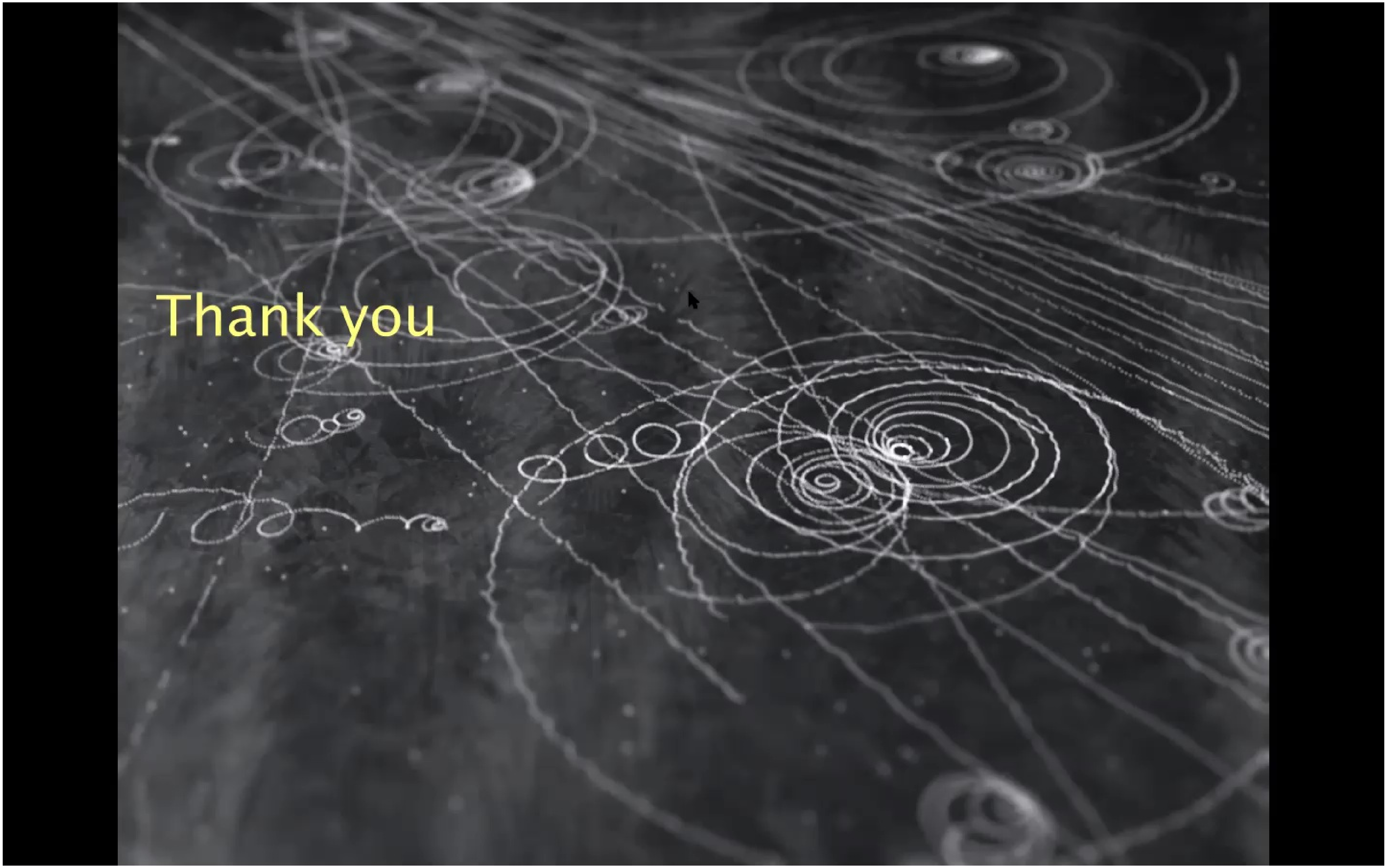
Summary

The "total energy pole" is a generic feature of NG

- The cosmological analogue of energy conservation
- Quantum vacuum fluctuations only have these
- Classical fluctuations also have physical poles

This result is a consequence of locality

A detection of non-Gaussianity enables a Bell test


Thank you