Title: Anomalies in gravitational charge algebras of null boundaries and black hole entropy

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Abstract: I will discuss how central extensions of charge algebras in gravitational theories with null boundaries arise from an anomalous transformation of the boundary term in the gravitational action. This parallels the way in which the holographic Weyl anomaly appears in AdS/CFT, with the ambiguity in the normalization of the null generator being the analogue of the choice of Weyl frame. I will discuss a way to define the gravitational charges unambiguously within the covariant phase space formalism. As an application, I will analyze the near-horizon Virasoro symmetries of black holes and give a systematic derivation of the central charges. I will show that the central extension computed by the charge algebra is directly related to the Bekenstein-Hawking entropy via the Cardy formula.

Anomalies in Gravitational Charge Algebras of Null Boundaries and Black Hole Entropy

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Based on 2009.10739 with AJ Speranza

The many faces of gravitational entropy

Black hole entropy:

$$S = \frac{A}{4G}$$

Can be derived from myriad approaches

- Semi-classical gravity path integral Gibbons, Hawking (1977); Lewkowycz, Maldacena (2013)
- Stringy microstate counting Strominger, Vafa (1996)
- Cardy formula, BTZ black hole Strominger (1997)

Hints at more universal description of microstates Carlip (1994, 1999)

Boundary degrees of freedom

The overarching story:

- No spacetime boundaries $\Rightarrow$ *all* diffeomorphisms are gauge
- Treat gravitational subregions as boundaries
  - Black hole horizons, entangling surfaces, causal diamonds...
  - Boundaries in what sense? Causal horizon acts as effective spacetime boundary to exterior observer
- Boundary manifolds are fixed partitions of spacetime
  - Boundary preserving diffeomorphisms become physical symmetries
  - Non-trivial surface charges $\sim$ boundary degrees of freedom

Can such boundary excitations account for black hole entropy?

Virasoro symmetries

In some cases, near horizon symmetries seem to determine BH entropy:

- Strominger, entropy of BTZ Strominger (1997)
- Carlip, entropy of Killing horizons Carlip (1999)
- Kerr/CFT, extremal Kerr black holes Guica et al (2009)
- Hidden conformal symmetry of Kerr Haco et al (2018); Castro, Maloney, Strominger (2010)
- Extended to general axisymmetric Killing horizons Chen et al (2020)

Generally find one or two copies of Virasoro algebra

- Central extension obtained from classical gravity

Assume "dual" CFT_2 description of horizon, apply Cardy formula

$$S = \frac{\pi^2}{3}\left(cT + \bar{c}\bar{T}\right) = \frac{A}{4G}$$

Going beyond numerology

These approaches suffer from one or more of the following:

- Unresolved ambiguities
- Restrictive boundary conditions
- Magic?

This talk: resolve these issues using covariant phase space formalism

Pinning down the setup

Goal: calculate central extension of gravitational charge algebra

$$\{H_\xi, H_\zeta\} = -\left(H_{[\xi,\zeta]} + K_{\xi,\zeta}\right)$$

Most constructions impose very restrictive boundary conditions at horizon

- Gives boundary status as a physical barrier

Restrictive boundary conditions not natural when discussing subregions

- Boundary is just an arbitrary partition of spacetime

Haco, Hawking, Perry, and Strominger (HHPS), Virasoro symmetries:

- no such boundary conditions
- **non-integrable charges**

Necessary ingredients

Main tool: covariant phase space of GR with a null boundary $\mathcal{N}$

Three main issues to resolve with non-integrable charges

- Fix ambiguity in obtaining quasilocal charges
  - Can affect final result for entropy
- Define algebra of quasilocal charges
- Obtain symplectic potential on $\mathcal{N}$ without restrictive bdy conditions

Once these are in place, we can relate Cardy formula for HHPS symmetries to BH entropy

Anomaly interpretation

Central theme: interpret $K_{\xi,\zeta}$ as arising from a type of anomaly

Comes from fixing a **scaling frame** for null normal of $\mathcal{N}$

$$l_a \to e^f l_a$$

. . . analogous to fixing a Weyl frame in AdS/CFT

- May explain why many different symmetry algebras all lead to entropy through Cardy formula
- Get one step closer to making sense of Cardyology $\to$ anomaly is the relevant feature

Covariant phase space [Lee, Wald (1990); Iyer, Wald (1994)]

Kinematics and Dynamics

- Starting point: field space of metrics g on a manifold with boundary $\mathcal{N}$
- Project onto on-shell metrics to get covariant phase space
- Diffeomorphism invariance $g \to g + \pounds_\xi g$
- Boundary conditions on $g \sim$ partial gauge fixing

Covariant phase space [Lee, Wald (1990); Iyer, Wald (1994)]

Basic objects

- $\delta L = E(g) \cdot \delta g + \boxed{d\theta(\delta g)} \rightarrow$ presymplectic potential
- Noether's theorem: $j_\xi = \theta(\pounds_\xi g) - i_\xi L = \boxed{dQ_\xi} \rightarrow$ Noether charge 2-form
- Presymplectic form: $\boxed{\Omega = \int_\Sigma \delta\theta}$
- Linearized charges: $\boxed{\delta H_\xi \stackrel{?}{=} \Omega(\delta g, \pounds_\xi g) = \int_{\partial\Sigma} \delta Q_\xi - i_\xi \theta}$

Quasilocal charges [Wald, Zoupas (1999); Harlow, Wu (2019)]

Charges generally *not* integrable due to flux:

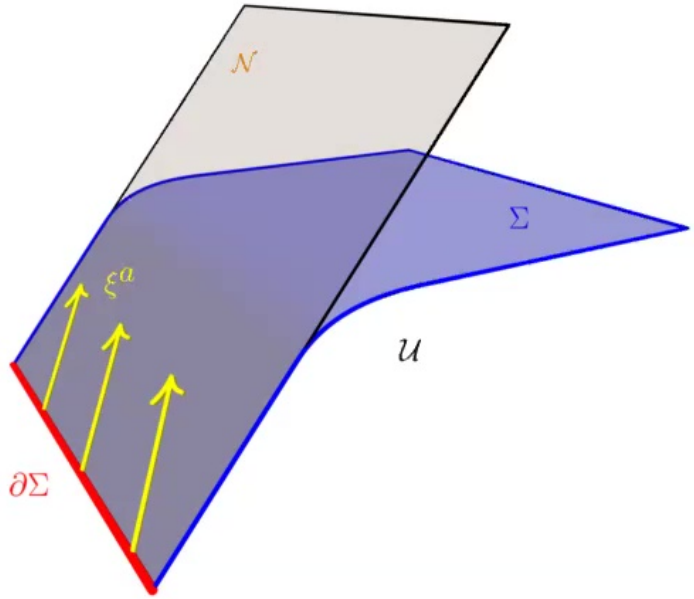
$$\underbrace{\boldsymbol{\theta}}_{\text{pullback}} = \underbrace{-\delta\ell}_{\text{boundary term}} + \underbrace{d\beta}_{\text{corner term}} + \underbrace{\mathcal{E}}_{\text{flux}}$$

Instead define manifestly integrable quasilocal charges:

$$H_\xi = \int_{\partial\Sigma} Q_\xi + i_\xi \ell - \beta(\pounds_\xi g)$$

Relation to variationl principle:

$$\delta\left(\int_{\mathcal{U}} L + \int_{\mathcal{N}} \ell\right) = \int_{\mathcal{U}} E\cdot\delta g + \int_{\mathcal{N}} \mathcal{E} + \int_{\partial\mathcal{N}} \beta$$



Dirichlet flux condition

Flux fixed to be in Dirichlet form:

$$\mathcal{E} = \pi^{ij}\delta g_{ij}$$

Resolves ambiguity in obtaining quasilocal charges (up to small caveats)

Arguments for Dirichlet form:

- Interpretation of conjugate momenta as Brown-York stress tensor
- Well motivated by holography (extrapolate dictionary)
- Gluing subregions $\mathcal{U}, \overline{\mathcal{U}}$ using path integral; combined action gives:

Dirichlet: match g_{ij}	Neumann: match π^{ij}
$\int_{\mathcal{N}} (\pi^{ij} - \bar{\pi}^{ij})\delta g_{ij}$	$\int_{\mathcal{N}} (g_{ij} - \bar{g}_{ij})\delta \pi^{ij}$
Metric is continuous in path integral	Off-shell metrics are ill-defined

Continuity equation

Can derive continuity eq for quasilocal charge

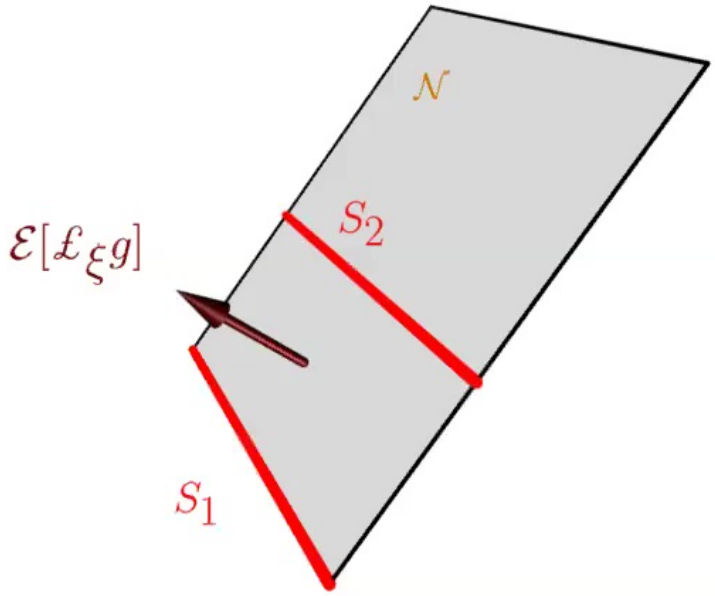
$$H_\xi(S_2) - H_\xi(S_1) = \int_{\mathcal{N}_{12}} \mathcal{E}(\pounds_\xi g) - \Delta_\xi \ell$$

First term is just the flux; Δ_ξ is anomaly operator for diffeomorphisms

Second term is an explicit violation of continuity eq

$$\Delta_\xi \ell = \delta_\xi \ell - \pounds_\xi \ell$$

when ℓ transforms anomalously $\sim$ anomalous Ward identity



Barnich-Troessaert bracket [Barnich, Troessaert (2011)]

Bracket for quasilocal charges is *not* Poisson bracket, since we have an open system

Instead, define modified bracket

$$\{H_\xi, H_\zeta\} = \delta_\xi H_\zeta + \int_{\partial\Sigma} i_\xi \mathcal{E}(\pounds_\xi g)$$

Modified bracket results in diffeomorphism algebra, up to central extension

$$\{H_\xi, H_\zeta\} = -\left(H_{[\xi,\zeta]} + K_{\xi,\zeta}\right)$$

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GR with null boundary

Field space = metrics g_{ab} on a manifold with fixed boundary $\mathcal{N}$

- Normal l_a specified independently of metric $\Rightarrow \delta l_a = 0$
- Impose nullness dynamically $l^a l^b \delta g_{ab} = 0$
- No fluctuations in scaling frame of $l_a \rightarrow$ sole source of anomaly

No other (local) boundary conditions on variations:

- Would be incompatible with HHPS symmetries
- Would eliminate physical excitations on null boundaries

Decomposition of symplectic potential

In vacuum GR,

$$\theta = \frac{1}{16\pi G}\epsilon_a \left(g^{bc}\delta\Gamma^a_{bc} - g^{ac}\delta\Gamma^b_{bc} \right)$$

On $\mathcal{N}$, decompose

$$\boldsymbol{\theta} = -\delta\ell + d\beta + \pi^{ij}\delta g_{ij} + \pi_i \delta l^i$$

Dirichlet form of flux: on null boundary, both induced metric g_{ij} and null generator l^i are intrinsic. . . Boundary term:

$$\ell = -\frac{k\eta}{8\pi G}$$

$\eta =$ induced volume element; $k =$ inaffinity, $l^b\nabla_b l^a = kl^a$

Computing anomalies

For any vector field ξ^a preserving $\mathcal{N}$,

$$\pounds_\xi l_a = w_\xi l_a$$

Therefore, anomalous transformation of l_a is

$$\Delta_\xi l_a = -w_\xi l_a$$

All other anomalous transformations determined by w_ξ. In particular,

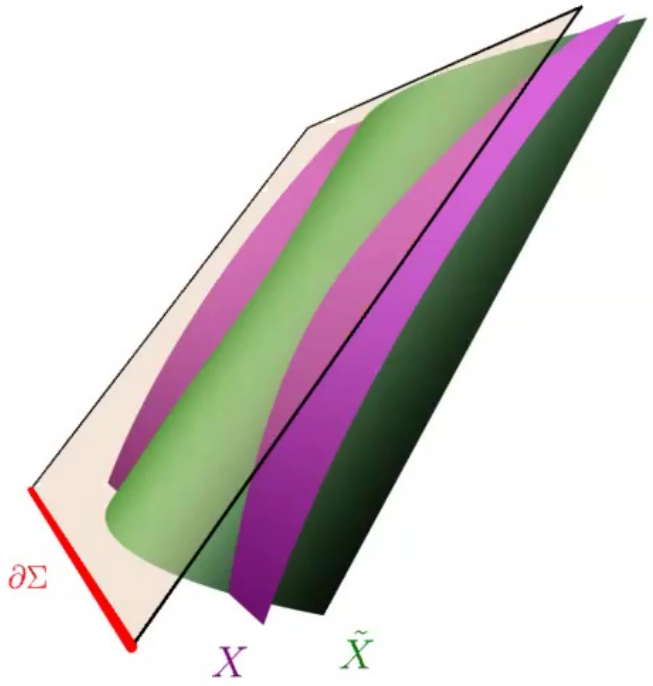
$$\Delta_\xi \ell = -\frac{1}{8\pi G}\left(l^a \nabla_a w_\xi\right)\eta$$

Stretched horizon and scaling frame

Background scaling frame of l_a determined by sequence of stretched horizons:

$$l_a = \nabla_a X$$

- Reparametrizing $X \to \tilde{X}(X, x^i)$ rescales l_a pointwise by $\tilde{X}'(0, x^i)$
- Analogous to AdS/CFT: changing radial cutoff = different Weyl frame of boundary CFT



Near horizon coordinates [HHPS (2018); Chen et al (2020)]

Restrict to axisymmetric Killing horizons: l^a = horizon Killing vector, $\partial\Sigma$ = bifurcation surface

Near horizon metric can be written in Rindler form

$$ds^2 = -\kappa^2 x^2 dt^2 + dx^2 + \psi^2 \phi^2 + q_{AB} d\theta^A d\theta^B + \dots$$

Can transform to "conformal coordinates"

$$w^+ = x e^{\alpha\phi + \kappa t}, \; w^- = x e^{\beta\phi - \kappa t}, \; y = e^{(\alpha+\beta)\phi/2}$$

Metric can then be written in terms of AdS_3 factor

$$ds^2 = \frac{dw^+ dw^-}{y^2} + \frac{4\psi^2}{(\alpha+\beta)^2}\frac{dy^2}{y^2} + q_{AB} d\theta^A d\theta^B + \dots$$

HHPS symmetries = vector fields which preserve the form of this metric

Virasoro vector fields [HHPS (2018); Chen et al (2020)]

Vector fields which preserve AdS_3 factor:

$$\xi = \varepsilon(w^+)\partial_+ + \frac{1}{2}\varepsilon'(w^+)y\partial_y \qquad \text{(future horizon)}$$

$$\bar{\xi} = \bar{\varepsilon}(w^-)\partial_- + \frac{1}{2}\bar{\varepsilon}'(w^-)y\partial_y \qquad \text{(past horizon)}$$

Expanding in modes, Lie bracket leads to $\text{Diff}(S^1) \times \text{Diff}(S^1)$ algebra

$$[\xi_m, \xi_n] = i(n-m)\xi_{n+m} \qquad [\bar{\xi}_m, \bar{\xi}_n] = i(n-m)\bar{\xi}_{n+m}$$

Central charges

Apply general formula for central extension

$$K_{\xi,\zeta} = \int_{\partial\Sigma} i_\xi \Delta_\zeta \ell - i_\zeta \Delta_\xi \ell$$

Results in Vir $\times$ Vir algebra with central charges

$$c = \bar{c} = \frac{3A}{\pi G(\alpha + \beta)}$$

Cardy formula

Gravitational charge algebra on Killing horizon resembles right moving and left moving sectors of a 2D CFT. . .apply Cardy formula

Determine temperatures of left and right sectors [HHPS (2018)]

$$T = \frac{\alpha}{2\pi} \qquad \overline{T} = \frac{\beta}{2\pi}$$

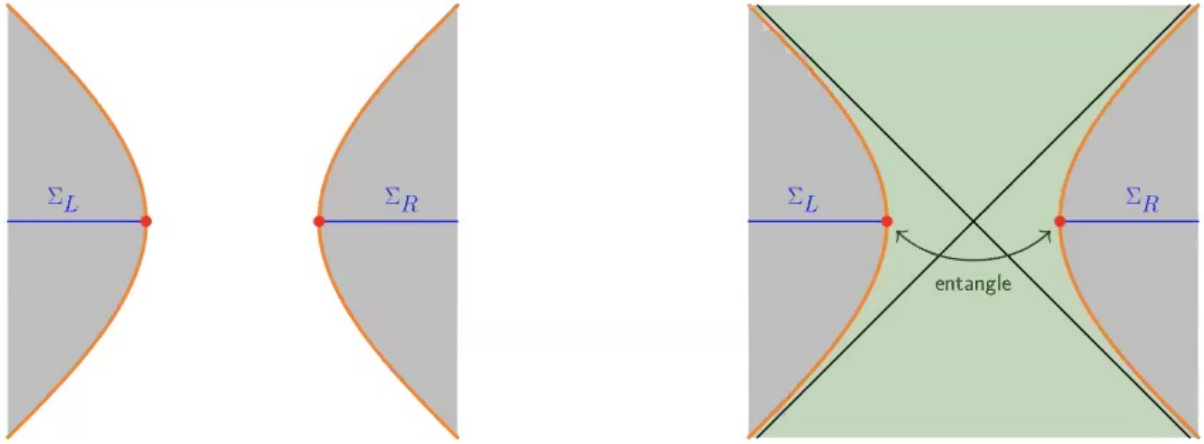
Canonical Cardy formula

$$S = \frac{\pi^2}{3}(cT + \bar{c}\overline{T}) = 2\left(\frac{A}{4G}\right)$$

Entropy is **twice** the Bekenstein-Hawking entropy!

Edge modes and gluing

Conjecture: BT bracket computes central charge of entangled CFTs $\sim$ gluing edge modes



Thermal Cardy calculation would then give $2S_{\mathsf{BH}}$, but entropy of one side after tracing out complement would be

$$S = \frac{A}{4G}$$

Integrable charges

By contrast, algebra of integrable charges should not include complement edge modes

HHPS charges integrable only for special values of α, β [Chen et al (2020)]

BT bracket reduces to Poisson bracket so $c, \bar{c}$ can be directly computed from symplectic form

Agrees with our general formula iff boundary term is *halved* $\Rightarrow$ Cardy formula gives $A/4G$

Microcanonical Cardy formula with halved boundary term:

$$S = 2\pi \left(\sqrt{\frac{L_0 c}{6}} + \sqrt{\frac{\bar{L}_0 \bar{c}}{6}} \right) = \frac{A}{4G}$$

Halved boundary term $\neq$ Dirichlet flux $\rightarrow$ some other principle to fix flux in general case?

Summary

- Dirichlet flux condition fixes ambiguity in covariant phase space construction of quasilocal charges
- Central charge arises from anomalous transformation of boundary term
- For null boundary, scaling anomaly $\leftrightarrow$ Weyl anomaly in AdS/CFT
- Anomaly is what's important, not specific symmetry algebras (universality)
- Cardy formula gave *twice* BH entropy $\rightarrow$ possible explanation in terms of entangled edge modes
- Edge mode picture consistent with validity of microcanonical Cardy formula in integrable case

Concluding remarks

- Factor of 2 in entropy: need a first principles derivation of BT bracket, possibly from gluing construction
- Central charge for more general null boundaries
- Physical signifiance of HHPS vector fields
- Horizon analogue of modular invariance?
- Why does the Cardy formula work so well and what is the connection to scaling anomaly?