Title: Quantum preparation games

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Abstract: To analyze the performance of adaptive measurement protocols for the detection and quanti cation of state resources, we introduce the framework of quantum preparation games. A preparation game is a task whereby a player sequentially sends a number of quantum states to a referee, who probes each of them and announces the measurement result. The measurement setting at each round, as well as the final score of the game, are decided by the referee based on the past history of settings and measurement outcomes. We show how to compute the maximum average score that a player can achieve under very general constraints on their preparation devices and provide practical methods to carry out optimizations over n-round preparation games. We apply our general results to devise new adaptive protocols for entanglement detection and quanti cation. Given a set of experimentally available local measurement settings, we provide an algorithm to derive, via convex optimization, optimal n-shot protocols for entanglement detection using these settings. We also present families of non-trivial adaptive protocols for multiple-target entanglement detection with arbitrarily many rounds. Surprisingly, we find that there exist instances of entanglement detection problems with just one target entangled state where the optimal adaptive protocol supersedes all non-adaptive alternatives.

Quantum Preparation Games

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Framework: Quantum Preparation Games

Basic setting: player prepares resources, referee scores player's resources after n rounds

Player's strategy $\mathcal{P}$

- Prepare quantum systems from $\mathcal{C}$

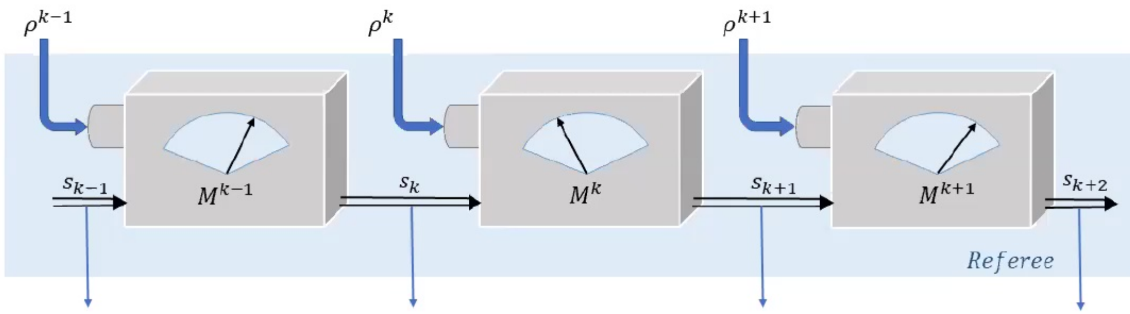
Referee's strategies

- Measuring devices $\mathcal{M}$
- Classical memory with states $\mathcal{S}$
- Scoring rule g

Expected score for a player with strategy $\mathcal{P}$

$$G(\mathcal{P}) = \sum_{s \in \mathcal{S}} p(s|\mathcal{P}) \langle g(s) \rangle .$$

Framework: *n*-round Maxwell demon games



- Full information about previous states of the experiment in memory $\mathcal{S}_k$ (demon).
- Assumption: The set $\mathcal{C}$ is closed under post-selection with $\mathcal{M}$ by the referee.
- Recursive computation of the score of a player with strategy $\mathcal{P}$:

$$\mu_s^{(n)} = \max_{\rho\in\mathcal{C}} \sum_{s'} \mathrm{tr}(M_{s'|s}^{(n)}\rho)\langle g(s')\rangle,$$

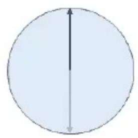
$$\mu_s^{(k)} = \max_{\rho\in\mathcal{C}} \sum_{s'} \mathrm{tr}(M_{s'|s}^{(k)}\rho)\mu_{s'}^{(k+1)},$$

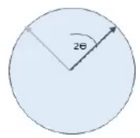
$$G_{\max} = \mu^{(1)}.$$

Example: preparation games inspired by gradient descent

Task: quantify the entanglement of $|\psi_\theta\rangle = \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$ in n-rounds, i.e., find a game with $G(\mathcal{P}) \leq \delta\ \forall \mathcal{P} \in \mathcal{E}_{\mathrm{SEP}}$ and $G(\theta) >> \delta$ for strategy with $|\psi_\theta\rangle$ (for most θ).

- Alice and Bob (= referees) can perform local measurements and communicate inputs and outcomes.





$$|\psi_\theta\rangle = \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$$
$$= |+\rangle \left(\frac{\cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle}{\sqrt{2}} \right) + |-\rangle \left(\frac{\cos(\theta)\,|0\rangle - \sin(\theta)\,|1\rangle}{\sqrt{2}} \right)$$

$$W(\theta) = \frac{1}{2}[\ |0\rangle\langle 0| \otimes Z + |1\rangle\langle 1| \otimes (-Z)$$
$$+ |+\rangle\langle +| \otimes (\sin(2\theta)X + \cos(2\theta)Z) + |-\rangle\langle -| \otimes (-\sin(2\theta)X + \cos(2\theta)Z)]$$

- Gradient

$$\frac{\partial}{\partial\theta} W(\theta) = |+\rangle\langle +| \otimes (\cos(2\theta)X - \sin(2\theta)Z) - |-\rangle\langle -| \otimes (\cos(2\theta)X + \sin(2\theta)Z)\,.$$

Example: preparation games from gradient descent

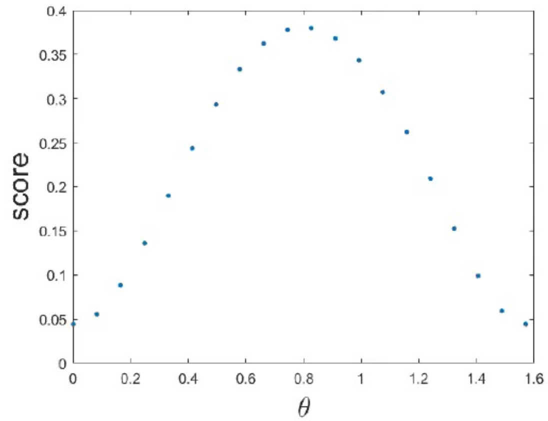
- Quantify entanglement of a player who prepares $|\psi_\theta\rangle\langle\psi_\theta|^{\otimes n}$ and compare it to other players.

- Referees' strategy for round k (starting at $k=0$)
  - Measure $M^0 = \{M^0_{-1}(\theta_k), M^0_1(\theta_k)\}$ or $M^1 = \{M^1_{-1}(\theta_k), M^1_1(\theta_k)\}$

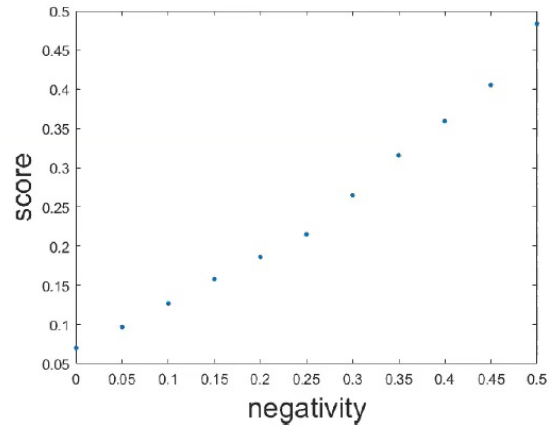
    (where $M^0_1(\theta_k) - M^0_{-1}(\theta_k) = W(\theta_k)$ and $M^1_1(\theta_k) - M^1_{-1}(\theta_k) = \frac{\partial}{\partial\theta} W(\theta_k)$).
  - Update memory state $\begin{pmatrix} s_k^0 \\ s_k^1 \end{pmatrix} \in \{-(k-1), \ldots, (k-1)\}^2$.
  - Update $\theta_{k+1} = \theta_0 + \epsilon s_k^1$ for the next round.
  - If $k=n$, score the player with $g(\theta_n, s_n^0) = h\left(\cos^2(\theta_n)\right) \Theta\left(s_n^0 - \delta(\theta_n)\right)$.

Example: preparation games from gradient descent

Player preparing states $|\psi_\theta\rangle\langle\psi_\theta|^{\otimes n}$



Player preparing states $\sigma \in \mathcal{E}_{\mathcal{N}}$



Negativity $\mathcal{N}(\sigma) = \frac{\|\sigma^{T_B}\|_1 - 1}{2}$.

Entanglement certification with various types of measurements

Task: certify entanglement of a family of states $\rho \in E$ in n rounds. Distinguish $\mathcal{E}_{\mathrm{ENT}} = \{\rho^{\otimes n} | \rho \in E\}$ from $\mathcal{E}_{\mathrm{SEP}}$ with various types of measurements.

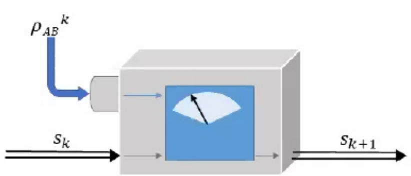
- Binary final outcome $s \in \{\mathrm{ent}, \mathrm{sep}\}$ with $g(\mathrm{ent}) = 1$, $g(\mathrm{sep}) = 0$.
- Quality of the test given by the worst-case errors

$$e_I = \max_{\mathcal{P} \in \mathcal{E}_{\mathrm{SEP}}} p(\mathrm{ent}|\mathcal{P})$$

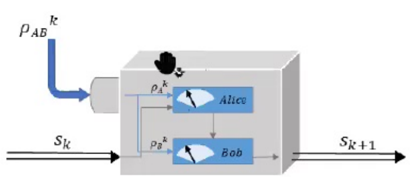
$$e_{II} = \max_{\mathcal{P} \in \mathcal{E}_{\mathrm{ENT}}} p(\mathrm{sep}|\mathcal{P})$$

- Goal: find best strategies for different types of measurements and compare them.

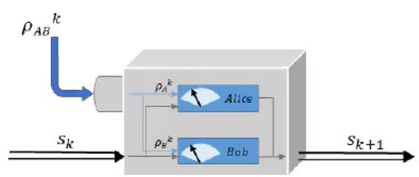
Global measurements



Adaptive measurements



"Fixed" local measurements



Can we optimise over preparation games?

- General optimisation over protocols (1-round)

$$
\begin{aligned}
&\min_{(M_s)_s} e_{II} \\
\text{s.t. } & 1 - \sum_s \operatorname{tr}(M_s \rho)\langle g(s)\rangle \leq e_{II} \qquad \forall \rho \in E, \\
& \sum_s \operatorname{tr}(M_s \sigma)\langle g(s)\rangle \leq e_{I} \qquad \forall \sigma \in \mathcal{C}, \\
& (M_s)_s \in \mathcal{M},
\end{aligned}
$$

Can we optimise over preparation games?

- General optimisation over protocols (1-round)

$$\min_{(M_s)_s} e_{II}$$

$$\text{s.t. } 1 - \sum_s \mathrm{tr}(M_s \rho)\langle g(s) \rangle \leq e_{II} \qquad \forall \rho \in E,$$

$$e_I \mathbb{I} - \sum_s M_s \langle g(s) \rangle \in \mathcal{C}^*,$$

$$(M_s)_s \in \mathcal{M},$$

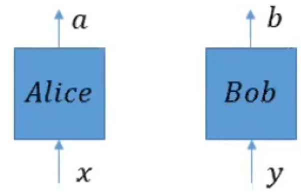
- For $\mathcal{C} = \mathcal{E}_{\mathrm{SEP}}$, the dual to the Doherty-Parillo-Spedalieri hierarchy approximates $\mathcal{E}^*_{\mathrm{SEP}}$ (from the inside).
- Similar hierarchies for other $\mathcal{C}$, e.g. for entanglement dimension (see article).

Doherty, Parillo, Spedalieri, PRL 88, 2002 and PRA 69, 2004.

Entanglement certification with various types of measurements

Example: single-shot entanglement certification of $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |1+\rangle)$.

$$
\begin{aligned}
&\min_{M_{ent}} e_{II} \\
\text{s.t. } & 1 - \mathrm{tr}(M_{ent}\,|\phi\rangle\langle\phi|) \leq e_{II}, \\
& e_I \mathbb{I} - M_{ent} = V_0 + V_1^{T_B}, \\
& V_0, V_1 \geq 0, \\
& \{M_{ent}, \mathbb{I} - M_{ent}\} \in \mathcal{M},
\end{aligned}
$$



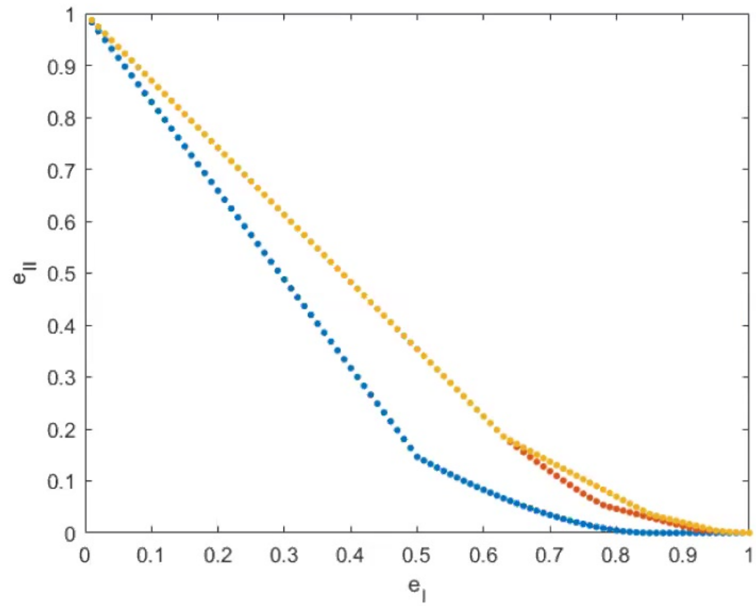
- $\mathcal{M}_1$ global measurements: any POVM $\{M_{ent}, \mathbb{I} - M_{ent}\}$
- $\mathcal{M}_2$ adaptive Pauli measurements $M_{ent} = \sum_{x,y} P(x, y, ent|a, b)\sigma_{x,a} \otimes \sigma_{y,b}$ s.t.

$$\sum_{y,s} P(x, y, s|a, b) = P(x) \quad \text{and} \quad \sum_{s} P(x, y, s|a, b) = P(x, y|a).$$

- $\mathcal{M}_3$ "fixed" Pauli measurements $M_{ent} = \sum_{x,y} P(x, y, ent|a, b)\sigma_{x,a} \otimes \sigma_{y,b}$ s.t.

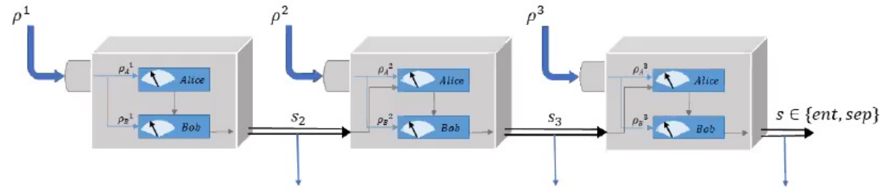
$$\sum_{s} P(x, y, s|a, b) = P(x, y).$$

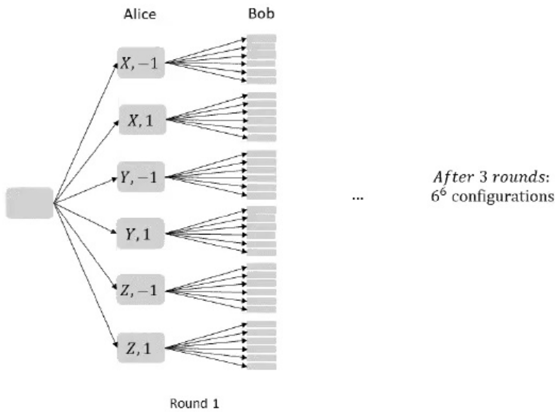
Entanglement detection with various types of measurements

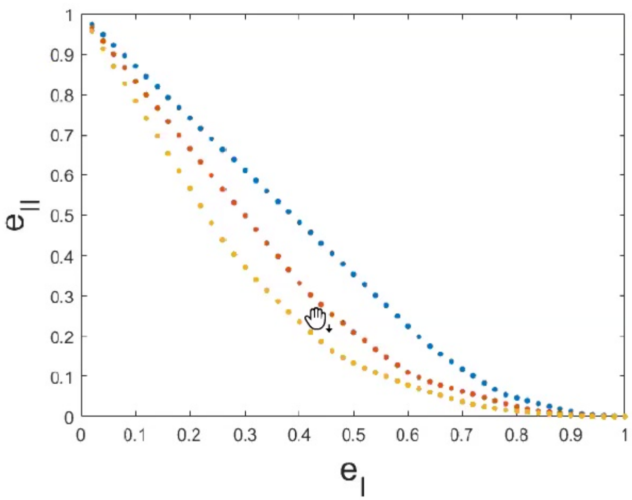


Entanglement certification with various types of measurements

Example: 3-round entanglement detection of $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |1+\rangle)$ with adaptive Pauli measurements.

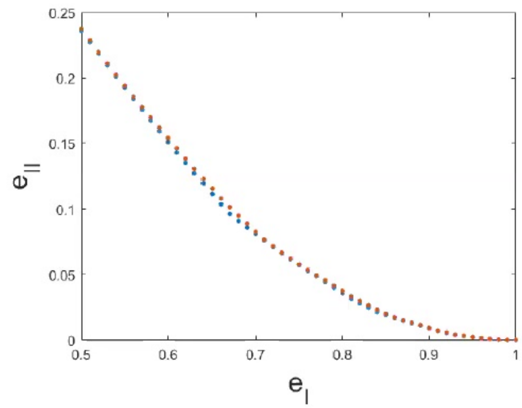






Entanglement certification with various types of measurements

- For single-shot entanglement detection the minimal total error $e_I + e_{II}$ often close together for $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$.
- Adaptiveness between rounds helps for n round entanglement detection even for $\mathcal{E}_{\mathrm{ENT}} = \{|\phi\rangle\langle\phi|^{\otimes n}\}$



- Recover previous results on certifying entanglement of $\rho^{\otimes n}$ in few rounds by requiring $e_{II} = 0$ and minimizing e_I (with player without access to s_k).

Dimić and Dakić, npj Quantum Information 4, 2018.
Saggio et al., Nature Physics 15, 2019.

Preparation games during interaction with an environment

- During each preparation the honest player's device interacts with an environment A, preparing

$$\mathrm{tr}_A\left[\sum_i K_i \rho_A^* K_i^\dagger\right],$$

where ρ_A^* is the current state of the environment and $K_i : \mathcal{H}_A \to \mathcal{H}_A \otimes \mathcal{H}$.

- Score of the player $G = \mathrm{tr}[\Omega \rho_A^*]$.
- Optimisation

$$\begin{aligned}
\min_{\{M_{s_k}^{(k)}\}_{k,s_k}} \quad & e_{II} \\
\text{s.t.} \quad & e_I \geq \mathrm{tr}[M(\{M_{s_k}^{(k)}\}_{k,s_k})\sigma] \quad \forall \sigma \in \mathcal{C} \\
& e_{II} \geq 1 - \mathrm{tr}[\Omega(\{M_{s_k}^{(k)}\}_{k,s_k})\rho_A] \quad \forall \rho_A
\end{aligned}$$

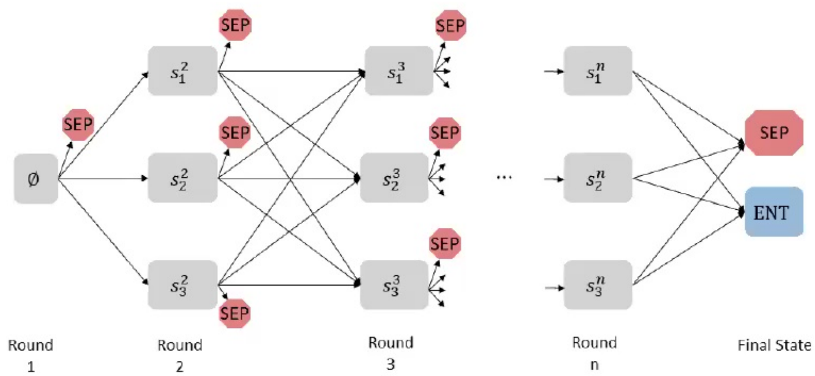
with recursive decomposition of Ω into $\{\Omega_{s_k}^{(k)}\}_{k,s_k}$ and of $M(\{M_{s_k}^{(k)}\}_{k,s_k})$ as before.

Example: entanglement detection when interacting with an environment

- Task: certify the entanglement of $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ in $n = 30$ rounds interacting (for $t = 0.1$) with an environment according to

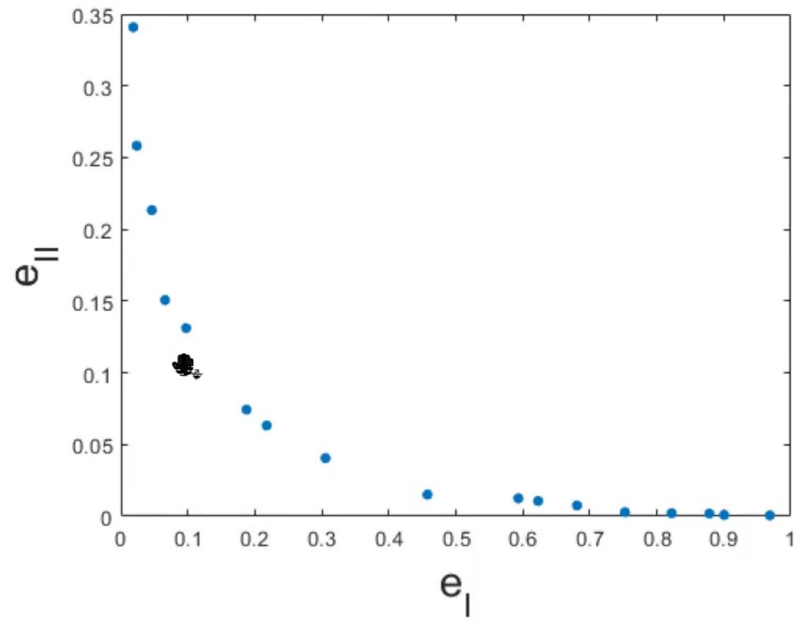
$$H_I = a^\dagger \otimes (\mathbb{I} \otimes |0\rangle\langle 1| + |0\rangle\langle 1| \otimes \mathbb{I}) + a \otimes (\mathbb{I} \otimes |1\rangle\langle 0| + |1\rangle\langle 0| \otimes \mathbb{I}) .$$

- Style of the memory architecture



- Measurements: See-saw optimisation over POVMs.

Example: entanglement detection when interacting with an environment



Further research and open questions

- Implications resources other than entanglement ? (Multidimensional entanglement, multi-party entanglement, magic states)
- Remove assumption of closure under post-selection with $\mathcal{M}$.
- Application for modelling NISQ devices (allow referee with quantum memory of fixed dimension) ?

Thank you for your attention!