Title: Controlling Majorana zero modes with machine learning

Speakers: Luuk Coopmans

Series: Machine Learning Initiative

Date: October 02, 2020 - 11:00 AM

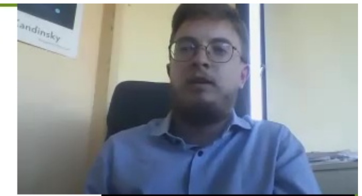
URL: http://pirsa.org/20100025

Abstract: Majorana zero modes have attracted much interest in recent years because of their promising properties for topological quantum computation. A key question in this regard is how fast two Majoranas can be exchanged giving rise to a unitary gate operation. In this presentation I will first explain that the transport of Majoranas in one-dimensional topological superconductors can be formulated as a â€œsimpleâ€• optimal control optimization problem for which we propose several different control regimes. Next I will discuss the optimization methods, Differential Programming and Natural Evolution Strategies, that were applied to the Majorana control problem and came up with a counter-intuitive transport strategy. This strategy, which we dubbed jump-move-jump, will form the focus of the last part of the presentation in which I explain the key underlying mechanisms behind the strategy by reformulating the motion of Majoranas in a moving frame. I will conclude by arguing that these results demonstrate that machine learning for quantum control can be applied efficiently to quantum many-body dynamical systems with performance levels that make it relevant to the realization of large-scale quantum technology.

DIAS

Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

sfi Science Foundation Ireland



Controlling Majoranas with Machine Learning

Luuk Coopmans

Supervisor:
Graham Kells

Collaborators:
Juan Carrasquilla (Toronto)
Di Luo (Illinois)
Bryan Clark (Illinois)

Arxiv: 2008.09128

October 2, 2020

Seminar Perimeter Institute Quantum Intelligence Lab (Canada)

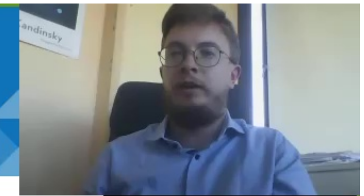
Outline

1. Motivation and background
2. Control of Majoranas formulated as a game
3. Differential Programming (DP) and Natural Evolution Strategies (NES)
4. Jump-move-jump strategy for Majorana quantum control
5. Outlook and conclusion

Trinity College Dublin
The University of Dublin

Motivation I

Quantum Technologies, Quantum Control and Decoherence



Outline

Motivation

Setup

- Majorana Control
- Moving Frame

Methods

Results

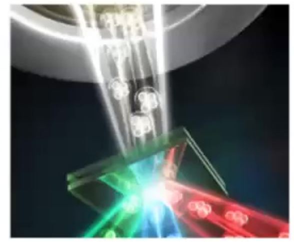
- ML Results
- Analysis

Conclusion

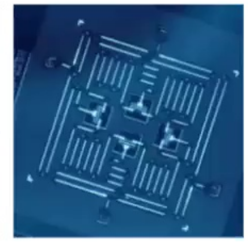
Applications of Quantum Technologies:

Artificial Intelligence

Cryptography

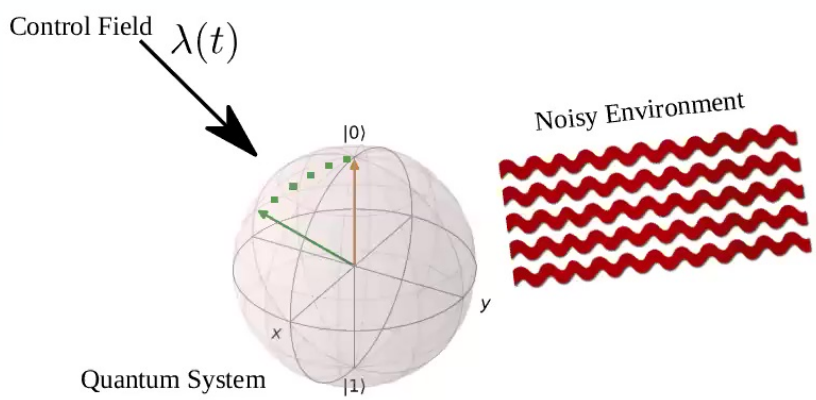


Sensing

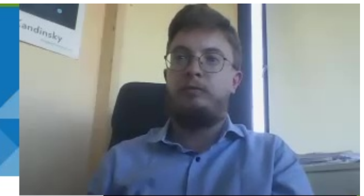


Computation

Quantum Control:



Trinity College Dublin
The University of Dublin



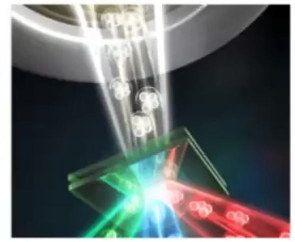
Motivation I

Quantum Technologies, Quantum Control and Decoherence

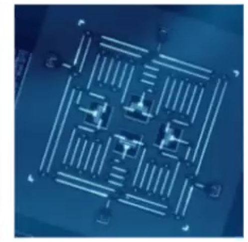
Applications of Quantum Technologies:

Artificial Intelligence

Cryptography



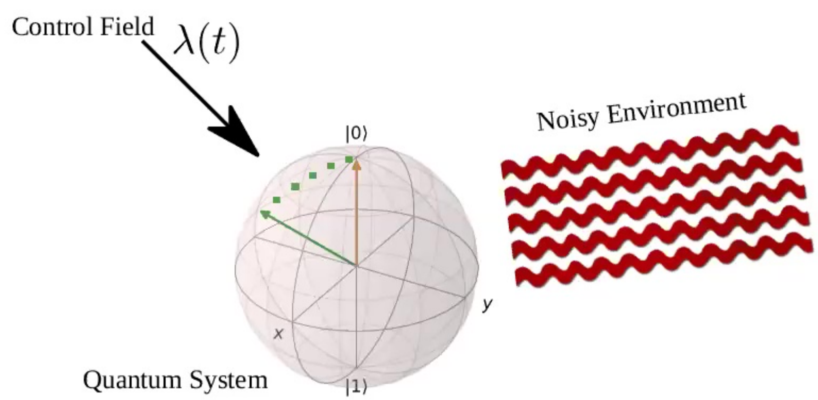
Sensing



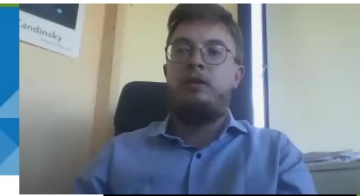
Computation

Standard schemes suffer from decoherence

Quantum Control:



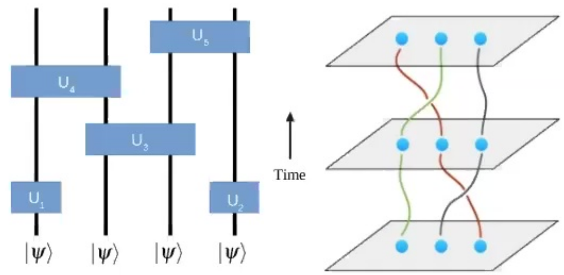
Trinity College Dublin
The University of Dublin



Motivation II

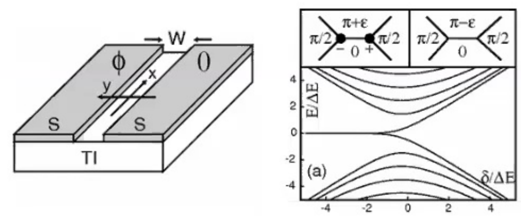
Majorana Zero Modes and Topological Quantum Computation

Trinity College Dublin
The University of Dublin



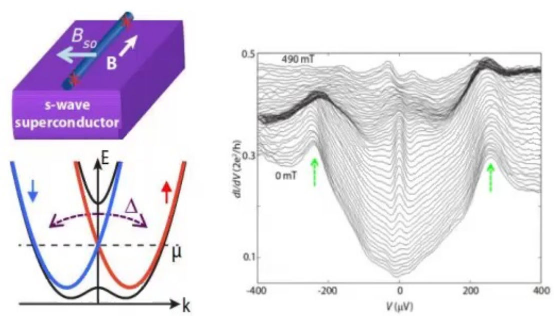
Non-abelian Anyons and Topological Quantum Computation

Kitaev (2003), **Nayak et al.** (2008)



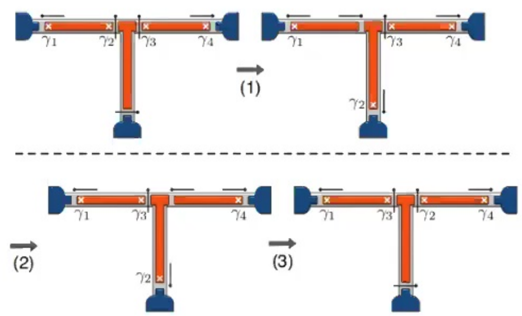
Superconducting Proximity Effect

Fu and Kane (2008), **Oreg et al.** (2010), **Lutchyn et al.** (2010)



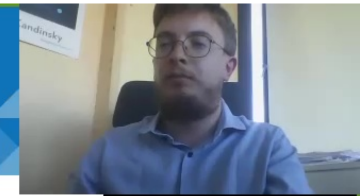
Signatures of Majorana Zero Modes

V. Mourik et al. (2012), **Zhang et al.** (2018)



Braiding Majorana Zero Modes

Alicea et al. (2011), **Aasen et al.** (2016)



The Kitaev Chain Kitaev (2001)

An effective toy model for topological superconductivity

$$\mathcal{H} = -\sum_{x=1}^{N}[(\mu(x) - V(x))c_x^\dagger c_x - wc_x^\dagger c_{x+1} + \Delta c_x^\dagger c_{x+1}^\dagger] + h.c. \qquad (1)$$

Majorana Zero Modes

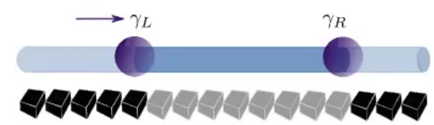
$$\mathcal{H} = \sum_n \epsilon_n(\beta_n^\dagger \beta_n - \frac{1}{2}) \qquad (2)$$

$$\beta_0^\dagger = \gamma_R - i\gamma_L, \quad \beta_0 = \gamma_R + i\gamma_L \qquad (3)$$

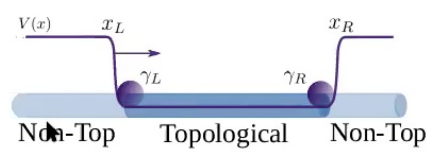
$$\phi(x) \propto e^{-x/\xi}\sin\left(\sqrt{k_F^2 + 1/\xi^2}x\right) \qquad (4)$$

Moving the Majoranas

'Keyboard' Potential (Alicia et al. (2011)):



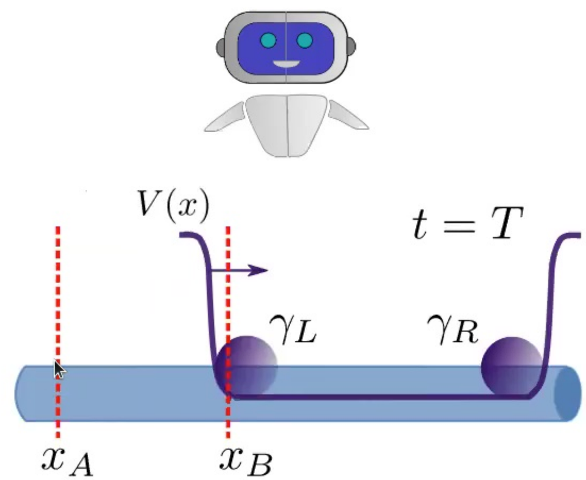
Smooth Potential Profile:



$$V(x,t) = V_{\text{height}}[f(x - x_L(t)) + f(x_R - x)] \qquad (5)$$

The Majorana Game

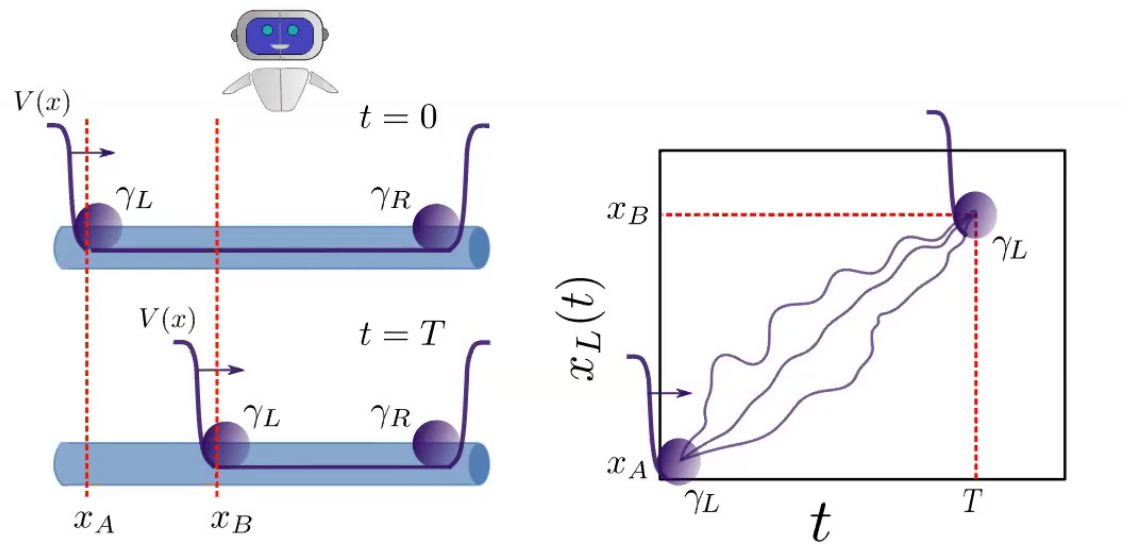
Controlling Majoranas with Machine Learning



Trinity College Dublin
The University of Dublin

The Majorana Game

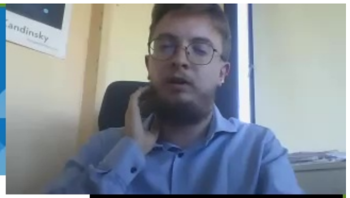
Controlling Majoranas with Machine Learning

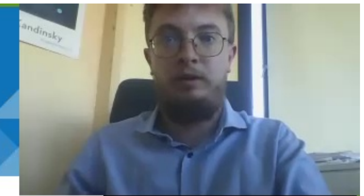


Quantitative measure for the quality of the path:

$$\mathcal{I}(T) \equiv 1 - \mathcal{F}(T) = 1 - |\langle\psi_B| \mathcal{T} e^{-i\int_0^T \mathcal{H}(t)dt} |\psi_A\rangle|^2 \quad (6)$$

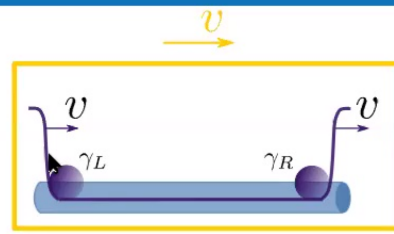
Constraint: $v_{\text{avg}} = (x_B - x_A)/T \equiv l/T$ fixed.





Motion of Majoranas in a moving frame

Critical velocity and the energy gap



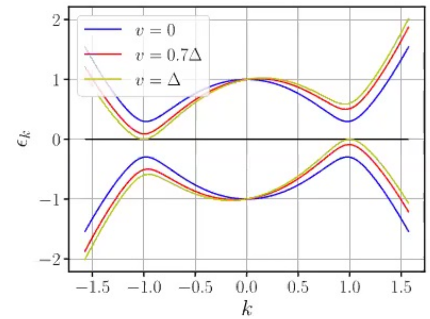
Moving Frame Hamiltonian:

Scheurer et al. (2013), ***Karzig et al.*** (2013)

$$H_v(t) = U^\dagger(t)\mathcal{H}U(t) + i\frac{dU^\dagger}{dt}U(t) \tag{7}$$

$$\epsilon_k = \pm\sqrt{(\frac{k^2}{2m} - \mu)^2 + \Delta^2 k^2} + vk \tag{8}$$

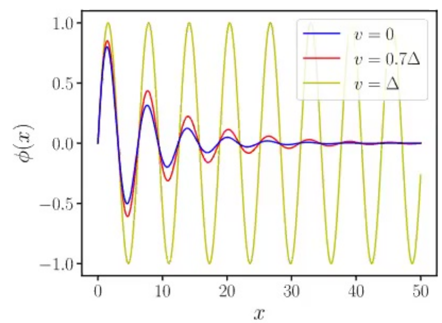
$$v_{\text{crit}} = \Delta \tag{9}$$



Resonance Frequency/Time-scale:

Conlon et al. (2019)

$$\omega_{\text{res}} = \Delta k_F \tag{10}$$



Trinity College Dublin
The University of Dublin

Different Motion (Control) Regimes

Outline

Motivation

Setup

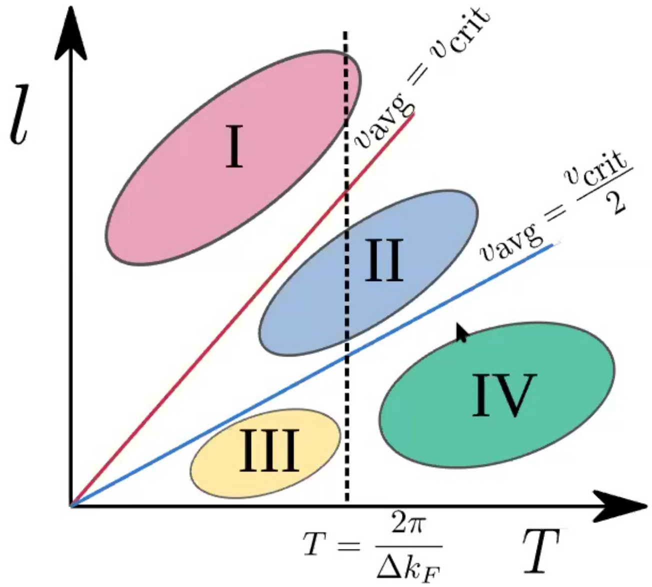
- Majorana Control
- Moving Frame

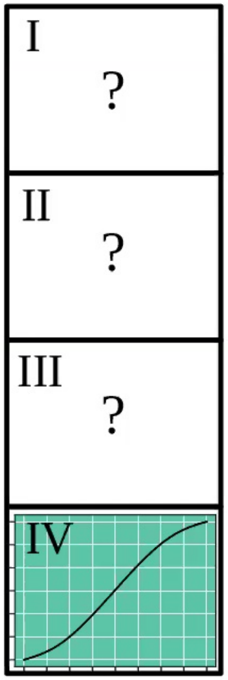
Methods

Results

- ML Results
- Analysis

Conclusion

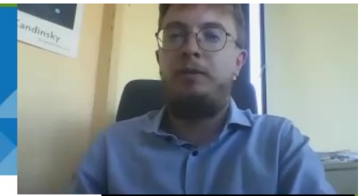


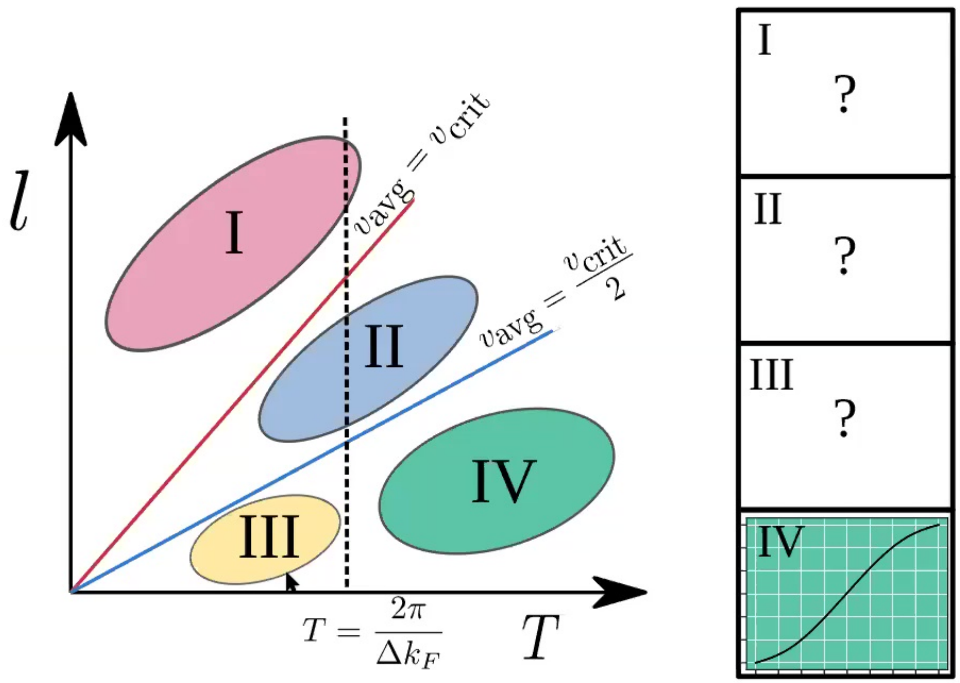


Trinity College Dublin

The University of Dublin

Different Motion (Control) Regimes





Minimize the infidelity $\mathcal{I}(T)$ in each regime

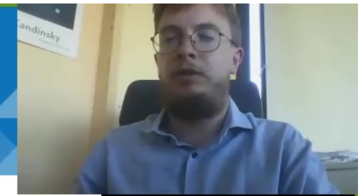
Trinity College Dublin
The University of Dublin

Differential Programming (DP)

A programming paradigm to efficiently (back-propagation) obtain numerically exact gradients of complete computer programmes.

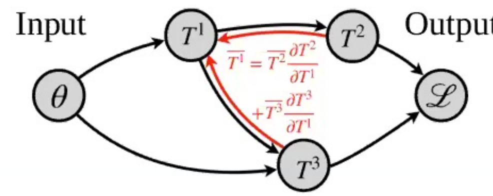
Trinity College Dublin
The University of Dublin

Differential Programming (DP)

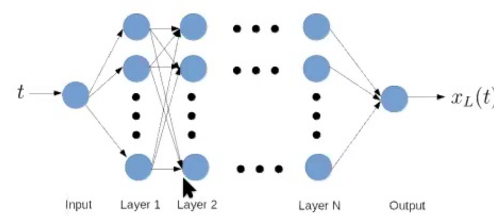


A programming paradigm to efficiently (back-propagation) obtain numerically exact gradients of complete computer programmes.

- Computer programmes as computational graphs
- $\mathcal{I}(T) = f_n \circ f_{n-1} \circ \cdots \circ f_1(x_L(t))$
- Derivative of the infidelity with respect to the control $\frac{d\mathcal{I}}{dx_L}$
- Jax Library for automatic differentiation **Bradbury et al.** (2018)
- Updates with Gradient Descent $x_L(t)_{i+1} = x_L(t)_i - \nabla_{x_L(t)_i}\mathcal{I}(T)$
- Allows to use neural networks $x_L(t) = \mathrm{NN}_\theta(t)$

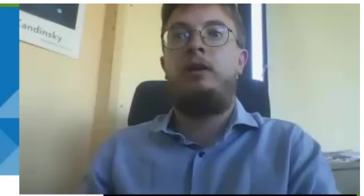


Liao et al. (2019)



Trinity College Dublin
The University of Dublin

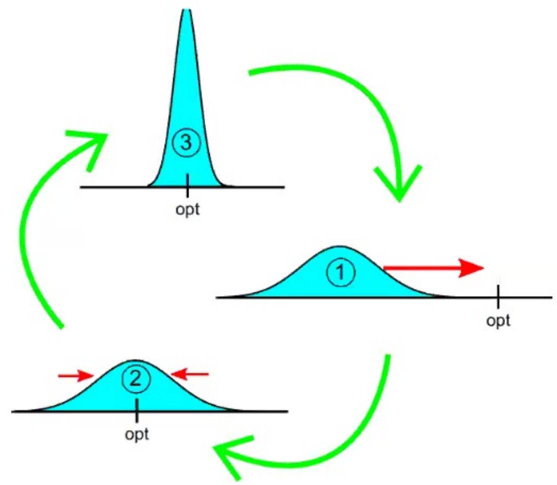
Natural Evolution Strategies (NES)



Black-box optimization method inspired by Natural Evolution

- Generations, mutations, fitness and survival
- Draw parameters $x_L(t)$ from Gaussian distribution
- Optimize distribution
- Allows to optimize our cost function $\mathcal{I}(T)$ without knowing the exact gradient
- Efficient because highly parallellizable

Wierstra et al. (2014), **Salimans et al.** (2017)

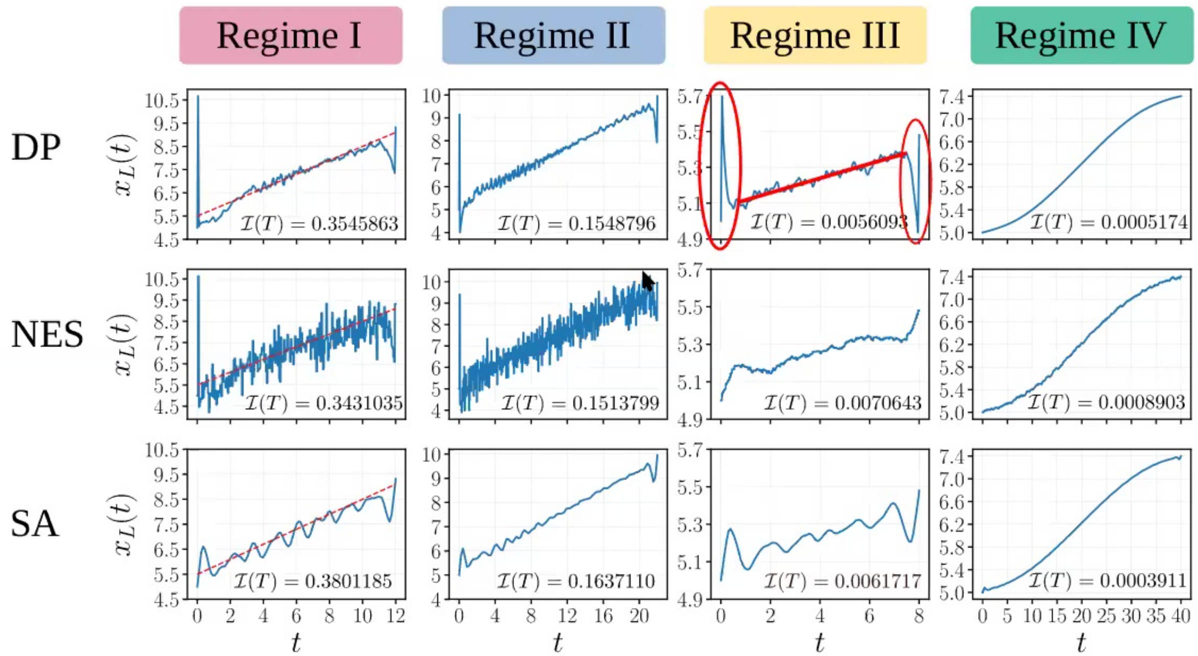


Trinity College Dublin
The University of Dublin

Numerical Results

Machine Learned Strategies for Majorana Control

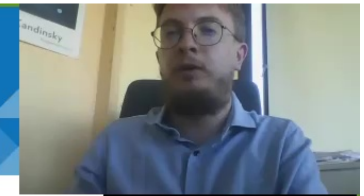
- **Outline**
- **Motivation**
- **Setup**
  - Majorana Control
  - Moving Frame
- **Methods**
- **Results**
  - ML Results
  - Analysis
- **Conclusion**



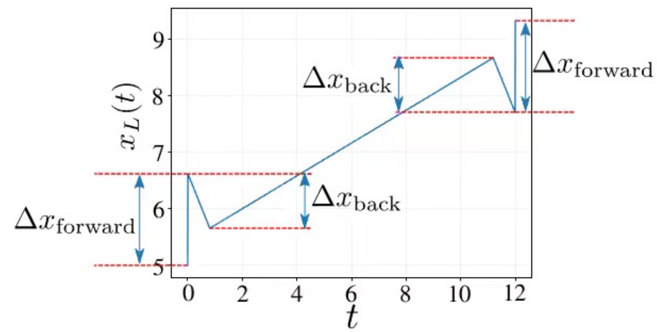
Use the main features in regimes I-II-III as starting point for our analysis.

Trinity College Dublin
The University of Dublin

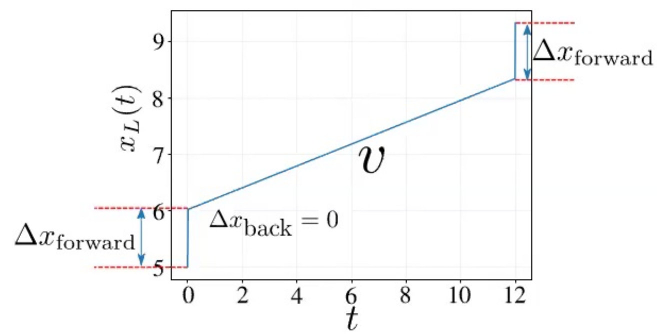
The Jump-Move-Jump (JMJ) strategy



- Inspired by ML results
- Bare JMJ: $\Delta x_{\text{back}} = 0$
- Dressed JMJ: $\Delta x_{\text{back}} \neq 0$



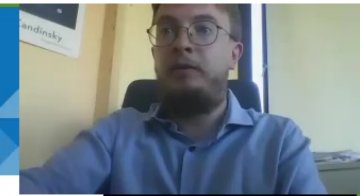
Bare JMJ:

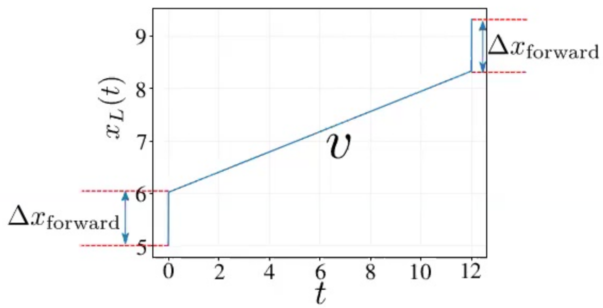


Trinity College Dublin
The University of Dublin

Bare Jump-Move-Jump Strategy

Evaluation of the Majorana motion in the moving frame





Maximization of two contributions:

$$\mathcal{I}(T) = 1 - \mathcal{F}(T) \approx 1 - O_\delta^2 O_v^2 \quad (11)$$

Boundary Jumps:

$$\Delta x_{\text{forward}} = (l - vT)/2 \equiv \delta \quad (12)$$

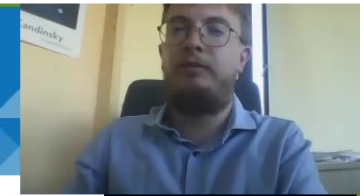
$$O_\delta = |\left\langle \psi_{x_L} \middle| \psi_{x_L+\delta} \right\rangle|^2 \sim \exp\left(-\delta^2/s^2\right) \quad (13)$$

Constant velocity *move* part:

$$O_v = |\left\langle \psi_{x_L} \middle| \psi_v \right\rangle|^2 \sim 1 - \beta \left(\frac{v}{\Delta}\right)^2 \gamma \quad (14)$$

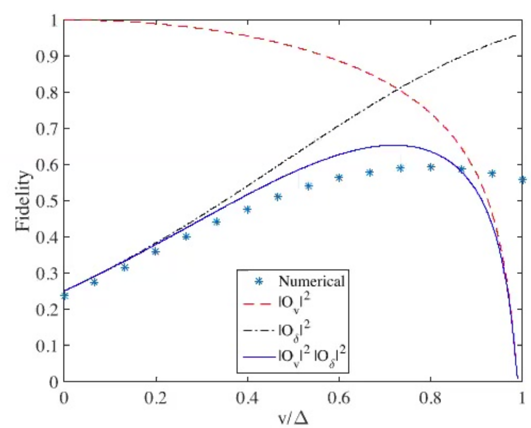
$$\gamma = 1/\sqrt{1 - v^2/\Delta^2} \quad (15)$$

Trinity College Dublin
The University of Dublin



Bare Jump-Move-Jump Strategy

Evaluation of the Majorana motion in the moving frame



Maximization of two contributions:

$$\mathcal{I}(T) = 1 - \mathcal{F}(T) \approx 1 - O_\delta^2 O_v^2 \qquad (16)$$

Boundary Jumps:

$$\Delta x_{\text{forward}} = (l - vT)/2 \equiv \delta \qquad (17)$$

$$O_\delta = |\left\langle \psi_{x_L} \middle| \psi_{x_L+\delta} \right\rangle|^2 \sim \exp\left(-\delta^2/s^2\right) \qquad (18)$$

Constant velocity *move* part:

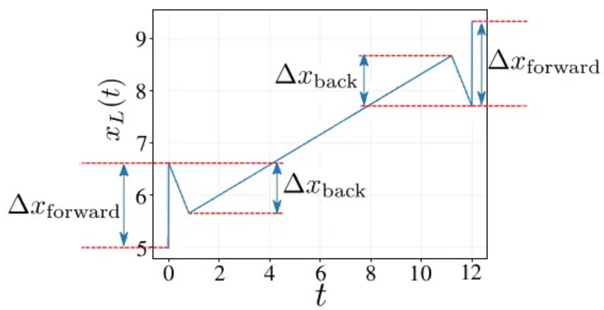
$$O_v = |\left\langle \psi^0_{x_L} \middle| \psi^0_v \right\rangle|^2 \sim 1 - \beta \left(\frac{v}{\Delta}\right)^2 \gamma \qquad (19)$$

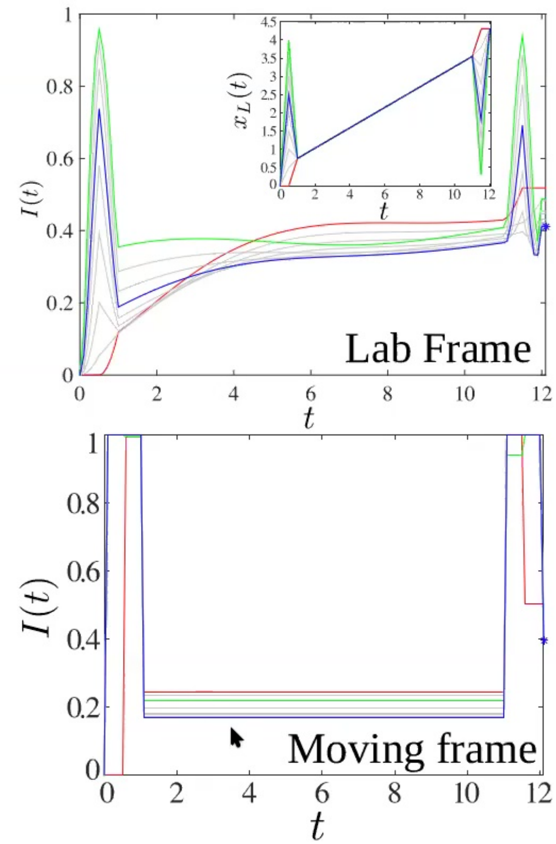
$$\gamma = 1/\sqrt{1 - v^2/\Delta^2} \qquad (20)$$

Trinity College Dublin
The University of Dublin

Dressed Jump-move-jump strategy

Better targeting the moving frame groundstate

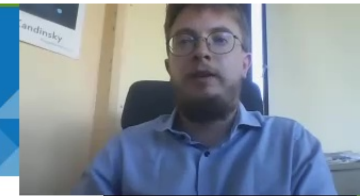




Instantaneous infidelity:

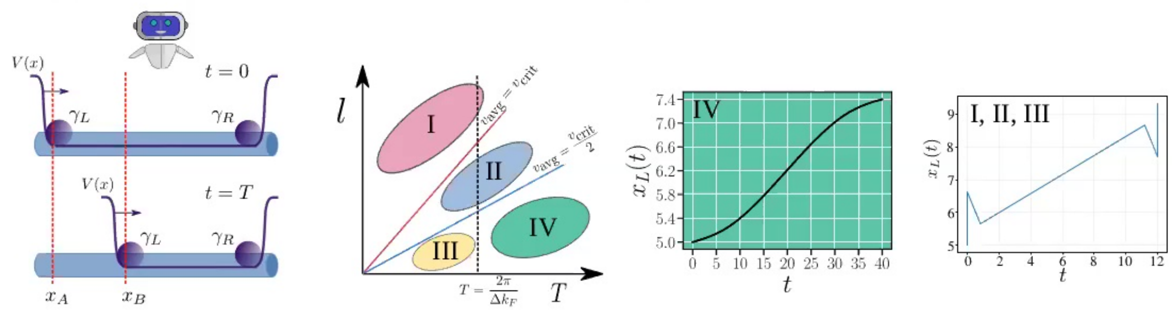
$$I(t) = 1 - |\langle \psi(t) | \psi_{\text{ins}} \rangle|^2 \qquad (21)$$

Trinity College Dublin
The University of Dublin



Summary and further work

- Formulated the Majorana control problem as a game to which ML techniques can be applied
- Successfully applied DP and NES and benchmarked them with SA
- Recovered the expected smooth protocols in the adiabatic regime IV
- Found a new strategy, *jump-move-jump*, in the non-adiabatic regimes
- Developed a theoretical understanding for this strategy by analyzing it in the moving frame
- Look at models even closer to experiment, include disorder
- Use the machine learning techniques to overcome key challenges in the realization of large scale quantum technology



Trinity College Dublin
The University of Dublin

Thank You.

DIAS | Coláiste na Tríonóide, Baile Átha Cliath Trinity College Dublin Ollscoil Átha Cliath | The University of Dublin | sfi Science Foundation Ireland

Supervisor:
Graham Kells

arxiv.org/abs/2008.09128
github.com/LuukCoopmans/majorana_game

Collaborators:
Juan Carrasquilla (Toronto)
Di Luo (Illinois)
Bryan Clark (Illinois)

Luuk Coopmans