Title: Quasimaps and BPS counts

Speakers: Henry Liu

Series: Mathematical Physics

Date: October 08, 2020 - 1:30 PM

URL: http://pirsa.org/20100022

Abstract: The theory of quasimaps to Nakajima quiver varieties X has recently been used very effectively by Aganagic, Okounkov and others to study symplectic duality. For certain X, namely Hilbert schemes of ADE surfaces, it turns out quasimap theory is equivalent to a particular flavor of Donaldson-Thomas theory on a related threefold Y. I will explain this equivalence and how it intertwines concepts and tools from the two sides. For example, symplectic duality has something to say about the crepant resolution conjecture for Y.

Quasimaps and BPS counts - Microsoft OneNote

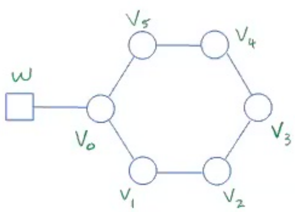
File Home Insert Share Draw Review View Pages

Quasimaps and BPS counts

(based on [arXiv: 2006.14695])

October-02-20
10:32 PM

3d $\mathcal{N}=4$ quiver gauge theories and quasimaps



Quiver Q

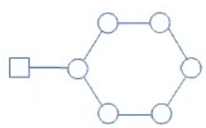
Higgs branch $\mathcal{M}^H_\theta(Q)$ = Nakajima quiver variety

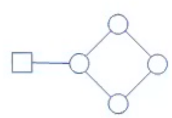
(an algebraic symplectic reduction $T^* \mathrm{Rep}(Q) /\!\!/\!\!/_\theta \, GL(\vec{v})$)

Carries an action of $GL(W) \times \mathbb{C}^\times_\hbar \supset A \times \mathbb{C}^\times_\hbar$

Coulomb branch $\mathcal{M}^C(Q)$ = BFN construction.

3d mirror symmetry (going back to [Intrilligator - Seiberg]) :





Quiver $Q \longleftrightarrow Q^\vee$

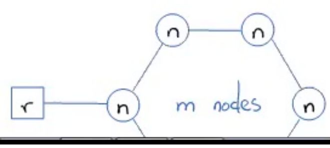
Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

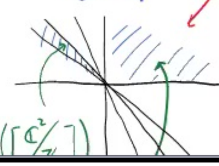
3d mirror symmetry (going back to [Intriligator - Seiberg]) :

Quiver	Q	$\longleftrightarrow$	$Q^\vee$	
Higgs branch	$\mathcal{M}^H(Q)$		$\mathcal{M}^H(Q^\vee)$	
Coulomb branch	$\mathcal{M}^C(Q)$		$\mathcal{M}^C(Q^\vee)$	
FI parameters	$K \ni \vec{\theta} = (\theta_1, \dots, \theta_m)$		$(\theta_1^\vee, \dots, \theta_m^\vee) = \vec{\theta}^\vee \in K^\vee$	"Kähler torus"
mass parameters	$A \ni (a_1, \dots, a_r)$		$(a_1^\vee, \dots, a_r^\vee) \in A^\vee$	"Equivariant torus"

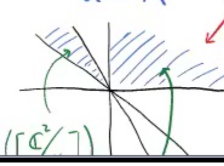
Our main example (affine type A quiver) :



$\vec{\theta} \in K$ "GIT chamber"



$\vec{a} \in A$ "attracting chamber"

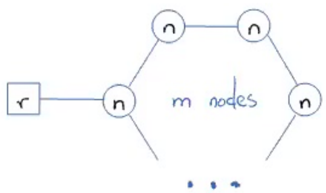


$\mathcal{M}^H_{\vec{\theta}} = \mathcal{M}_{r,n}([\mathbb{C}^2/\mathbb{Z}_m])$ $\quad$ $\mathcal{M}^C_{\vec{a}} = \mathcal{M}_{\dots}([\mathbb{C}^2/\dots])$

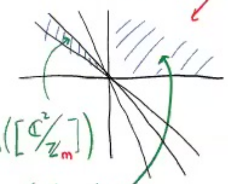
FI parameters $K \ni \vec{\theta} = (\theta_1, \ldots, \theta_m) \longleftrightarrow (\theta_1^\vee, \ldots, \theta_m^\vee) = \vec{\theta}^\vee \in K^\vee$ "Kähler torus"

mass parameters $A \ni (a_1, \ldots, a_r) \longleftrightarrow (a_1^\vee, \ldots, a_r^\vee) \in A^\vee$ "Equivariant torus"

Our main example (affine type A quiver):



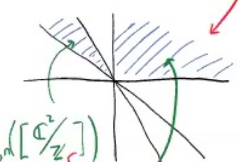
$\vec{\theta} \in K$ "GIT chamber"



$\mathcal{M}^H_{\vec{\theta}} = \mathcal{M}_{r,n}([\mathbb{C}^2/\mathbb{Z}_m])$

$\mathcal{M}^H_{\vec{\theta}} = \mathcal{M}_{r,n}(\mathcal{A}_{m-1})$

$\vec{a} \in A$ "attracting chamber"



$\mathcal{M}^C_{\vec{a}} = \mathcal{M}_{m,n}([\mathbb{C}^2/\mathbb{Z}_r])$

$\mathcal{M}^C_{\vec{a}} = \mathcal{M}_{m,n}(\mathcal{A}_{r-1})$

($\mathcal{M}_{r,n}(X)$ = moduli of rank r instantons on X of instanton number n.)

$r \leftrightsquigarrow m$ is like level-rank duality.

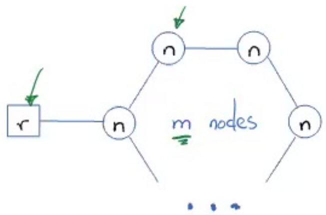
Can compute SUSY indices for these 3d theories.

Quasimaps and BPS counts - Microsoft OneNote

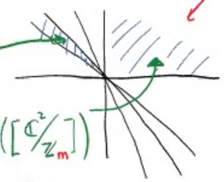
FI parameters $K \ni \vec{\theta} = (\theta_1, \dots, \theta_m) \longleftrightarrow (\theta_1^\vee, \dots, \theta_m^\vee) = \vec{\theta}^\vee \in K^\vee$ "Kähler torus"

mass parameters $A \ni (a_1, \dots, a_r) \longleftrightarrow (a_1^\vee, \dots, a_r^\vee) \in A^\vee$ "Equivariant torus"

Our main example (affine type A quiver):



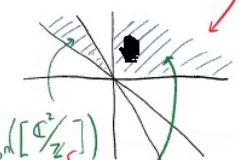
$\vec{\theta} \in K$ "GIT chamber"



$\mathcal{M}^H_{\vec{\theta}} = \mathcal{M}_{r,n}([\mathbb{C}^2/\mathbb{Z}_m])$

$\mathcal{M}^H_{\vec{\theta}} = \mathcal{M}_{r,n}(\mathcal{A}_{m-1})$

$\vec{a} \in A$ "attracting chamber"



$\mathcal{M}^C_{\vec{a}} = \mathcal{M}_{m,n}([\mathbb{C}^2/\mathbb{Z}_r])$

$\mathcal{M}^C_{\vec{a}} = \mathcal{M}_{m,n}(\mathcal{A}_{r-1})$

($\mathcal{M}_{r,n}(X)$ = moduli of rank r instantons on X of instanton number n.)

$r \longleftrightarrow m$ is like level-rank duality.

Can compute SUSY indices for these 3d theories.

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

Can compute SUSY indices for these 3d theories.

 $\times$ 

space C time S^1

Do it in the IR limit, where $\mathrm{vol}(C) \gg 0$.

Contributions localize to (after a topological twist, see [Bullimore - Ferrari - Kim])

$$\Big[\ \text{(space } C\text{)} \xrightarrow{f} \mathcal{M}^H \Big] \in \mathrm{QMaps}(C \to \mathcal{M}^H)$$

space C

These are quasimaps (to $\mathcal{M}^H$).

(Twisted) SUSY index $=$ Index of a virtual Dirac operator on $\mathcal{M}^H$ $= \chi\big(\mathrm{QMaps}(C \to \mathcal{M}^H), \hat{\mathcal{O}}^{\mathrm{vir}}\big)$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

$$\left[\ C \xrightarrow{f} \mathcal{M}^H \right] \in QMaps(C \to \mathcal{M}^H)$$

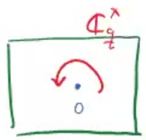
space C

These are quasimaps (to $\mathcal{M}^H$).

(Twisted) SUSY index $=$ Index of a virtual Dirac operator on $\mathcal{M}^H$ $= \chi\left(QMaps(C \to \mathcal{M}^H), \hat{\mathcal{O}}^{vir}\right)$

(symmetrized) virtual structure sheaf

By degenerating C, get pieces



$$\left[\ \mathbb{C} \xrightarrow{f} \mathcal{M}^H \right] \in QMaps(\mathbb{P}^1 \to \mathcal{M}^H)_{\text{nonsing } \infty}$$

$\mathbb{C}^\times_q$, 0

$\mathbb{C} \subset \mathbb{P}^1$

(These depend on a boundary condition $\mathcal{E} \in K(\mathcal{M}^H)$ (at $\infty \in \mathbb{P}^1$))

equivariant

These are quasimaps (to $\mathcal{M}^H$).

(Twisted) SUSY index $=$ Index of a virtual Dirac operator on $\mathcal{M}^H$ $= \chi\left(\mathrm{QMaps}(C \to \mathcal{M}^H), \hat{\mathcal{O}}^{vir}\right)$

(symmetrized) virtual structure sheaf

By degenerating C, get pieces

$$\left[\; \mathbb{C}^\times_q \curvearrowright \mathbb{C} \subset \mathbb{P}^1 \xrightarrow{f} \mathcal{M}^H \; \right] \in \mathrm{QMaps}(\mathbb{P}^1 \to \mathcal{M}^H)_{\text{nonsing } \infty}$$

(These depend on a boundary condition $\mathcal{E} \in K(\mathcal{M}^H)$ (at $\infty \in \mathbb{P}^1$))

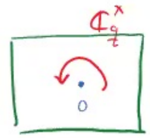
$\Rightarrow$ indices are built from equivariant K-theoretic integrals on

using the symmetrized virtual structure sheaf $\hat{\mathcal{O}}^{vir}$

$(\mathcal{M}^H)$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages



$$\left[\mathbb{C} \subset \mathbb{P}^1 \xrightarrow{f} \mathcal{M}^H \right] \in QMaps(\mathbb{P}^1 \to \mathcal{M}^H)_{\text{nonsing } \infty}$$

(These depend on a boundary condition $\mathcal{E} \in K(\mathcal{M}^H)$ (at $\infty \in \mathbb{P}^1$))

$\Rightarrow$ indices are built from equivariant K-theoretic integrals on

using the symmetrized virtual structure sheaf $\hat{\mathcal{O}}^{vir}$

(and possibly more insertions from $K(\mathcal{M}^H)$)

Quasimap vertex $V^{\mathcal{E}} = \chi\Big(\underset{\text{with b.d. cond. } \mathcal{E}}{QMaps(\mathbb{P}^1 \to \mathcal{M}^H)_{\text{nonsing } \infty}},\ \hat{\mathcal{O}}^{vir} \cdot z^{\deg} \Big)$

Highly nontrivial power series in $\vec{z} = (z_1, \dots, z_m)$ ("Kähler variables")

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

(and possibly more insertions from $K(\mathcal{M}^H)$)

Quasimap vertex $V^{\mathcal{E}} = \chi\Big(\underset{\text{with b.d. cond. } \mathcal{E}}{QMaps(\mathbb{P}^1 \to \mathcal{M}^H)_{\text{nonsing } \infty}},\ \hat{\mathcal{O}}^{vir} \cdot z^{\deg}\Big)$

Highly nontrivial power series in $\vec{z} = (z_1, \dots, z_m)$ ("Kähler variables")

valued in rational functions of $\vec{a} = (a_1, \dots, a_r)$, ("equivariant variables")

q and $\hbar$.

"CY limit"

Useful: $q\hbar = 1 \Rightarrow$ all equivariant variables disappear.

Quasimaps and Donaldson-Thomas theory

Quasimaps and BPS counts - Microsoft OneNote

using the symmetrized virtual structure sheaf $\hat{\mathcal{O}}^{\mathrm{vir}}$

(and possibly more insertions from $K(\mathcal{M}^H)$)

Quasimap vertex $V^{\mathcal{E}} = \chi\Big(\underset{\text{with b.d. cond. } \mathcal{E}}{QMaps(\mathbb{P}^1 \to \mathcal{M}^H)_{\text{nonsing } \infty}}, \hat{\mathcal{O}}^{\mathrm{vir}} \cdot z^{\deg}\Big)$

Highly nontrivial power series in $\vec{z} = (z_1, \dots, z_m)$ ("Kähler variables")

valued in rational functions of $\vec{a} = (a_1, \dots, a_r)$, ("equivariant variables") q and t.

"CY limit"

Useful: $qt = 1 \Rightarrow$ all equivariant variables disappear.

Quasimaps and Donaldson-Thomas theory

Henry Liu

Quasimaps and BPS counts - Microsoft OneNote

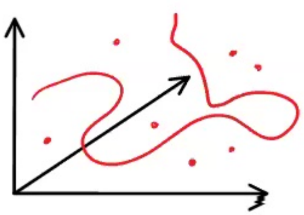
File Home Insert Share Draw Review View Pages

Quasimaps and Donaldson-Thomas theory

Q = affine ADE quiver $\rightsquigarrow$ $\mathcal{M}^H$ = some flop of Hilb(ADE surface)

(1-dim framing, for simplicity)

Graph Γ of a quasimap f = 1-dim. subscheme embedded in ADE $\times\, \mathbb{C}$!



Counting embedded subschemes in 3-folds = (some flavor of) Donaldson-Thomas theory

moduli Hilb(X, curves) of 1-dim subschemes in X.

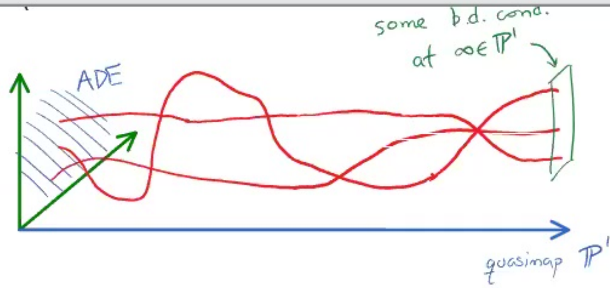
not necessarily ~~pure~~ 1-dim.

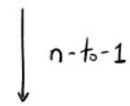
Quasimaps and BPS counts - Microsoft OneNote

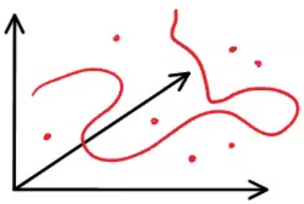
File Home Insert Share Draw Review View Pages

Graph Γ of a quasimap f

$=$ 1-dim. subscheme embedded in $ADE \times \mathbb{C}$!







Counting embedded subschemes in 3-folds

$=$ (some flavor of) Donaldson-Thomas theory

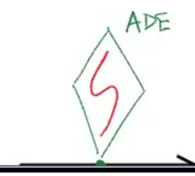
moduli $\mathrm{Hilb}(X, \text{curves})$ of 1-dim subschemes in X.

not necessarily ~~pure~~ 1-dim.

BUT ! DT $\neq$ Quasimaps

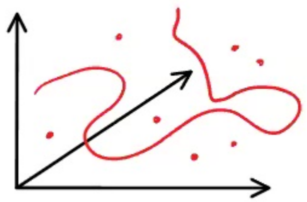
(I.) has free-floating points

(II) curves may have components lying completely in a fiber



Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages



Counting embedded subschemes in 3-folds

$=$ (some flavor of) Donaldson-Thomas theory

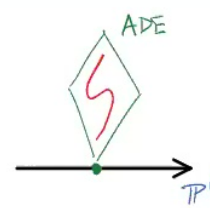
not necessarily pure 1-dim.

moduli $\mathrm{Hilb}(X, \text{curves})$ of 1-dim subschemes in X.

BUT ! DT $\neq$ Quasimaps

(I.) has free-floating points

(II.) curves may have components lying completely in a fiber



Same principle as for quasimap vertex

$$V_{DT}(ADE \times \mathbb{C}) \neq V_{QMaps}(ADE \times \mathbb{C})$$

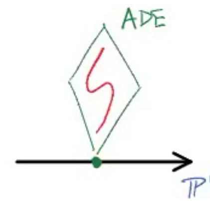
Removing unwanted contributions :

Quasimaps and BPS counts - Microsoft OneNote

BUT ! DT $\neq$ Quasimaps

(I.) has free-floating points

(II.) curves may have components lying completely in a fiber



Same principle as for quasimap vertex

$$V_{DT}(ADE \times \mathbb{C}) \neq V_{QMaps}(ADE \times \mathbb{C})$$

Removing unwanted contributions :

Option A: use this

$$\frac{V_{original}}{V_{unwanted}} = V'$$

Option B: construct theory immediately producing

Donaldson-Thomas $\rightsquigarrow$ Pandharipande-Thomas $\rightsquigarrow$ Bryan-Steinberg

[MNOP '03] remove (I.) [PT '09] remove (II.) [BS '16]

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

Henry Liu

Removing unwanted contributions :

Option A: use this

$$\frac{V_{\text{original}}}{V_{\text{unwanted}}} = V'$$

Option B: construct theory immediately producing

Donaldson-Thomas [MNOP '03] V_{DT} $\rightsquigarrow$ remove I Pandharipande-Thomas [PT '09] V_{PT} $\rightsquigarrow$ remove II Bryan-Steinberg [BS '16] V_{BS}

for crepant resolutions only

$V_{PT} \underset{\text{(conjectural)}}{=} V_{DT} / V_{DT,\,pts}$

$V_{BS} \underset{\text{(conjectural)}}{=} V_{PT} / V_{PT,\,fibers}$

MAIN THEOREM [H.L. '20] : (only type A in detail)

Whenever comparable (ADE surface fibrations over curves) :

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

(conjectural) / $V_{DT,\,pts}$ (conjectural) / $V_{PT,\,fibers}$

MAIN THEOREM [H.L. '20] : (only type A in detail)

Whenever comparable (ADE surface fibrations over curves) :

Bryan - Steinberg theory = quasimap theory

isomorphic moduli spaces

+ equivalent deformation theories (for producing $\hat{\mathcal{O}}^{vir}$)

at least in K-theory.

E.g.

$$V_{QMaps}\left(\mathrm{Hilb}(\mathcal{A}_{m-1})\right) = V_{BS}\left(\mathcal{A}_{m-1} \times \mathbb{C}\right)$$

minor change of variables

$$\overset{m=0}{\Rightarrow} \quad V_{QMaps}\left(\mathrm{Hilb}(\mathbb{C}^2)\right) = V_{BS}\left(\mathbb{C}^2 \times \mathbb{C}\right) = V_{PT}\left(\mathbb{C}^3\right)$$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

isomorphic moduli spaces

+ equivalent deformation theories (for producing $\hat{\mathcal{O}}^{vir}$)

at least in K-theory.

E.g.

$$V_{QMaps}\left(Hilb(\mathcal{A}_{m-1})\right) = V_{BS}\left(\mathcal{A}_{m-1} \times \mathbb{C}\right)$$

minor change of variables

$m=0$

$$\Rightarrow \quad V_{QMaps}\left(Hilb(\mathbb{C}^2)\right) = V_{BS}\left(\mathbb{C}^2 \times \mathbb{C}\right) = V_{PT}\left(\mathbb{C}^3\right)$$

previously known (straightforward)

= (1-legged) topological vertex in CY limit

all equivariant variables disappear

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

MAIN THEOREM [H.L. '20] : (only type A in detail)

Whenever comparable (ADE surface fibrations over curves) :

Bryan-Steinberg theory = quasimap theory

isomorphic moduli spaces

+ equivalent deformation theories (for producing $\hat{\mathcal{O}}^{vir}$)

at least in K-theory.

E.g.

$$V_{QMaps}\left(Hilb(A_{m-1})\right) = V_{BS}\left(A_{m-1} \times \mathbb{C}\right)$$

minor change of variables

$m=0$

$$\Rightarrow V_{QMaps}\left(Hilb(\mathbb{C}^2)\right) = V_{BS}\left(\mathbb{C}^2 \times \mathbb{C}\right) = V_{PT}\left(\mathbb{C}^3\right)$$

previously known

= (1-legged) topological vertex

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

$$V_{QMaps}(X)$$

$\updownarrow$ 3d mirror sym.

$$V_{QMaps}(X^\vee)$$

$$\xLeftrightarrow{QMaps/BS}$$

$$V_{BS}(Y)$$

$\updownarrow$???

$$V_{BS}(``Y^\vee")$$

<u>Thm</u> [Aganagic - Okounkov] :

choice of attracting chamber

If $(X, \mathcal{C})$ is 3d mirror to $(X^\vee, \mathcal{C}^\vee)$,

Kähler $\leftrightarrow$ equivariant

elliptic stable envelope.

appropriate normalization

$$\widetilde{V}_{QMaps}(X) = \widetilde{\mathrm{Stab}}^{Ell}_{\mathcal{C}^\vee} \cdot \widetilde{V}_{QMaps}(X^\vee)$$

simplify by taking the CY limit on X

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

3d mirror sym.

$$V_{QMaps}(X^\vee) \qquad \Longleftrightarrow \qquad V_{BS}(\text{"}Y^\vee\text{"})$$

???

Thm [Aganagic - Okounkov] :

choice of attracting chamber

If $(X, \mathcal{C})$ is 3d mirror to $(X^\vee, \mathcal{C}^\vee)$,

Kähler ↔ equivariant

elliptic stable envelope.

appropriate normalization

$$\widetilde{V}_{QMaps}(X) = \widetilde{\mathrm{Stab}}^{Ell}_{\mathcal{C}^\vee} \cdot \widetilde{V}_{QMaps}(X^\vee)$$

simplify by taking the CY limit on X

no equivariant variables on X

$$V^{CY}_{QMaps}(X) = S^\bullet\left(T^{1/2, \mathcal{C}^\vee} X^\vee\right) \cdot 1$$

no Kähler variables on $X^\vee$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

$V_{QMaps}(X^\vee)$ $\qquad$ $V_{BS}(``Y^\vee")$

Thm [Aganagic-Okounkov] : choice of attracting chamber

If $(X, \mathfrak{C})$ is 3d mirror to $(X^\vee, \mathfrak{C}^\vee)$,

Kähler ↔ equivariant $\qquad$ elliptic stable envelope.

appropriate normalization

$$\widetilde{V}_{QMaps}(X) = \widetilde{\mathrm{Stab}}^{Ell}_{\mathfrak{C}^\vee} \cdot \widetilde{V}_{QMaps}(X^\vee)$$

simplify by taking the CY limit on X

no equivariant variables on X

$$V^{CY}_{QMaps}(X) = S^\bullet\left(T^{1/2,\mathfrak{C}^\vee} X^\vee\right) \cdot 1$$

no Kähler variables on $X^\vee$

plethystic exp. : $S^\bullet(x) = \frac{1}{\;\cdot\;}$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

Thm [Aganagic - Okounkov] :

choice of attracting chamber

If $(X, \mathcal{C})$ is 3d mirror to $(X^\vee, \mathcal{C}^\vee)$,

Kähler ↔ equivariant

elliptic stable envelope.

appropriate normalization

$$\widetilde{V}_{QMaps}(X) = \widetilde{\mathrm{Stab}}^{Ell}_{\mathcal{C}^\vee} \cdot \widetilde{V}_{QMaps}(X^\vee)$$

simplify by taking the CY limit on X

no equivariant variables on X

$$V^{CY}_{QMaps}(X) = S^\bullet(T^{1/2,\mathcal{C}^\vee} X^\vee) \cdot 1$$

no Kähler variables on $X^\vee$

plethystic exp. : $S^\bullet(x) = \frac{1}{1-x}$

Example 1:

a bunch of topological vertices

$$V^{CY}(\mathrm{Hilb}(A_{m-1})) \overset{QMaps/BS}{=} V^{CY}_{..}(A_{m-1} \times \mathbb{C})$$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

no equivariant variables on X

$$V^{CY}_{QMaps}(X) = S^\bullet\left(T^{1/2, C^\vee} X^\vee\right) \cdot 1$$

no Kähler variables on $X^\vee$

plethystic exp. : $S^\bullet(x) = \frac{1}{1-x}$

Example 1:

$$V^{CY}_{QMaps}\left(Hilb(A_{m-1})\right) \overset{QMaps/BS}{=} V^{CY}_{BS}\left(A_{m-1} \times \mathbb{C}\right)$$

a bunch of topological vertices

3d mirror sym. ||

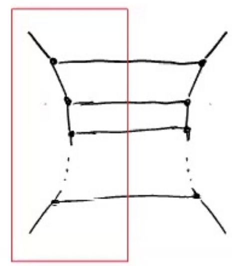
$$S^\bullet\left(T^{1/2, C^\vee} \mathcal{M}_m(\mathbb{C}^2)\right)$$

"half" of Nekrasov partition function.

//

geometric engineering of pure $SU(m)$ 4d gauge theory

[Iqbal - Kashani-Poor]



Example 2:

$$V^{CY}\left(Hilb(A_{\ldots})\right) = V^{CY}_{\ldots}\left(A_{m-1} \times \mathbb{C}\right)\left(\ldots \frac{V^{CY}_{PT}\left(A_{m-1} \times \mathbb{C}\right)}{\ldots}\right)$$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

Example 2:

$$V^{CY}_{QMaps}(\mathrm{Hilb}(A_{m-1})) = V^{CY}_{BS}(A_{m-1}\times\mathbb{C}) \quad \left(= \frac{V^{CY}_{PT}(A_{m-1}\times\mathbb{C})}{V^{CY}_{PT}(A_{m-1})} \right)$$

(as rational functions) $/\!/\!/$

??

$$V^{CY}_{QMaps}(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])) = V^{CY}_{PT}([\mathbb{C}^2/\mathbb{Z}_m]\times\mathbb{C})$$

DT crepant resolution conjecture [Bryan-Cadman-Young]

GIT chamber $\mathcal{C}_-^\vee$

$$V^{CY}_{QMaps}(\mathrm{Hilb}(A_{m-1})) = S^\bullet(T^{1/2}, \mathcal{C}_-^\vee \mathcal{M}_m(\mathbb{C}^2))$$

absorbed here

equal up to $\underbrace{\text{overall}}$ monomial

GIT chamber $\mathcal{C}_+^\vee$

$$V^{CY}_{QMaps}(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])) = S^\bullet(T^{1/2}, \mathcal{C}_+^\vee \mathcal{M}_m(\mathbb{C}^2))$$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

$$V_{QMaps}(\mathrm{Hilb}(A_{m-1})) = S^\bullet(T^{1/2, C_-^\vee} \mathcal{M}_m(\mathbb{C}^2))$$

equal up to overall monomial

GIT chamber $C_+^\vee$

$$V^{CY}_{QMaps}\left(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])\right) = S^\bullet\left(T^{1/2, C_+^\vee} \mathcal{M}_m(\mathbb{C}^2)\right)$$

in CY limit

$$T\mathcal{M}_m(\mathbb{C}^2) = \mathcal{G} + \mathcal{G}^\vee$$

(a very desirable property of K-theory classes called self-duality)

$\Rightarrow$ changing $C_+^\vee \leftrightarrow C_-^\vee$ is like

$$S^\bullet(x) \leftrightarrow S^\bullet(x^{-1})$$

$$\frac{1}{1-x} = (-x)\frac{1}{1-x^{-1}} \qquad \frac{1}{1-x^{-1}}$$

Lift from CY limit:

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

$V^{CY}_{PT}(A_{m-1})$

(as rational functions)

??

$$V^{CY}_{QMaps}\left(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])\right) = V^{CY}_{PT}\left([\mathbb{C}^2/\mathbb{Z}_m]\times\mathbb{C}\right)$$

DT crepant resolution conjecture [Bryan-Cadman-Young]

absorbed here

GIT chamber $\mathcal{C}_-^\vee$

$$V^{CY}_{QMaps}\left(\mathrm{Hilb}(A_{m-1})\right) = S^\bullet\left(T^{1/2,\mathcal{C}_-^\vee}\mathcal{M}_m(\mathbb{C}^2)\right)$$

equal up to overall monomial

GIT chamber $\mathcal{C}_+^\vee$

$$V^{CY}_{QMaps}\left(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])\right) = S^\bullet\left(T^{1/2,\mathcal{C}_+^\vee}\mathcal{M}_m(\mathbb{C}^2)\right)$$

$$T\mathcal{M}_m(\mathbb{C}^2) \overset{\text{in CY limit}}{=} \mathcal{G} + \mathcal{G}^\vee$$

(a very desirable property of K-theory classes called self-duality)

$\Rightarrow$ changing $\mathcal{C}_+^\vee \leftrightarrow \mathcal{C}_-^\vee$ is like

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

Example 2:

$$V^{CY}_{QMaps}(\mathrm{Hilb}(A_{m-1})) = V^{CY}_{BS}(A_{m-1} \times \mathbb{C}) \quad \left(= \frac{V^{CY}_{PT}(A_{m-1} \times \mathbb{C})}{V^{CY}_{PT}(A_{m-1})} \right)$$

(as rational functions)

??

$$V^{CY}_{QMaps}(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])) = V^{CY}_{PT}([\mathbb{C}^2/\mathbb{Z}_m] \times \mathbb{C})$$

DT crepant resolution conjecture [Bryan-Cadman-Young]

absorbed here

GIT chamber $\mathcal{C}_-^\vee$

$$V^{CY}_{QMaps}(\mathrm{Hilb}(A_{m-1})) = S^\bullet\left(T^{1/2,\, \mathcal{C}_-^\vee} \mathcal{M}_m(\mathbb{C}^2)\right)$$

equal up to overall monomial

GIT chamber $\mathcal{C}_+^\vee$

$$V^{CY}_{QMaps}(\mathrm{Hilb}([\mathbb{C}^2/\mathbb{Z}_m])) = S^\bullet\left(T^{1/2,\, \mathcal{C}_+^\vee} \mathcal{M}_m(\mathbb{C}^2)\right)$$

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

Henry Liu

K-theory classes called
self-duality)

→ changing $C_+^\vee \leftrightarrow C_-^\vee$ is like

$$S^\bullet(x) \leftrightarrow S^\bullet(x^{-1})$$

$$\frac{1}{1-x} = (-x)\frac{1}{1-x^{-1}} \qquad \frac{1}{1-x^{-1}}$$

Lift from CY limit:

Conjecture / Thm: (K-theoretic DT CRC)

$$V_{PT}([\mathbb{C}^2/\mathbb{Z}_m]\times\mathbb{C}) = \tilde{R}^{Ell}_{C_+^\vee \leftarrow C_-^\vee} \cdot V_{BS}(A_{m-1}\times\mathbb{C})$$

elliptic R-matrix $\widetilde{Stab}^{Ell}_{C_+^\vee} \cdot (\widetilde{Stab}^{Ell}_{C_-^\vee})^{-1}$

i.e. DT crepant resolutions governed by $\tilde{R}^{Ell}$!

Proof of MAIN THM

Quasimaps and BPS counts - Microsoft OneNote

File Home Insert Share Draw Review View Pages

$$T\mathcal{M}_m(\mathbb{C}^2) \overset{\text{in CY limit}}{=} \mathcal{G} + \mathcal{G}^\vee$$

(a very desirable property of K-theory classes called self-duality)

changing $C_+^\vee \leftrightarrow C_-^\vee$ is like

$$S^\bullet(x) \leftrightarrow S^\bullet(x^{-1})$$

$$\frac{1}{1-x} = (-x)\frac{1}{1-x^{-1}} \qquad \frac{1}{1-x^{-1}}$$

Lift from CY limit:

Conjecture / Thm: (K-theoretic DT CRC)

$$V_{PT}\left([\mathbb{C}^2/\mathbb{Z}_m]\times\mathbb{C}\right) = \tilde{R}^{Ell}_{C_+^\vee \leftarrow C_-^\vee} \cdot V_{BS}\left(A_{m-1}\times\mathbb{C}\right)$$

elliptic R-matrix $\widetilde{\mathrm{Stab}}^{Ell}_{C_+^\vee} \cdot \left(\widetilde{\mathrm{Stab}}^{Ell}_{C_-^\vee}\right)^{-1}$

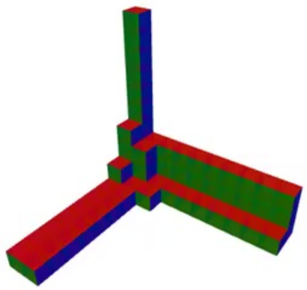
elliptic R-matrix $\mathrm{Stab}^{Ell}_{C_+^\vee} \cdot (\mathrm{Stab}_{C_-^\vee})$

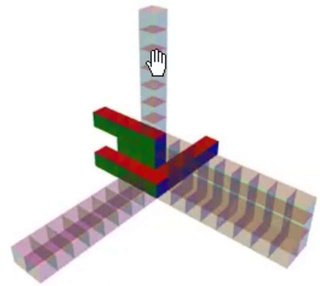
elliptic R-matrix $\mathrm{Stab}_{C_+^\vee} \cdot (\mathrm{Stab}_{C_-^\vee})$

Quasimaps and BPS counts - Microsoft OneNote

2. Show that Φ preserves deformation theory.

Fixed points: DT PT BS





???

Note: Quasimap vertex $\rightsquigarrow$ DT/PT/BS 1-leg vertex

PT 1-leg vertex $(\mathbb{C}^3)$

BS 1-leg vertex $(\mathcal{A}_3 \times \mathbb{C})$