Title: Renormalisation and momentum dependence in Quantum Gravity

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Series: Quantum Gravity

Date: October 01, 2020 - 2:30 PM

URL: http://pirsa.org/20100012

Abstract: Renormalisation in curved spacetimes is an involved subject. In contrast to renormalisation in a flat spacetime, the standard momentum representation is not directly available. Nevertheless, the momentum dependence of correlation functions is crucial to deciding whether a theory is unitary and causal. I will discuss how to define a notion of momentum dependence in gravity on a fundamental level. With this at hand, one can discuss an important quantum field theory observable: scattering cross sections. Taking the example of gravity-mediated scalar scattering, I will discuss conditions that a quantum field theory of gravity has to fulfil to have a well-behaved scattering amplitude. These can be satisfied without the introduction of massive higher spin modes as is done in string theory. Finally, I will review the status of first principle calculations of the non-perturbative momentum dependence of quantum gravity correlation functions.

RENORMALISATION AND MOMENTUM DEPENDENCE IN QUANTUM GRAVITY

BENJAMIN KNORR

2007.00733 (TO APPEAR IN PRL), 2007.04396 AND WORK IN PROGRESS

PERIMETER INSTITUTE

WHAT ARE THE CONSTRAINTS IMPOSED BY UNITARITY AND CAUSALITY ON A QFT OF QUANTUM GRAVITY?

(Can they be fulfilled?)

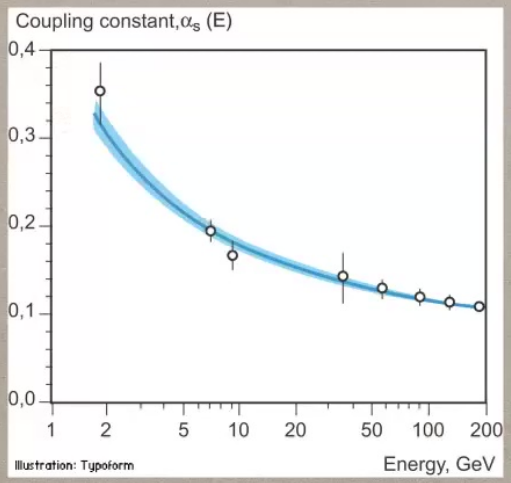
OVERVIEW

- Running couplings in a covariant theory
- Gravity-mediated scattering of scalar fields
- Computing amplitudes from first principles

RUNNING COUPLINGS IN A COVARIANT THEORY

RUNNING COUPLING CONSTANTS

- established experimental fact: coupling constants "run with energy"



Nobel prize in Physics 2004
(Gross, Politzer, Wilczek)
"for the discovery of asymptotic freedom in the theory of strong interaction"

nobelprize.org

RUNNING COUPLING CONSTANTS

- established experimental fact: coupling constants “run with energy”
- measure scattering cross sections and compare them to theoretical predictions - coupling “constants” depend on energy scale dictated by their beta functions

$$\beta_{\alpha_s} = -\left(11 - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

RUNNING COUPLING CONSTANTS

- What is the fundamental meaning of “running coupling constants”?
  - “fundamental”: discuss in terms of QFT concepts using the language of the effective action Γ
- How do we generalise this notion to a curved spacetime?

FORM FACTORS

- RG running = dependence of a coupling in the effective action on covariant derivatives

EM/YM: $$\Gamma = \int d^4x \sqrt{-g} \left[-\frac{1}{4} \mathscr{F}^{\mu\nu} \frac{1}{\alpha_s(\Delta)} \mathscr{F}_{\mu\nu} + \mathcal{O}\left(\mathscr{F}^3\right) \right]$$

gravity: $$\Gamma = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[-R + 2\Lambda - \frac{1}{6} R f_R(\Delta) R + \frac{1}{2} C^{\mu\nu\rho\sigma} f_C(\Delta) C_{\mu\nu\rho\sigma} + \mathcal{O}\left(\mathscr{R}^3\right) \right]$$

FORM FACTORS

- interaction terms are more complicated, e.g. three-point function:

$$\Gamma^{(3)} \supset \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, f_{R^3}(\Delta_1, \Delta_2, \Delta_3) R_1 R_2 R_3$$

- four-point function and higher: operator ordering needs convention (difference is of higher order)

$$\Gamma^{(4)} \supset \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, f_{R^4}\Big(D_1 \cdot D_2, D_1 \cdot D_3, D_1 \cdot D_4, D_2 \cdot D_3, D_2 \cdot D_4, D_3 \cdot D_4\Big) R_1 R_2 R_3 R_4$$

FORM FACTORS

- RG running of couplings generically depends on several momentum scales - there is no unique scale in many processes
- based on curvature/field strength expansion - can access momentum dependence by considering n-point function around vanishing field configuration
- Asymptotic Safety with FRG: "running" $G_{N,k}$, Λ_k mimic physical momentum dependence of form factors f_R,f_C, actual G_N, Λ don't run

FORM FACTOR EXPANSION VS DERIVATIVE EXPANSION

- alternative basis: derivative expansion

$$\Gamma = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left[-R + 2\Lambda + g_r R^2 + g_c C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \mathcal{O}(R^3, D^2R^2)\right]$$

- downside: finite orders necessarily introduce spurious poles in the propagator even if full theory is well-behaved
- not suitable to assess degrees of freedom and unitarity of a theory

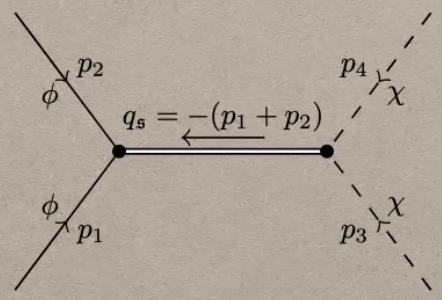
GRAVITY-MEDIATED SCATTERING OF SCALAR FIELDS

GRAVITY-MEDIATED SCATTERING CROSS SECTIONS

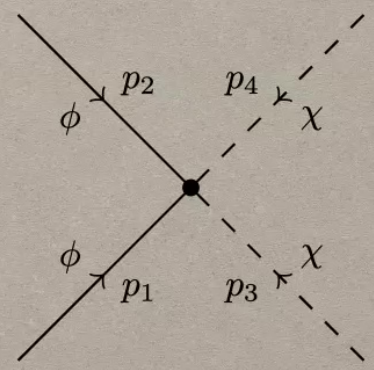
- for this part: assume that effective action is given - *what to do with it?*
- back to QFT 1: calculate scattering cross sections
- strategy: ansatz for effective action that includes all form factors that contribute to a given scattering in flat background
- only tree-level Feynman diagrams are necessary with effective action

GRAVITY-MEDIATED SCALAR SCATTERING - DIAGRAMS

- tree-level diagrams for $\phi\phi \to \chi\chi$ scattering mediated by gravity:



s-channel



(renormalised) vertex

GRAVITY-MEDIATED SCALAR SCATTERING - EFFECTIVE ACTION

Draper, BK, Ripken, Saueressig 2007.04396

- most general action that contributes to this scattering:

$$\Gamma \simeq \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left[-R-\frac{1}{6}R f_R(\Delta)\,R+\frac{1}{2}C^{\mu\nu\rho\sigma}f_C(\Delta)\,C_{\mu\nu\rho\sigma}\right]$$
$$+\int d^4x\sqrt{-g}\left[\frac{1}{2}\phi f_\phi(\Delta)\,\phi+f_{R\phi\phi}(\Delta_1,\Delta_2,\Delta_3)\,R\,\phi\,\phi+f_{Ric\phi\phi}(\Delta_1,\Delta_2,\Delta_3)\,R^{\mu\nu}\left(D_\mu D_\nu\phi\right)\phi\right]$$
$$+(\phi\to\chi)+\frac{1}{(2!)^2}\int d^4x\sqrt{-g}\,f_{\phi\chi}(\{-D_{ij}\})\phi\phi\chi\chi$$

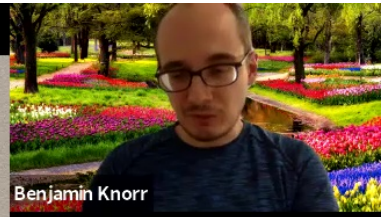
- assumption: single pole defines mass of normalised scalar fields

$$f_\phi(m_\phi^2)=0\,,\qquad f'_\phi(m_\phi^2)=1$$

GRAVITY-MEDIATED SCALAR SCATTERING - WORK FLOW

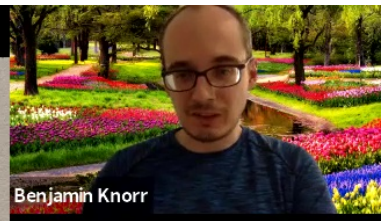
Draper, BK, Ripken, Saueressig 2007.04396

- workflow:
  - calculate vertices and propagators (= Feynman rules)
  - glue them together, imposing momentum conservation
  - impose on-shell conditions on external legs
- use tensor algebra package (xAct, FORM, …)



GRAVITY-MEDIATED SCALAR SCATTERING - AMPLITUDES

Draper, BK, Ripken, Saueressig 2007.04396



- gravity-mediated amplitude:

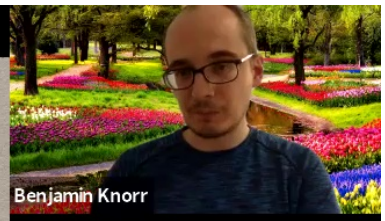
$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Bigg[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s})\left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$
$$+\left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$
$$\times\left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Bigg]$$

$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$\mathfrak{s} = (p_1+p_2)^2$$
$$p_1^2 = p_2^2 = m_\phi^2 \qquad \mathfrak{t} = (p_1+p_3)^2$$
$$p_3^2 = p_4^2 = m_\chi^2 \qquad \mathfrak{u} = (p_1+p_4)^2$$

GRAVITY-MEDIATED SCALAR SCATTERING - AMPLITUDES

Draper, BK, Ripken, Saueressig 2007.04396



- gravity-mediated amplitude:

vertex factors — *graviton propagator* — *contraction factor*

$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Bigg[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \cdot \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$

$$+\left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$

$$\times\left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Bigg]$$

spin 2

spin 0

$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

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$$p_1^2 = p_2^2 = m_\phi^2 \qquad \mathfrak{t} = (p_1+p_3)^2$$
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GRAVITY-MEDIATED SCALAR SCATTERING - AMPLITUDES

Draper, BK, Ripken, Saueressig 2007.04396

- self-interaction amplitude:

$$\mathcal{A}_4^{\phi\chi} = f_{\phi\chi}\left(\frac{\mathfrak{s}-2m_\phi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{s}-2m_\chi^2}{2}\right)$$

$$\mathfrak{s} = (p_1+p_2)^2$$

$$p_1^2 = p_2^2 = m_\phi^2 \qquad \mathfrak{t} = (p_1+p_3)^2$$

$$p_3^2 = p_4^2 = m_\chi^2 \qquad \mathfrak{u} = (p_1+p_4)^2$$

PHYSICS OF S-CHANNEL SCATTERING

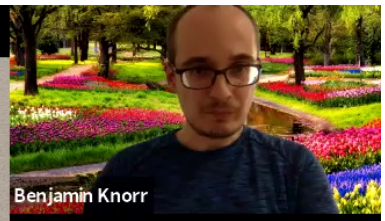
Draper, BK, Ripken, Saueressig 2007.04396

- partial wave decomposition:

$$a_l(\mathfrak{s}) = \frac{1}{16\pi\mathfrak{s}} \int_{-\mathfrak{s}}^{0} dt\, P_l\left(-1 - \frac{2t}{\mathfrak{s}}\right) \mathscr{A}(\mathfrak{s}, t)$$

P_l: Legendre polynomial

- applying to general result:
  - spin 0 graviton contributes to a_0
  - spin 2 graviton contributes to a_2
  - self-interaction = resummation of ladder diagrams (k-graviton exchange), contributes to a_{2k}



PHYSICS OF S-CHANNEL SCATTERING

Draper, BK, Ripken, Saueressig 2007.00733, 2007.04396

- conditions on (partial wave) amplitude:
  - unitarity (Froissart bound):

    Froissart 1961

    $$|a_l(\mathfrak{s})| \leq 1$$

  - causality (forward scattering):

    Camanho, Edelstein, Maldacena, Zhiboedov 2016

    $$\lim_{\mathfrak{s}\to\infty} \mathscr{A}(\mathfrak{s},\mathfrak{t})\Big|_{\mathfrak{t}} = o(\mathfrak{s}^2)$$

  - Cerulus-Martin bound: for large $\mathfrak{s}$ at fixed $\mathfrak{t}$, amplitude cannot fall faster than $e^{-\sqrt{\mathfrak{s}}\ln\mathfrak{s}}$

    Cerulus, Martin 1964

PHYSICS OF S-CHANNEL SCATTERING

Draper, BK, Ripken, Saueressig 2007.00733, 2007.04396

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  - causality (forward scattering): $\lim_{\mathfrak{s}\to\infty} \mathscr{A}(\mathfrak{s},\mathfrak{t})\Big|_{\mathfrak{t}} = o(\mathfrak{s}^2)$ **X** $(\phi\chi \to \phi\chi)$

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PHYSICS OF S-CHANNEL SCATTERING

Draper, BK, Ripken, Saueressig 2007.00733

- Is there a choice of form factors satisfying all requirements and:
  - without extra degrees of freedom (no massive poles),
  - action is local (i.e. power law) in the UV?

PHYSICS OF S-CHANNEL SCATTERING

Draper, BK, Ripken, Saueressig 2007.00733

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  - action is local (i.e. power law) in the UV!
- graviton propagator:

$$f_{R,C}(x) = c_{R,C} G_N \tanh\left(c_{R,C} G_N x\right)$$

- scalar self-interaction:

$$f_{\phi\chi} \text{ s.t. } \mathscr{A}_4 = 4\pi G_N G_C(\mathfrak{s})(\mathfrak{t}^2 + \mathfrak{u}^2) \frac{c_t G_N^2 \mathfrak{s} \tanh\left(c_t G_N^2 \mathfrak{s}\right)}{1 + c_t G_N^2 \mathfrak{s} \tanh\left(c_t G_N^2 \mathfrak{s}\right)}$$

PHYSICS OF S-CHANNEL SCATTERING

Draper, BK, Ripken, Saueressig 2007.00733

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tames PWA, no extra real poles

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ensures causality in crossed channel

PHYSICS OF S-CHANNEL SCATTERING

Draper, BK, Ripken, Saueressig 2007.00733

- causality needs non-trivial relation between scalar self-interaction and graviton propagator/graviton-scalar vertex at high energies
- extra condition for Asymptotic Safety, beyond existence of fixed point
- "momentum locality" and "effective universality": results point towards that such relations might indeed be realised

Christiansen, Denz, Dona, Eichhorn, BK, Labus, Lippoldt, Litim, Meibohm, Pawlowski, Percacci, Reichert, Schiffer, Skrinjar, … 2015+

COMPUTING AMPLITUDES FROM FIRST PRINCIPLES

CALCULATING FORM FACTORS WITH FRG

Draper, BK, Ripken, Saueressig 2007.04396

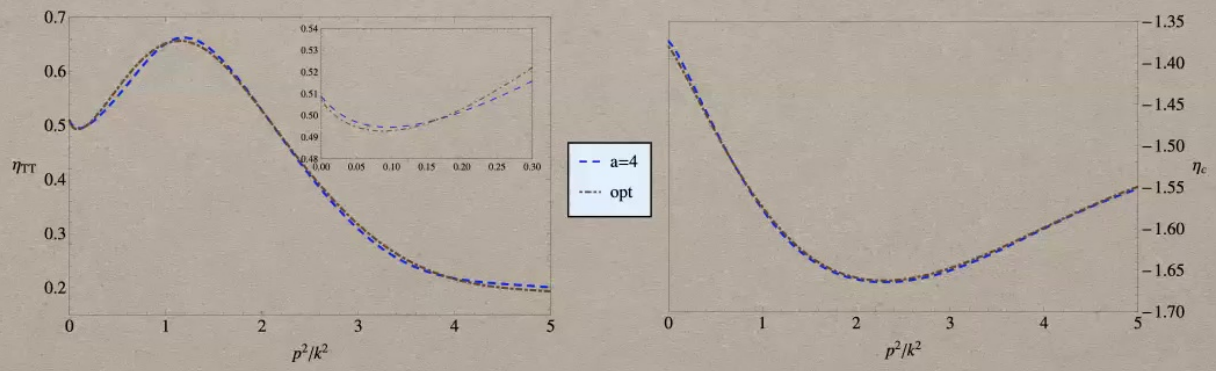
- "running" Newton's constant: identifies RG running of several form factors

$$\mathscr{A}_{\mathfrak{s}}^{\phi\chi} \simeq V_{\phi\phi h}(\mathfrak{s}) V_{\chi\chi h}(\mathfrak{s}) G(\mathfrak{s}) \mathscr{T}(\mathfrak{t}, \mathfrak{u}) \equiv \frac{G_{N,k\sim\sqrt{\mathfrak{s}}}}{\mathfrak{s}} \mathscr{T}(\mathfrak{t}, \mathfrak{u})$$

- assumes similar behaviour of spin 2 and spin 0 sector
- can work qualitatively, but might not work quantitatively

CALCULATING FORM FACTORS WITH FRG

- full momentum dependence of TT-graviton propagator (flat fluctuation calculation), related to f_C



$\eta = -\partial_t \ln Z$

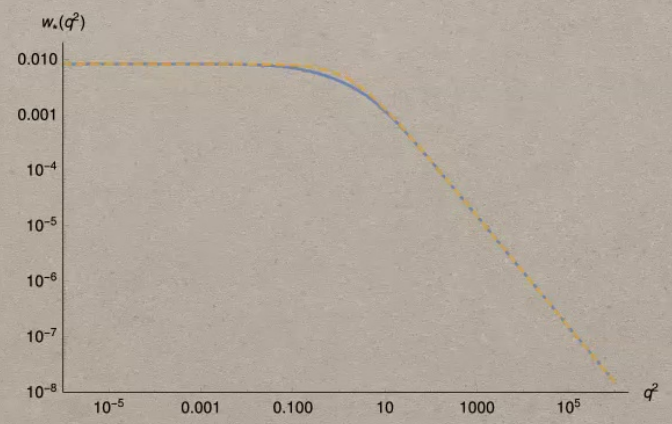
Christiansen, BK, Pawlowski, Rodigast 2014

CALCULATING FORM FACTORS WITH FRG

- full momentum dependence of TT-graviton propagator (flat fluctuation calculation), related to f_C
- resolution of full graviton propagator work in progress (BK, M. Schiffer): access to both f_C and f_R
- mid-term goal: compute complete three-graviton vertex

CALCULATING FORM FACTORS WITH FRG

- direct calculation of f_C (conformally reduced approximation)

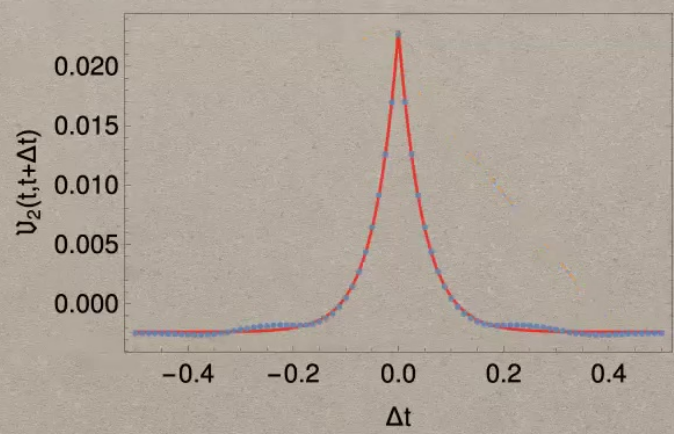


Bosma, BK, Saueressig 2019

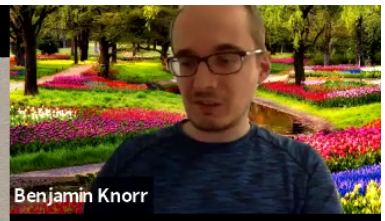
CALCULATING FORM FACTORS BY MATCHING CDT DATA

- use correlation function for which CDT data is available, e.g.

$$\mathfrak{V}_2 = \langle \delta V_3(t) \delta V_3(t + \Delta t) \rangle$$



BK, Saueressig 2018



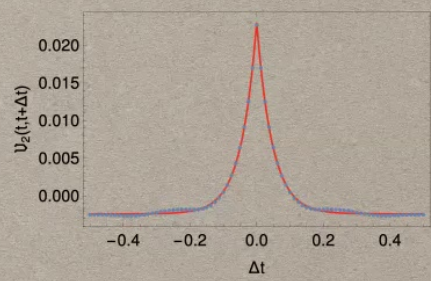
CALCULATING FORM FACTORS BY MATCHING CDT DATA

- use correlation function for which CDT data is available, e.g.

$$\mathfrak{V}_2 = \langle \delta V_3(t) \delta V_3(t + \Delta t) \rangle$$

- match to numerical data (only sensitive to small x):

$$f_R(x) \propto \frac{1}{x^2}$$



- gauge-invariant mass term, interesting for cosmology

see e.g. Belgacem, Dirian, Foffa, Maggiore 2018

BK, Saueressig 2018

SUMMARY

- on our way to calculate form factors from first principles
  - FRG: direct access to momentum-dependent correlation functions
  - CDT: reconstruct form factors from correlation functions
  - so far only propagators, need vertices for scattering amplitudes

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