

Title: Astrophysical backgrounds of gravitational waves: an overview

Speakers: Giulia Cusin

Series: Strong Gravity

Date: October 08, 2020 - 1:00 PM

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Abstract: The astrophysical background of gravitational waves (AGWB) is composed by the incoherent superposition of gravitational wave signals emitted by resolved and unresolved astrophysical sources from the onset of stellar activity until today. In this talk, I will present a theoretical framework to characterize the AGWB in terms of energy density and polarization and I will show predictions for the angular power spectra of the background anisotropy and for its cross-correlations with electromagnetic observables, in the frequency bands accessible by LIGO/Virgo and LISA. I will then discuss the astrophysical content of these observables, give an overview of the state of the art of observations, and highlight future observational challenges.

Astrophysical background of gravitational waves: from theoretical characterization to prospects for detection

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in collaboration with C. Pitrou, J.P. Uzan, I. Dvorkin,
R. Durrer, P. Ferreira, D. Alonso, C. Contaldi, A. Renzini

based on

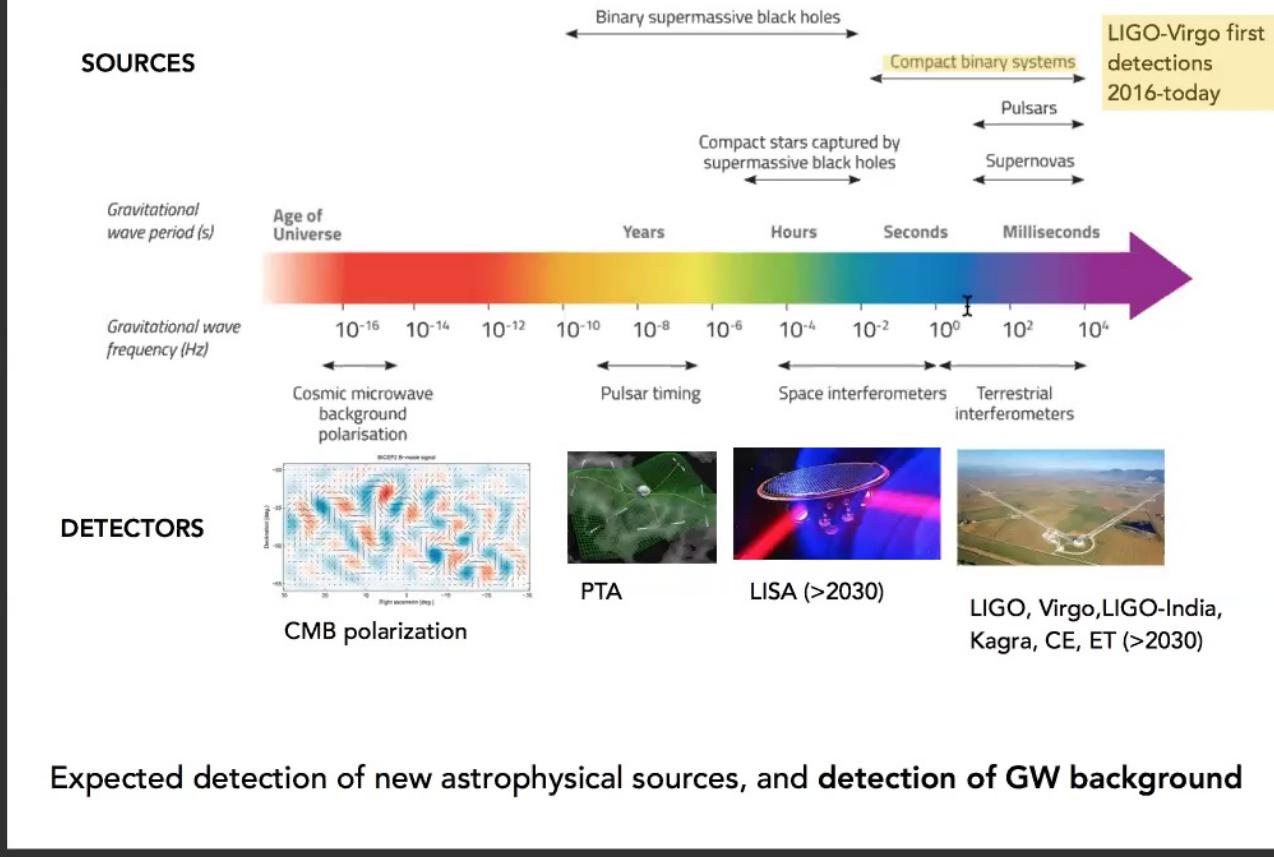
- GC et al. Phys.Rev. D96 (2017) 103019
- GC et al. PRL 120 (2018) 231101
- GC et al. Phys.Rev. D97 (2018) 123527
- GC et al. Phys.Rev. D99 (2019) 023534
- GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004
- GC, Dvorkin et al. MNRAS Lett (2019)
- Pitrou, GC, Uzan, Phys.Rev.D 101 (2020), 081301
- Alonso, GC, Pitrou, Ferreira, Phys.Rev.D 101 (2020)
- Alonso, GC, Ferreira ++ Phys.Rev.D 102 (2020)

Outline of this talk

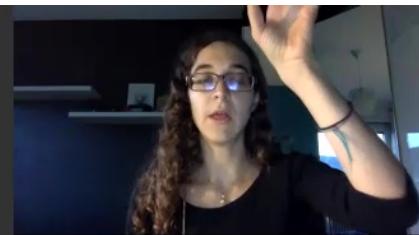
- **Introduction.** Astrophysical gravitational wave background: what is it?
State of the art of observations & theory
- **Theoretical framework** to study anisotropies and polarization
I
- **Numerical predictions** in LISA and LIGO/Virgo bands
- **Astrophysical interest:** content of this new observable
and what we can learn out of it
- **Bridging theory and observations:** characterization of different
noise components and prospects of detection

from theory to
forecasts of
detectability

The new era of gravitational wave astronomy



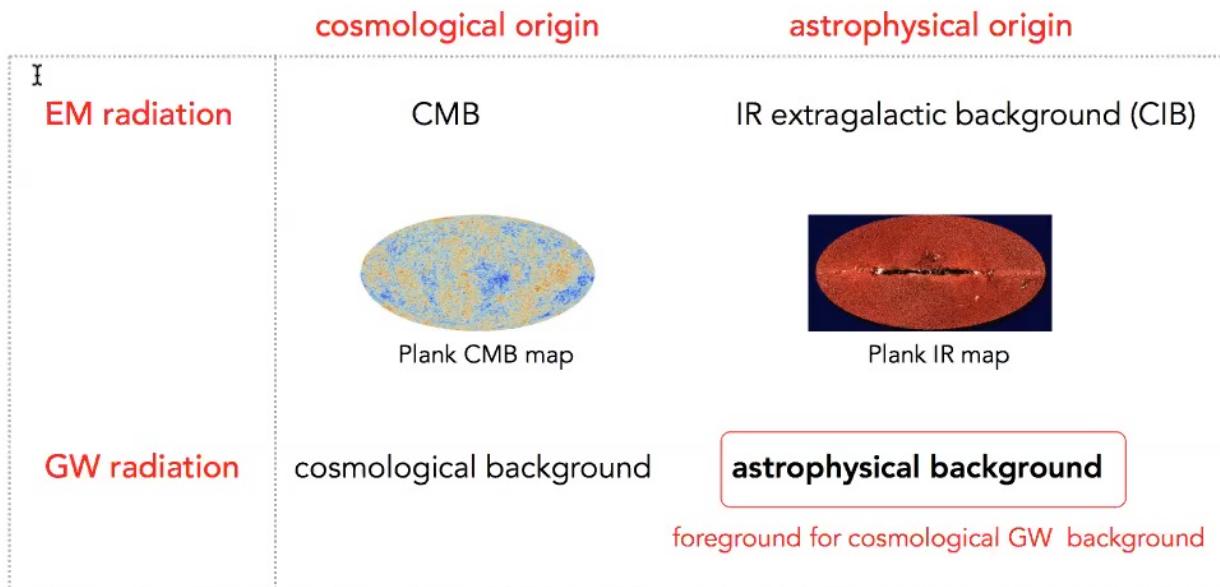
Two types of astrophysical GW observables



astrophysical observable	what we measure	what we can learn
GW from resolvable sources 	waveform as a function of time and frequency	properties of the source (e.g. nature, mass, spin, distance...)
Astrophysical background of GW 	intensity and polarization as functions of direction and frequency	collective properties of a population of sources (redshift distribution, time evolution, distribution of masses...)

Stochastic backgrounds of radiation

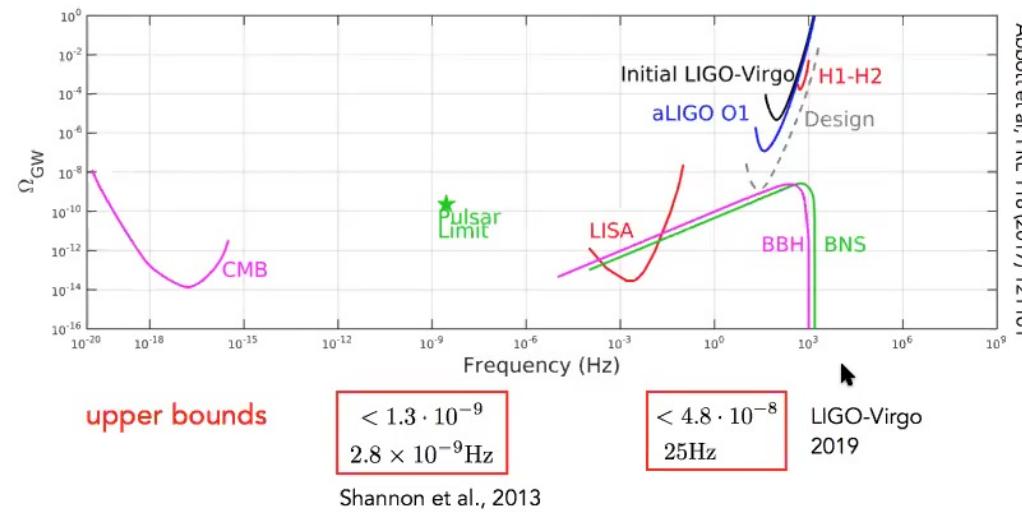
Stochastic background: incoherent superposition of signals from all sources



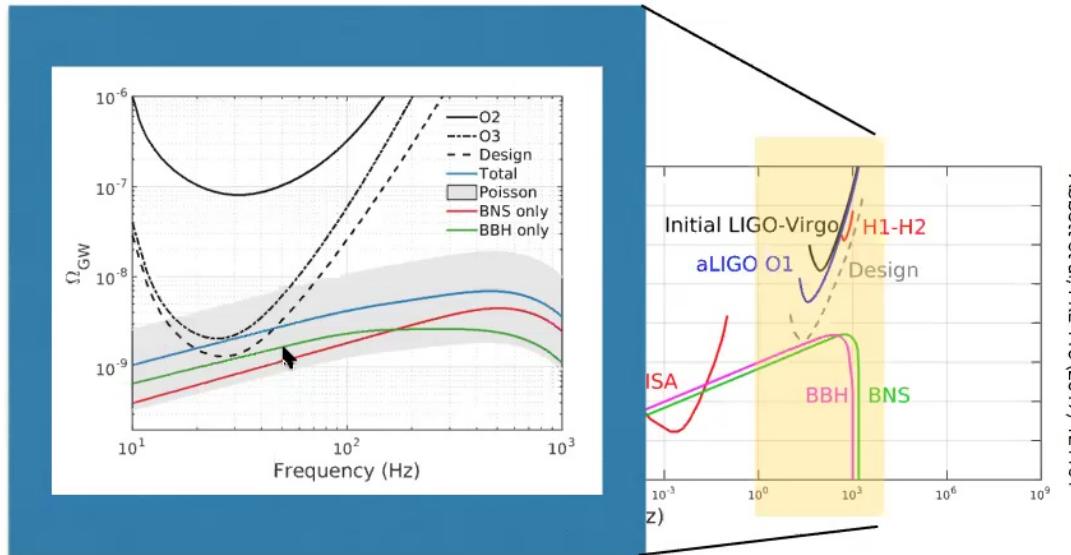
Astrophysical stochastic background

$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}(f)}{df} = \frac{f}{\rho_c} \int dz \frac{dE(z)}{df} R(z) \frac{n(z)}{H(z)}$$

spectrum rate of events density of sources



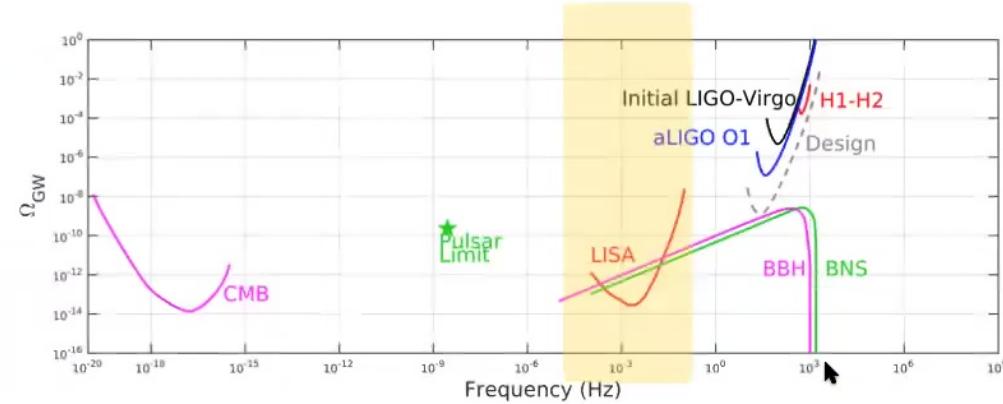
Expected future detection (3 yrs)



Abbott et al., PRL 118 (2017) 121101

Detection background (monopole) very probable as the designed sensitivity is reached!

LISA: inspiralling phase of mergers



LISA will see a continuous background (stationary) from inspiralling phase of binary systems

(vs LIGO will see merger phase of binary evolution. Popcorn-like signal)

Pagina 11 di 65

Angular searches: sky map

$$\Omega_{GW}(f) = \int d^2\mathbf{e} \boxed{\Omega_{GW}(f, \mathbf{e})}$$

$$\Omega_{GW}(f, \mathbf{e}) = \frac{f}{\rho_c} \frac{d^3\rho_{GW}(f, \mathbf{e})}{d^2\mathbf{e} df}$$

LIGO/Virgo: directional searches implemented. SNR consistent with gaussian noise

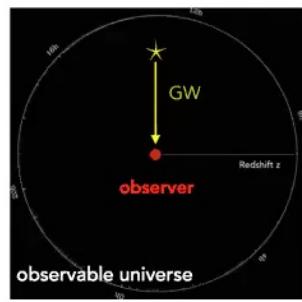
(map reconstruction methods similar interferometric reconstruction for CMB or HI emission)



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Theoretical modeling and numerical predictions

Beyond the usual modeling: a realistic description



Usual modeling:
sources isotropically distributed,
propagation along straight line

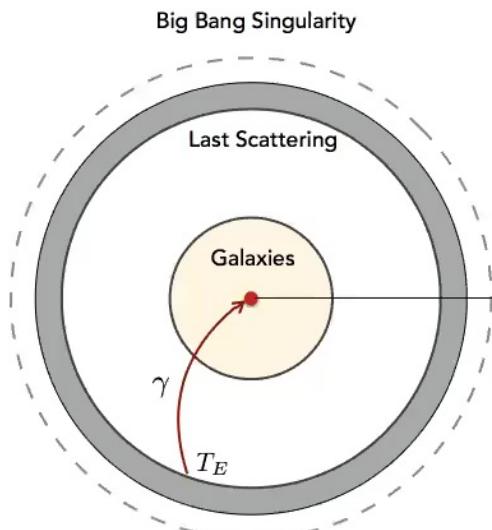
- no anisotropies in the received flux
- no generation of polarization

More realistic description:
including effects of inhomogeneities,
lensing, distortion

**Accurate characterization of
anisotropies and polarization**



CMB: a case study

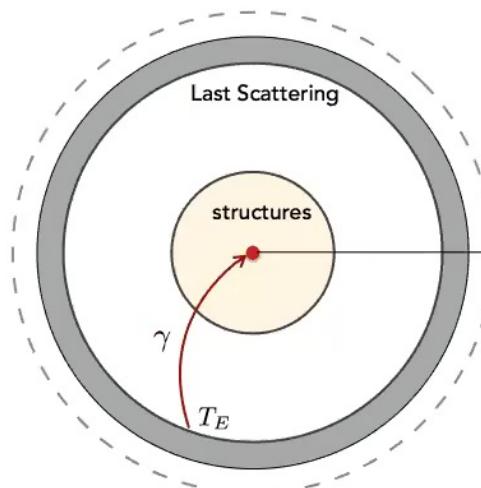


Temperature observed \leftrightarrow
temperature at Large Scattering Surface

$$\frac{T_O}{T_E} = \frac{E_O}{E_E} = \frac{(k^\mu u_\mu)_O}{(k^\mu u_\mu)_E}$$

The diagram illustrates the decomposition of the temperature ratio into wave vector and velocity components. A blue dashed arrow points from the term $(k^\mu u_\mu)_O$ to the label "wave vector". A red dotted arrow points from the term $(k^\mu u_\mu)_E$ to the label "velocity comoving observer".

CMB: effects of structures



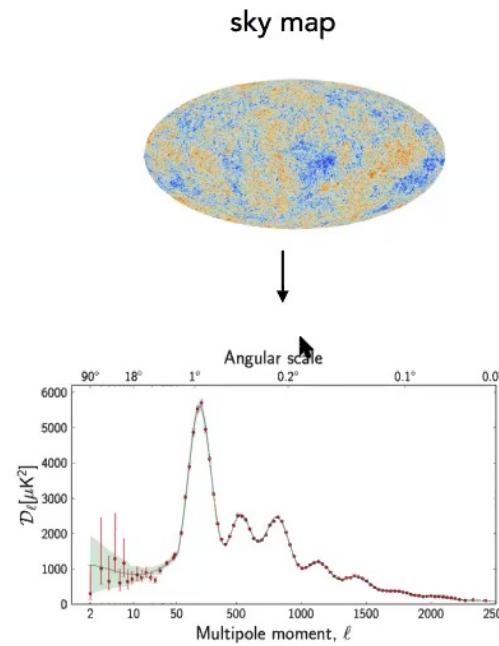
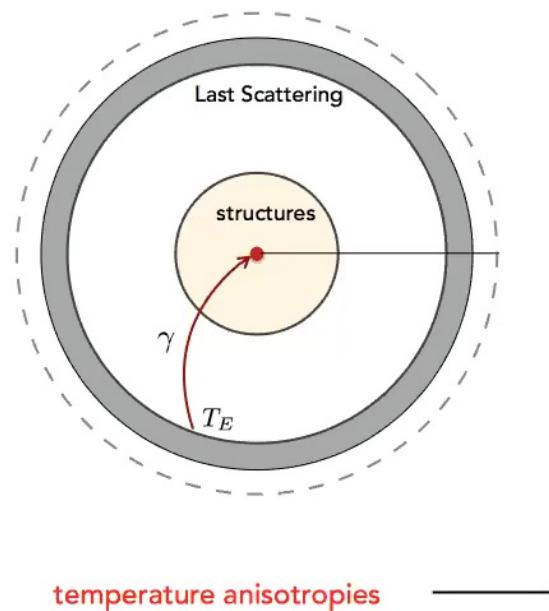
$$ds^2 = a^2 [-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j]$$

$$T_O(\mathbf{e}) = \bar{T}_O(\eta_O)(1 + \Theta(\mathbf{e}))$$

$$\Theta(\mathbf{e}, \mathbf{x}_O, \eta_O) = \left(\frac{1}{4}\delta_\gamma + \Phi - \mathbf{e} \cdot \mathbf{v} \right) (\mathbf{x}_E, \bar{\eta}_E) + \int_E^O (\Phi' + \Psi') d\eta \quad (\text{Sachs-Wolfe formula '67})$$

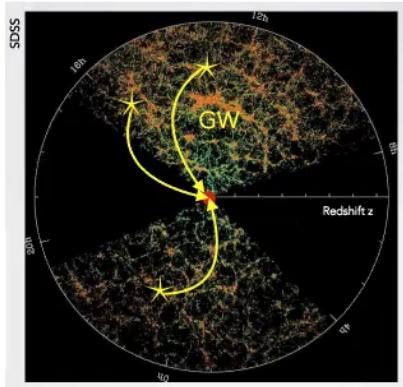


CMB : a more realistic modeling



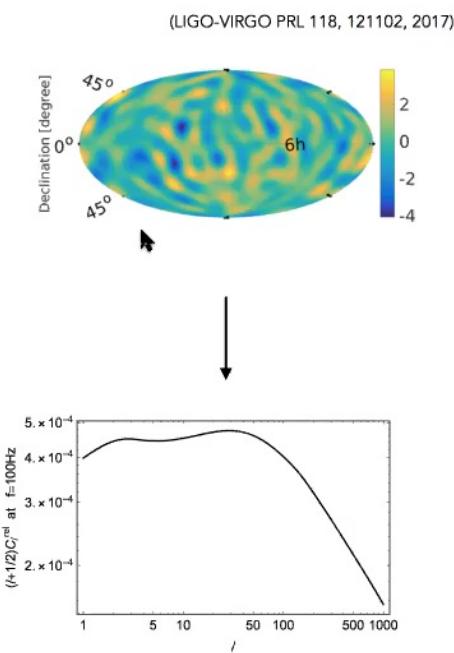
GW background case

Predictions



$$\Omega_{GW}(f, \mathbf{e}) \longrightarrow$$

sky map (future)



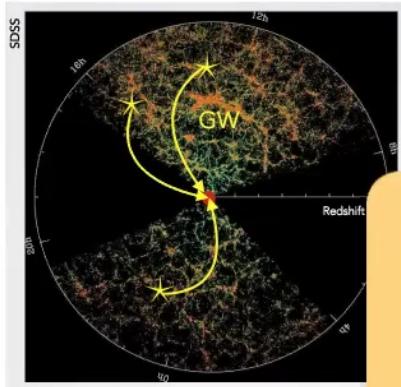
Idea: set constraints on astro functions from comparison theory-data

[Cusin, Dvorkin, Uzan, Pitrou 2018]

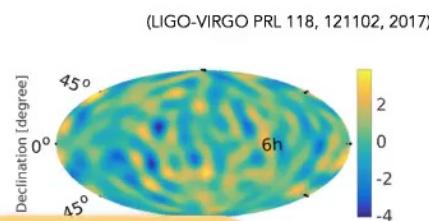


GW background case

Predictions



sky map (future)

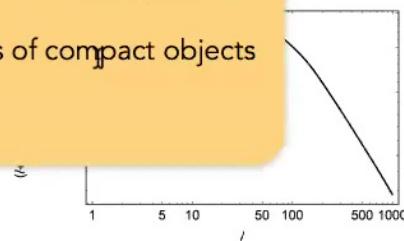


what we can learn

- stellar evolution model
- fraction black holes in binary systems
- distribution masses of compact objects

...

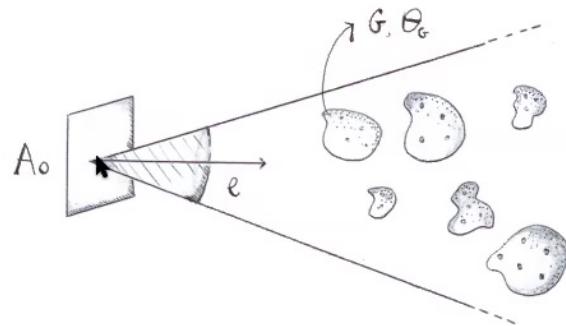
$$\Omega_{GW}(f, \mathbf{e})$$



[Cusin, Dvorkin, Uzan, Pitrou 2018]

Scheme of our approach

Observer looks at the sky in a given direction

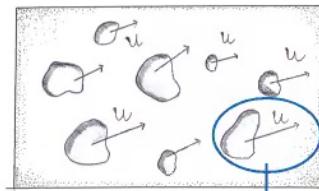


$$\Phi(\mathbf{e}, z_G, \theta_G) = \frac{\text{Energy}}{A_O \Delta t_O}$$

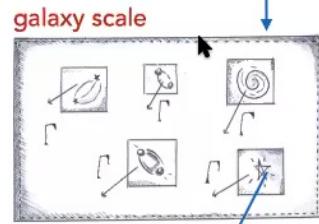
Total flux received: **sum the contributions** from all the galaxies in the solid angle of observation



Three scales in the problem



cosmological scale. Galaxies: point-like sources moving with the cosmic flow



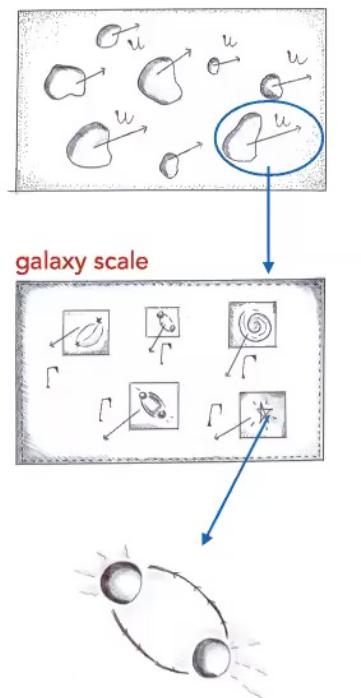
galactic scale. Effective luminosity of a galaxy defined taking into account the various contributions of the sources



local scale: single GW sources inside a galaxy



From cosmological to local scale



cosmological scale

galaxy scale

local scale

$$\Phi = \frac{(1 + z_G)}{D_L^2} \mathcal{L}_G$$

function local
quantities at sources

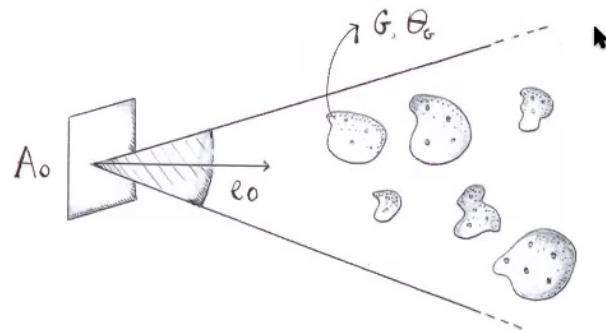


Final parametrization

$$\Omega_{GW}(f, \mathbf{e}) = \frac{f}{\rho_c} \int dz_G \int d\theta_G \Phi[z_G, f, \theta_G] \boxed{\frac{d^3 \mathcal{N}_G}{dz_G d^2 \mathbf{e}}(z_G, \theta_G)}$$

↑
integral over halo mass function

flux from one galaxy # galaxies in
comoving volume



Final parametrization

$$\Omega_{GW}(f, \mathbf{e}) = \frac{f}{\rho_c} \int dz_G \int d\theta_G \Phi[z_G, f, \theta_G] \frac{d^3 \mathcal{N}_G}{dz_G d^2 \mathbf{e}}(z_G, \theta_G)$$

↑
integral over halo mass function

flux from one galaxy # galaxies in comoving volume

↓
rewritten in terms of GW luminosity rewritten in terms of comoving density and comoving volume



Results in cosmological context

astrophysical part cosmological part

$$\delta\Omega_{GW}(\mathbf{e}, f) = \frac{f}{4\pi\rho_c} \int \frac{dz}{H(z)} \mathcal{A}(z, f) \left[b\delta_m + 4\Psi - 2\mathbf{e} \cdot \nabla v + 6 \int^z \frac{dz'}{H(z')} \dot{\Psi} \right]$$

anisotropic part



$$\mathcal{A}(z, f) = \frac{1}{(1+z)^4} \int d\theta_G \bar{n}_G(z) \mathcal{L}_G(z, f_G, \theta_G)$$

$$\Omega_{GW}(f, \mathbf{e}) = \bar{\Omega}_{GW}(f) + \boxed{\delta\Omega_{GW}(f, \mathbf{e})}$$



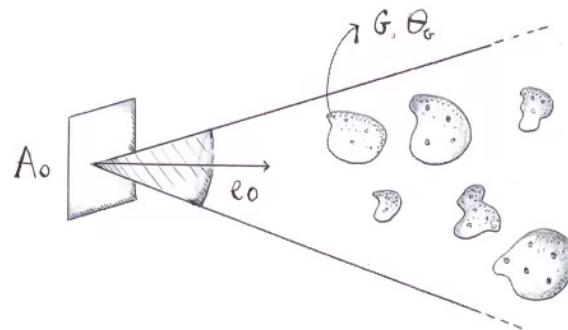
Final parametrization

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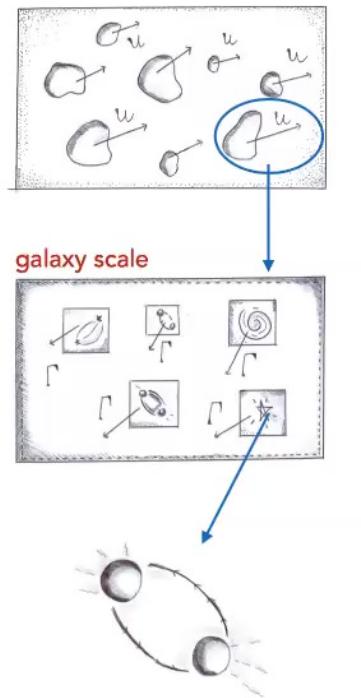
↑

integral over halo mass function

flux from one galaxy # galaxies in comoving volume



From cosmological to local scale



cosmological scale

galaxy scale

local scale

$$\Phi = \frac{(1 + z_G)}{D_L^2} \boxed{\mathcal{L}_G}$$

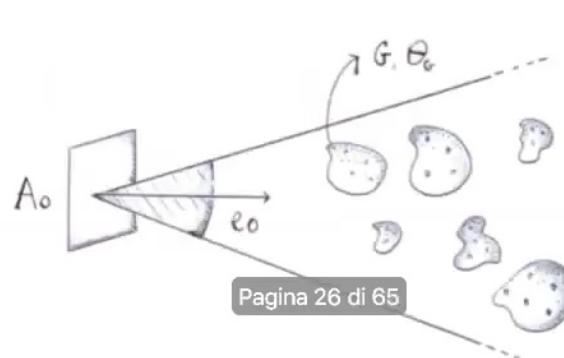
function local
quantities at sources

Final parametrization

$$\Omega_{GW}(f, \mathbf{e}) = \frac{f}{\rho_c} \int dz_G \int d\theta_G \Phi[z_G, f, \theta_G] \boxed{\frac{d^3 \mathcal{N}_G}{dz_G d^2 \mathbf{e}}(z_G, \theta_G)}$$

↑
integral over halo mass function

flux from one galaxy # galaxies in
comoving volume



Results in cosmological context

astrophysical part cosmological part

$$\delta\Omega_{GW}(\mathbf{e}, f) = \frac{f}{4\pi\rho_c} \int \frac{dz}{H(z)} \mathcal{A}(z, f) \left[b\delta_m + 4\Psi - 2\mathbf{e} \cdot \nabla v + 6 \int^z \frac{dz'}{H(z')} \dot{\Psi} \right]$$

anisotropic part



$$\mathcal{A}(z, f) = \frac{1}{(1+z)^4} \int d\theta_G \bar{n}_G(z) \mathcal{L}_G(z, f_G, \theta_G)$$

$$\Omega_{GW}(f, \mathbf{e}) = \bar{\Omega}_{GW}(f) + \boxed{\delta\Omega_{GW}(f, \mathbf{e})}$$



Astrophysical model: ingredients

$$(1) \quad \mathcal{L}_G = \int dm dm' da_f \frac{dE}{df} \times \mathcal{R}_m[m, m', a_f, t]$$

↓ ↓
spectrum merger rate

star formation rate
stellar evolution model
fraction of black holes in binary systems
mass range of compact objects ...

- (2) sum over galaxy population using the halo mass function calibrated with simulations

GC, Dvorkin, Pitrou, Uzan PRL 120 (2018) 231101
Dvorkin, Uzan, Vangioni, Silk, Phys.Rev.D94 (2016), 103011



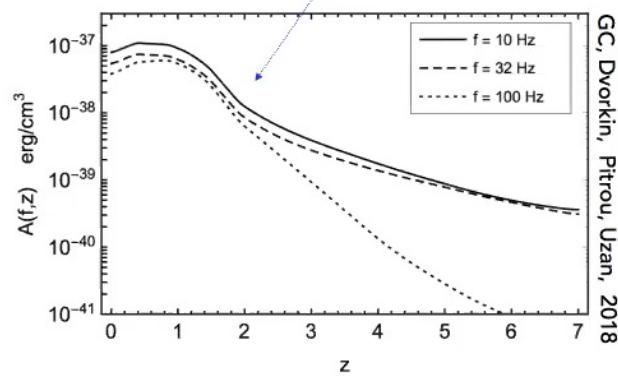
Astrophysical kernel for a reference model

astrophysical part cosmological part

$$\delta\Omega_{\text{GW}}(\mathbf{e}, f) = \frac{f}{4\pi\rho_c} \int \frac{dz}{H(z)} \mathcal{A}(z, f) \left[b\delta_m + 4\Psi - 2\mathbf{e} \cdot \nabla v + 6 \int^z \frac{dz'}{H(z')} \dot{\Psi} \right]$$



$$\mathcal{A}(z, f) = \frac{1}{(1+z)^4} \int d\theta_G \bar{n}_G(z) \mathcal{L}_G(z, f_G, \theta_G)$$



Angular power spectrum

Non-vanishing auto-correlation linked to **correlation of large scale structures**
(with modulation from local physics)

$$C(f, \theta) = \langle \delta\Omega_{GW}(f, \mathbf{e}_1) \delta\Omega_{GW}(f, \mathbf{e}_2) \rangle$$

$$= \sum_{\ell} \frac{2\ell + 1}{2\pi} C_{\ell}(f) P_{\ell}(\mathbf{e}_1 \cdot \mathbf{e}_2)$$

depends on frequency

$$C_{\ell}(f) = \frac{2}{\pi} \int dk k^2 |\hat{\delta\Omega}_{\ell}(k, f)|^2$$

Correlation with other cosmological observables



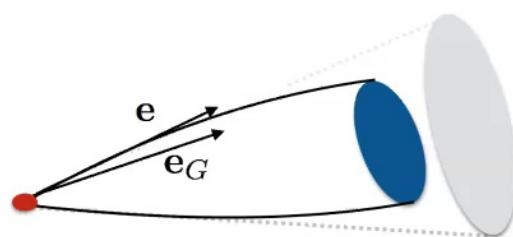
Number counts: number of galaxies as a function of direction and redshift

(see e.g. Bonvin & Durrer 2011)

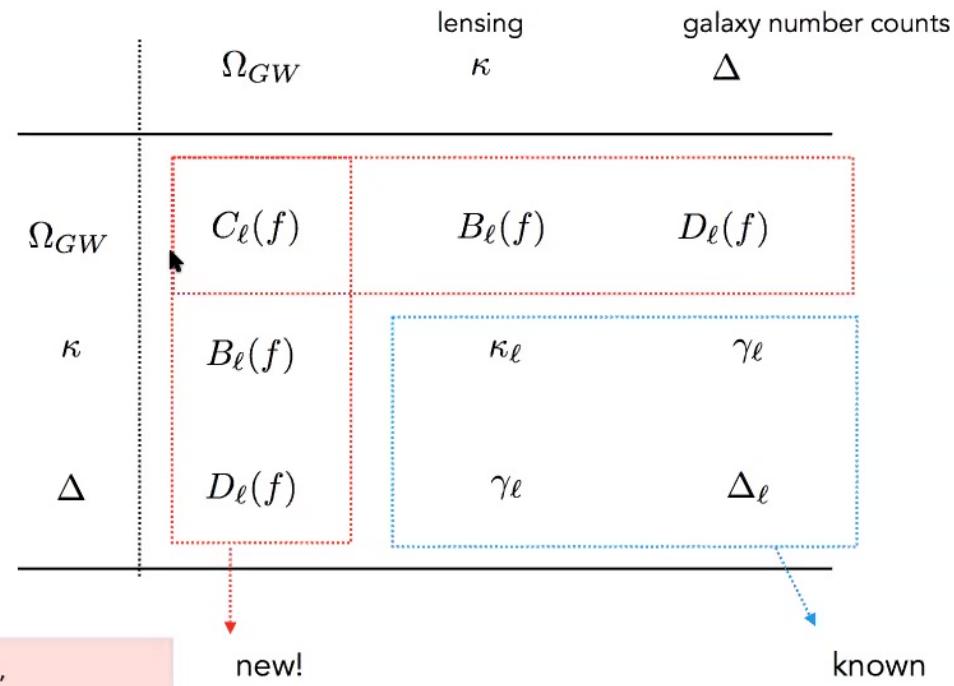
Weak lensing describes the deformation of the shape of a given galaxy by the gravitational potential of the large scale structures

$$\mathbf{e}_G = \mathcal{A} \cdot \mathbf{e}$$

↓
amplification matrix



Summary of correlations

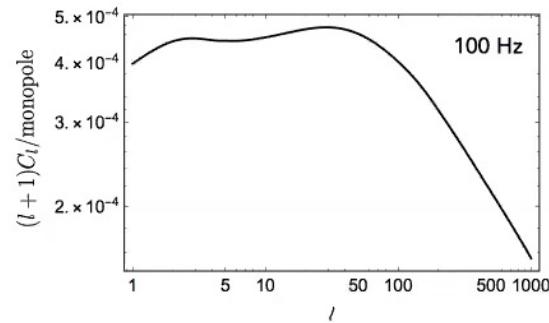


GC, Pitrou, Uzan,
Phys. Rev. D96 (2017) 10, 103019

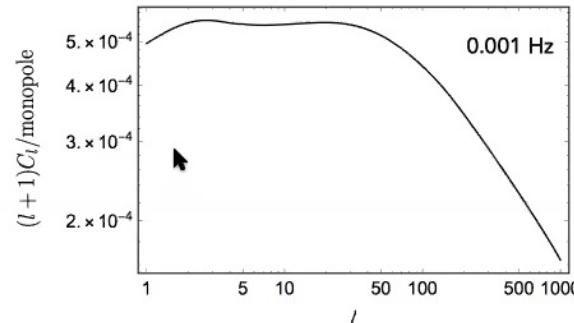


Angular power spectrum

LIGO-Virgo band



LISA band



detectable by Einstein Telescope/Cosmic Explorer (—see final part talk)

$$\left(\ell + \frac{1}{2}\right) C_\ell(f) \simeq \int dk P_\delta(k) \quad \text{for large angular scales}$$

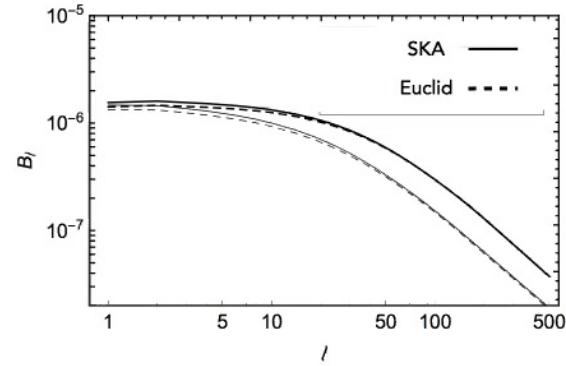
GC, Dvorkin et al. PRL 120 (2018) 231101

GC, Dvorkin et al. MNRAS Lett (2019)

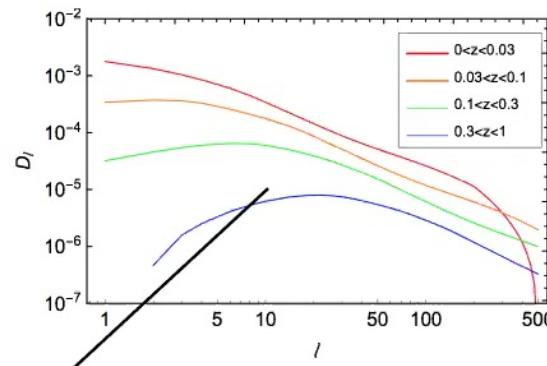


Cross-correlations

cross correlation with weak lensing



cross correlation with galaxy number counts



window function at different redshifts

allows tomographic reconstruction: contribution from different redshifts

allows distinguish astrophysical GW background from cosmological one



Correlation with other cosmological observables



Number counts: number of galaxies as a function of direction and redshift

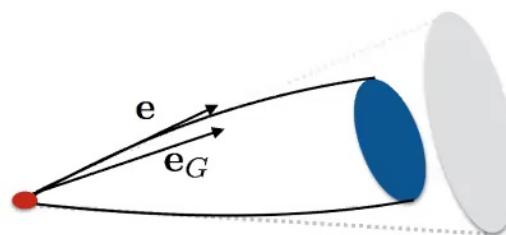
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(see e.g. Bonvin & Durrer 2011)

Weak lensing describes the deformation of the shape of a given galaxy by the gravitational potential of the large scale structures

$$\mathbf{e}_G = \mathcal{A} \cdot \mathbf{e}$$

↓
amplification matrix



Correlation with other cosmological observables



Number counts: number of galaxies as a function of direction and redshift

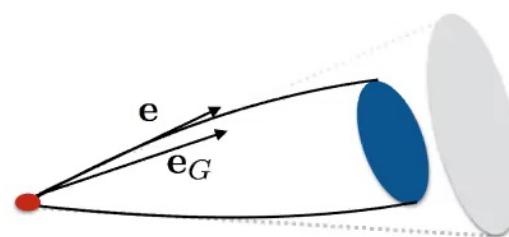
(see e.g. Bonvin & Durrer 2011)

I

Weak lensing describes the deformation of the shape of a given galaxy by the gravitational potential of the large scale structures

$$\mathbf{e}_G = \mathcal{A} \cdot \mathbf{e}$$

↓
amplification matrix



Relevance of this study for astrophysics



$$\delta\Omega_{\text{GW}}(\mathbf{e}, f) = \frac{f}{4\pi\rho_c} \int \frac{dz}{H(z)} \mathcal{A}(z, f) \left[b\delta_m + 4\Psi - 2\mathbf{e} \cdot \nabla v + 6 \int^z \frac{dz'}{H(z')} \dot{\Psi} \right]$$

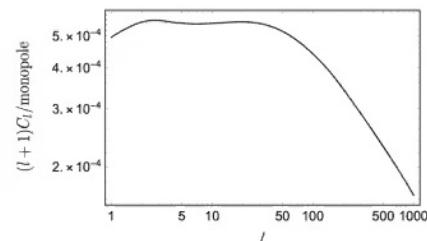
anisotropies per units
of frequency and
directions

astrophysical part

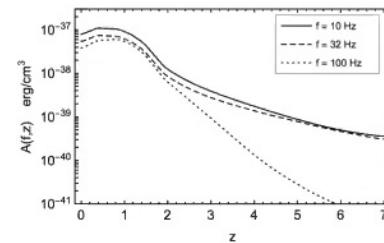
stellar evolution model
fraction compact objects in binaries
mass range
distribution parameters binary system...

cosmological part

metric perturbations
peculiar velocities
matter over density
galaxy bias ...



Astro kernel for a reference model



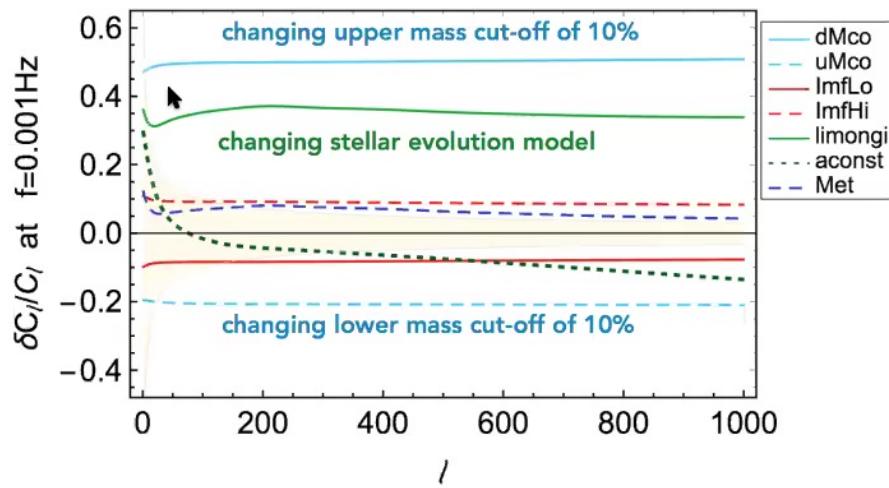
cosmological transfer
functions for perturbations

$T(z)$

+

Test astrophysical modeling: explorative approach

Fractional differences with respect to a reference model



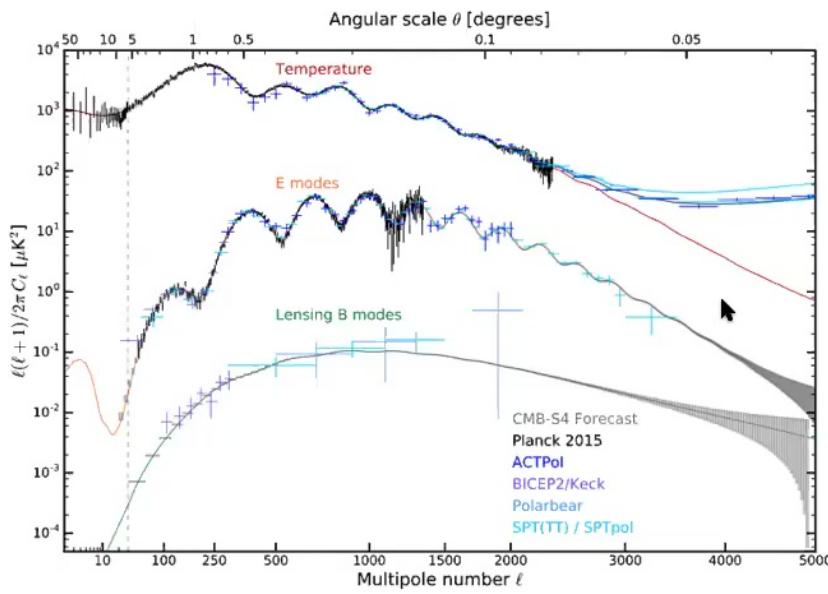
variation >40%

GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004
GC, Dvorkin et al. MNRAS Lett (2019)



What about polarization?

CMB analogy



Characterization of a GW background: Stokes parameters

Superposition signals in given direction and at a given frequency

strain

$$\tilde{h}_{ij}(f, \mathbf{n}) = \tilde{h}_+(f, \mathbf{n}) e_{ij}^+(\mathbf{n}) + \tilde{h}_\times(f, \mathbf{n}) e_{ij}^\times(\mathbf{n})$$

$$\tilde{\mathcal{P}}_{ab} = \tilde{h}_a^* \tilde{h}_b \quad \text{polarization tensor}$$

$a, b = +, \times$

It fully describes background

$$\tilde{\mathcal{P}}_{ab}(\mathbf{n}, f) = \frac{1}{2} \left[I(\mathbf{n}, f) 1_{ab} + U(\mathbf{n}, f) \sigma_{ab}^{(1)} + V(\mathbf{n}, f) \sigma_{ab}^{(2)} + Q(\mathbf{n}, f) \sigma_{ab}^{(3)} \right]$$

Proportional to background energy density $\Omega_{GW}(\mathbf{n}, f)$

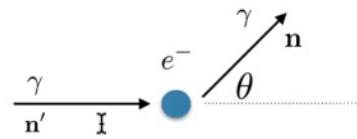


Generation of polarization: two ingredients needed



CMB photons

GW background



?

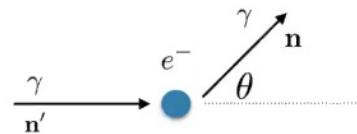
- $\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$
- anisotropy incoming radiation

Is there for a GW background a process
analogue to Thomson scattering for CMB?

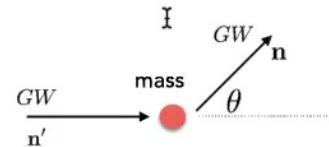
Generation of polarization: wave scattering of gravitons



CMB photons



GW background



- $\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$
- anisotropy incoming radiation

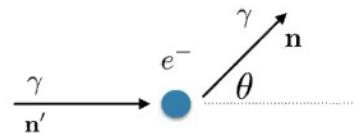
wave scattering !

GC, Durrer, Ferreira, Phys.Rev. D99 (2019)

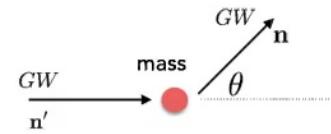
Generation of polarization: wave scattering of gravitons



CMB photons



GW background



Rutherford-like pre factor

- $\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$
- anisotropy incoming radiation

$$\bullet \frac{d\sigma}{d\Omega} = (MG)^2 \frac{1}{\sin^4 \theta/2} |\epsilon_{ij}(\mathbf{n})\epsilon_{ij}(\mathbf{n}')|^2$$

- anisotropy incoming radiation

I

Diffusion by distribution of lenses

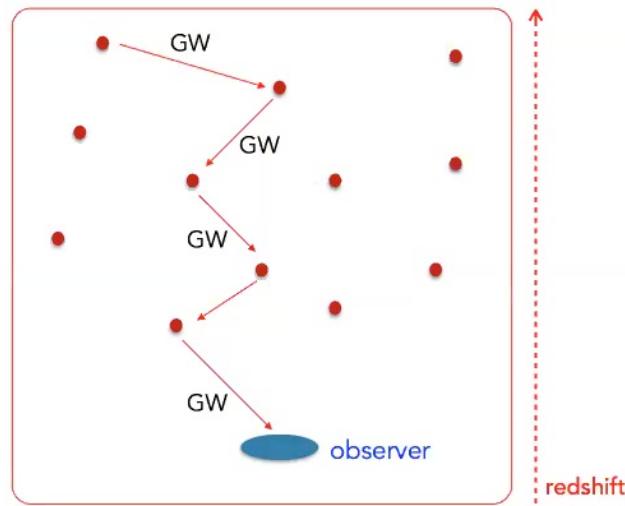
Multi-scattering process with **optical depth**

$$\tau(\eta, \eta_0) = \int_{\eta}^{\eta_0} d\eta' a(\eta') n_{\text{ph}}(\eta') \sigma(\eta')$$

number density
scattering centers

cross section

Visibility function for gravitons extends in redshift (vs CMB)



Predictions for the total amount of polarization produced

Typical suppression of a factor $10^{-4} - 10^{-3}$ wrt anisotropies [in LISA-PTA bands]

GC, Durrer, Ferreira, Phys.Rev. D99 (2019)



Making contact with observations:
characterization of different noise components



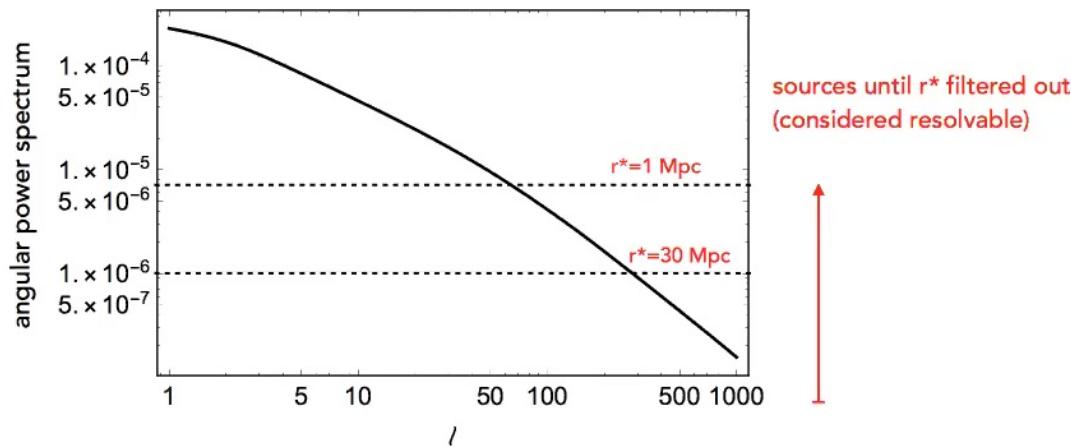
Inclusion of noise: (spatial) shot noise

We need to account for discreteness of sources in space

$$\hat{C}_\ell = \boxed{C_\ell} + \boxed{S_n} \quad \text{shot noise}$$

theoretical prediction

$$S_n = \frac{1}{(4\pi)^2} \int_{r^*} dr \left| \frac{\partial \bar{\Omega}_{GW}}{\partial r} \right|^2 \frac{1}{r^2} \frac{1}{\bar{n}_G(r)} \quad \begin{matrix} r^* \text{ corresponds to upper cut-off in flux} \\ (\text{threshold to resolve sources individually}) \end{matrix}$$



Inclusion of noise: popcorn noise in LIGO/Virgo band

We need to account for discreetness of emission in time

$$\hat{C}_\ell = \boxed{C_\ell} + \boxed{S_n}, \text{ popcorn noise}$$

theoretical prediction

$$S_n = \frac{1}{(4\pi)^2} \int_{r^*} dr \left| \frac{\partial \bar{\Omega}_{GW}}{\partial r} \right|^2 \frac{1}{r^2} \frac{1}{\bar{n}_G(r)}$$

\downarrow

$$\beta_T \cdot \bar{n}_G(r)$$

number of galaxies containing a merger
in observation time T

$$\beta_T = \frac{T}{a^3 \bar{n}_G} \times \frac{d\mathcal{N}}{dt dV} \quad \ll 1$$

In the frequency band of terrestrial interferometers, there is **large contribution from popcorn noise**

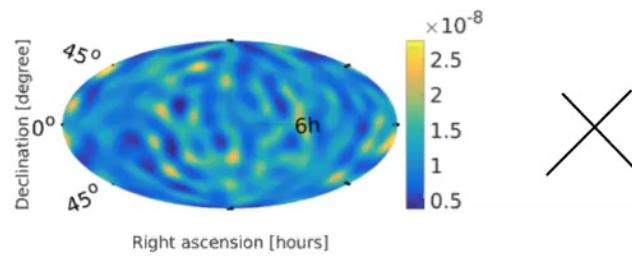
Important! This popcorn noise component **not present in LISA band**. Intrinsic (irreducible) background there

GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004



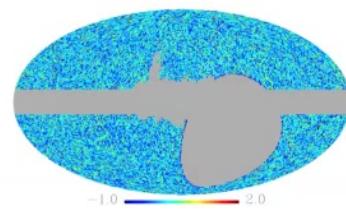
Cross-correlation very useful for popcorn backgrounds

Popcorn-noise dominated map



(LIGO-Virgo PRL 118, 121102,
2017)

Galaxy number counts map



e.g. map radio sources, from:
Bengaly, Maartens, Santos 2018

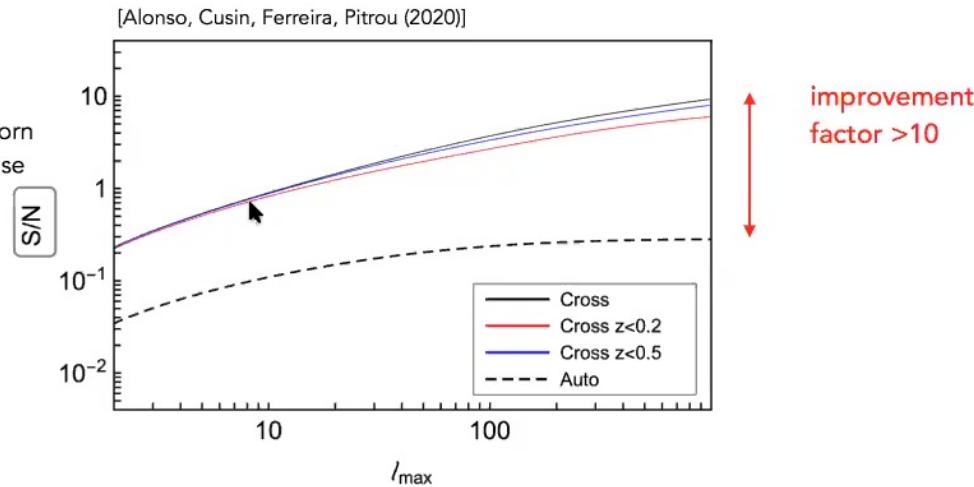
cross-correlating helps to extract GW signal from clustering
out of popcorn noise threshold



Power cross-correlation in Hz band: quantitative analysis



considering popcorn
noise as (only) noise
component



cross-correlation (potentially) allows to explore clustering properties of population sources

What about instrumental noise?

Different steps to take to get characterization:

- 1) assume network of detectors (response functions) and scan strategy
- 2) translate map noise into noise on power spectrum (build estimator for intensity and compute its covariance matrix)
- 3) effective formula of **instrumental noise** per multipole

like N_I to describe Planck noise

Analytic scaling for Earth-based detectors

$\alpha = 2/3$ for astro background

$\alpha = 0$ for cosmo background

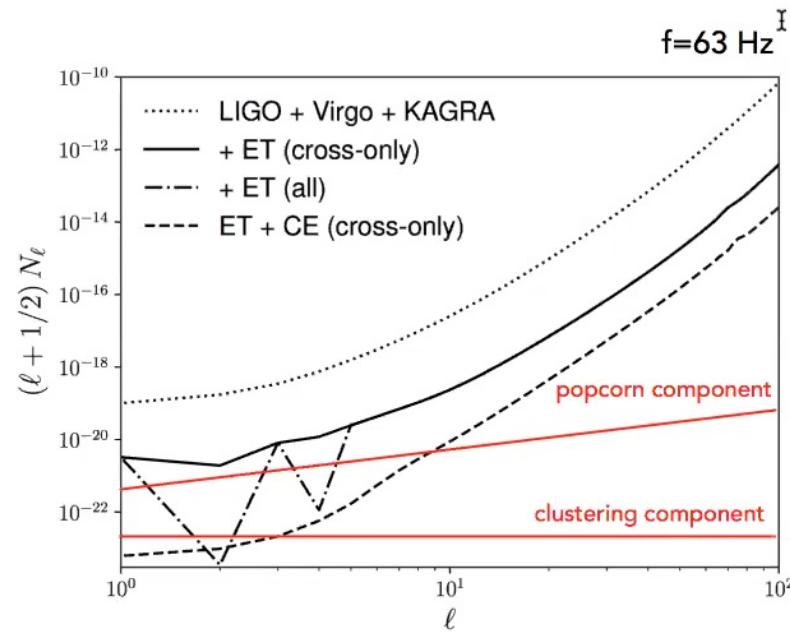
$$\Omega_{GW}(f, \mathbf{e}) = \left(\frac{f}{f_{\text{ref}}} \right)^\alpha D(\mathbf{e})$$

↳
$$(N_{AB})_\ell^{-1} = 10^{23} (2\pi)^{-2\alpha} \left(\frac{N_f^{\text{LIGO}}}{N_f^A} \right) \left(\frac{N_f^{\text{LIGO}}}{N_f^B} \right)$$
 noise power spectra density
$$\times \left(\frac{f_{\text{ref}}}{100\text{Hz}} \right)^{-2\alpha} \left(\frac{b_{AB}}{3000\text{km}} \right)^{-2\alpha+5} \left(\frac{T_{\text{obs}}}{\text{yr}} \right)$$
 baseline (distance A-B)
$$\times \int_{\beta_{\min}}^{\beta_{\max}} d\beta \beta^{-6+2\alpha} |j_\ell(\beta)|^2,$$
 observation time

where $\beta \equiv 2\pi b_{AB} f$



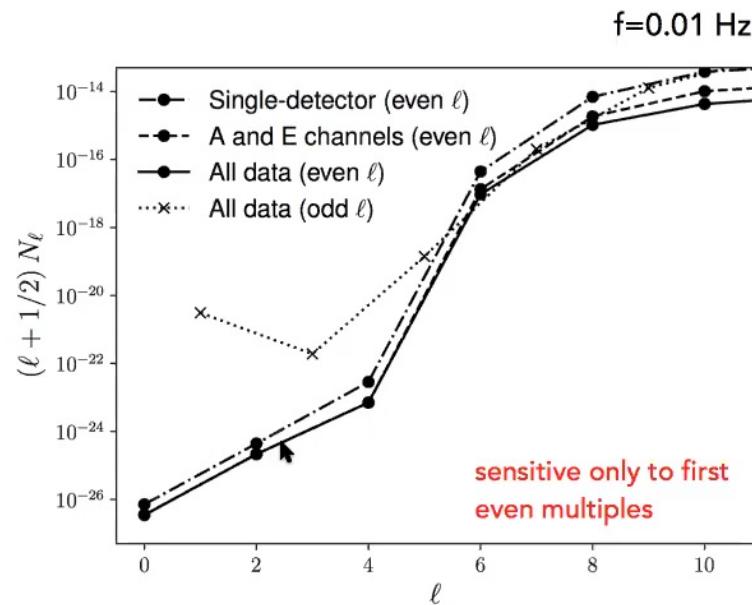
Instrumental noise: extended LIGO-Virgo network



[Alonso, Contaldi, **Cusin**, Ferreira, Renzini (2020)]



Instrumental noise LISA



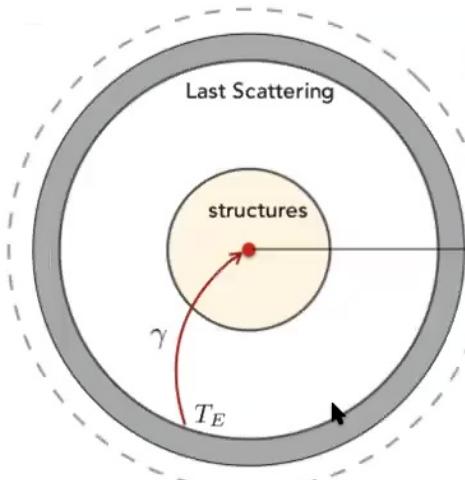
signal is down here! $(\ell + 1/2)C_\ell \sim 10^{-30}$

we will need to wait for LISA+ (ALIA)

[Alonso, Cusin, Ferreira + (2020)]

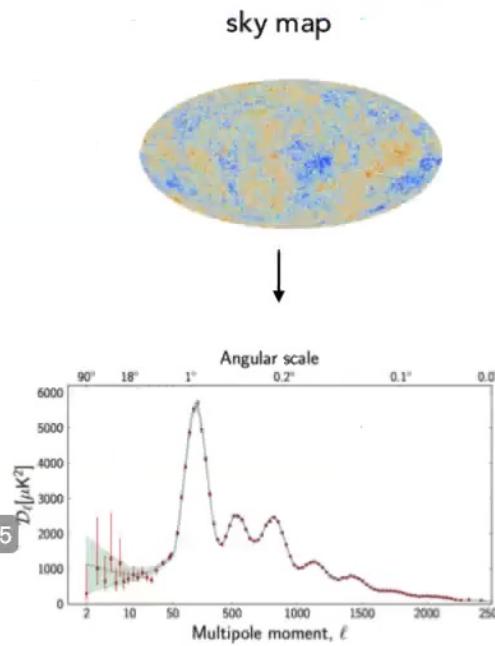


CMB : a more realistic modeling

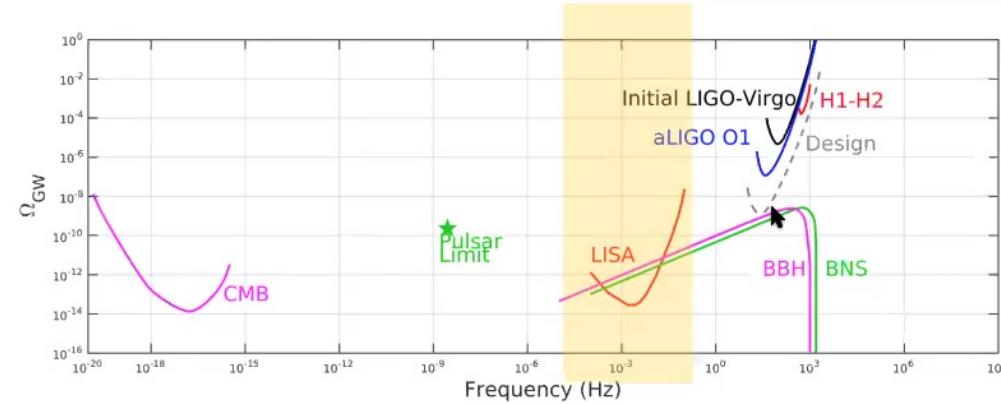


temperature anisotropies

Pagina 20 di 65



LISA: inspiralling phase of mergers



LISA will see a continuous background (stationary) from inspiralling phase of binary systems

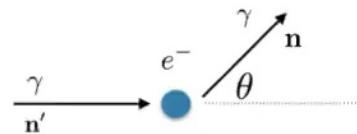
(vs LIGO will see merger phase of binary evolution. Popcorn-like signal)



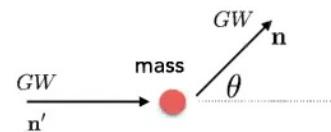
Generation of polarization: wave scattering of gravitons



CMB photons



GW background



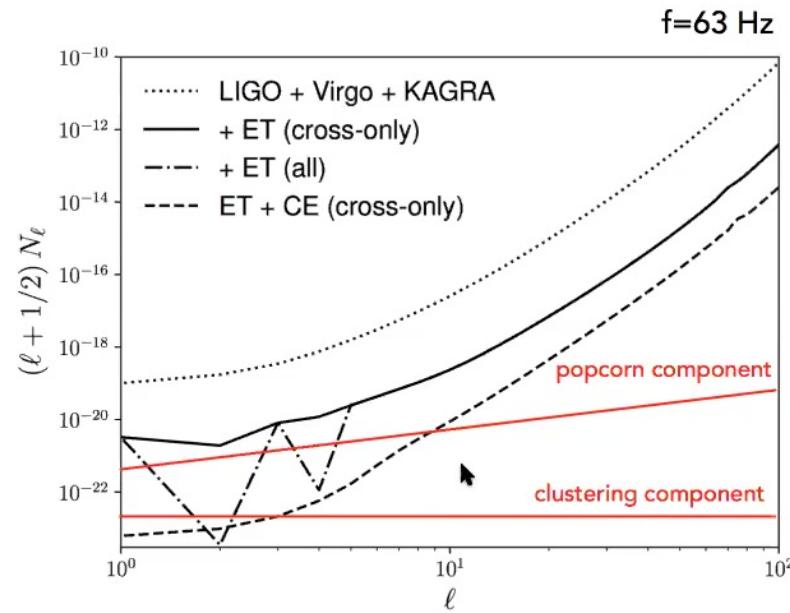
- $\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$
- anisotropy incoming radiation

wave scattering !

Pagina 41 di 65

GC, Durrer, Ferreira, Phys.Rev. D99 (2019)

Instrumental noise: extended LIGO-Virgo network

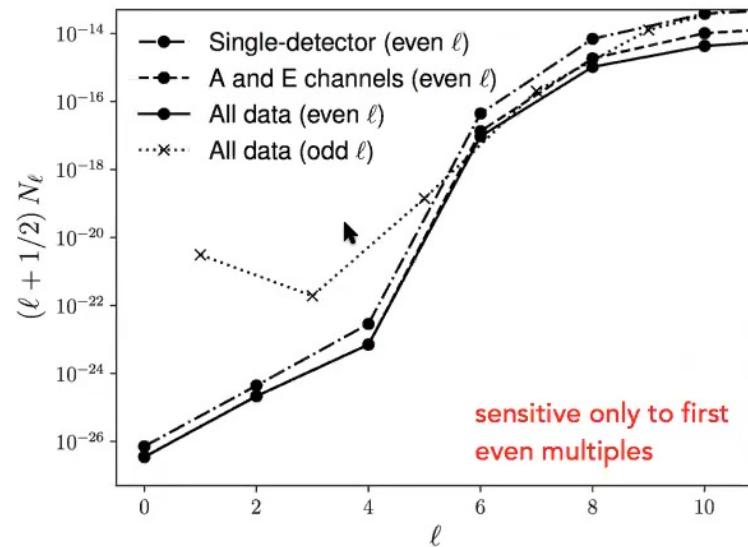


[Alonso, Contaldi, **Cusin**, Ferreira, Renzini (2020)]



Instrumental noise LISA

f=0.01 Hz



signal is down here! $(\ell + 1/2)C_\ell \sim 10^{-30}$

we will need to wait for LISA+ (ALIA)

[Alonso, Cusin, Ferreira + (2020)]

Summary: state of the art and work in progress



noise characterization, forecasts for
present and future missions

predictions ↔ data analysis

anisotropies



polarization



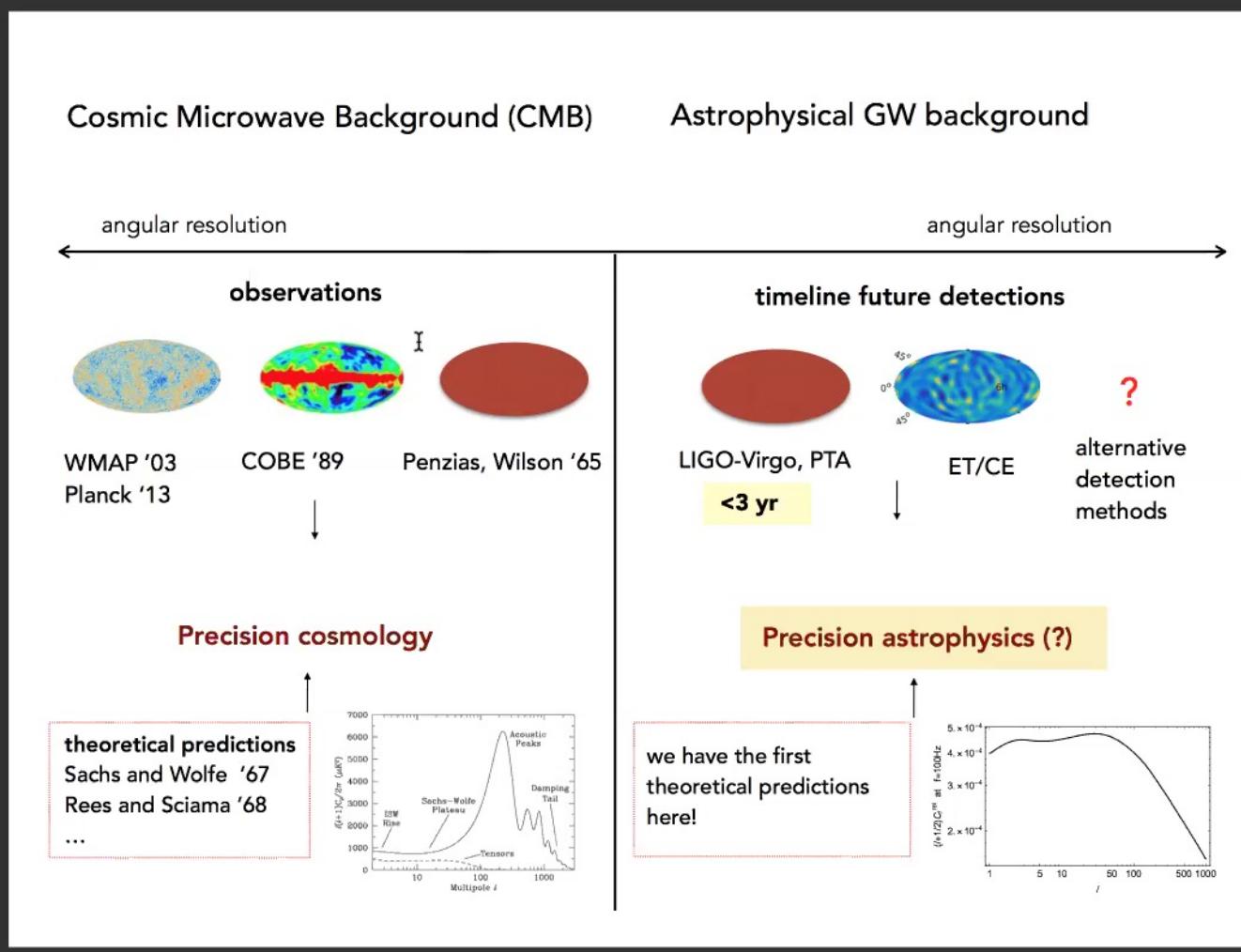
cross-correlation
(electromagnetic
counterparts)

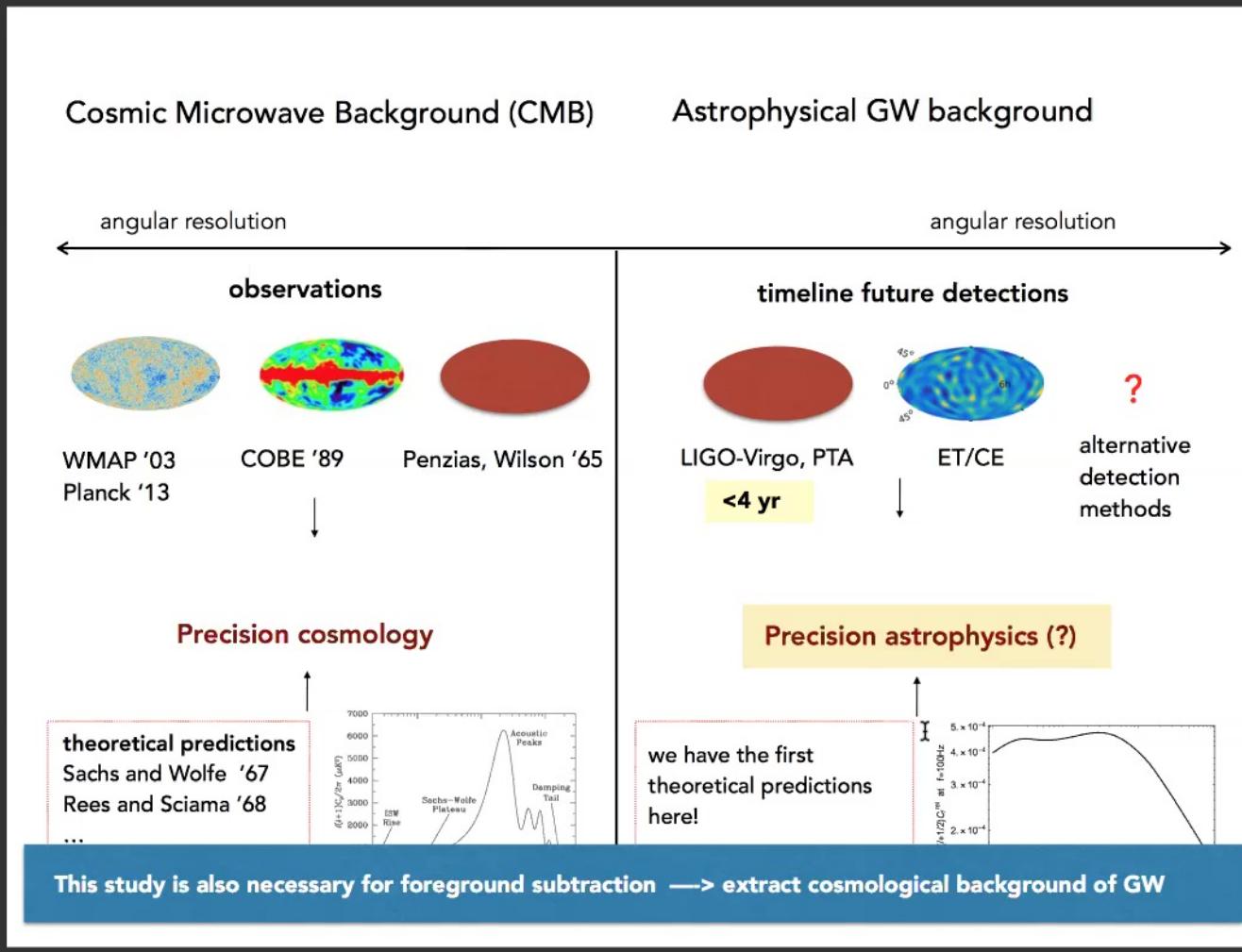


no existing
infrastructure for
cross-correlation

Future ...

first theoretical predictions
we know: there is astro info inside anisotropies





Diffusion by distribution of lenses

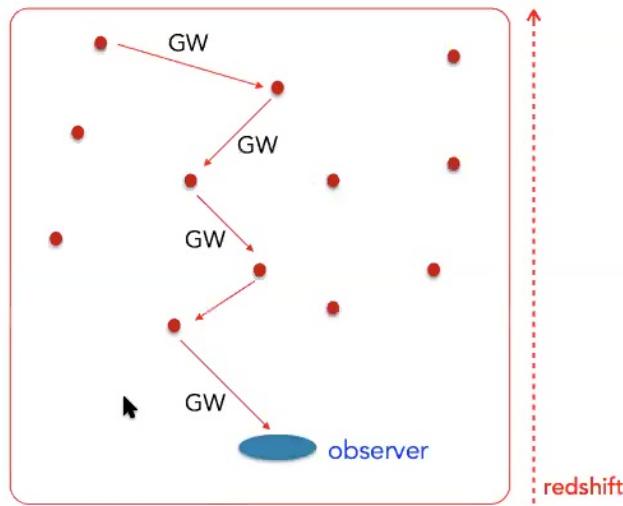
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Pagina 46 di 65

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GC, Durrer, Ferreira, Phys.Rev. D99 (2019)

