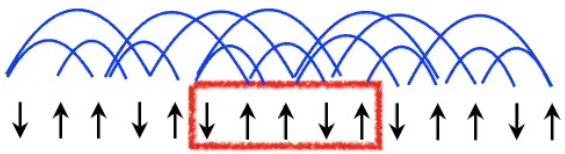
Title: Non-equilibrium quantum matter through the prism of quantum entanglement

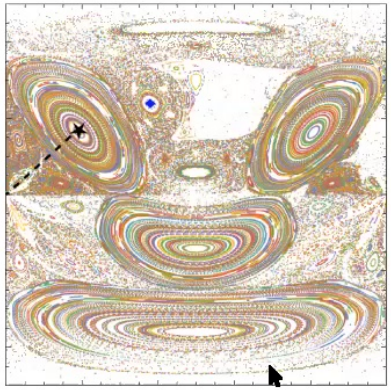
Speakers: Dima Abanin
Date: October 26, 2020 - 12:30 PM

URL: http://pirsa.org/20100003

Abstract: The remarkable experimental advances made it possible to create highly tunable quantum systems of ultracold atoms and trapped ions. These platforms proved to be uniquely suited for probing non-equilibrium behavior of interacting quantum systems. From statistical mechanics, we expect that a non-equilibrium system will thermalize, settling to a state of thermodynamic equilibrium. Surprisingly, there are classes of systems which do not follow this expectation. I will give examples of systems which avoid thermalization, thanks to disorder-induced localization and quantum scarring. While thermalization leads to â€œscramblingâ€• of quantum information, its absence may protect local quantum coherence. This enables non-equilibrium states of matter not envisioned within the framework of statistical mechanics. I will highlight the recent theoretical insights into the remarkable physical properties of such states, based on the underlying patterns of quantum entanglement. I will finally describe a possible theoretical route towards developing a classification of dynamical universality classes in many-body systems.

Non-Equilibrium Dynamics Through the Prism of Quantum Entanglement





Dima Abanin

UNIVERSITÉ DE GENÈVE

Condensed Matter Seminar
Perimeter Institute
26 October 2020

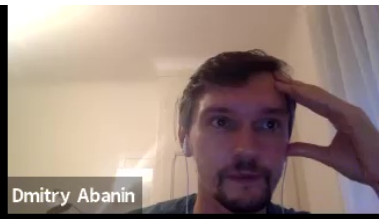
Quantum exponential wall

1 spin-1/2

Parameters: 2

$\downarrow \quad \uparrow$

$$|\psi\rangle = A_\uparrow|\uparrow\rangle + A_\downarrow|\downarrow\rangle$$



N spins-1/2

Parameters: 2^N

$\downarrow \uparrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow \downarrow \uparrow$

$\downarrow \uparrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow \downarrow \uparrow$

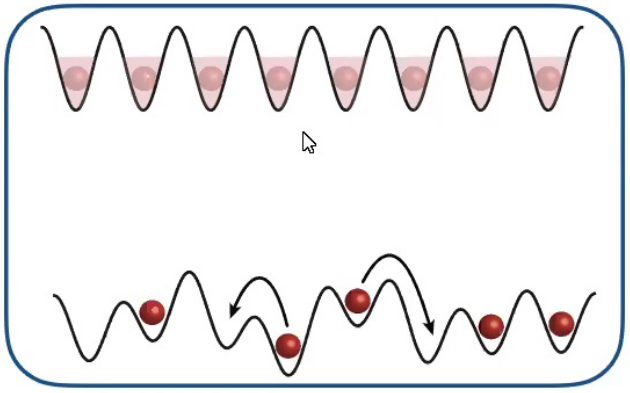
$$|\psi\rangle = \sum_{s_i=\uparrow,\downarrow} A_{s_1\ldots s_N}|s_1\ldots s_N\rangle$$

..if you want to make a simulation of nature, you'd better make it quantum-mechanical, and by golly it's a wonderful problem, because it doesn't look so easy

Feynman'82

Synthetic quantum systems

Ultracold atoms



Design dimensionality, lattice,interactions,..

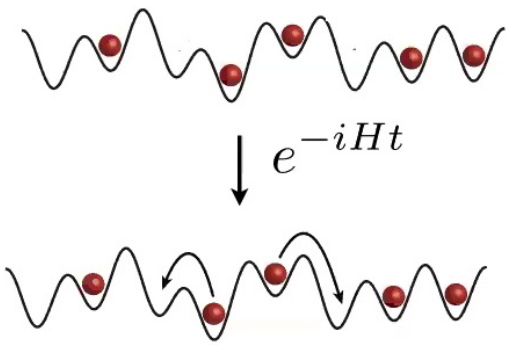
Unlike "conventional" materials

- Isolated (no phonon/heat bath)
- Long time scales $\sim 10^{-3}s$ vs $10^{-12}s$

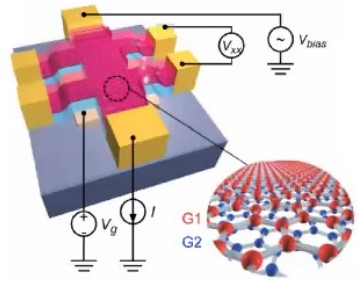
A platform to study non-equilibrium phenomena

New opportunities

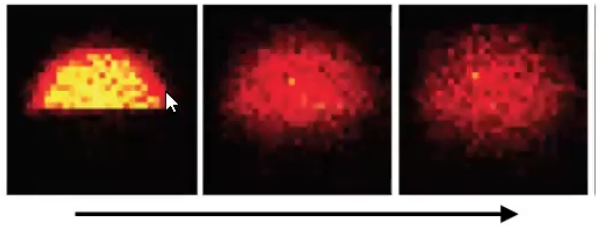
Synthetic systems
New setups, probes



Materials
conductivity,
heat capacity..



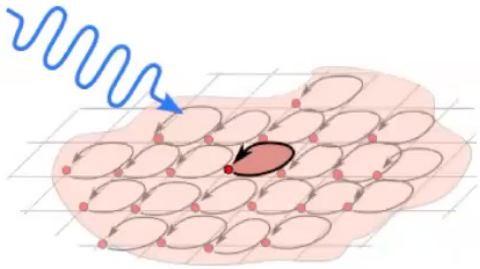
Single-site, real-time resolution
Harvard, Munich,..

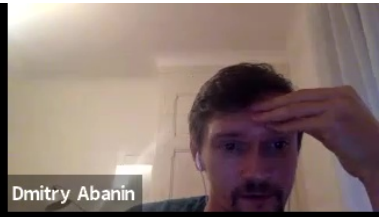


time

Bloch group'16

Periodically driven systems





Non-equilibrium quantum matter

Cold atoms

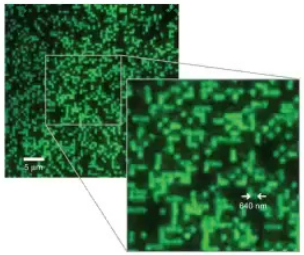


Figure: Greiner group (Harvard)

Rydberg atom arrays

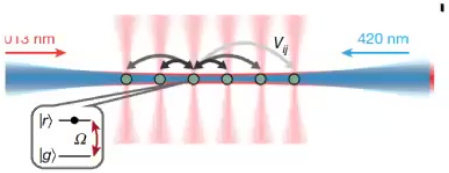


Figure: Bernien et al'17 (Harvard-MIT)

Superconducting qubits

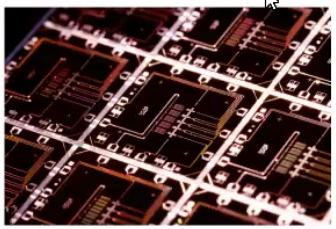


Figure: J. Martinis group (Google)

Nitrogen-vacancy spins in diamond

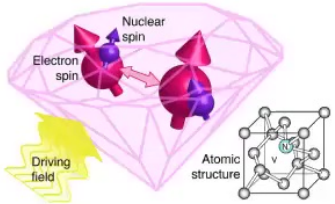
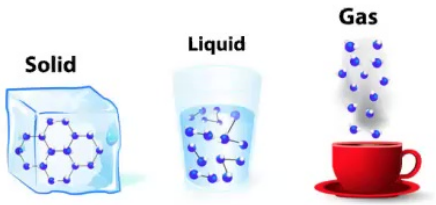
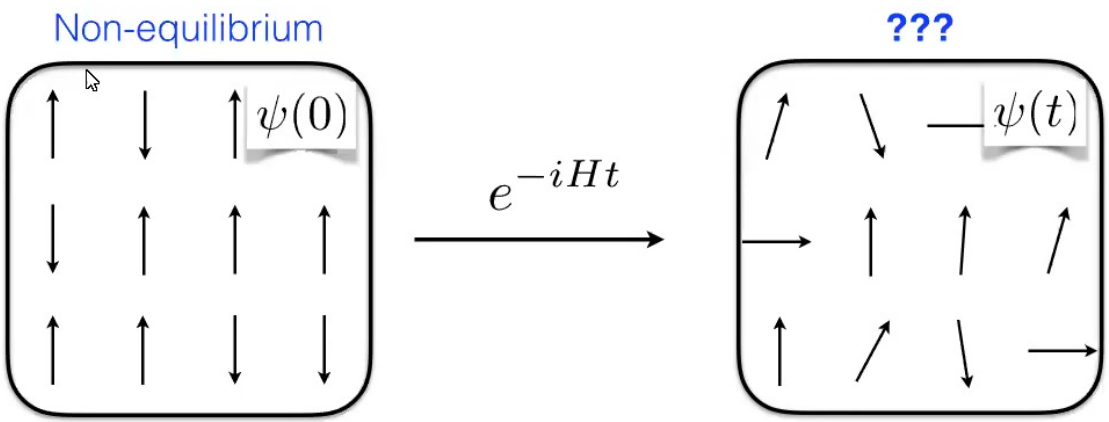


Figure: aps.org

Dynamics of isolated quantum systems

Thermal equilibrium: universality, phase of matter
Principle: symmetry breaking

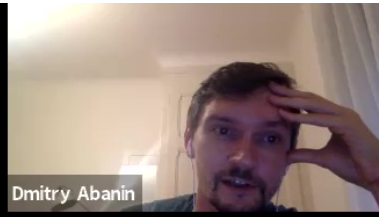




Universality? New out-of-equilibrium phases/properties?

↓

Efficient control of synthetic systems



Ergodicity

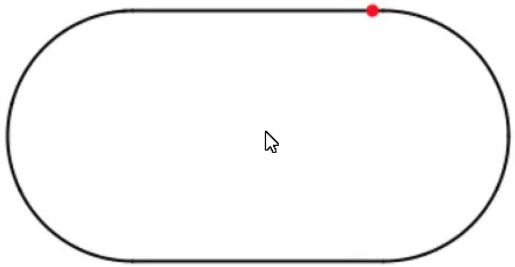
Non-equilibrium

Thermal equilibrium

Ergodicity

Non-equilibrium

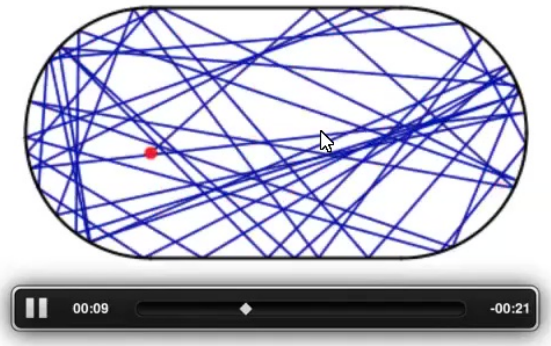
Thermal equilibrium

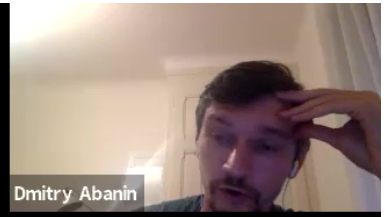


Ergodicity

Non-equilibrium

Thermal equilibrium

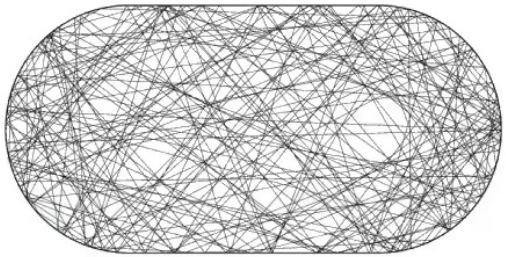


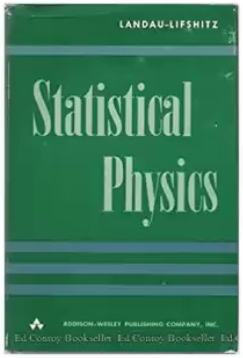


Ergodicity

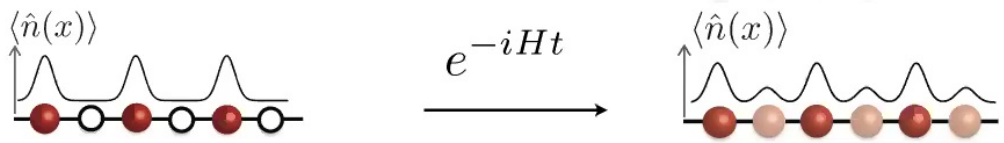
Non-equilibrium

Thermal equilibrium



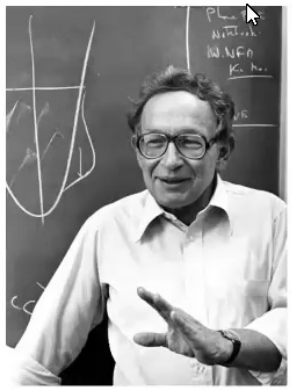


Ergodicity breakdown in disordered quantum systems



a system in which an approach to equilibrium is impossible!

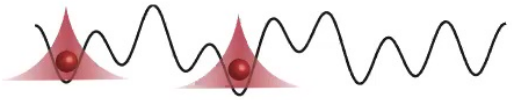
Anderson'58



New paradigm: many-body localization (MBL)

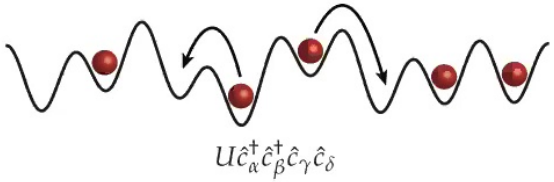
Anderson localization

Single-particle



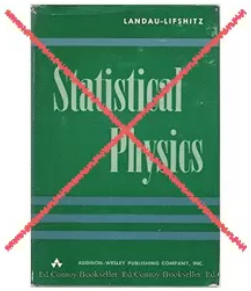
Many-body localization

Generic non-ergodic phase



Anderson Fleischmann'80; Gornyi et al'05; Barsko et al'05

Review: DA, Altman, Bloch, Serbyn, Rev. Mod. Phys. 2019



→ **New opportunities**

Difficult: Interactions + disorder + highly non-equilibrium

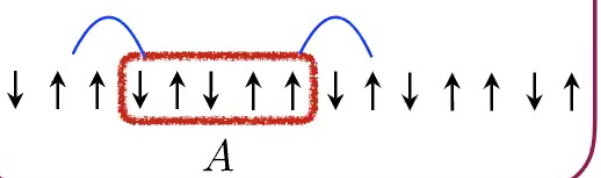
Complexity and entanglement

N spins wave function $|\psi\rangle$ $\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow$

How difficult to represent? Naively $\sim 2^N$ parameters

Weakly entangled states can be efficiently "compressed" $\sim N^a$

$S_{\rm ent}(A) \propto {\rm vol}(\partial A)$ "area-law"

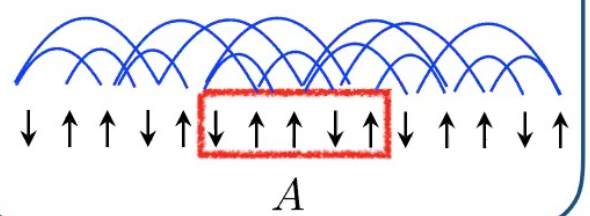


Ground states, "easy"

Tensor networks methods

Highly entangled states "hard"

$S_{\rm ent}(A) \propto {\rm vol}(A)$ "volume-law"

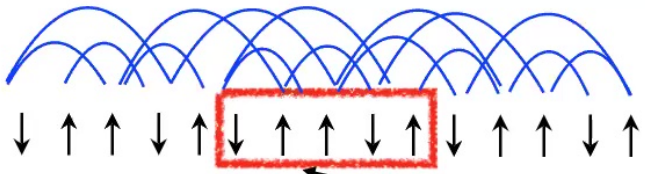


Non-equilibrium states, "hard"

Eigenstate thermalisation hypothesis (ETH)

Deutch'91, Srednicki'94

-Individual excited eigenstates of a quantum-ergodic Hamiltonian H have thermal observables



Eigenstate $|n\rangle$

A

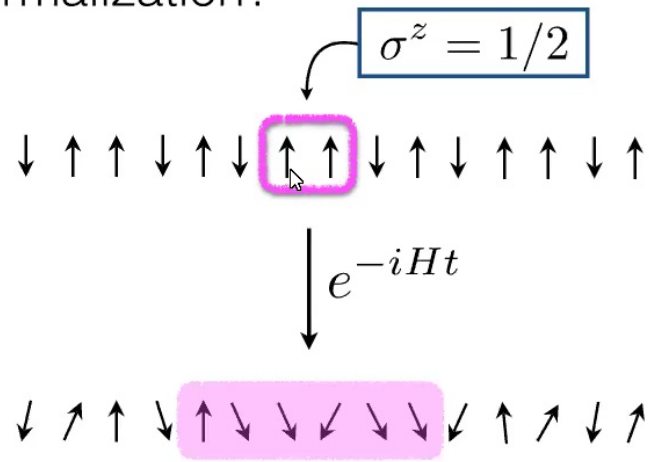
Reduced density matrix: $\rho_A(n) \approx \rho_A^{\text{Thermal}}$

-System acts as a heat bath for its subsystems

-"Volume-law entanglement" $S_A(n) \approx S_A^{\text{Thermal}} \propto \text{Vol}(A)$

Why avoid thermalization?

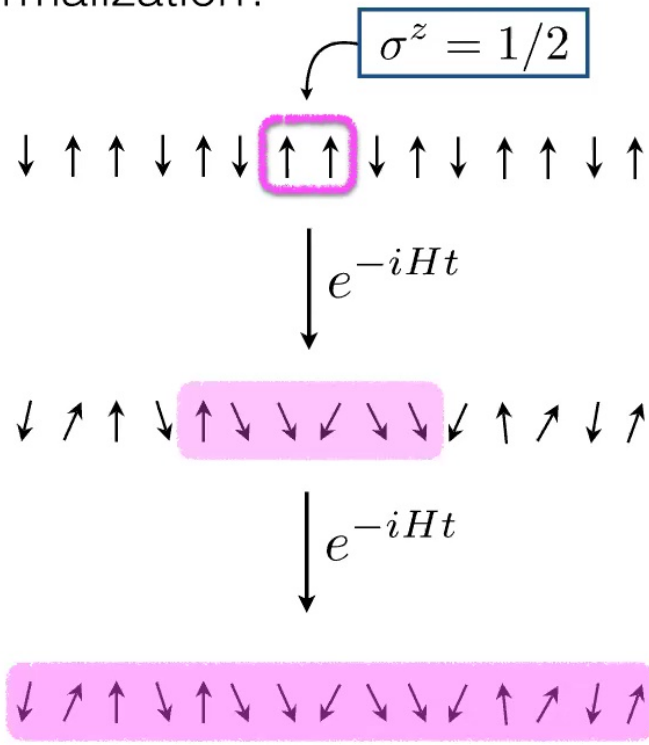
Write information into initial state



Quantum information spreads over many spins $\longleftrightarrow$ entanglement

Why avoid thermalization?

Write information into initial state



Quantum information spreads over many spins ⟷ entanglement

Becomes unrecoverable

$$\sigma^z(t) = e^{iHt}\sigma^z e^{-iHt}$$

Thermalisation ⟶ scrambling of quantum information

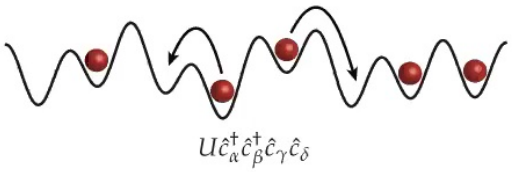
Protect coherence? New phases?

MBL phase: insights from entanglement

Models of many-body localisation (MBL)

MBL=strong disorder + short-ranged interactions
+highly non-equilibrium

Fermions



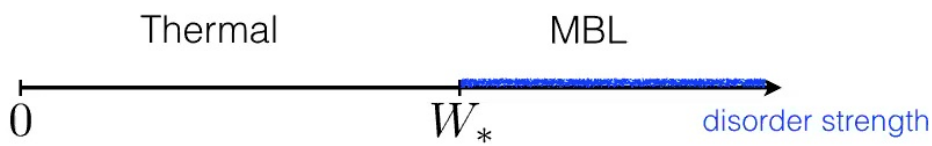
Spins

$$H = \sum h_i \sigma_i^z + J \sum_{\langle ij \rangle} \sigma_i^+ \sigma_j^- + V \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

disorder — interactions

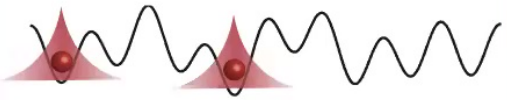
$h_i \in [-W, W]$

Dynamical phase diagram



Are interactions important?

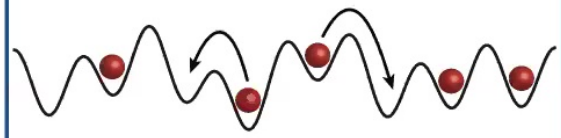
Anderson localization



single-particle

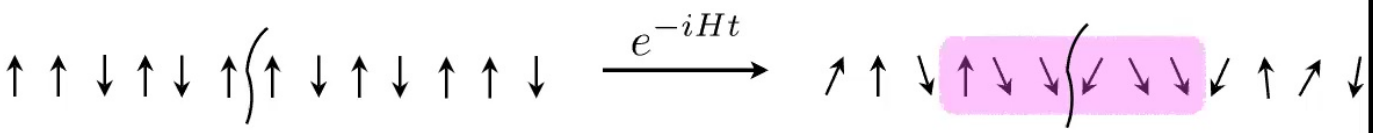
Conductivity $\sigma = 0$

Many-body localization



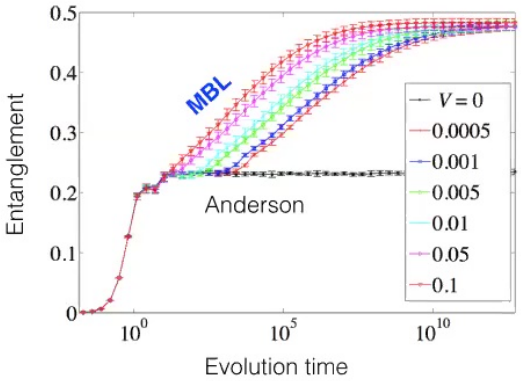
$U\hat{c}^\dagger_\alpha \hat{c}^\dagger_\beta \hat{c}_\gamma \hat{c}_\delta$

Conductivity $\sigma = 0$



MBL: no particle transport, but "glassy" spreading of quantum information

$$S_{\rm ent}(t) \sim \log(t)$$???



Bardarson,Pollmann, Moore'12, Serbyn, Papic, DA'13

Emergent integrability

Serbyn, Papic, DA'13; Oganesyan, Huse'14 Imbrie'16

Infinite-disorder limit

$$H_0 = \sum h_i \sigma_i^z + V \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

Trivially "integrable"

$$[\sigma_i^z, H_0] = 0$$

MBL phase, finite disorder

$$H = H_0 + J \sum_{\langle ij \rangle} \sigma_i^+ \sigma_j^-$$

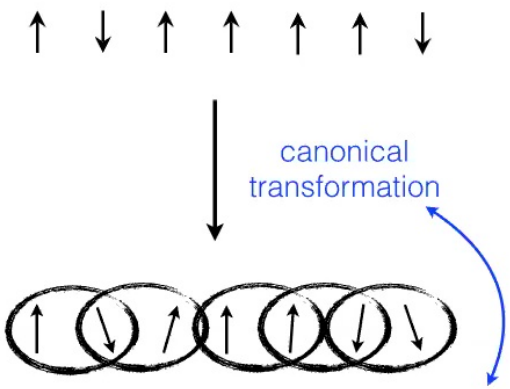
Local integrals of motion

$$[\hat{\tau}_i^z, \hat{H}] = 0$$

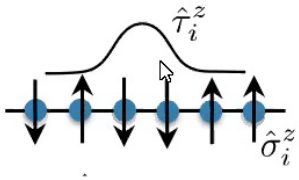
"Action-angle" variables

- quasi-local
- qubits with $T_1 = \infty$

Eigenstates: product states



Eigenstates: area-law entangled



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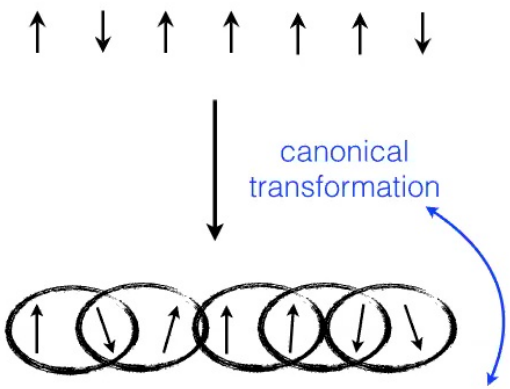
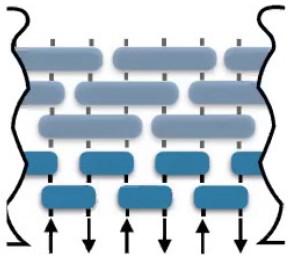


Eigenstates: area-law entangled

Compress efficiently by tensor networks - new algorithms

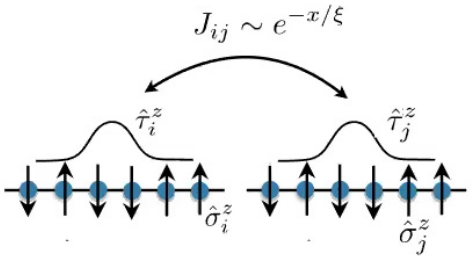


Dynamical properties of MBL phase

"Action-angle" variables

$$[\hat{\tau}_i^z, \hat{H}] = 0$$

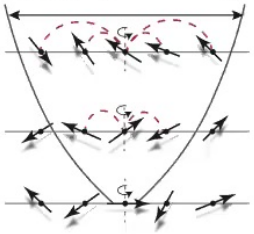
$$H = \sum_i \tilde{h}_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \sum J_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$



Solve for dynamics

entanglement time $t \sim \frac{1}{J_{ij}} \sim e^{x/\xi}$

$$x_{\text{ent}}(t) \sim \log t$$



Logarithmic spread of entanglement

ergodic: linear spread of entanglement

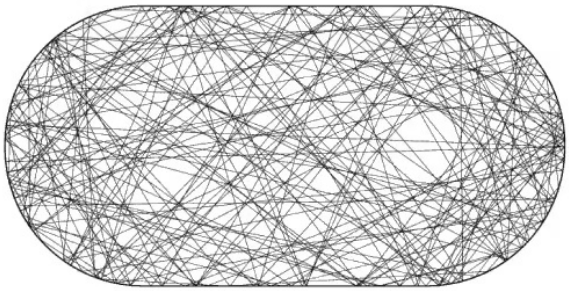
Equilibration to a highly non-thermal state

$$\langle \tau_i^z \rangle = \text{const}$$

Serbyn, Papic, DA'14

Ergodicity breaking: quantum vs classical

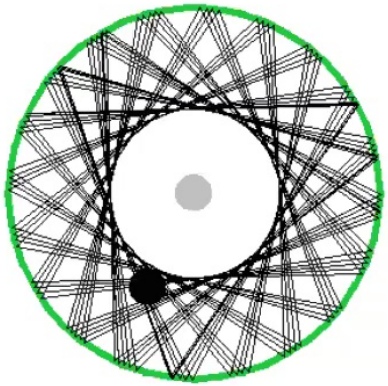
Ergodicity and integrability in classical physics



Ergodic

Chaotic motion

Ergodicity broken



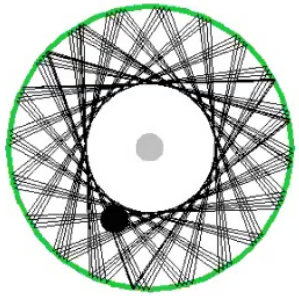
Integrable

Regular motion

Stability to perturbations?
Yes — Kolmogorov-Arnold-Moser (KAM)
Weak perturbations, few-body

Classical vs quantum

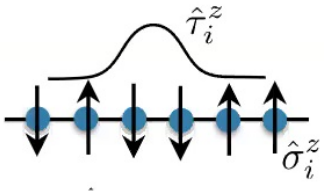
Classical: Integrable, regular motion



Kolmogorov-Arnold-Moser (KAM) theory

Robustness to weak perturbations
few-body only $N \sim 1$

Quantum



Disorder "protects" integrability

"Quantum KAM"

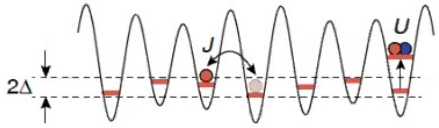
A stable phase of matter $N \to \infty$

MBL and thermal systems: comparison

	Ergodic	**MBL**
Eigenstate entanglement	Volume-law	Area-law
Conserved quantities	Only global (e.g. energy)	Local integrals of motion
Spreading of correlations	$x_{\mathrm{ent}} \sim vt$	$x_{\mathrm{ent}}(t) \sim \log t$

Experiments

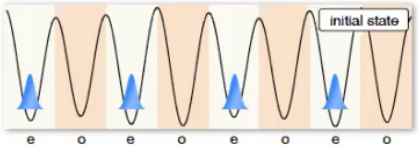
System: disorder+short-range interactions



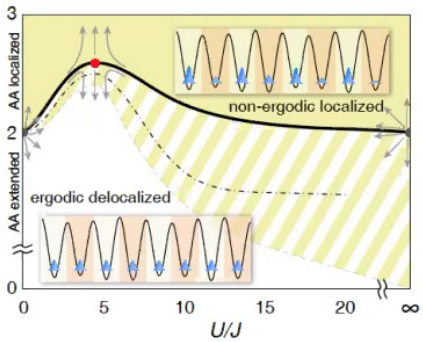
Munich group'15-

Quantum quench setup

Lack of relaxation signals MBL (exps in 1d, 2d)

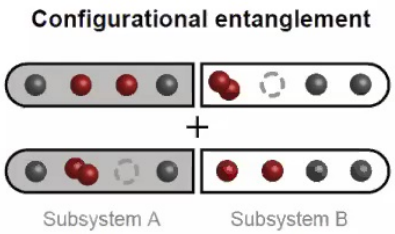


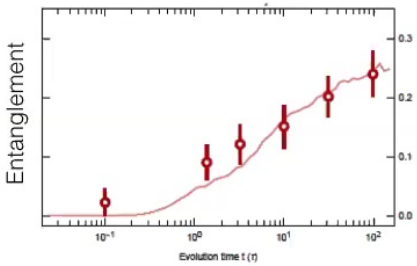
$$\xrightarrow{e^{-iHt}}$$



Recently: entanglement growth measured Harvard group'18-

Unique MBL signature





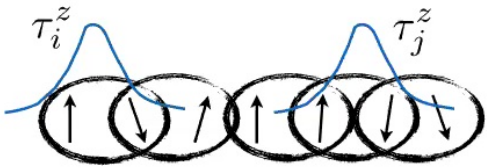
Evolution time

Implications of ergodicity breaking

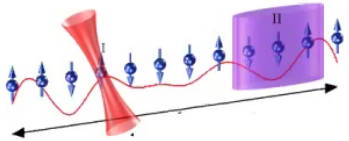
MBL protects quantum coherence

Bahri, Altman et al'14, Serbyn, Knap, Gopalakrishnan, DA et al'14

Slow decoherence

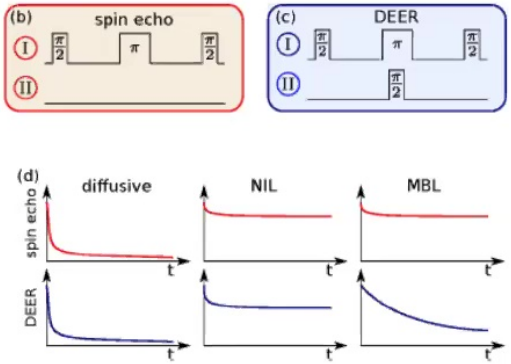


Restore coherence with spin echo



Double electron-electron resonance

- study interactions b/w remote q-bits
- smoking-gun signature of MBL



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Slow decoherence

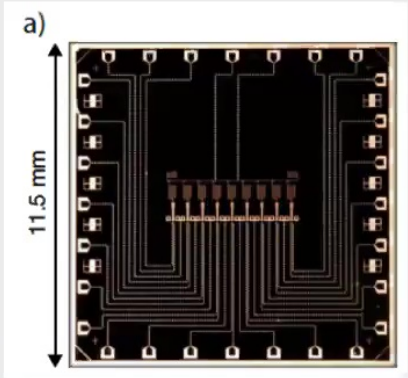
Restore coherence w

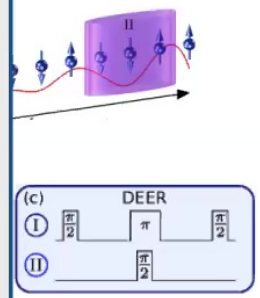
Double electron-elec

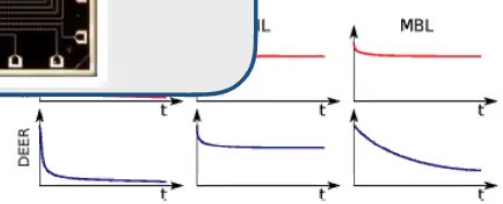
- study interaction
- smoking-gun sig

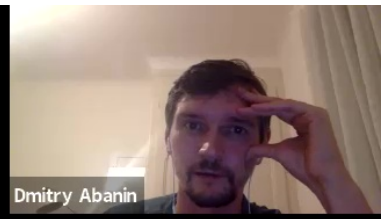
Recently: demonstrated experimentally w/superconducting qubits

collaboration with Google, TUM groups

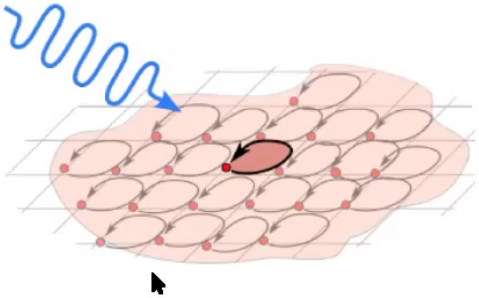








MBL enables new non-equilibrium phases

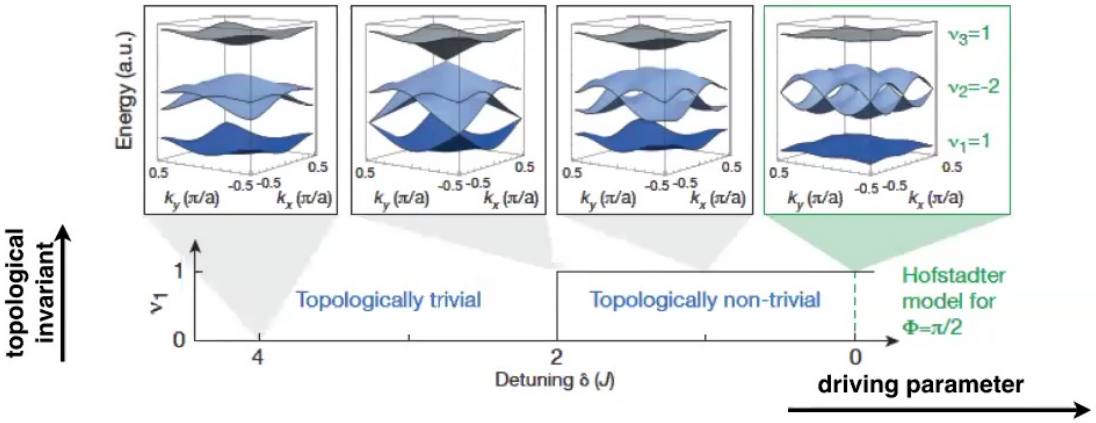


Engineering topological bands

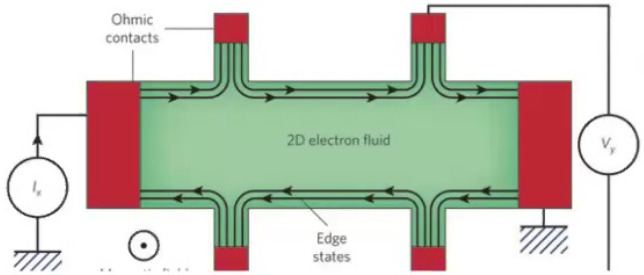
Driving: control topology of single-particle Bloch bands

Oka, Aoki'08, Lindner et al'11, Kitagawa et al'10-11, Experiments:Jotzu et al'14, Aidelsburger et al'14…

image: Aidelsburger et al'15



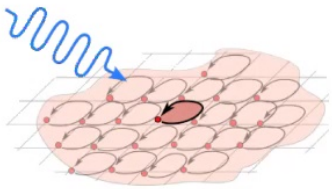
Quantum Hall-like band
no magnetic field



von Klitzing et al'80

Floquet operator and the heating problem

$$H(t+T) = H(t)$$

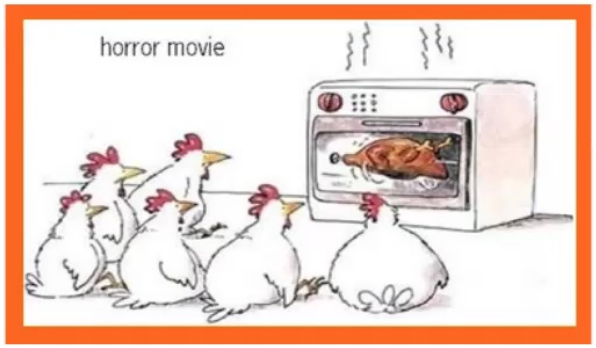


Floquet operator $$F = \mathcal{T} \exp\left(-i \int_0^T H(t)dt\right)$$

Floquet eigenstates, quasi-energies

Effective Hamiltonian $$F = e^{-iH_{\rm eff}T}$$

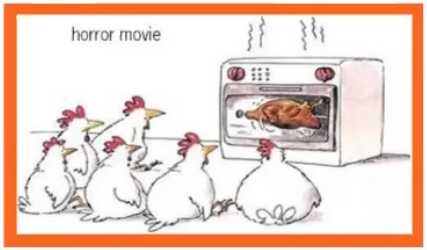
Heating $\longrightarrow$ $H_{\rm eff}$ nonlocal, non-unique (no energy conservation!)



New phases enabled by MBL

- **MBL in driven systems**

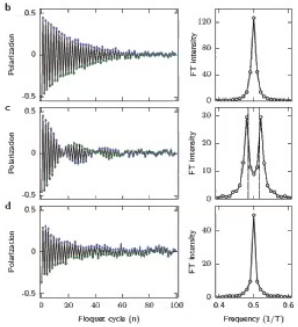
Solves heating problem



Ponte, Papic, Huveneers DA'15
Lazarides, Das, Moessner '15

- **Unique non-equilibrium phases**

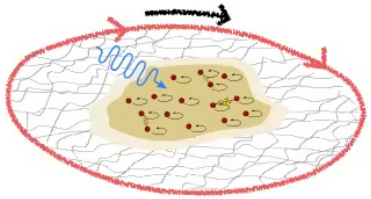
Time crystals



Khemani et al'16, Else et al'16

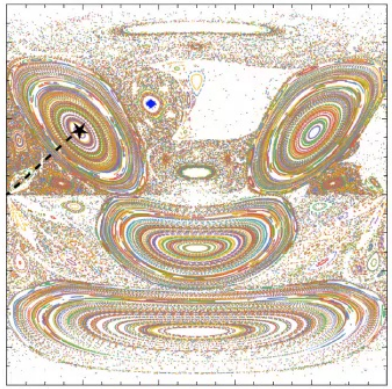
Experiment: Lukin group'17
Theory: Ho, Choi, Lukin, DA'18

New topological invariants



Rudner et al'13, Potter et al'16, Nathan, DA, Berg, et al'18,…

Beyond MBL



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Con qua		grals n
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Phases with intermediate properties?

Different entanglement scaling?

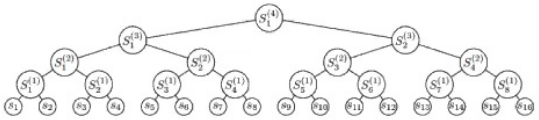
Non-ergodicity beyond MBL

Non-Abelian symmetries+disorder

New non-ergodic regime

- symmetry prohibits conventional MBL
- different entanglement pattern, scaling

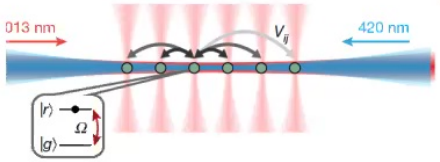
$$S_{\mathrm{ent}}(L) \sim \log(L)$$



Vasseur et al'17-, Protopopov, DA+Demler, Scardicchio group'18-

Non-ergodicity w/o disorder

Bernien et al'17

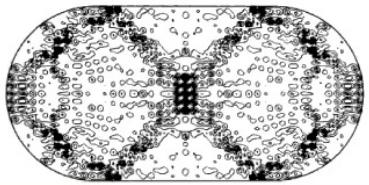


Rydberg atom simulator

- non-thermalising initial states
- wave function revivals

Turner et al'18; Ho et al'18

Quantum many-body scars



Prethermalization

Metastable non-thermal states

DA, Ho, Roeck, Huveneers'15-; Else, Bauer, Nayak'17-, Fendley et al'16-

Non-Abelian symmetries prohibit MBL

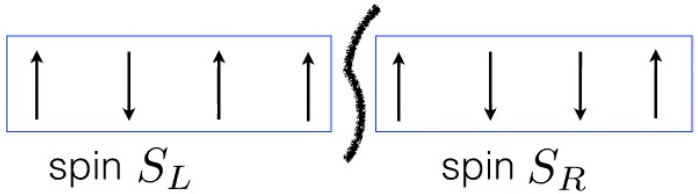
Vasseur et al'14-16, Protopopov, DA'16

Disordered Heisenberg model $H = \sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1}$

$SU(2)$ symmetry

$\downarrow$

multiplets



degeneracy $2S_L + 1$ $\quad$ $2S_R + 1$

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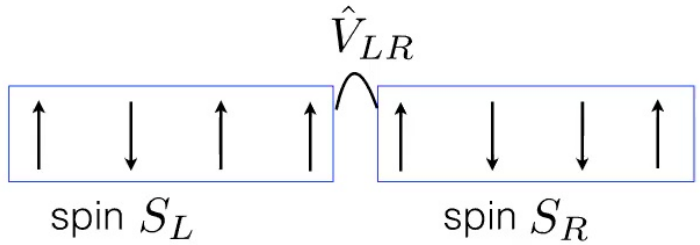


degeneracy $2S_L + 1 \quad \times \quad 2S_R + 1$

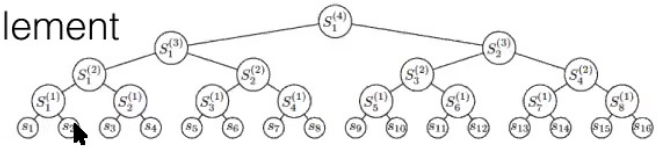
Add spins $|S, M\rangle = \sum_{M_L + M_R = M} C_{SM}^{S_L M_L; S_R M_R} |S_L M_L\rangle \otimes |S_R M_R\rangle$

Minimum amount of entanglement $S_{\text{ent}} \gtrsim \log L$ $\quad L$ system size

Some integrals of motion become non-local → no conventional MBL

A new non-ergodic phase?

Tree-like eigenstates, low entanglement



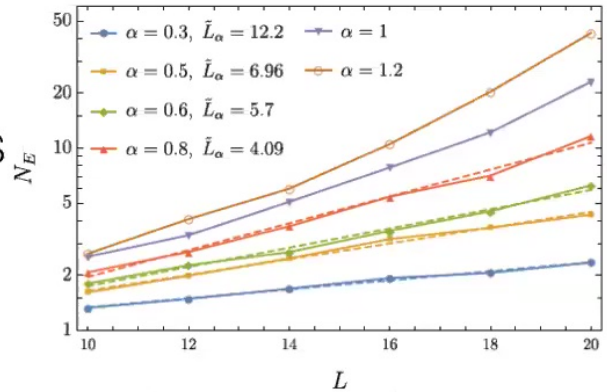
$$H = \sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1} \qquad P(|J|) \sim \frac{1}{|J|^{1-\alpha}}$$

$\alpha \ll 1$ strong disorder

$\alpha \approx 1$ weak disorder

Are trees close to real eigenstates?

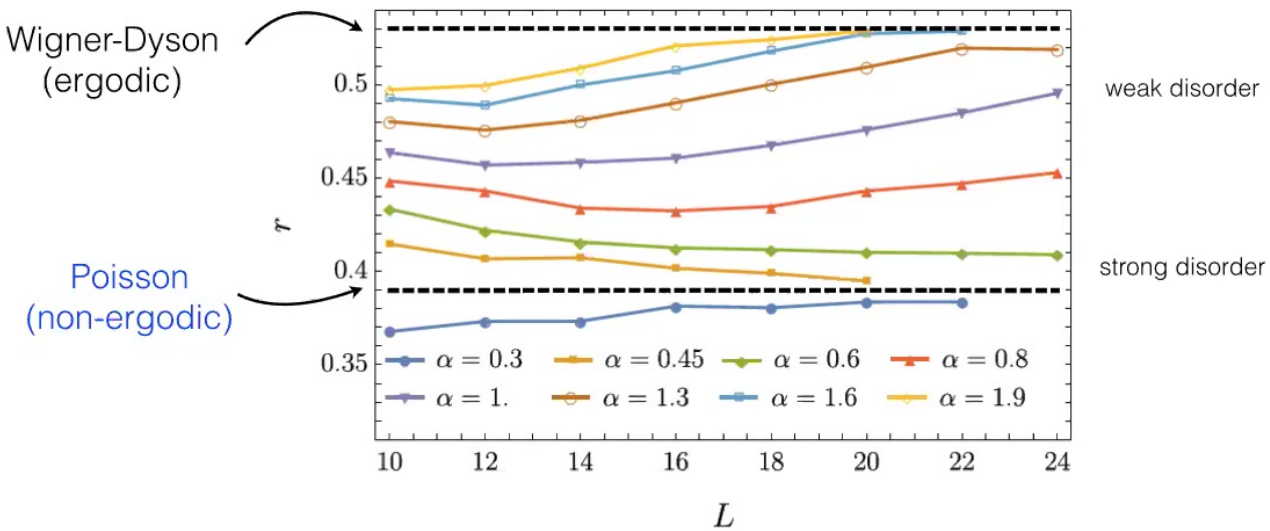
Participation ratio is small —> YES



Protopopov, Scardicchio group, Demler, DA, arXiv:1902.09236

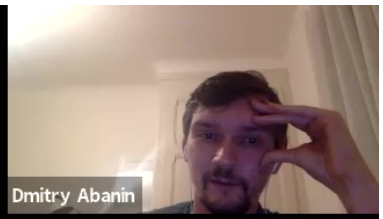
A new non-ergodic phase?

An apparent phase transition in level statistics



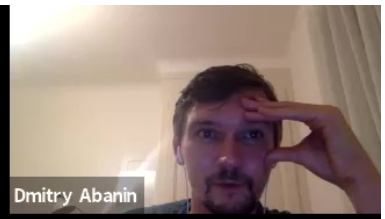
Consistent with other quantities (entanglement, ETH, etc)

Protopopov, Scardicchio group, Demler, DA, arXiv:1902.09236

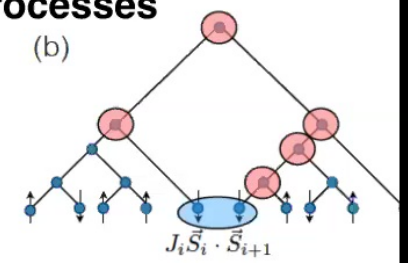


(In)stability due to multi-spin processes

Protopopov, Scardicchio group, Demler, DA, arXiv:1902.09236



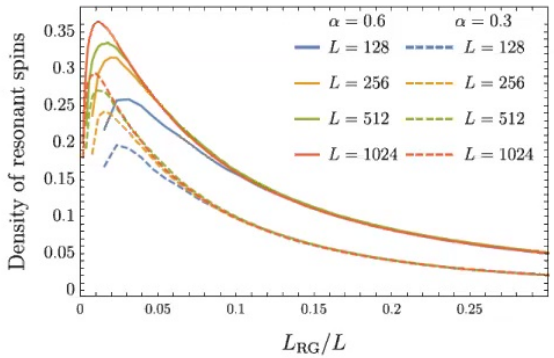
-A new approach: real-space RG+**many-body processes**

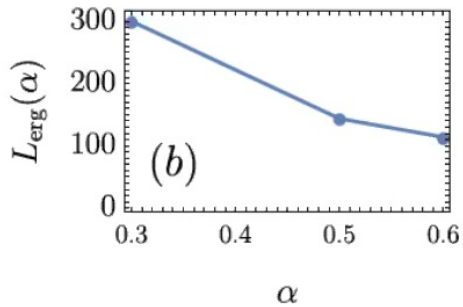


Accessible system sizes: $L \gtrsim 1000$

vs $L \approx 20$ *with exact diagonalization*

-Density of resonant spins grows as RG proceeds





<u>A broad new regime of non-ergodic dynamics</u>

Eventual thermalisation at large lengths/times

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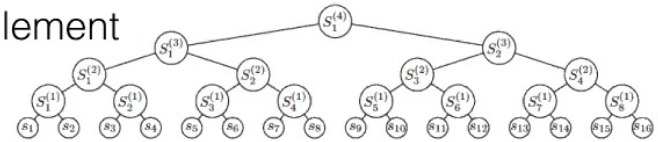

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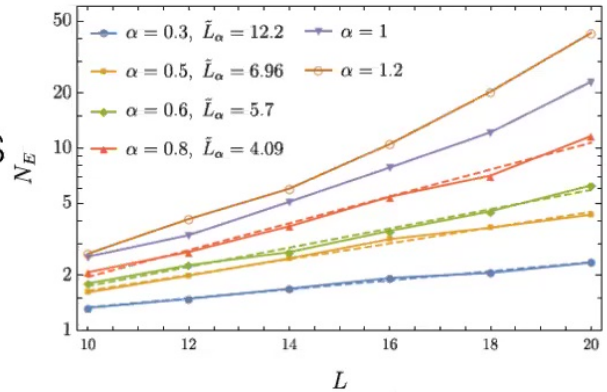

Protopopov, Scardicchio group, Demler, DA, arXiv:1902.09236

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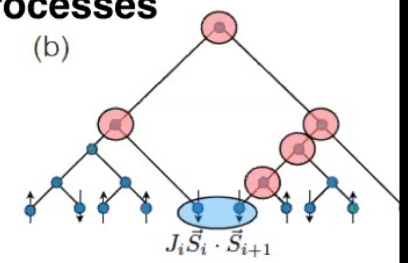

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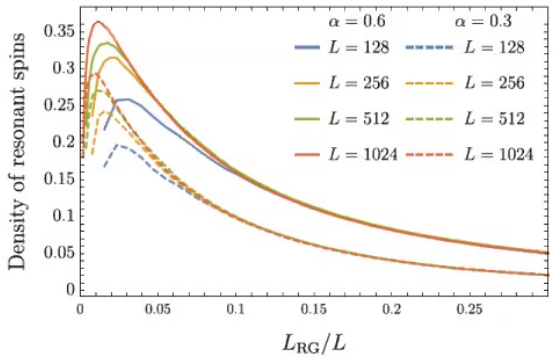

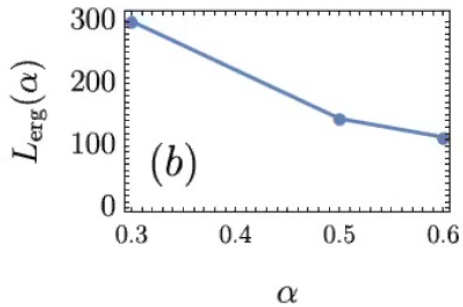

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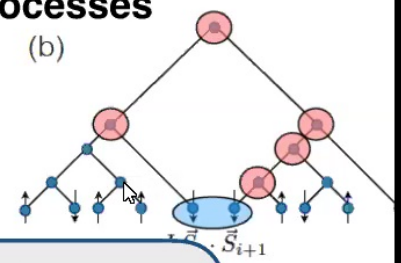

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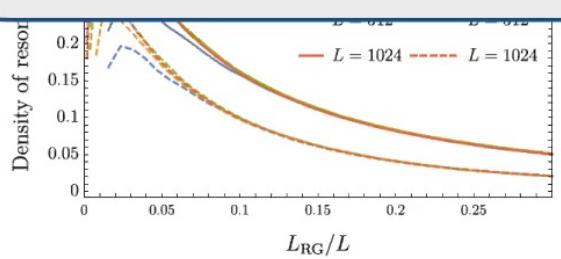
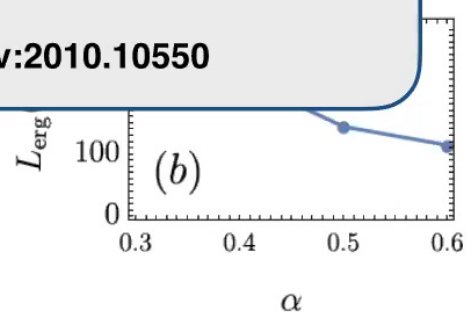
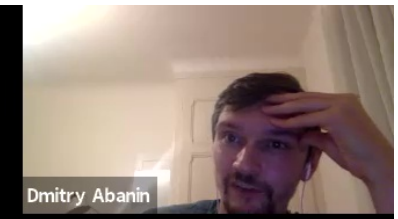
-D

Recently: extended to discrete non-Abelian groups

Ware, DA, Vasseur, arXiv:2010.10550







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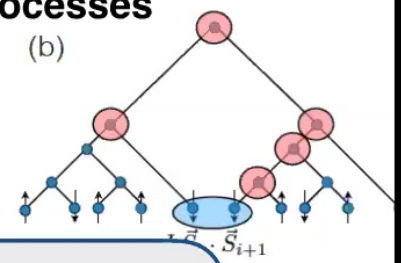

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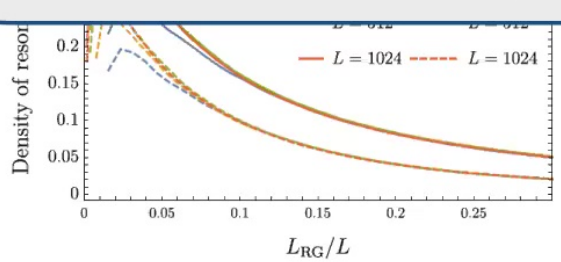
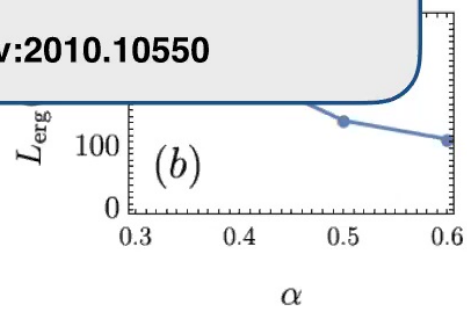
vs $L \approx 20$ *with exact diagonalization*

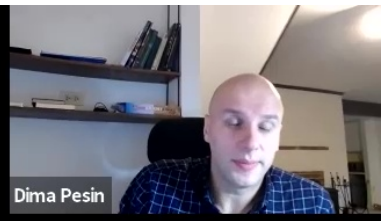
-D

Recently: extended to discrete non-Abelian groups

Ware, DA, Vasseur, arXiv:2010.10550







<u>A broad new regime of non-ergodic dynamics</u>

Eventual thermalisation at large lengths/times

Influence matrix approach to quantum dynamics

arXiv:2009.10105

Current approaches to non-equilibrium dynamics

-Construct eigenstates: perturbative (MBL) or exact (scars)

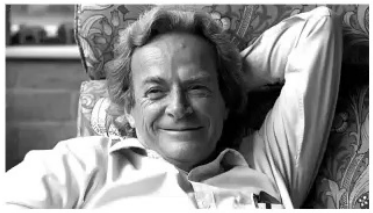
-Exact diagonalization (small system, long times)

-Tensor networks: time evolution, e.g. time-evolving block decimation; limited by spatial entanglement

Properties of a system as a quantum bath? New approaches?

Characterising a quantum bath by influence functional

ANNALS OF PHYSICS: **24**, 118–173 (1963)



The Theory of a General Quantum System Interacting with a Linear Dissipative System

R. P. FEYNMAN

California Institute of Technology, Pasadena, California

AND

F. L. VERNON, JR.*

Idea: integrate out bath (oscillators), obtain non-local action for system

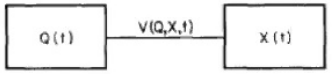


FIG. 1. General quantum systems Q and X coupled by a potential $V(Q, X,$

Effect of the bath: "influence functional"
Generating function of quantum noise

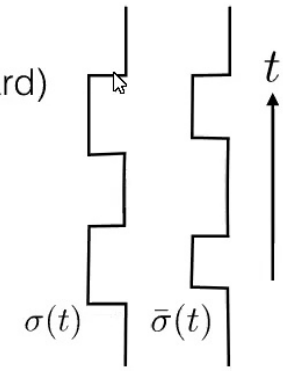
$$\mathfrak{F}(Q, Q') = \int \chi_f^*(X_T)\chi_f(X'_T) \cdot \exp\{(i/\hbar)[S(X) - S(X') + S_I(Q, X) - S_I(Q', X')]\} \times \chi_i^*(X'_\tau)\chi_i(X_\tau)\, dX_\tau\, dX'_\tau \mathfrak{D}X(t)\mathfrak{D}X'(t)$$

Influence functional (IF): intuition

Trajectory of a quantum spin $\sigma(t), \bar{\sigma}(t)$ (forward and backward)

Classical trajectories: $\sigma(t) = \bar{\sigma}(t)$

Quantum trajectories (interference contributions) $\sigma(t) \neq \bar{\sigma}(t)$



Example: Interaction with a classical noise

$$\delta H = \xi(t)\sigma^z(t)$$

$$\langle \xi(t)\xi(t')\rangle \propto F(t-t')$$

Influence functional:

$$\mathcal{I}(\sigma(t), \bar{\sigma}(t')) = e^{-\int dt dt' (\sigma(t)-\bar{\sigma}(t)) \boxed{G(t-t')} (\sigma(t')-\bar{\sigma}(t'))}$$

Intuition: "measurement" (by classical noise, or quantum bath) suppresses classical trajectories

Self-consistent influence functional for many-body dynamics

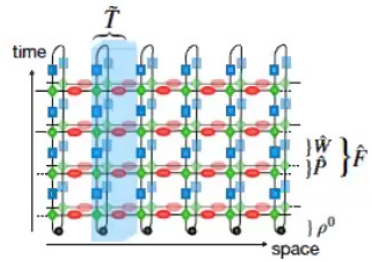
Floquet spin systems

Kick

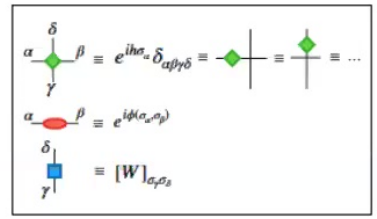
$$\hat{W} = e^{i\epsilon\sigma_x} e^{ih\sigma_z}$$

Interaction:

$$\prod_i e^{iJ\sigma_i^z \sigma_{i+1}^z}$$



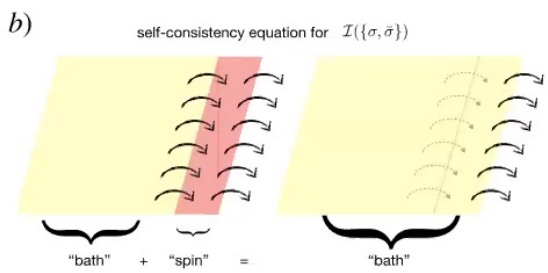
Path integral evolution (Keldysh)

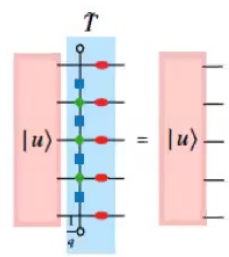


Model

Idea: Self-consistency equation for influence matrix

Lerose, Sonner, DA, arXiv'20





In tensor-network language

Perfect dephaser: exactly solvable chaotic dynamics

Simplest form of thermalising influence matrix (IM):

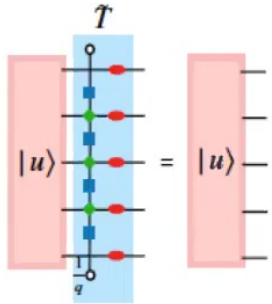
$$\mathcal{I}(\{s,\bar{s}\}) = \prod_{\tau=1}^{T-1} \delta_{s^\tau \bar{s}^\tau}$$

Spin "measured" by the system at each step

"Perfect dephaser"

Solution of the self-consistency equation for

$$|\epsilon| = |J| = \pi/4$$

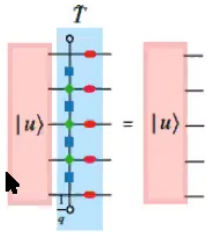


This model is chaotic (e.g. level repulsion)

New way to find solvable models of quantum dynamics

Away from solvable points: generic thermalising models

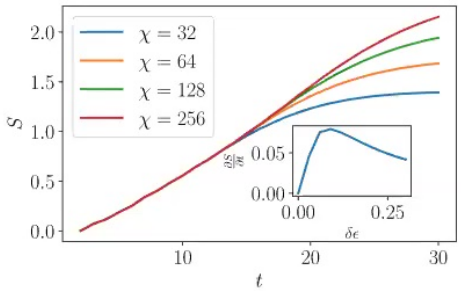
View influence matrix as a “wave function” (in time)



At solvable points, it is a product state!

Detune —> low temporal entanglement

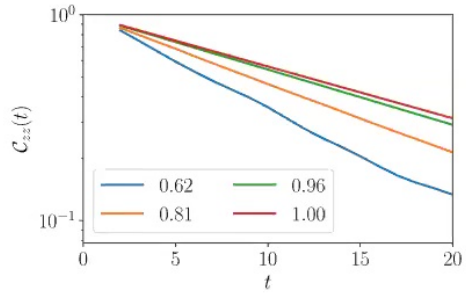
Matrix-product states efficient



Possible to describe correlation functions

Lerose, Sonner, DA, arXiv’20

temporal entanglement scaling



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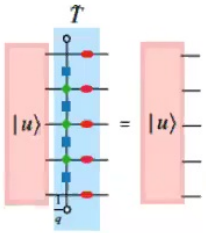

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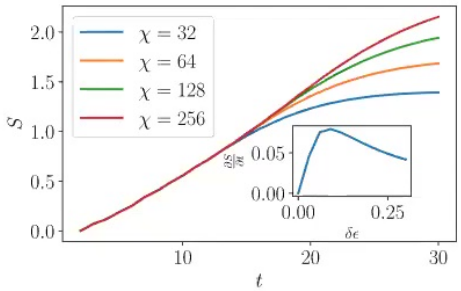

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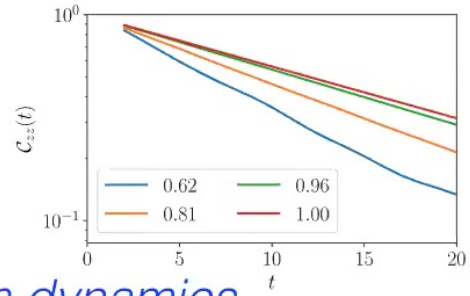
Further ideas: classes of self-consistent IM - universality classes of quantum dynamics?



A new approach to quantum dynamics

Summary

Many-body localisation: integrability, ergodicity breakdown
MBL protects coherence, enables new driven phases of matter

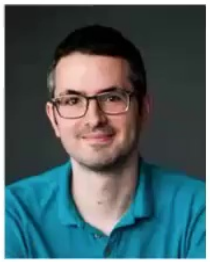
Quantum scars, non-Abelian symmetries

Influence matrix: a new approach to quantum dynamics

No thermalisation→new dynamical phases
New ways to control quantum matter

Acknowledgements

Maks Serbyn
(IST Vienna)



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Collaborators: Wen Wei Ho (Stanford), W. De Roeck (Leuven), M. Lukin (Harvard), A. Scardicchio (Trieste), E. Altman (Berkeley), I. Bloch (Munich), S. Choi (Berkeley), H. Pichler (Innsbruck), E. Demler (Harvard), M. Rudner (Copenhagen), F. Nathan (Copenhagen), E. Berg (Weizmann), N. Lindner (Technion), …

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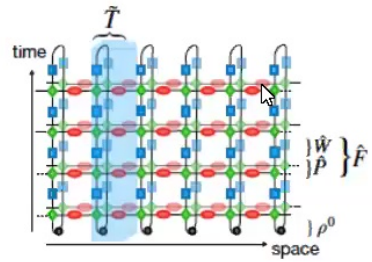

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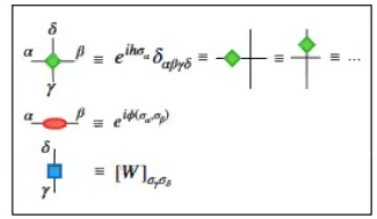

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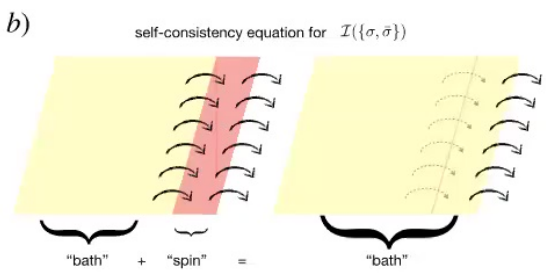

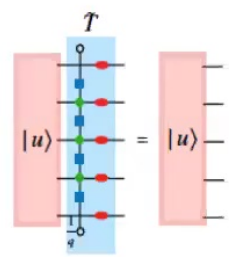

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