Title: The quasi-particle picture and its breakdown after local quenches in conformal field theories

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Abstract: We discuss the dynamics of (RÃ©nyi) mutual information, logarithmic negativity, and (RÃ©nyi) reflected entropy after exciting the ground state by a local operator in (1+1)d conformal field theories. In particular, we contrast "integrable" and "chaotic" conformal field theories, by looking at the quasi-particle picture and its possible breakdown. In comparing the calculations in the two classes of theories, we are able to identify the dynamical mechanism for the breakdown of the quasi-particle picture in 2D conformal field theories. Intriguingly, we also find preliminary evidence that our general lessons apply to quantum systems considerably distinct from conformal field theories, such as integrable and chaotic spin chains, suggesting a universality of entanglement dynamics in non-equilibrium systems.

The quasi-particle picture and its breakdown after global/local quenches in conformal field theories

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October 19, 2020

Introduction

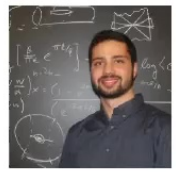
- Quantum many-body systems can exhibit rich and complicated dynamics, e.g., in quantum quench
- Roughly speaking, we can distinguish integrable and chaotic dynamics. This is so for a few body systems, but this distinction persists to many-body quantum systems and quantum field theories.
- Thermalization, quantum information scrambling are related to (signature of) chaotic dynamics.
- Unambiguously diagnosing chaoticity of many-body quantum systems remains a challenge; OTOC, spectral form factor, etc.
- Today, we will discuss integrable and chaotic dynamics in field theory context.

Introduction

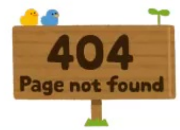
- My talk today is based on arXiv:1812.00013, arXiv:1906.07639, arXiv:1910.14575, arXiv:2001.05501, arXiv:2005.14243, arXiv:2008.11266.

- and in collaborations with:

Mao-Tian Tan
(Chicago)

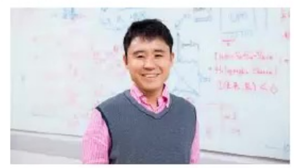


Jonah Kudler-Flam
(Chicago)



Yuya Kusuki
(Kyoto)

Laimei Nie
(Chicago→UIUC)

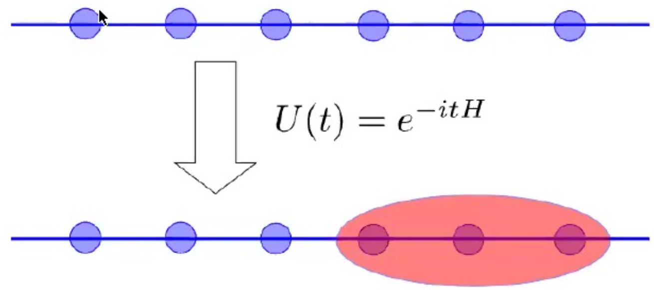


Masahiro Nozaki
(Chicago→RIKEN)

Thermalization and scrambling

- Thermalization is ubiquitous even in isolated quantum systems

$$|\Psi_{init}\rangle = |\uparrow\uparrow\cdots\rangle$$



$$U(t) = e^{-itH}$$

$$\rho_A \sim e^{-\beta_{\text{eff}} \int_A dx h(x)}$$

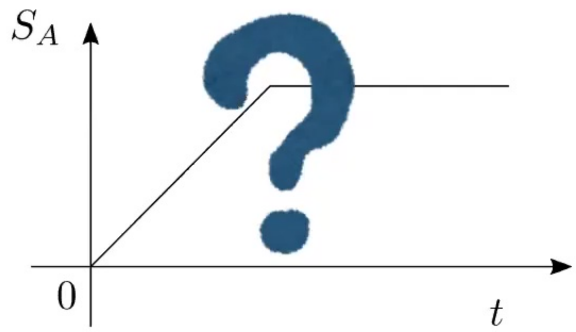
- Memory of initial states is lost, or inaccessible locally. Scrambling: (fast) delocalization of quantum information. [Hayden-Preskill, Sekino-Susskind, ...]
- Can happen for sufficiently complex dynamics

Entanglement as a probe

- Time evolution of von Neumann or Rényi entropy

$$S_A = -\mathrm{Tr}\,\rho_A \log \rho_A, \quad S_A^{(2)} = -\log \mathrm{Tr}\,\rho_A^2$$

after quantum quench: $|\Psi(t)\rangle = e^{-iHt}|\Psi_{init}\rangle$



- Eigen state thermalization, many-body localization, etc.
- Experimentally accessible

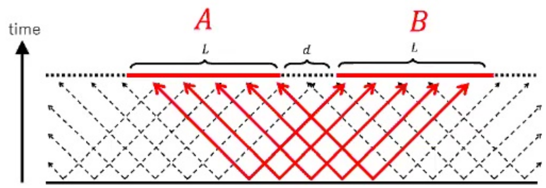
Systems and quantities of our interest

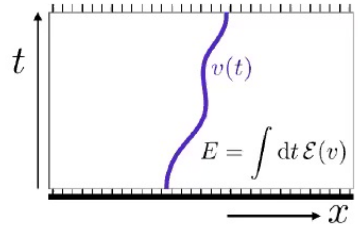
- Will study quantum quench in CFTs in (1+1) dimensions
  Infinite conformal symmetry, yet we have varieties of systems:
  - Rational CFTs (e.g., free fermion) finite number of "tower of states" (primary fields),
  - Pure CFT; $c > 1$, no extra current than the Virasoro current. In $c \to \infty$, may allow semiclassical holographic gravity description
- To distinguish these theories, we need more sophisticated probe than single-interval entanglement entropy.
- Interested in negativity $\mathcal{E}$ and reflected entropy S_R after (global/local) quantum quench

Two paradigms

- Interested in effective descriptions of entanglement propagation. C.f. "Hydrodynamics"
- There are two canonical models for entanglement propagation:

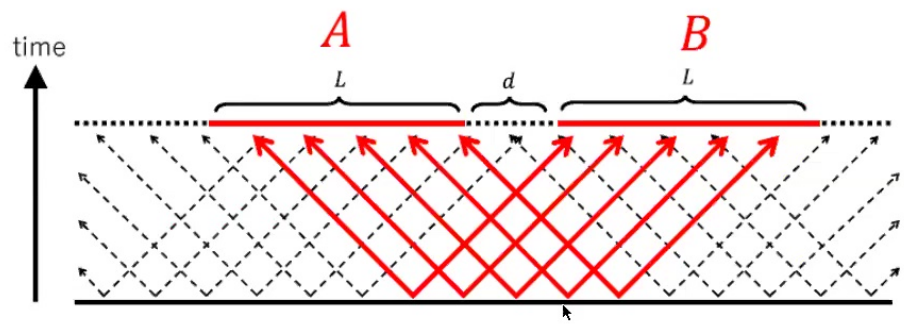
Quasi-particle picture v.s Membrane picture



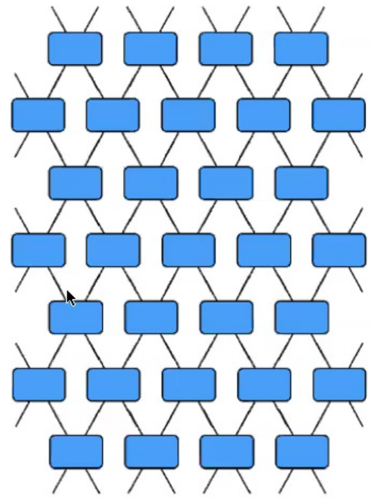


[Figures: Jonay-Huse-Nahum (18)]

Quasi particle picture



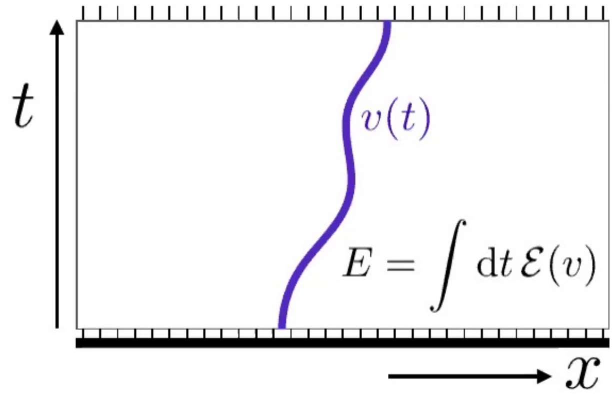
Random unitary circuit and line tension picture



- Toy model for quantum information spreading

  [Nahum-Ruhman-Vijay-Haah (16-18); Khemani-Vishwanath-Huse (18); Keyserlingk-Rakovszky-Pollmann-Sondhi (17); Zhou-Nahum (18); Joney-Huse-Nahum (18), ...]

Line-tension picture



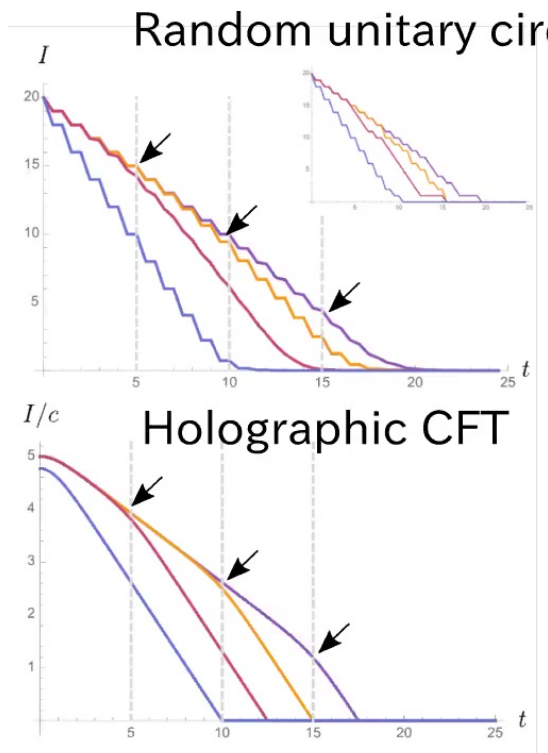
- A picture for generic non-integrable models, supported by some exact results and numerics
- Entanglement is given by minimal "energy" of a curve with tension $\mathcal{E}(v)$ ($v = dx/dt$ is the velocity of the curve).

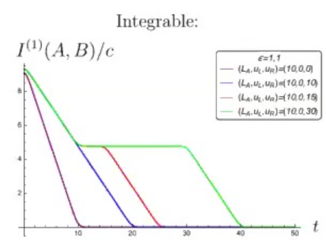
$$\mathcal{E}(v, x, t) = \begin{cases} \log q & v < 1 \\ v \log q & v > 1 \end{cases}$$

[Joney-Huse-Nahum (18),...]

Operator entanglement and line-tension picture

[Nie-Tan-Nozaki-SR (18)] [Kudler-Flam-Tan-Nozaki-SR (19)]





Outline

- In this talk, we will discuss various entanglement measures in (global/local) quantum quench [arXiv:2001.05501, arXiv:2008.11266]
- Mutual information, negativity, reflected entropy, and odd entropy.
- These are correlation measures for mixed quantum states.
- The latter three are related to entanglement wedge cross sections.
- What can we learn about dynamics from these different entanglement measures?
-

Setup (Global quench)

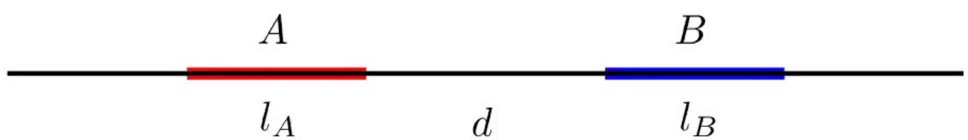
- Global quantum quench.

$$|\Psi(t)\rangle = e^{-itH}|\Psi_{init}\rangle \quad \text{with} \quad |\Psi_{init}\rangle = e^{-\beta H/4}|B\rangle$$

where $|B\rangle$ is a conformal boundary state [Calabrese-Cardy (05)]

- Mutual information, negativity, reflected entropy for two disjoint intervals A and B:

$$\rho_{AB} = \mathrm{tr}_{\overline{AB}}\, |\Psi(t)\rangle\, \langle\Psi(t)|\,,$$



Setup (Local operator quench)

- Time-dependent pure state prepared by inserting a local Virasoro primary operator on the vacuum [Nozaki-Numasawa-Takayanagi (14)]

$$|\Psi(t)\rangle = \sqrt{\mathcal{N}} e^{-\epsilon H - iHt} O(0) |0\rangle \,,$$

where $\mathcal{N}$ is the normalization and ϵ is a UV regulator.

- Mutual information, negativity, reflected entropy for two disjoint intervals A and B:

$$\rho_{AB} = \mathrm{tr}_{\overline{AB}} |\Psi(t)\rangle \langle \Psi(t)| \,,$$

Quantities of our interest (i)

- The **Rényi mutual information**:

$$I^{(n)}(A:B) \equiv S^{(n)}(A) + S^{(n)}(B) - S^{(n)}(A \cup B)$$

where $S^{(n)}(\Omega)$ is the Rényi entropy of the reduced density matrix ρ_Ω

$$S^{(n)}(\Omega) \equiv \frac{1}{1-n} \log \mathrm{Tr} \rho_\Omega^n.$$

Quantities of our interest (ii)

- The **logarithmic negativity**:

$$\mathcal{E}(A:B) \equiv \log \left|\rho_{AB}^{T_B}\right|_1$$

  where $|\cdot|_1$ is the trace norm and $\cdot^{T_B}$ is the partial transpose operation.

- Partial transpose M^{T_B} of an operator M:

$$\langle e_i^{(A)} e_j^{(B)} | M^{T_B} | e_k^{(A)} e_l^{(B)} \rangle := \langle e_i^{(A)} e_l^{(B)} | M | e_k^{(A)} e_j^{(B)} \rangle$$

  where $|e_i^{(A,B)}\rangle$ is the basis of $\mathcal{H}_{A,B}$.

- Detect only quantum correlations (positive partial transpose criterion).
- Monotone under LOCC

Quantities of our interest (iii)

- The **Rényi reflected entropy** is the Rényi entropy of a "canonical purification" [Dutta-Faulkner (19)]

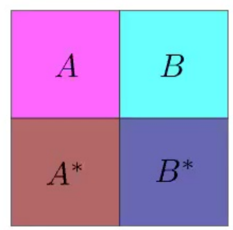
$$S_R^{(n)}(A:B) = S^{(n)}(AA^*),$$

where the purification

$$\rho_{AB} = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|_{AB}$$

$$\to \left|\sqrt{\rho_{AB}}\right\rangle_{AA^*BB^*} \equiv \sum_i \sqrt{p_i} \left|\psi_i\right\rangle_{AB} \left|\psi_i^*\right\rangle_{A^*B^*}$$

is defined on a doubled Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_{A^*} \otimes \mathcal{H}_B \otimes \mathcal{H}_{B^*}$



- Captures both quantum and classical correlations and satisfies $I(A:B) \leq S_R(A:B) \leq 2\min\left[S(A), S(B)\right]$.

Relation between measures

- Alba-Calabrese (18) derived simple relations among correlation measures assuming the quasi-particle picture.

$$\mathcal{E} = \frac{1}{2} I^{(1/2)}.$$

- Similarly for the reflected entropy [Kudler-Flam-Kusuki-SR] ,

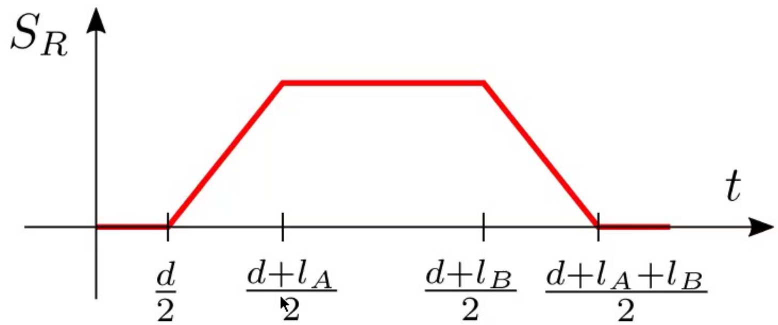
$$S_R^{(n)} = I^{(n)}$$

- In chaotic theories, these relations are expected to break down.

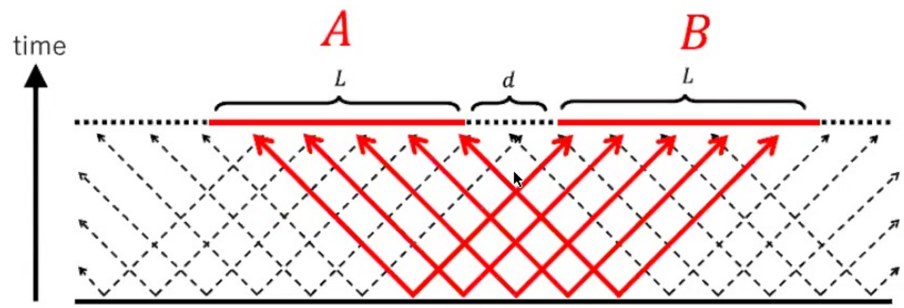
Results

- Results for global quench
  - Rational CFTs
  - Pure CFTs
- Results for local operator quench:
  - Rational CFTs
  - Pure CFTs

Result: rational CFTs



- Completely described by quasi-particle picture



time
A
B
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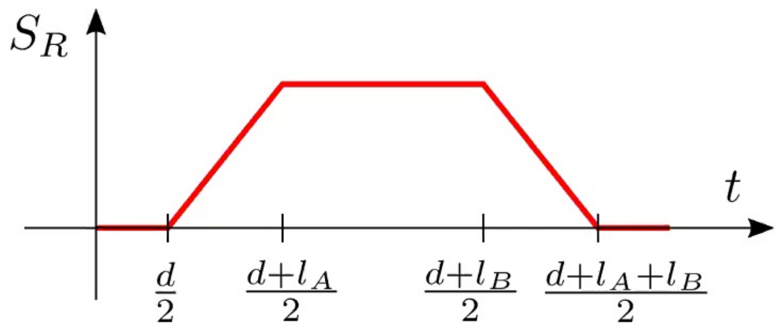
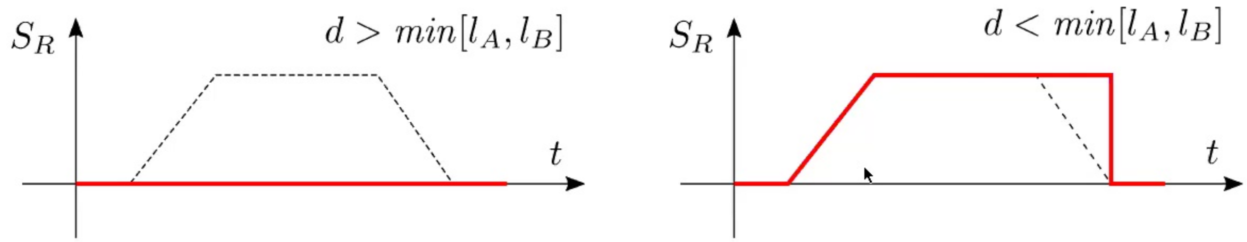
Result: rational CFTs



- Completely described by quasi-particle picture
- Rational CFTs have a finite number of primary fields, so they are sensitive to the ligh-cone singularity, giving the universal answer (= quasi-particle picture).

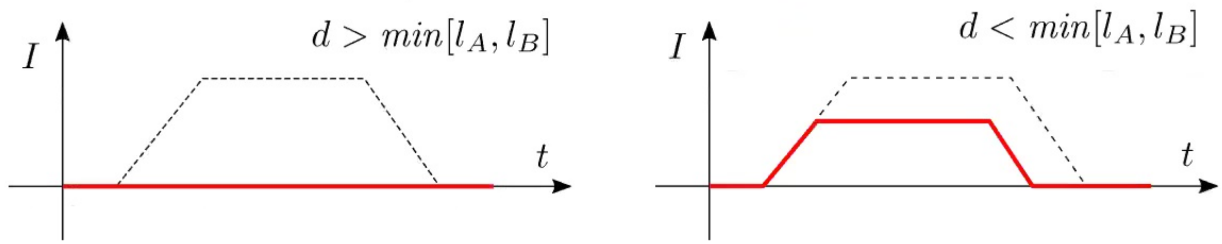
Result: pure CFTs



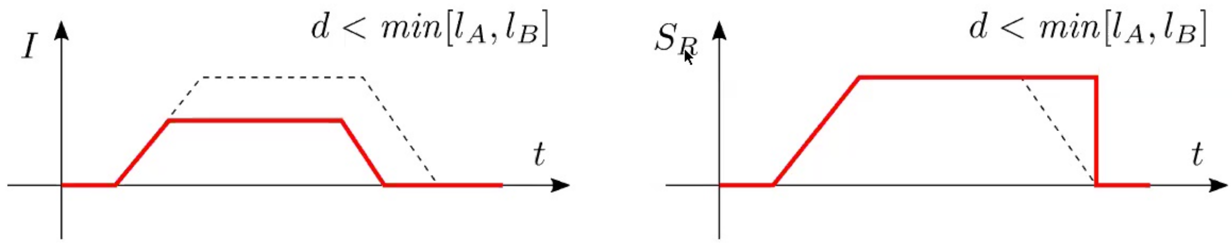
- Strong violation of the quasi-particle picture (Entanglement cannot be carried by quasi-local objects)

C.f. Mutual information (MI)

- For $d > min[l_A, l_B]$, MI $I_{AB} = S_A - S_B + S_{A \cup B}$ vanishes for sufficiently chaotic systems [Asplund et al. (15)]
- Even for $d < min[l_A, l_B]$, the quasi-particle picture is strongly violated, and information is rapidly scrambled



- MI for pure CFT is smaller than that for RCFT. For chaotic system, after $t = l_A/2$, part of the entanglement (as predicted from the quasi-particle picture) disappears. ("Missing entanglement")



- For $d > l_A, l_B$, the negativity/reflected entropy behave similarly to I. For $d < l_A, l_B$, we may have new information.
- While I is smaller in pure CFT, S_R does not show missing correlation, even after $t = l_A/2$, and develops mysterious correlation (Right).
- S_R is expected to be more sensitive to classical correlation than I. So, one may think mysterious correlation = classical correlation. Note: generic bound $S_R \geq I$.

Alba-Calabrese relation (global quench)

- Four point functions (and hence correlation measures) have universal and theory-dependent parts:

$$\langle \sigma_n(z_1)\bar{\sigma}_n(z_2)\sigma_n(z_3)\bar{\sigma}(z_4)\rangle_{\mathrm{UHP}}$$
$$= \frac{1}{\prod_{a=1}^{4}|z_a - \bar{z}_a|^{\Delta_n}} \frac{1}{\eta_{12}^{\Delta_n}\eta_{34}^{\Delta_n}} \left(\frac{\eta_{14}\eta_{23}}{\eta_{13}\eta_{24}}\right)^{-\Delta_n} \mathcal{G}(\{\eta_{jk}\})$$

where $\eta_{ij} = (z_i - z_j)(\bar{z}_i - \bar{z}_j)/(z_i - \bar{z}_j)(\bar{z}_i - z_j)$

- Focusing on the universal part, we recover the Alba-Calabrese relation:

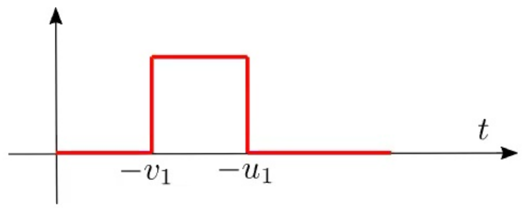
$$\mathcal{E} = -\frac{c}{8}\log\left(\frac{\eta_{14}\eta_{23}}{\eta_{13}\eta_{24}}\right), \quad I^{(n)} = S_R^{(n)} = -\frac{c(n+1)}{12n}\log\left(\frac{\eta_{14}\eta_{23}}{\eta_{13}\eta_{24}}\right),$$

Local operator quench

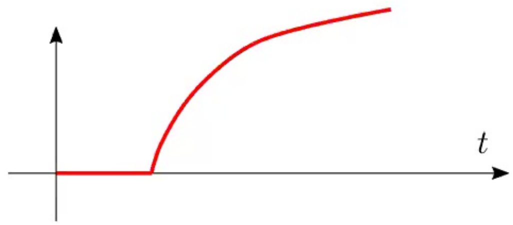
- For RCFT: square wave with hight given by quantum dimension:

$$\Delta I^{(n)}(A:B)[O]_{max} = 2\Delta\mathcal{E}(A:B)[O]_{max} = 2\log d_O$$

[He-Numasawa-Takayanagi-Watanabe (2014); Kusuki-Tamaoka (2019); Kudler-Flam-Kusuki-SR (20)]



- For pure CFT, logarithmic growth:



[Nozaki-Numasawa-Takayanagi (13); Caputa-Nozaki-Takayanagi (14); Asplund-Bernamonti-Galli-Hartman (14); Kusuki-Takayanagi (17), ...]

Local operator quench

- For RCFT: square wave with hight given by quantum dimension:

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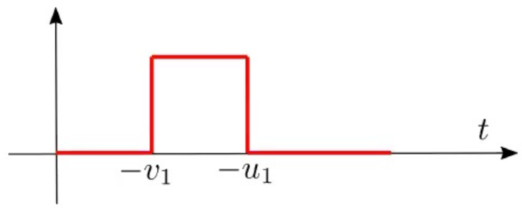

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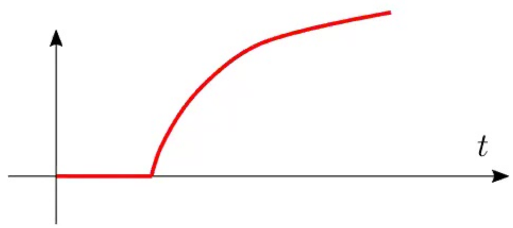
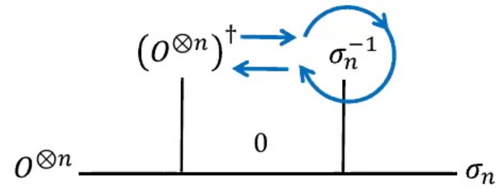


[Nozaki-Numasawa-Takayanagi (13); Caputa-Nozaki-Takayanagi (14); Asplund-Bernamonti-Galli-Hartman (14); Kusuki-Takayanagi (17), ...]

- These results can be obtained from the Regge limit. E.g. $S(A)[O]$ and $S(B)[O]$ in MI, the Regge limit of the relevant correlation functions is given by

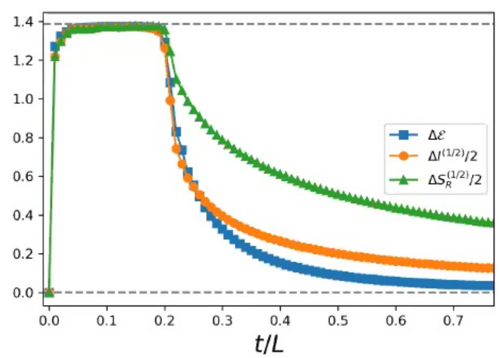


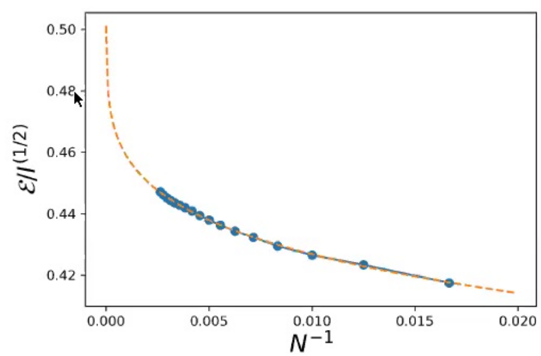
[Kusuki (2019); Kusuki-Miyaji (2019)]

- For RCFT, the monodromy matrix has a dominant contribution from the vacuum; for pure CFT, the dominant contribution comes from a non-vacuum state.

Free fermion chain

- Studied the free fermion chain $\hat{H} = (1/2)\sum_i^N \hat{c}_i^\dagger \hat{c}_{i+1} + h.c.$ Used adjacent intervals and inserted the operator at their interface.
- $\mathcal{E}$, $I^{(1/2)}/2$, and $S_R^{(1/2)}/2$ are nearly identical at early times, but diverge once the local quasi-particle excitation has left the subsystems.
- Finite scaling analysis in Fig. 30 to confirm that this late-time behavior of the quantities converges in the scaling limit.





Spin chains

- Computed the negativity and Rényi mutual information in the transverse field Ising model

$$H = \sum_i (-Z_i Z_{i+1} + gX_i + hZ_i).$$

- We probe integrable dynamics by taking $g = 1.0, h = 0$ and chaotic[1] dynamics by taking $g = -1.05, h = 0.5$.
- We simulate 31 lattice sites with 5 sites separating "semi-infinite" subsystems. The local operator used is a Pauli Z.

[1] Here, by chaotic, we mean that the level statistics of the Hamiltonian mimic those of random matrix theory.

- Integrable spin chain (Left): rapid growth and saturation, reminiscent of the RCFT picture (quasi-particles produces a step function in the correlation measures), but with non-conformal dispersion (and finite size effect).
- Chaotic spin chain (Right): a logarithmic growth, similar to the pure CFTs.