Title: Electrodynamics of Thin Sheets of Twisted Material

Speakers: Dam Thanh Son
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Abstract:

We construct a minimal theory describing the optical activity of a thin sheet of a twisted material, the simplest example of which is twisted bilayer graphene. We introduce the notion of "twisted electrical conductivity" which parametrizes the parity-odd response of a thin film to a perpendicularly falling electromagnetic wave with wavelength larger than the thickness of the sheet, and relate the chiral response to this kinetic coefficient. We show that the low-frequency Faraday rotation angle has different behaviors in different phases: \omega^2 for insulators and \omega^0 for superconductors. In both cases the frequency dependence of the Faraday rotation angle can be obtained from a simple power counting in an effective field theory. In the metallic state, the twisted conductivity is proportional to the "magnetic helicity" (scalar product of the velocity and the magnetic moment) of the quasiparticle, averaged around the Fermi surface. Many aspects of the theory are general and applicable to strongly correlated phases.

Reference: Dung X. Nguyen and DTS, arXiv:2008.02812.

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Electrodynamics of thin sheets of twisted materials

Dam T. Son (University of Chicago)

Ref: Dung X. Nguyen & DTS, arXiv:2008.02812

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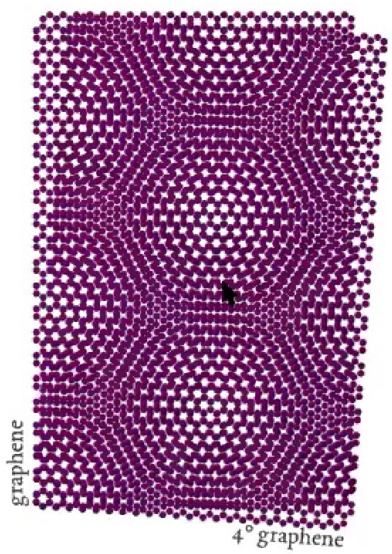
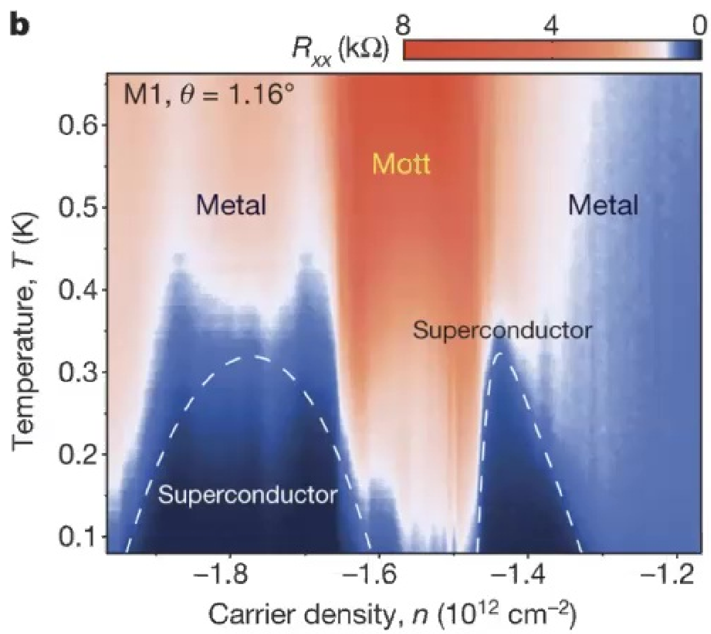
Motivation: twisted bilayer graphene





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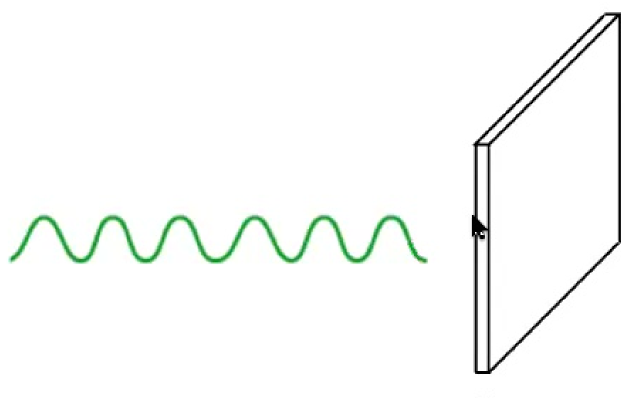
Other thin twisted layer materials

- Twisted double bilayer graphene
- Twisted trilayer graphene

- What can one learn by scattering light on a thin sheet of twisted material?



$$\lambda \gg d$$

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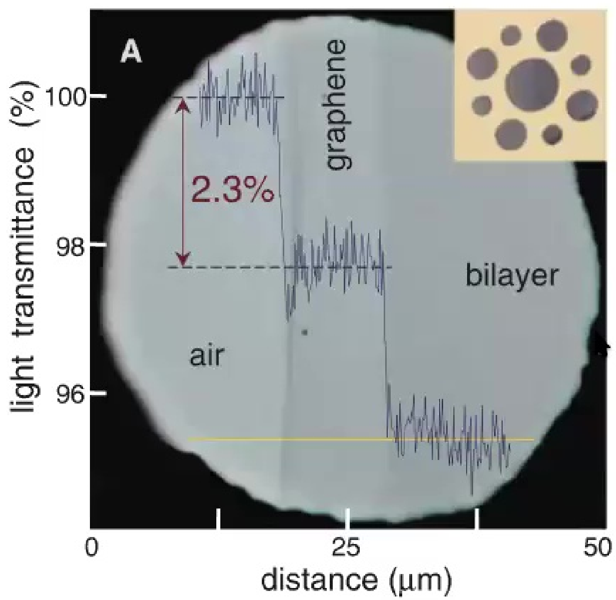
Scattering of light on single-layer graphene



$2.3\% \approx \pi\alpha$

Nair,…Geim, Science 120 (2008)

Mak… Heinz, PRL 101 (2008)

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Surface conductivity and light scattering

- Start from Maxwell equations

$$-\partial_z^2 \mathbf{A} - \frac{\omega^2}{c^2}\mathbf{A} = \frac{4\pi}{c}\mathbf{J} \qquad \mathbf{J} = \mathbf{j}_{2\mathrm{D}}\delta(z) = \sigma \mathbf{E}\delta(z)$$

~ scattering on delta-functional potential in QM
A is continuous, A' is discontinuous at z=0

$$\text{Absorption} = \frac{4\pi}{c}\sigma = \pi\alpha \qquad \sigma = \frac{\pi e^2}{2h}$$

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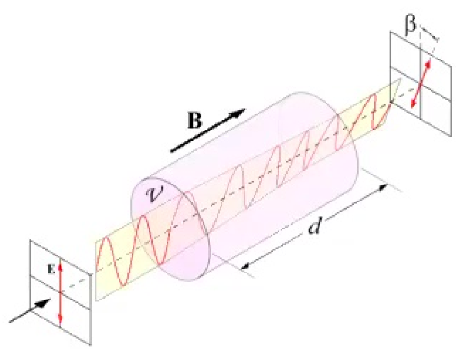
More possibilities?

- Except for electrical conductivity, can one learn more information?
- Twisted systems: optical activity



Faraday rotation

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Measures of optical activity

- Faraday rotation
- Chiral dichroism

$$A_\pm(z) = \begin{cases} e^{ikz} + R_\pm e^{-ikz}, & z < 0, \\ T_\pm e^{ikz}, & z > 0. \end{cases}$$

±: two circular polarizations

$$\theta_\mathrm{F} = \frac{1}{2} \arg \frac{T_+}{T_-} \qquad \mathrm{CD} = \frac{\mathcal{A}_+ - \mathcal{A}_-}{2(\mathcal{A}_+ + \mathcal{A}_-)}$$

Previous works

- Suárez Morell, Chico, Brey 2017
- Stauber, Low, Gómez-Santos 2018
- Ochoa, Asenjo-Garcia 2020
- Wang, Morimoto, Moore 2020

- We construct a simple 2D theory, then concentrate on the low-energy regime

What determines optical activity?

- The surface conductivity σ does not break spatial reflection
- Does one need to have a fully 3D treatment to understand chiral effects?
    - no, a slight extension of the 2D calculation suffices

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Symmetries

- Assume no symmetry under 2D reflection
  - $x \to x, y \to -y$
- No symmetry under exchanging two layers
  - $z \to -z$
- Symmetry under 2D reflection and exchanging two layers
  - $x \to x, y \to -y, z \to -z \; (C_{2x})$

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Symmetries

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  - $x \to x, y \to -y, z \to -z \; (C_{2x})$
- Time reversal

Minimal theory

- Defines two moments of the current

$$\mathbf{j}(x, y) = \int dz\, \mathbf{J}(x, y, z)$$

$$\zeta(x, y) = \int dz\, z\, \mathbf{J}(x, y, z)$$

↑

"electric dipole current"

For systems consisting of well-separated two layers:

$$\mathbf{j} = \mathbf{j}_1 + \mathbf{j}_2$$

$$\zeta = \frac{d}{2}(\mathbf{j}_1 - \mathbf{j}_2)$$

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“Twisted conductivity”

Entropy production

$$\dot{S} = \int dz\, \mathbf{J}(z) \cdot \mathbf{E}(z) = \mathbf{j} \cdot \mathbf{E}(0) + \boldsymbol{\zeta} \cdot \partial_z \mathbf{E}(0)$$

Generalized Ohm’s law, consistent with spatial symmetries

$$\mathbf{j} = \sigma \mathbf{E} - \sigma_1 \partial_z \mathbf{E} \times \hat{\mathbf{z}}$$

$$\boldsymbol{\zeta} = \tilde{\sigma}_1 \mathbf{E} \times \hat{\mathbf{z}} + \sigma_2 \partial_z \mathbf{E}$$

Onsager

$\sigma_1(\omega) = \tilde{\sigma}_1(\omega)$

“twisted conductivity”

We expect $\sigma_1 \sim \sigma d$

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Boundary conditions

$$-\partial_z^2 \mathbf{A} - \frac{\omega^2}{c^2}\mathbf{A} = \frac{4\pi}{c}\mathbf{J}$$

$$\int_{-\epsilon}^{\epsilon} dz \qquad -\mathbf{A}'\big|_{-\epsilon}^{\epsilon} = \frac{4\pi}{c}\mathbf{j} = \frac{4\pi i\omega}{c^2}(\sigma\mathbf{A} - \sigma_1 \mathbf{A}' \times \hat{\mathbf{z}})$$

$$\int_{-\epsilon}^{\epsilon} dz\, z \qquad \mathbf{A}\big|_{-\epsilon}^{\epsilon} = \frac{4\pi}{c}\zeta = \frac{4\pi i\omega}{c^2}(\sigma_1 \mathbf{A} \times \hat{\mathbf{z}} + \sigma_2 \mathbf{A}')$$

ambiguous boundary conditions

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- Similar ambiguity in QM with $\delta'(x)$ potential
- D. Griffiths's prescription (J.Phys.A 1993)

$$-\mathbf{A}'\big|_{-\epsilon}^{\epsilon} = \frac{4\pi}{c}\mathbf{j} = \frac{4\pi i\omega}{c^2}(\sigma\mathbf{A} - \sigma_1\mathbf{A}'\times\hat{\mathbf{z}})$$

$$\mathbf{A} \leftarrow \tfrac{1}{2}[A(\epsilon)+A(-\epsilon)] \qquad \mathbf{A}' \leftarrow \tfrac{1}{2}[A'(\epsilon)+A'(-\epsilon)]$$

Well-defined and consistent with unitarity

Results

$$T_\pm = \frac{1 \pm \frac{4\pi}{c} k\sigma_1}{1 + \frac{2\pi\sigma}{c}} \qquad R_\pm = -\frac{\frac{2\pi}{c}\sigma}{1 + \frac{2\pi}{c}\sigma}$$

$$\theta_{\rm F} = \frac{1}{2} \arg \frac{T_+}{T_-} = \frac{4\pi\omega}{c^2}\, \mathrm{Im}\, \sigma_1 .$$

(assume $k\sigma_1 \ll c$
but σ can be $\sim c$)

$$\frac{|T_+|^2}{|T_-|^2} = 1 + \frac{16\pi\omega}{c^2}\, \mathrm{Re}\, \sigma_1 .$$

More honest approach

$$j_a(z) = \int dz'\, \sigma_{ab}(z, z') E_b(z')$$

Solving the Maxwell equation in the limit $kd \ll 1$, perturbatively in k, exercise with Green's functions

$$\frac{1}{T} = \left(1 + \frac{\tilde{\sigma}}{2}\right)(1 - k\tilde{\sigma}_1) \qquad \frac{R}{T} = -\frac{\tilde{\sigma}}{2}(1 - k\tilde{\sigma}_1)$$

$$\tilde{\sigma} = \frac{4\pi}{c}\sigma$$

reproduces the results of the simplified approach (extra k correction in σ)

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Results

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Low-energy regime

- We want to understand what chiral activity reveals about the low-energy modes of the material
- Limit ourselves to low-energy photon:
  - < gap for insulator
  - < ϵ_F for metals
  - < BCS gap for superconductors
- How do the Faraday rotation angle depend on ω?

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Insulator

- Bulk action $$S = \frac{1}{8\pi}\int d^4x\,(\mathbf{E}^2 - \mathbf{B}^2)$$

"Brane" action

$$S_{\rm brane} = \beta\int d^4x\,\delta(z)\epsilon^{ab}E_a\partial_z E_b$$

$$+\#\int d^4x\,\delta(z)E^2$$

Current:

$$j^a = \frac{\delta S}{\delta A_a} = \beta\partial_t(\epsilon^{ab}\partial_z E_b)\delta(z) + \beta\partial_t[\epsilon^{ab}\partial_z(E_b)\delta(z))]$$

$$\sigma_1 \sim \beta\omega \to \theta_F \sim \beta\omega^2$$

Power counting

$$S_{\text{brane}} = \beta \int d^4x\, \delta(z) \epsilon^{ab} E_a \partial_z E_b$$

β	$\delta(z)$	E_a	∂_z	E_b
-2	1	2	1	2

Faraday rotation angle $\sim \beta\omega^2$

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Interpretation: Quadrupole moment

- Turning on E_x leads to quadrupole moment Q_{yz}

$$Q^{az} = \int dz\, d^2x\, z x^a \frac{\delta S}{\delta A_0} = -\beta S \epsilon^{ab} E_b,$$

Power counting: superconductor

- Superconductor: there is a phase φ on the brane
  - $S = \int d^4x\, \delta(z) f_s[(D_0\varphi)^2 - v_s^2(D_i\varphi)^2]$
- A new term allowed: "Lifshitz invariant"

$$S_{\text{brane}} = \alpha \int d^4x\, \delta(z)(\partial_a\varphi - A_a)B_a \qquad [\alpha] = 0$$

Faraday rotation angle ~ $\alpha\omega^0$ $\qquad E \ll \Delta_{BCS}$

Metallic phase

- In metals quasiparticles form a Fermi liquid
- consider scattering of low-energy photon ($\omega \ll \epsilon_F$), typically in the infrared range
- quasiparticles carry orbital magnetic moment
  - move in spiral-like trajectories, jumping between layers

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Metallic phase

- Quasiparticles carry orbital magnetic moment

$$E_{\mathbf{p}} = \varepsilon_{\mathbf{p}} - \boldsymbol{\mu}_{\mathbf{p}} \cdot \mathbf{B}$$

Boltzmann equation

$$\frac{\partial f_{\mathbf{p}}}{\partial t} - e\mathbf{E} \cdot \frac{\partial f_{\mathbf{p}}}{\partial \mathbf{p}} = -\frac{f_{\mathbf{p}} - f_0(E_{\mathbf{p}})}{\tau}$$

Solve the equation and put into the current

$$\mathbf{j} = -e \int_{\mathbf{p}} \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{p}} (f_0 + \delta f_{\mathbf{p}}),$$

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"Magnetic helicity"

- Solving the Boltzmann equation one finds

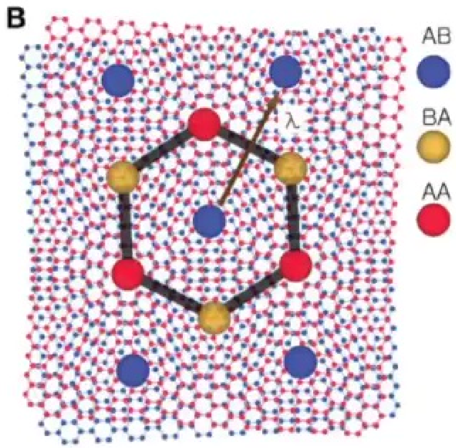
$$\sigma = \frac{1}{2}\frac{e^2\tau\nu(\varepsilon_F)}{1-i\omega\tau}\langle \boldsymbol{v}_\mathbf{p}^2\rangle \qquad \sigma_1 = -\frac{1}{2}\frac{ec\tau\nu(\varepsilon_F)}{1-i\omega\tau}\langle \boldsymbol{v}_\mathbf{p}\cdot\boldsymbol{\mu}_\mathbf{p}\rangle$$

$$\mathrm{CD} = \frac{\omega}{e}\frac{\langle \boldsymbol{v}_p\cdot\boldsymbol{\mu}_\mathbf{p}\rangle}{\langle \boldsymbol{v}_\mathbf{p}^2\rangle}$$

$$\theta_F = -\frac{2\pi e}{c}\nu(\varepsilon_F)\langle \boldsymbol{v}_\mathbf{p}\cdot\boldsymbol{\mu}_\mathbf{p}\rangle \qquad \omega\tau \gg 1$$

"magnetic helicity"

TBG: Moiré scales



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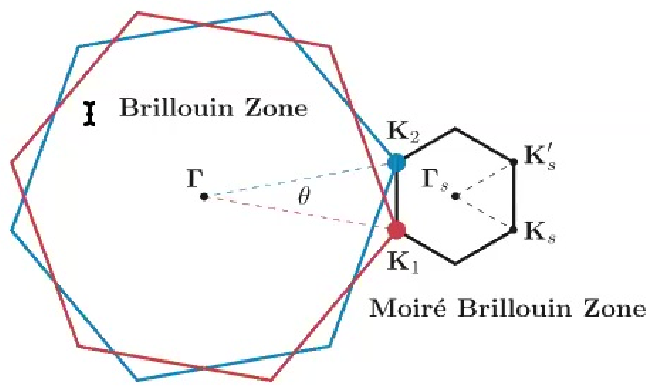


$$k_\theta = 2k_D \sin\frac{\theta}{2}$$

Continuum model

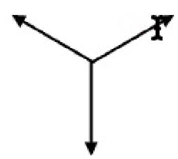
Bistritzer, MacDonald 2011

$$H = \begin{pmatrix} -iv_0 \boldsymbol{\sigma}_{\theta/2} \cdot \boldsymbol{\nabla} & T(\mathbf{x}) \\ T^\dagger(\mathbf{x}) & -iv_0 \boldsymbol{\sigma}_{-\theta/2} \cdot \boldsymbol{\nabla} \end{pmatrix}$$

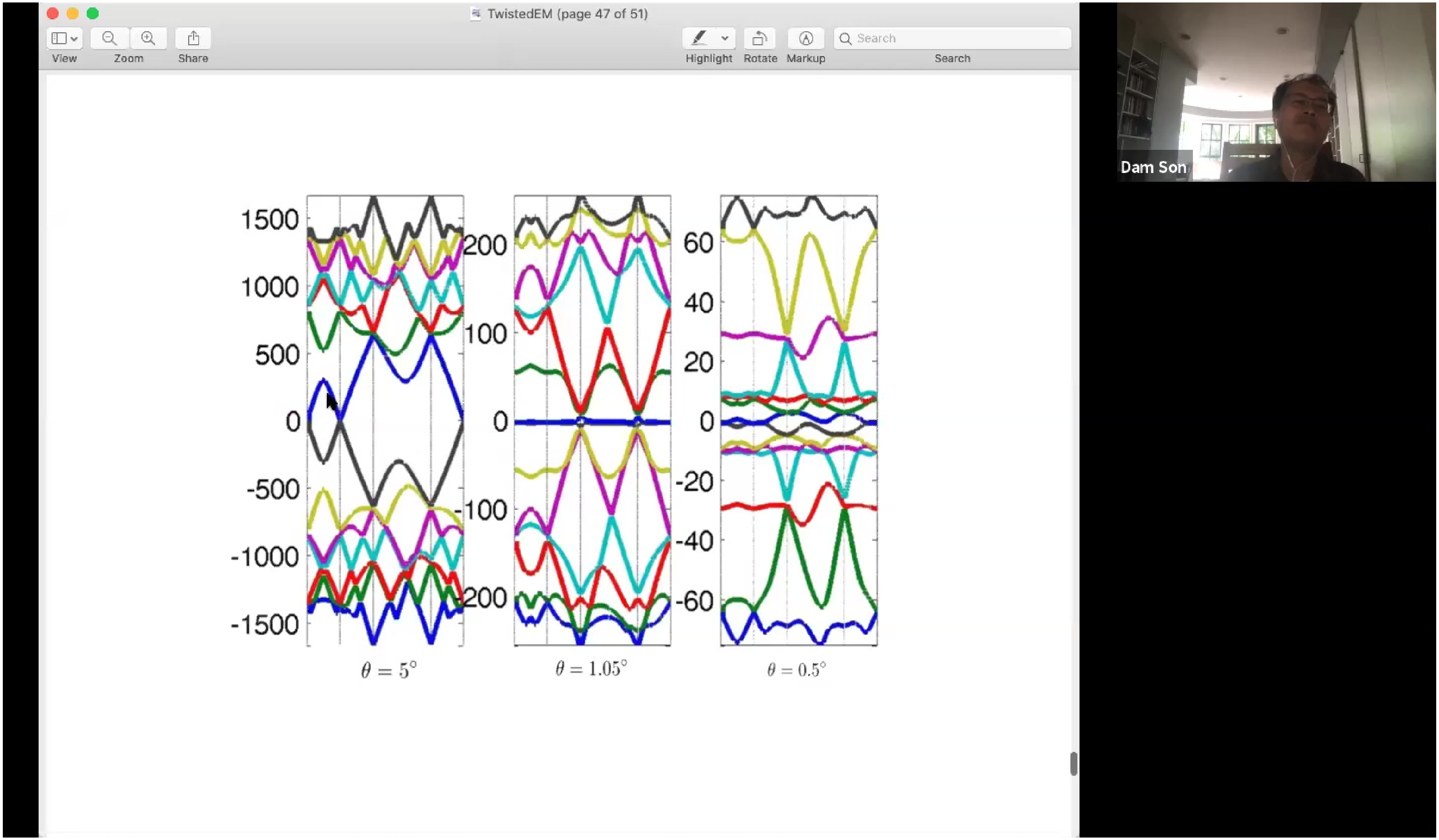
$$\boldsymbol{\sigma}_{\theta/2} = e^{-i\theta\sigma_z/4} \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix} e^{i\theta\sigma_z/4}$$

$$T(\mathbf{x}) = \sum_{a=1}^{3} T_a e^{-i\mathbf{q}_a \cdot \mathbf{x}}$$

$$T_a = w_{\mathrm{AA}} + w_{\mathrm{AB}} (\hat{\mathbf{z}} \times \hat{\mathbf{q}}_a) \cdot \boldsymbol{\sigma}$$



$$\mathbf{q}_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix} k_\theta$$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View
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Dam Son
1500
1000
500
0
-500
-1000
-1500
$\theta = 5^\circ$
200
100
0
-100
-200
$\theta = 1.05^\circ$
60
40
20
0
-20
-40
-60
$\theta = 0.5^\circ$

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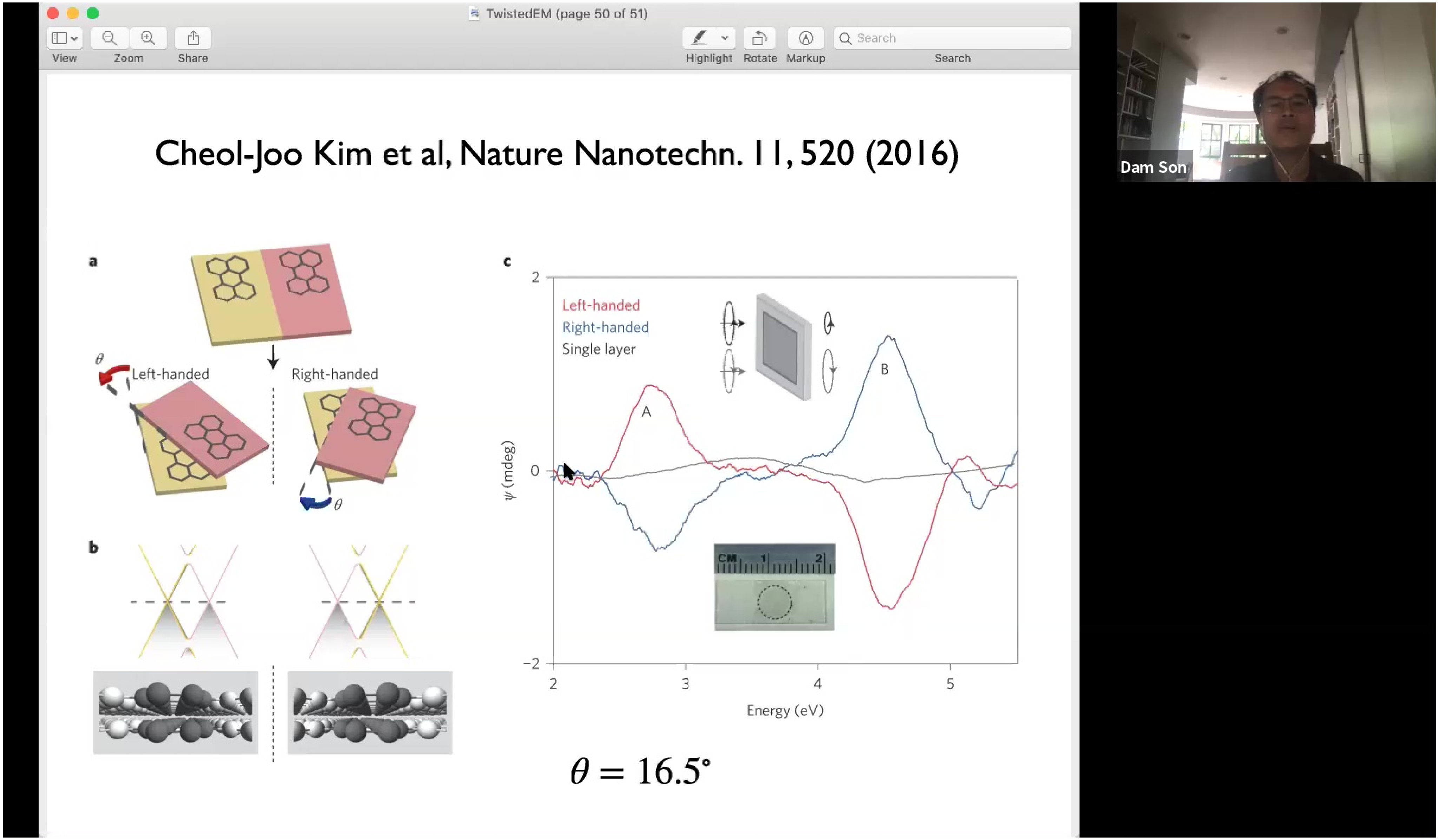
Faraday rotation by TBG

$$\theta_{\rm F} = -24\alpha \frac{v_0}{c} \frac{k_\theta d}{\hbar} \left(\frac{p_{\rm F}}{k_\theta}\right)^2 \frac{w_{\rm AA} w_{\rm AB}}{(v_0 k_\theta)^2}$$

For $\theta \sim 2^\circ, \theta_F$ can be 10^{-5}

CD can reach 10^{-4}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Cheol-Joo Kim et al, Nature Nanotechn. 11, 520 (2016)
a
θ
Left-handed
Right-handed
θ
b
c
Left-handed
Right-handed
Single layer
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ψ (mdeg)
2
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−2
2
3
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5
Energy (eV)
CM 1 2
θ = 16.5°
Dam Son

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Conclusions

- Scattering of photons can reveal properties of low-energy excitations
- Magnitudes of effects small but not beyond the sensitivity of experiments
- New experimental probes?
- Theoretical predictions for strongly-correlated states?

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