Title: Effective entropy of quantum fields coupled with gravity

Speakers: Xiaoliang Qi

Series: Quantum Fields and Strings

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Abstract: Entanglement entropy quantifies the amount of uncertainty of a quantum state. For quantum fields in curved space, entanglement entropy of the quantum field theory degrees of freedom is well-defined for a fixed background geometry. In this work, we propose a generalization of the quantum field theory entanglement entropy by including dynamical gravity. The generalized quantity named effective entropy, and its Renyi entropy generalizations, are defined by analytic continuation of a gravitational path integral on replica geometry with a co-dimension-2 brane at the boundary of region we are studying. We discuss different approaches to define the region in a gauge invariant way, and show that the effective entropy satisfies the quantum extremal surface formula. When the quantum fields carry a significant amount of entanglement, the quantum extremal surface can have a topology transition, after which an entanglement island region appears. Our result generalizes the Hubeny-Rangamani-Takayanagi formula of holographic entropy (with quantum corrections) to general geometries without asymptotic AdS boundary, and provides a more solid framework for addressing problems such as the Page curve of evaporating black holes in asymptotic flat spacetime. We apply the formula to two example systems, a closed two-dimensional universe and a four-dimensional maximally extended Schwarzchild black hole. We discuss the analog of the effective entropy in random tensor network models, which provides more concrete understanding of quantum information properties in general dynamical geometries. By introducing ancilla systems, we show how quantum information in the entanglement island can be reconstructed in a state-dependent and observer-dependent map. We study the closed universe (without spatial boundary) case and discuss how it is related to open universe.

Effective entropy of quantum fields coupled with gravity

Xiao-Liang Qi

Stanford University

Perimeter Institute 9/22/2020

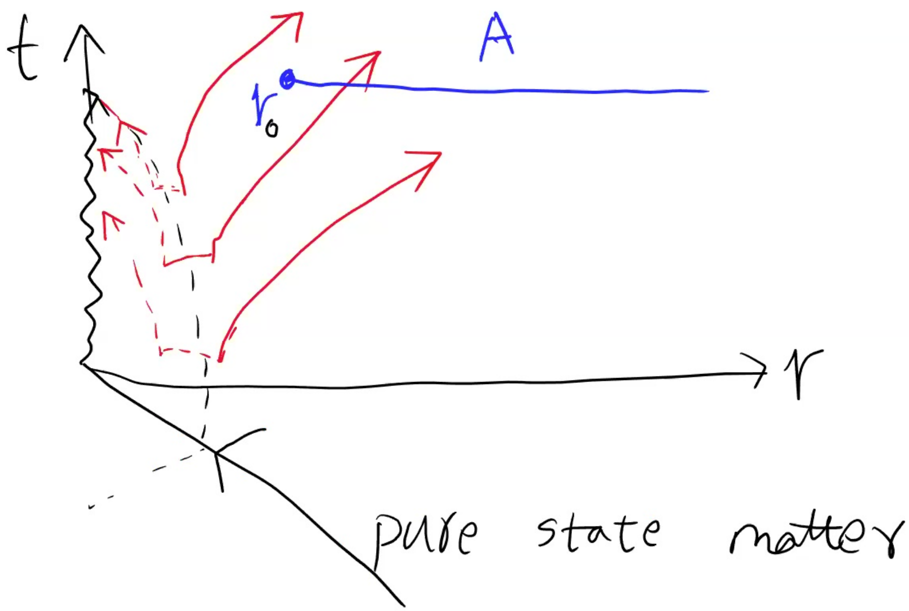
Outline

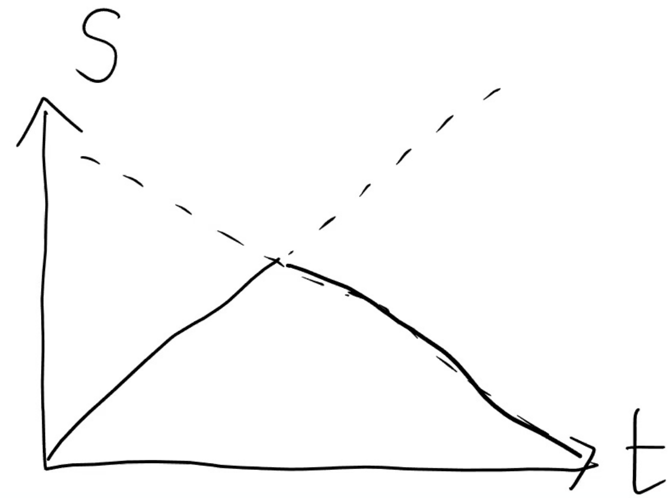
- Motivation: generalization of quantum field theory entanglement entropy to gravitational systems
- Review of QFT entropy
- Definition of the generalization—effective entropy
- Quantum extremal surface and entanglement island
- Example 1: Eternal black hole
- Example 2: 2d closed universe
- Random tensor network model
- Discussion and open questions

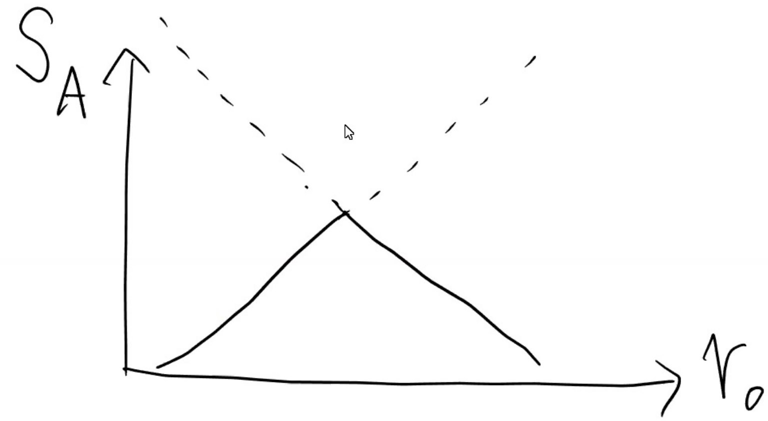
- Ref: Xi Dong, XLQ, Zhou Shangnan, Zhenbin Yang arxiv 2007.02987

Motivation

- Entropy of an evaporating black hole
- Hawking radiation seems to have an increasing entropy, but unitary requires the entropy to decrease back to zero.
- Page curve.

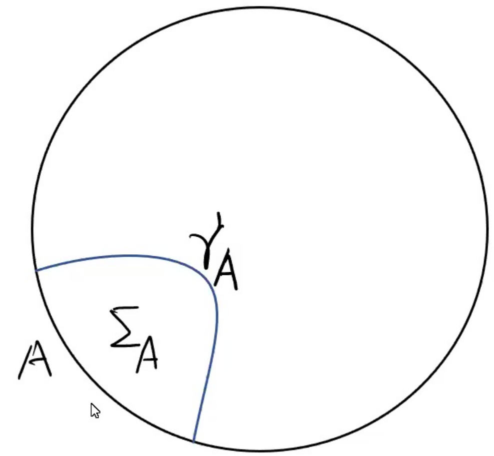






Motivation

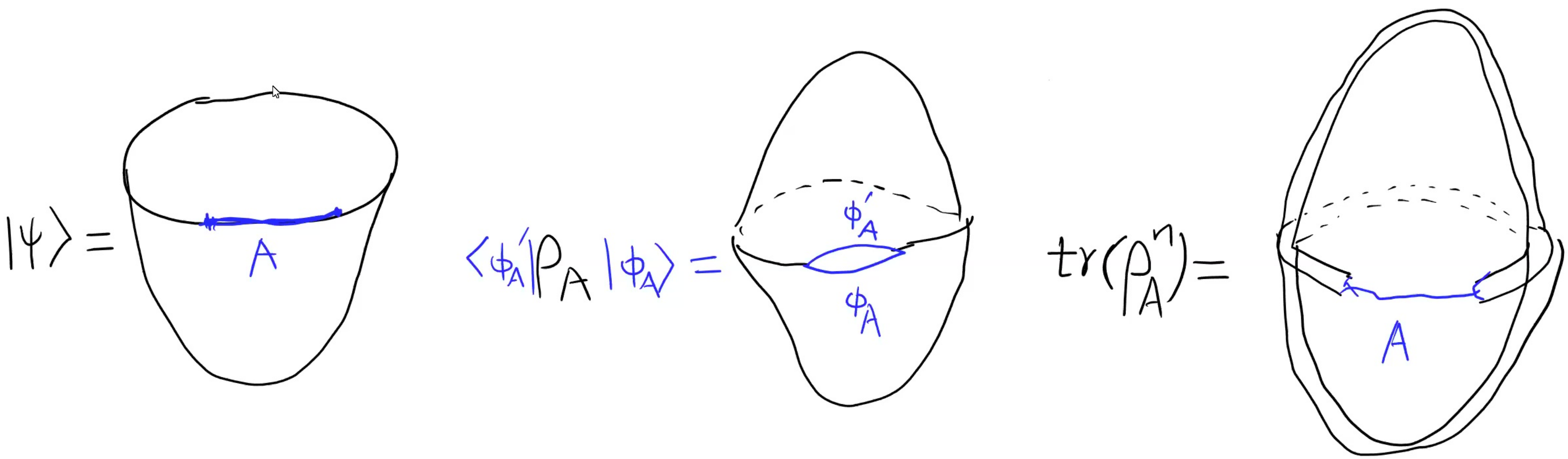
- Physically this means the Hawking radiation looks thermal but actually has less entropy, just like a pure state could look thermal on a subsystem (eigenstate thermalization hypothesis)
- Entropy is related to geometry (area) in Bekenstein -Hawking formula, and Ryu-Takayanagi formula.
- $S_A = \frac{|\gamma_A|}{4G_N} + S_{bulk}\,(\Sigma_A)$
  (Ryu-Takayanagi '06, Faulkner-Lewkowycz-Maldacena '13)
- Recent excitement: the Page curve has a geometrical interpretation
  (Penington, '19, Almheiri, Engelhardt, Marolf, Maxfield '19 and more recent works)



- The relation between entropy and geometry seems to go beyond holographic duality and anti-de Sitter space.
- We want to define subregion entropy in a gravitational system, and study its properties.

Entanglement entropy in quantum field theory

- Replica trick $S_n(A) = -\frac{1}{n-1}\log tr(\rho_A^n)\,, S = \lim_{n\to 1} S_n(A)$
- ρ can be expressed by a path integral
- $e^{-(n-1)S_n} = tr(\rho^n) = \frac{\mathrm{Z}_{\mathcal{M}_n(A)}}{\mathrm{Z}_{\mathcal{M}}^n}$ path integral on branch covering manifold.



Entanglement entropy in quantum field theory

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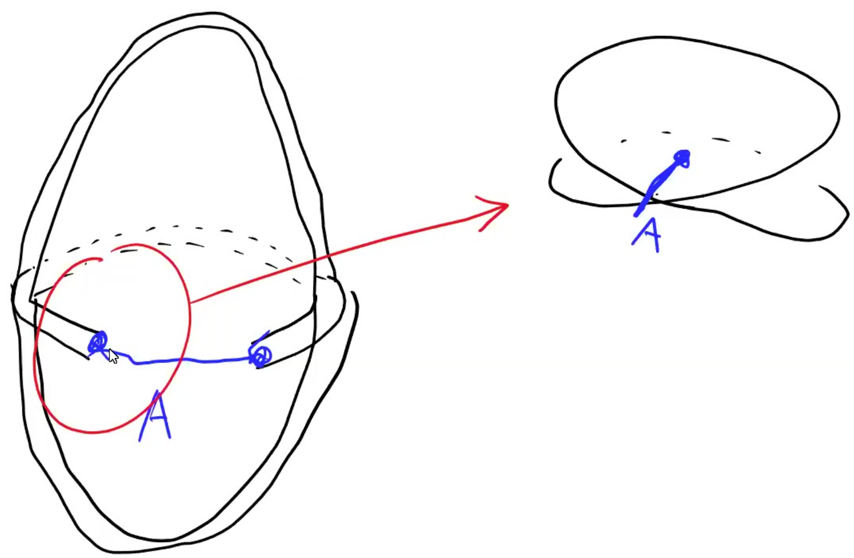
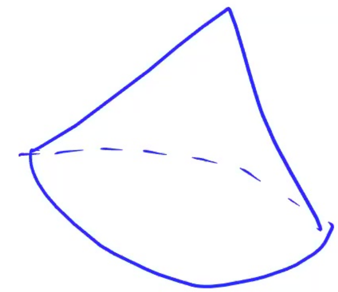


like



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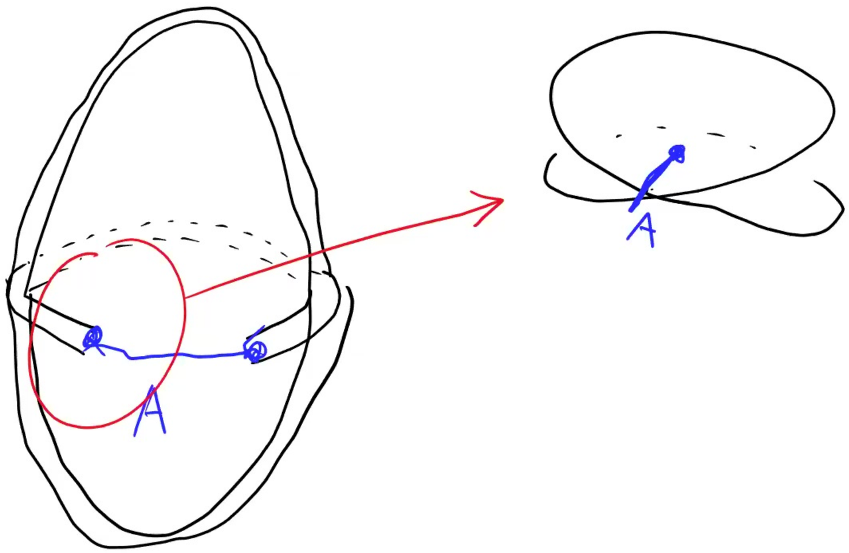

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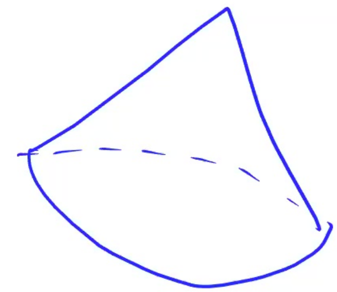

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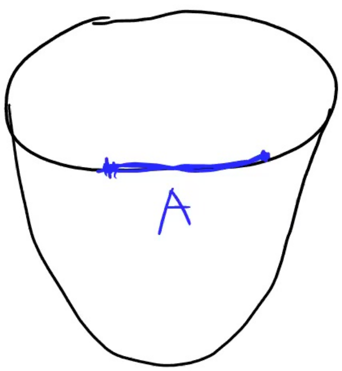
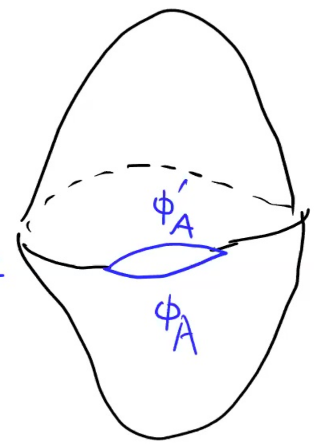


$\langle \phi_A' | \rho_A | \phi_A \rangle =$



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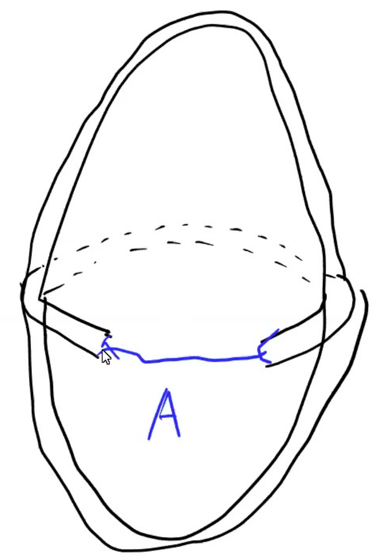
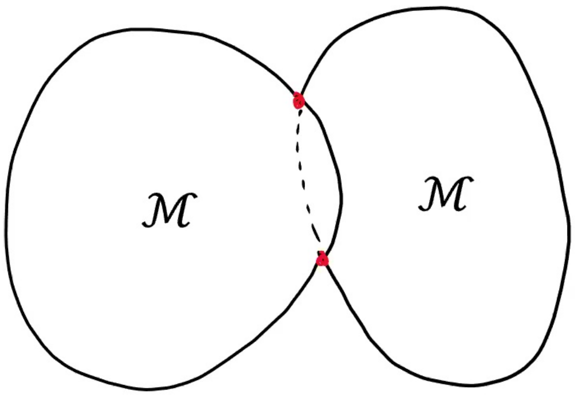


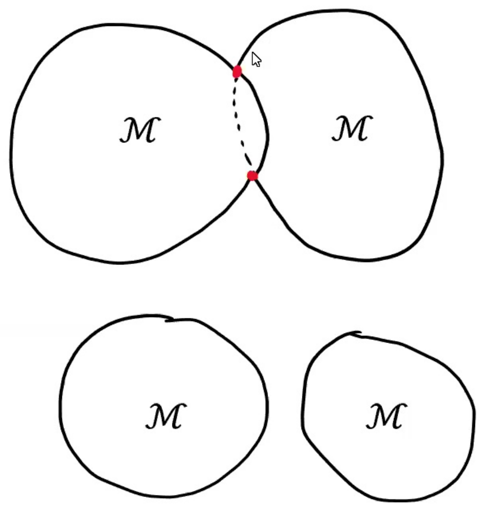
Generalization to systems with dynamical gravity

- Intuition: at least when quantum fluctuation of geometry is weak (like where we live now), the generalization of quantum field theory entropy should exist.
- Naïve guess: take QFT formula and integrate over geometry.
- $e^{-(n-1)S_n} = \frac{Z_{\mathcal{M}_n(A)}}{Z^n_{\mathcal{M}}} = \frac{\int_{\mathcal{M}_n(A)} D\phi e^{-S[\phi]}}{\int_{\mathcal{M}^n} D\phi e^{-S[\phi]}} \to \frac{\int DgD\phi e^{-S[\phi,g]-S_{grav}}}{\int DgD\phi e^{-S_0[\phi,g]-S_{grav}}}$
- Problem: (1) what gravitational action should we choose? What determines the difference between numerator and denominator? (2) the ratio is not very natural.
- If we use the metric of branch covering manifold $\mathcal{M}_n(A)$, denoted $\tilde{g}$, and add $S_{grav} = S_{EH}[\tilde{g}]$ the conical singularity will be resolved. Even in $G_N \to 0$ limit the entropy does not return to the QFT value.

Our proposal: effective entropy

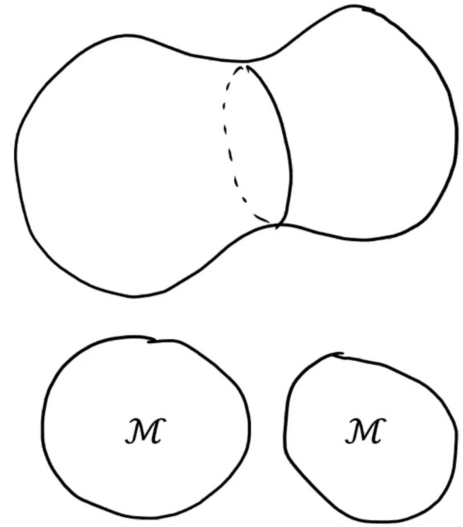


$\mathcal{M}_n(A)$ contributing for QFT entropy ($n = 2$)



Saddle point for our action

$$S_{QFT}[\phi, \tilde{g}] + S_{EH}[\tilde{g}] + \frac{1-n}{4G_N}|\partial A|$$



Saddle point if there is no brane term

$$S_{QFT}[\phi, \tilde{g}] + S_{EH}[\tilde{g}]$$

No conical singularity

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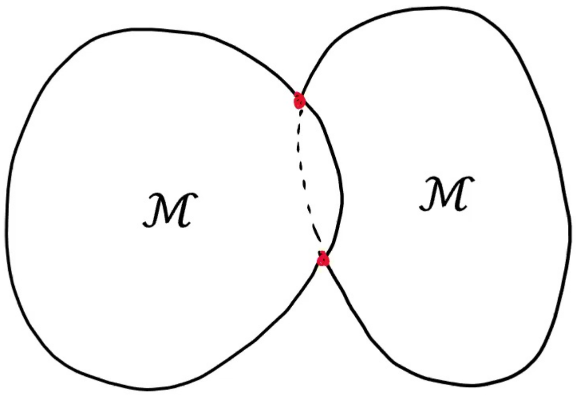

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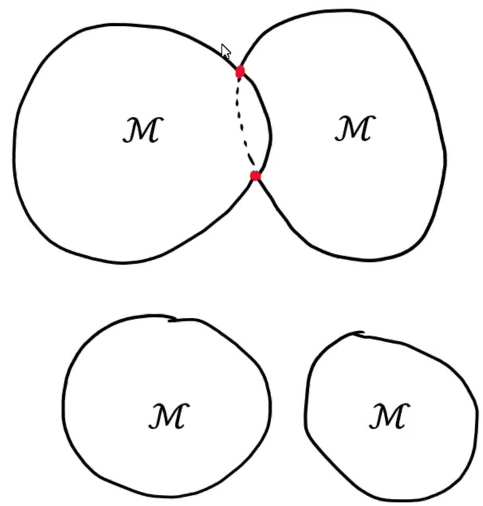

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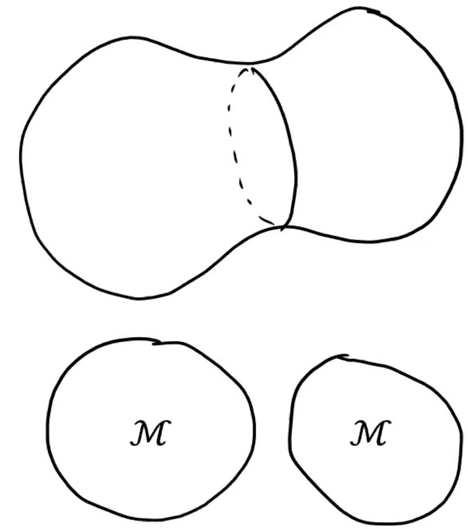

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- In other words, we add the gravity action $S_{EH}[g] \equiv S_{EH}[\tilde{g}] + \frac{1-n}{4G_N}|\partial A|$
- The last term is a brane source term that cancels the effect of the conical singularity.
- Another replica trick:

$$e^{-(n-1)S_n} = \lim_{\mathrm{m}\to-\mathrm{n}} \int_{n,m} D\tilde{g}D\phi e^{-S_{QFT}[\phi,\tilde{g}]-S_{EH}[\tilde{g}]+\frac{n-1}{4G_N}|\partial A|}$$

- $n+m$ copies of the system. Brane at the boundary of A.
- Trivial saddle point $\mathcal{M}_n(A) \times \mathcal{M}^m$

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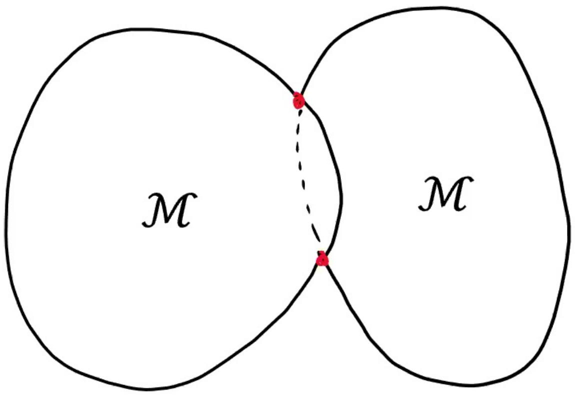

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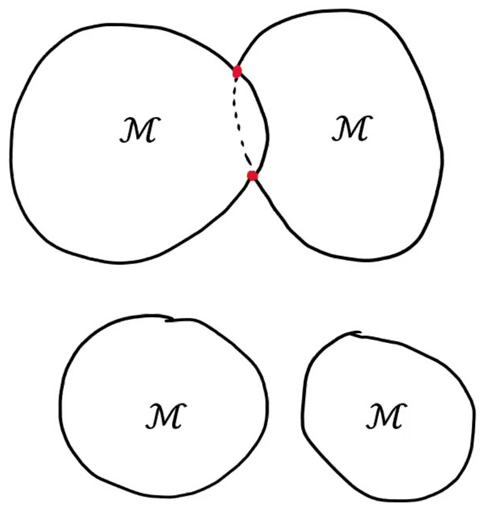

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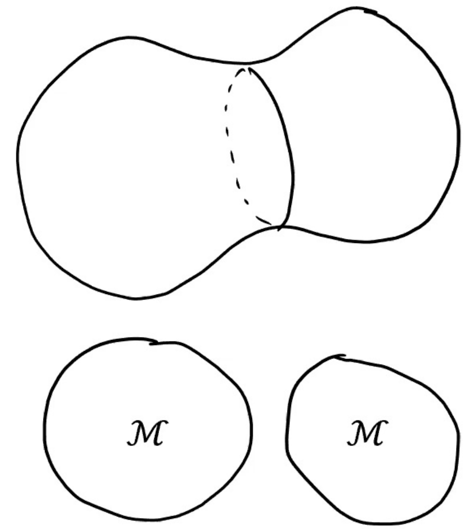

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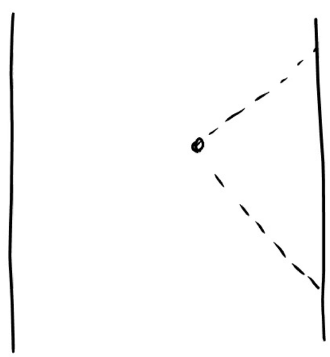
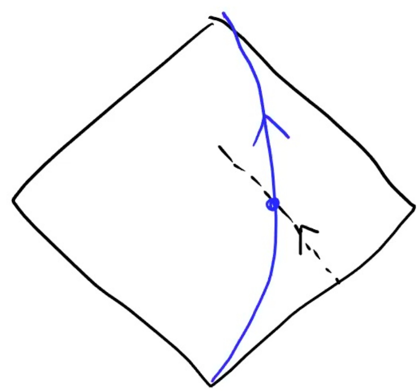
Gauge invariance

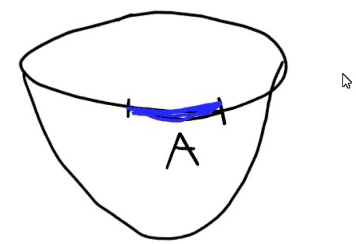
- In dynamical geometry, the location of region A needs to be decided in a gauge invariant way
- There are different physical ways of fixing the region.



AdS



asymptotically flat

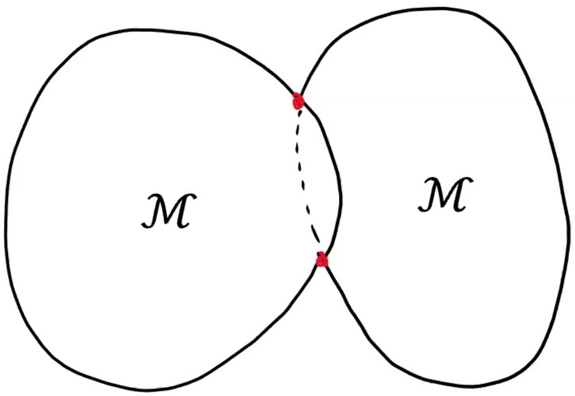


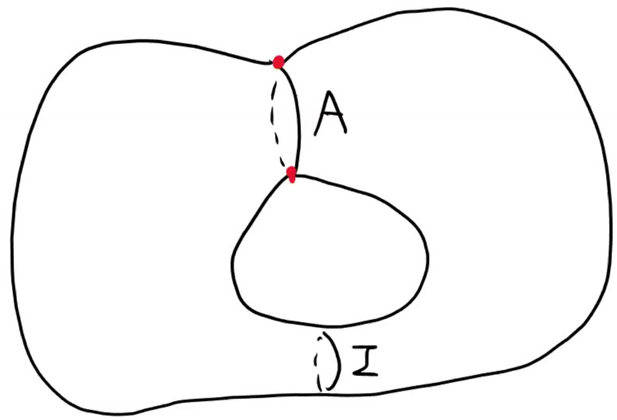
no boundary

Fix spatial metric of A

Quantum extremal surface

- The branch covering manifold $\mathcal{M}_n(A)$ is a saddle point, but may not be the dominant one
- Other possibilities: replica wormholes (Penington et al)





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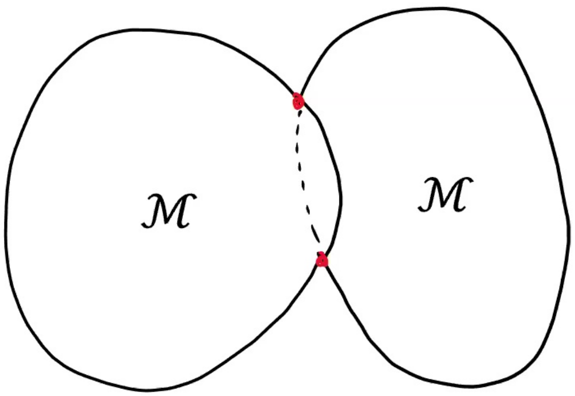
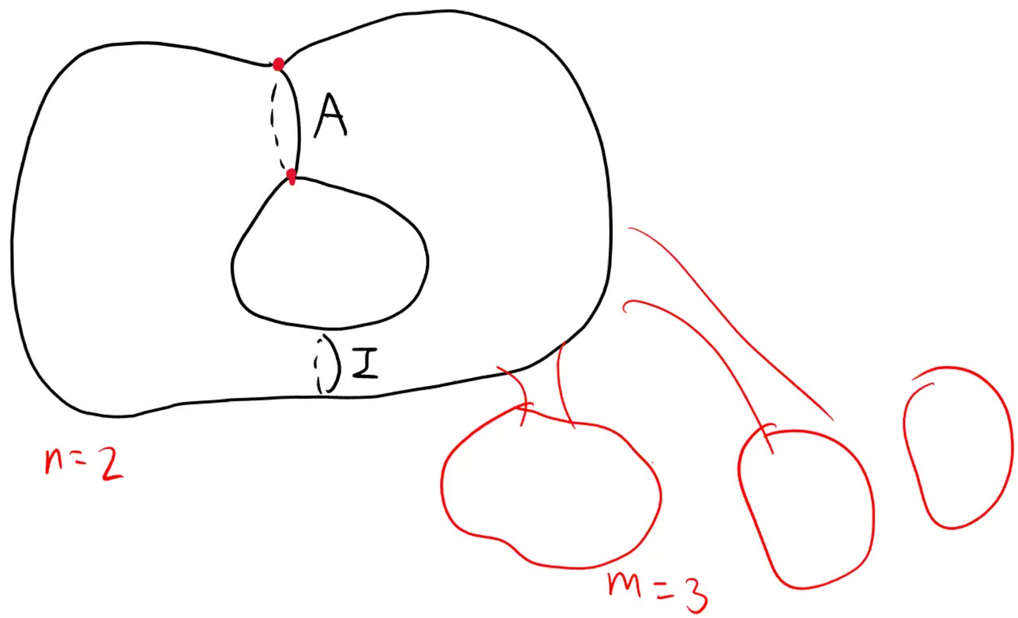
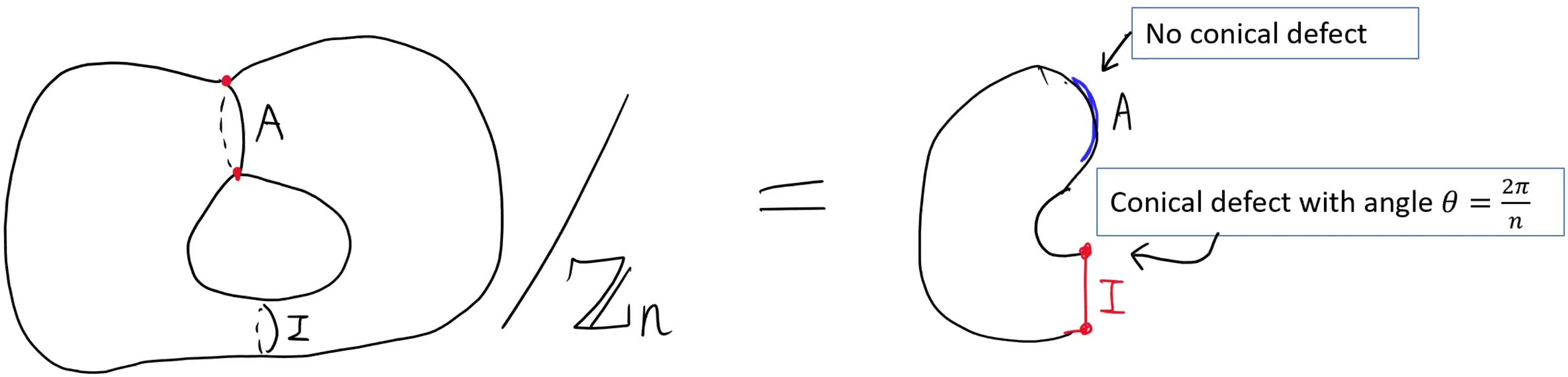




Quantum extremal surface (QES)

- If the replica geometry preserves Z_n symmetry, we can do a quotient similar to the AdS/CFT case (Lewkowycz-Maldacena '13)



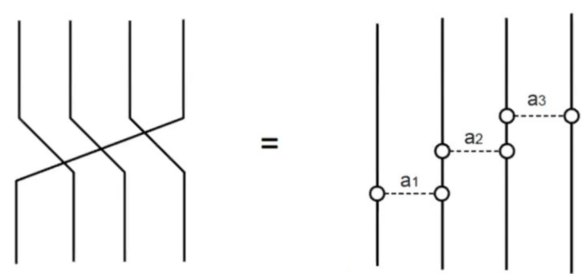
- $n \to 1$ leads to the quantum extremal surface formula

$$S\left(\bar{\mathcal{M}}\right) \simeq \left(1-\frac{1}{n}\right)\left[\frac{|\partial I|}{4G_N} + S^{\mathrm{QFT(n)}}_{A\cup I}\right] \Longrightarrow S_A = \mathrm{ext}_I\left[\frac{|\partial I|}{4G_N} + S^{\mathrm{QFT}}_{A\cup I}\right]$$

Physical interpretation

- When geometry is dynamical, density operator ρ_A of the QFT is not well-defined.
- Correlation functions are still well-defined (if we define location of the region in a gauge invariant way).
- We can view the effective entropy as a measure of correlation functions in A for low energy operators below a cutoff.
- Renyi entropy is related to an average of correlators.
- $e^{-S_A^{(2)}} = \sum_a \langle T_a \rangle^2$ average over 1-point functions, for an orthonormal basis of operators in A.

$$e^{-S_A^{(n)}} \equiv \sum_{a_1, a_2, \ldots, a_{n-1}} \langle T_{a_1} \rangle \langle T_{a_1} T_{a_2} \rangle \ldots \langle T_{a_{n-1}} T_{a_{n-2}} \rangle \langle T_{a_{n-1}} \rangle$$



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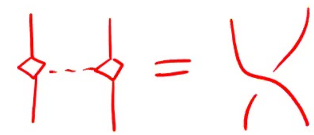

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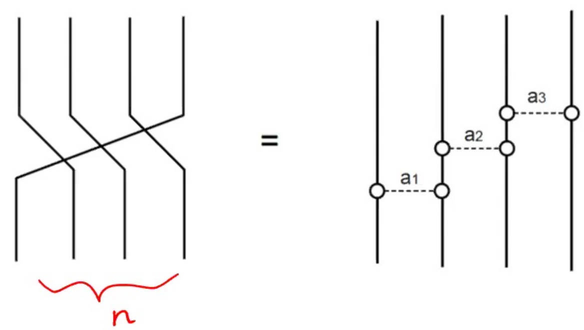

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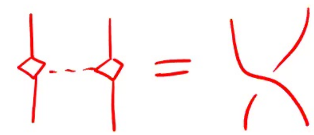

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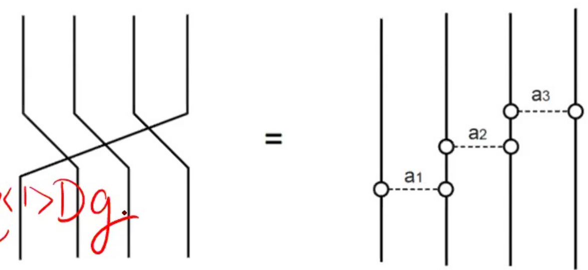

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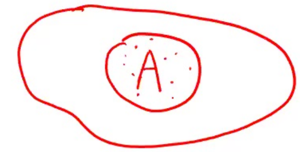

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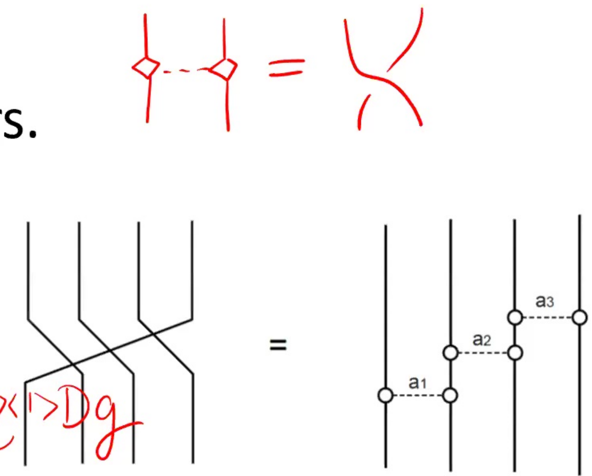

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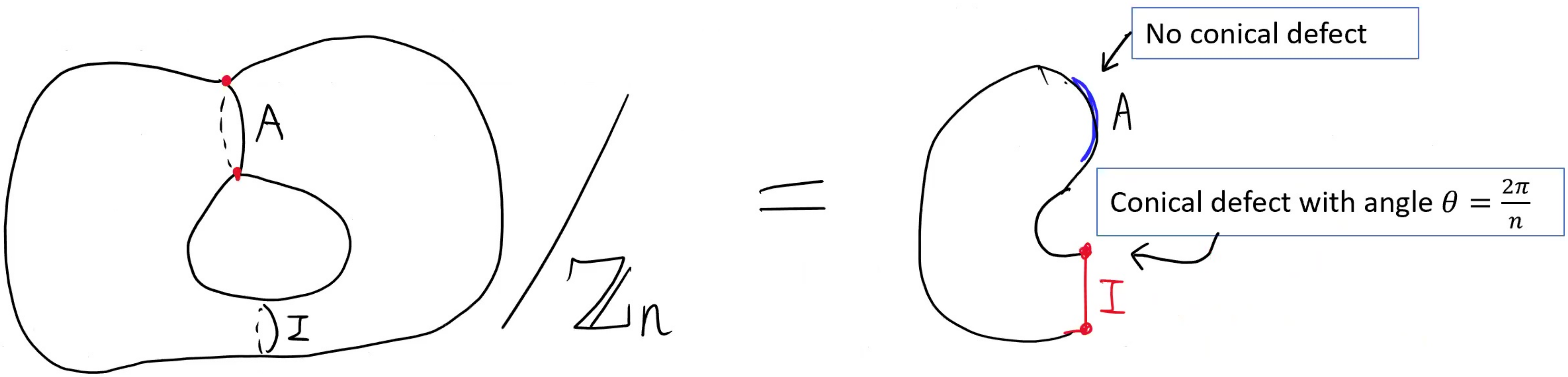

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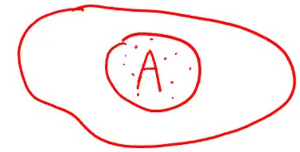

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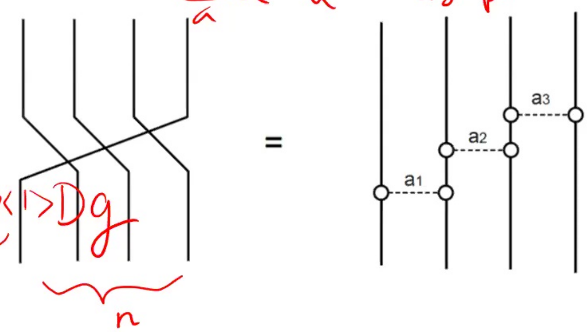
- Correlation functions are still well-defined (if we define location of the region in a gauge invariant way).
- We can view the effective entropy as a measure of correlation functions in A for low energy operators below a cutoff.
- Renyi entropy is related to an average of correlators.
- $e^{-S_A^{(2)}} = \sum_a \langle T_a \rangle^2$ average over 1-point functions, for an orthonormal basis of operators in A.

QFT: $\sum_a T_a^{\alpha\beta} T_a^{\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma}$

$$\langle X_n \rangle = e^{-S_A^{(n)}} \equiv \sum_{a_1,a_2,\ldots,a_{n-1}} \int \langle T_{a_1}\rangle_g \langle T_{a_1} T_{a_2}\rangle_g \ldots \langle T_{a_{n-1}} T_{a_{n-2}}\rangle_g \langle T_{a_{n-1}}\rangle_g \underbrace{\langle 1\rangle\langle 1\rangle\langle 1\rangle}_{m} Dg$$



Further comments

- The extremal surface formula

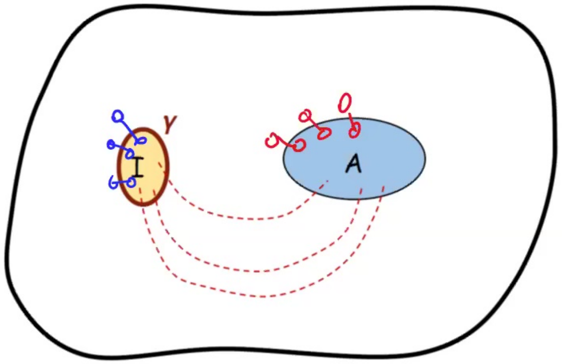
$$S_A = \mathrm{ext}_I \left[\frac{|\partial I|}{4G_N} + S^{\mathrm{QFT}}_{A\cup I} \right]$$

  depends on UV cutoff at the boundary of A, but not that of I. If we change the UV cutoff at ∂I, there should be a renormalization of G_N correspondingly, which preserves S_A

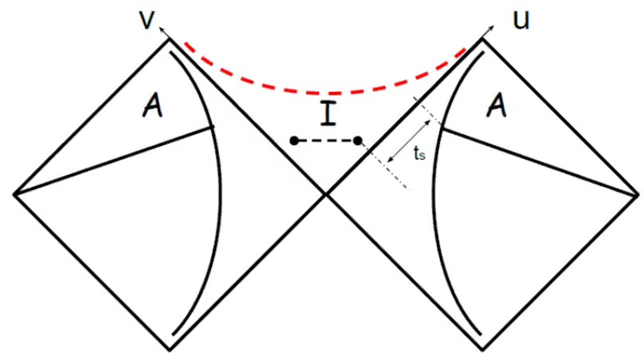
- This is consistent with the behavior of QFT entropy. The cutoff dependence is physical.

- A nontrivial extremal surface requires

$$\frac{|\partial I|}{4G_N} + S^{QFT}_{A\cup I} < S^{QFT}_A \Rightarrow S^{QFT}_I \geq S^{QFT}_A - S^{QFT}_{A\cup I} > \frac{|\partial I|}{4G_N}$$



Example 1: eternal black hole



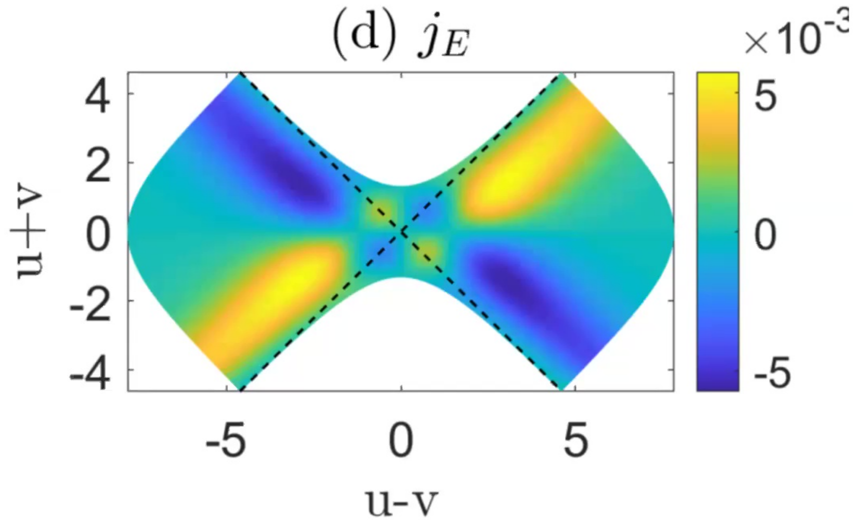
- Maximally extended blackhole geometry in 4d flat space
- Region A is the exterior of a spherical shell around the black hole
- Only considering s-wave mode described by a 2d CFT
- Qualitatively similar to other recent works (Gautason et al, Anegawa et al, Hashimoto et al, Hartman et al) but for a different state.
- Choice of the state: Weyl transformation

$$ds^2 = -\frac{4r_H^3}{r}\exp\left(-\frac{r}{r_H}\right)dUdV \xrightarrow{U=\sinh u,\, V=\sinh v}$$

$$ds^2 = -g(u,v)dudv; \quad g(u,v) = \frac{4r_H^3}{r}\exp\left(-\frac{r}{r_H}\right)\cosh u \cosh v$$

Example 1: eternal black hole

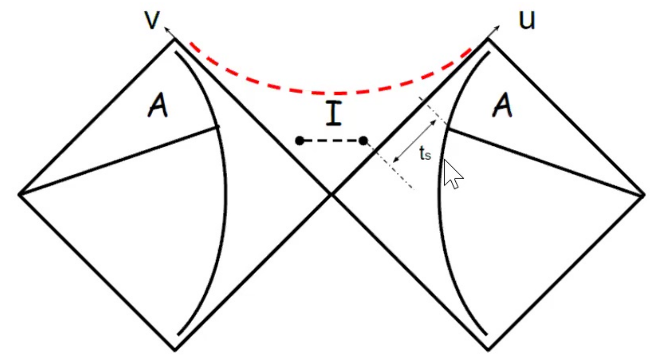
- The coordinate transformation leads to a state that approaches vacuum for large u or v. This avoided back-reaction to lose control.
- An entanglement island appears inside the horizon.
- Distance to the horizon $\propto cG$



$$v_I \simeq e^{-t} \simeq -v_A$$

$$u_I \sim t + \log \frac{ceG}{12\pi r_H^2} \simeq u_A - \left(\log \frac{12\pi r_H^2}{cG} - 1 + \log(2R) \right)$$

Example 1: eternal black hole



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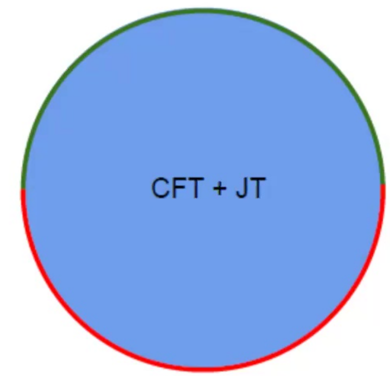
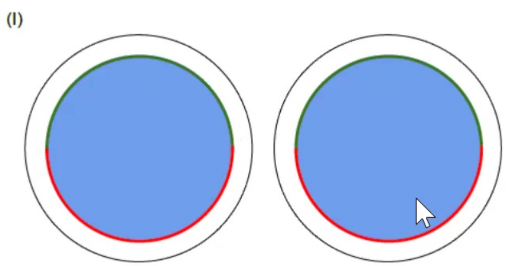
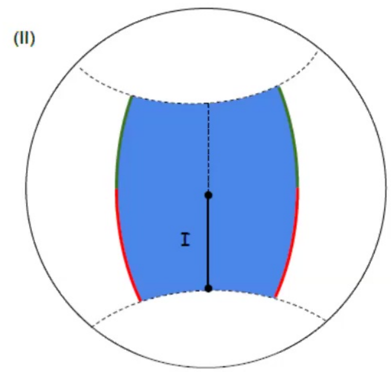
Example 2: 2d closed universe

- Euclidean AdS path integral as a density matrix ρ_A
- CFT matter field
- For single replica, dominant geometry is a disk
- With multiple replica, there could be an





Trivial geometry
$n = 2$



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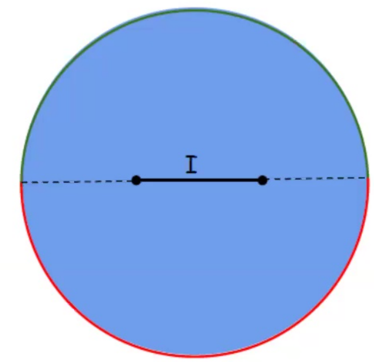
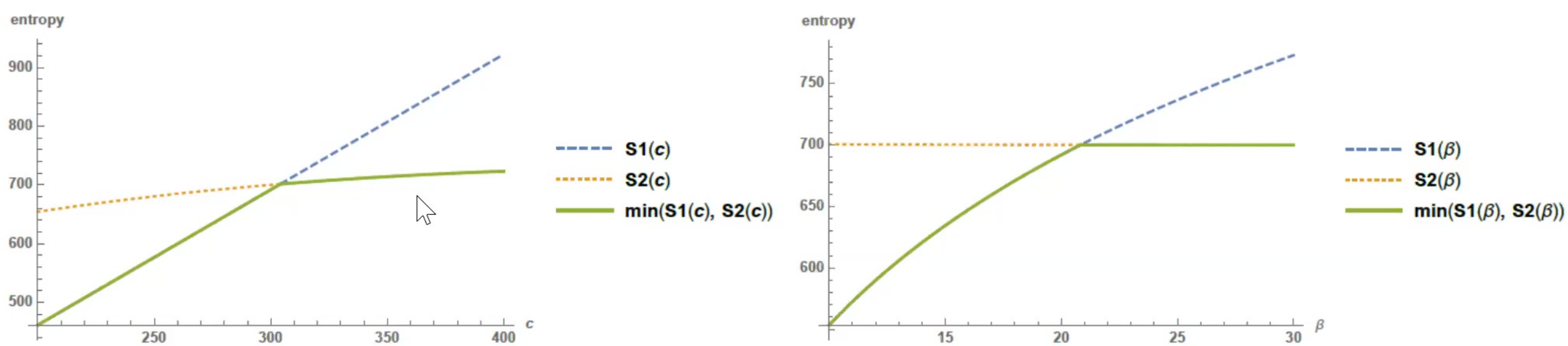


Entanglement island
in $n \to 1$

Example 2: 2d closed universe

- Page-like transition as a function of matter central charge c or boundary length β (in the unit of AdS radius)

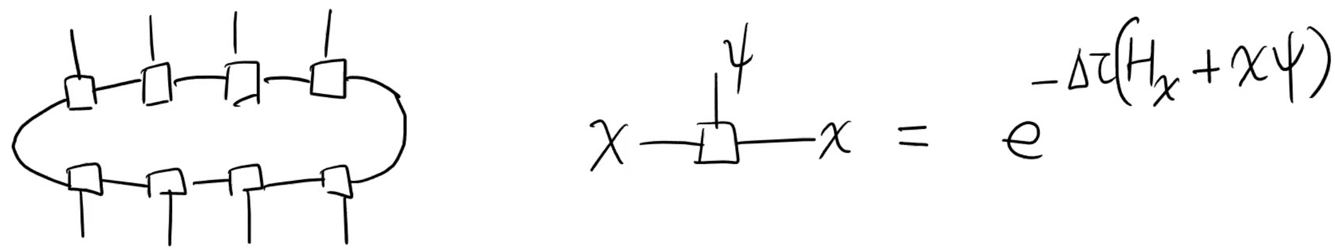


(a) $\tilde{\epsilon} = 0.1$, $S_0 = 200$, $2\pi\phi_b = 20$, $\beta = 20$.

(b) $\tilde{\epsilon} = 0.1$, $S_0 = 200$, $2\pi\phi_b = 20$, $c = 300$.

Example 2: 2d closed universe

- Physical interpretation: Sachdev-Ye-Kitaev model (SYK) turned sideways
- SYK model thermal partition function $Z = tr\left(e^{-\beta H}\right)$
- Approximately dual to AdS JT gravity couples with fermions
- Now we want to obtain a state with the SYK time direction viewed as space.



$$\rho(\psi_+, \psi_-) = \mathrm{Tr}\mathcal{P}\left(e^{-\beta/2 H_{SYK} - \int_0^{\beta/2} du \chi(u)\psi_+(u)} e^{-\beta/2 H_{SYK} - \int_{\beta/2}^{\beta} du \chi(u)\psi_-(u)}\right)$$

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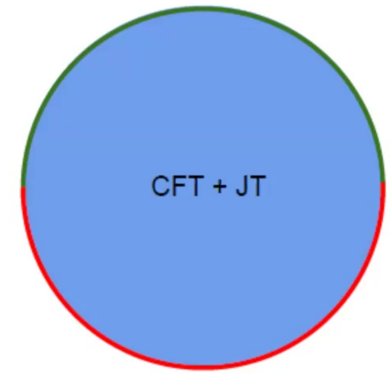

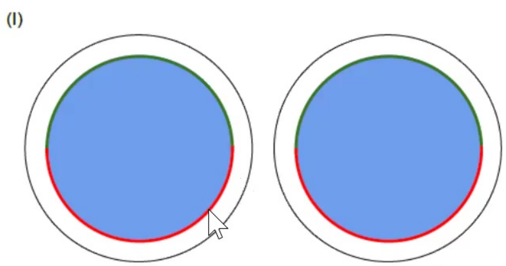

Trivial geometry
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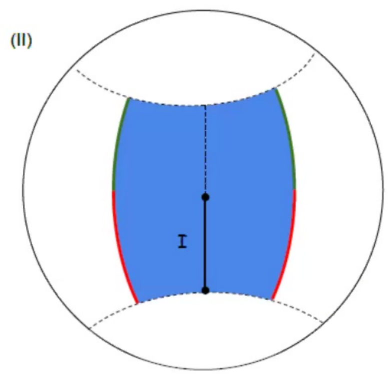

Nontrivial geometry
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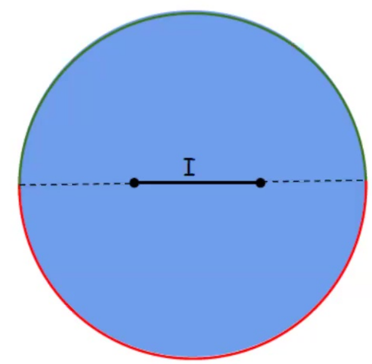

Entanglement island
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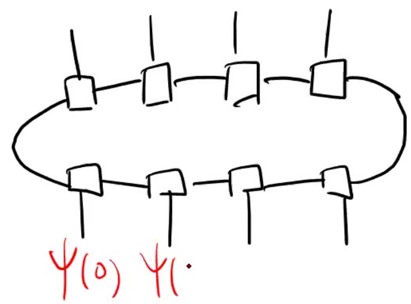
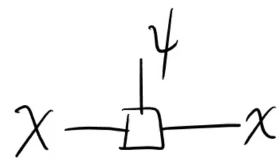


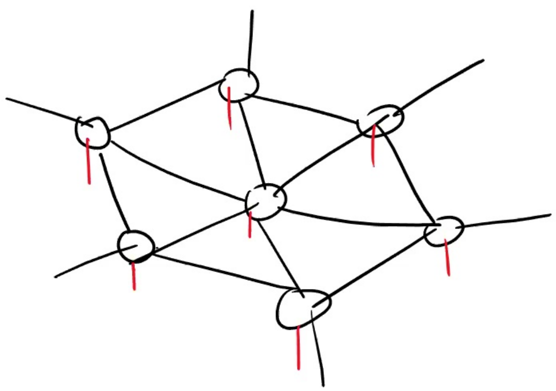


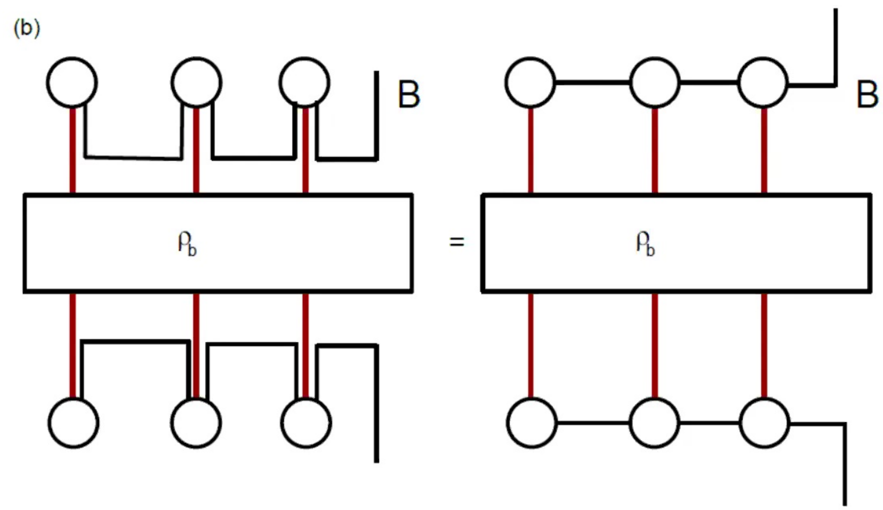
$= e^{-\Delta\tau(H_\chi + \chi\psi)}$

$$\rho(\psi_+,\psi_-) = \mathrm{Tr}\mathcal{P}\left(e^{-\beta/2H_{SYK}-\int_0^{\beta/2}du\chi(u)\psi_+(u)}e^{-\beta/2H_{SYK}-\int_{\beta/2}^{\beta}du\chi(u)\psi_-(u)}\right)$$

Random tensor network models

- Purpose: understand more explicitly the quantum information properties related to effective entropy
- Random tensor networks in holographic duality (Hayden et al 2016)
- A map from bulk to boundary

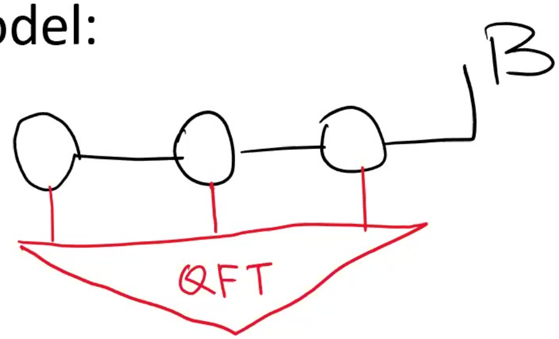


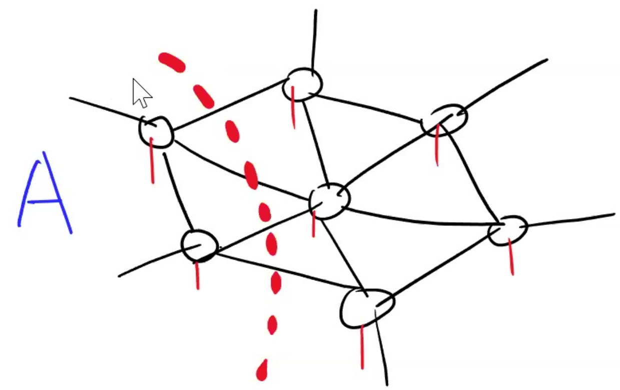


- Random tensors are random projections. $\mathbb{H}_b \otimes \mathbb{H}_g \rightarrow \mathbb{H}_B$

Random tensor network models

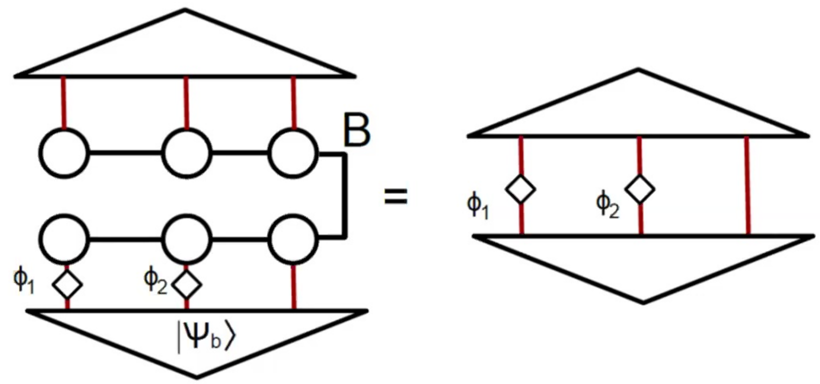
- Each random tensor is a random state with unitary invariant measure.
- Renyi entropy can be computed by averaging over randomness.
- In large bond dimension limit, tensor network models satisfy quantum extremal surface formula
- $S_A^{(n)} \simeq \log D\,|\gamma_A| + S_{QFT}^{(n)}(\Sigma_A)$
- QFT refers to the bulk matter state.
- 3-tensor model:

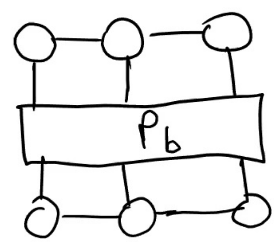




Random tensor network models

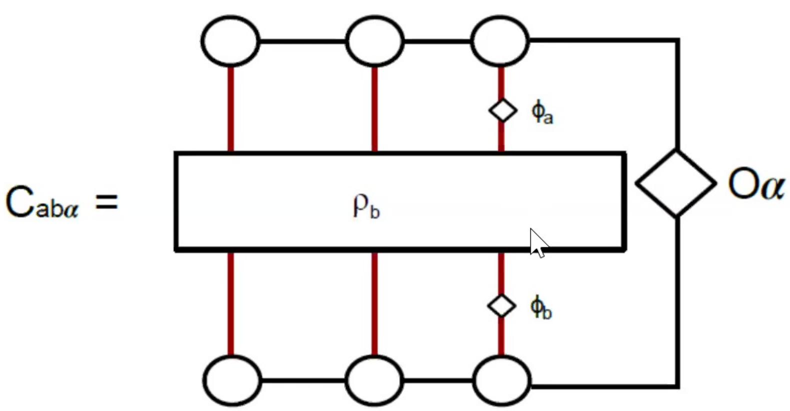
- In the case analogous to AdS/CFT, there is an isometry from bulk to boundary. All bulk states of the QFT has a boundary dual.
- Tensor network states can be defined on general graph geometry.
- In general geometry, there is no isometry to the boundary
- There may be even no boundary.

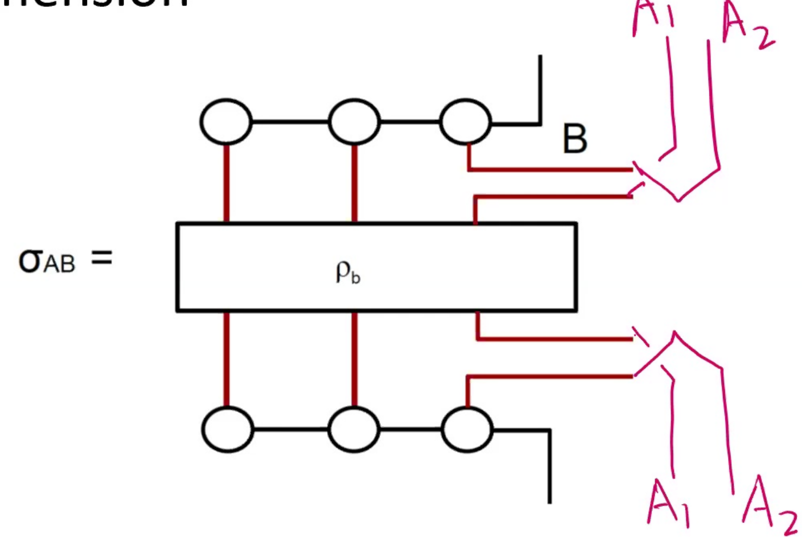




Correlation functions and super-density operator

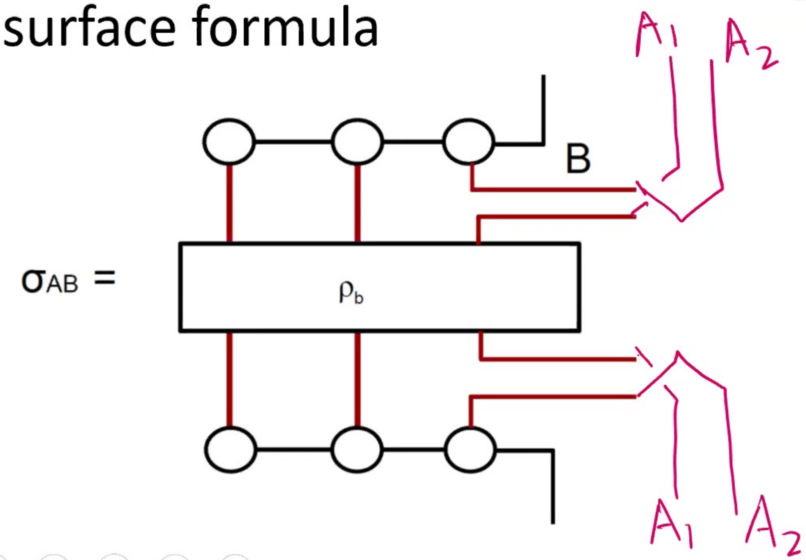
- Without bulk-to-boundary isometry, the tensor network model is like post-selected quantum mechanics
- Correlation functions can not be determined by a density operator
- If we want to determine the most general correlation function, we need a “super-density operator” of bigger dimension

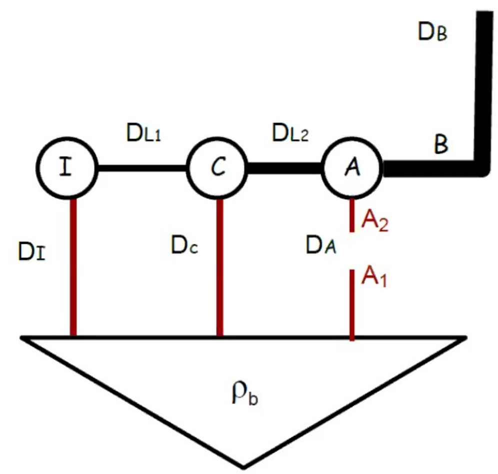




Super-density operator

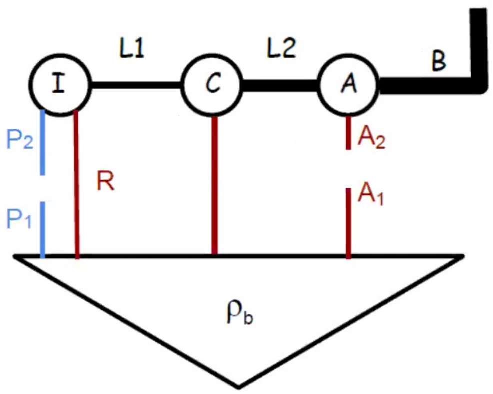
- Physically, the superdensity operator is the density operator of the system coupled with ancilla. (J Cotler, C M Jian, XLQ, F Wilczek '18)
- Defining the super-density operator enables us to compute entropy.
- In large bond dimension limit, $S_{A_1}^{(n)}$ satisfies the quantum extremal surface formula

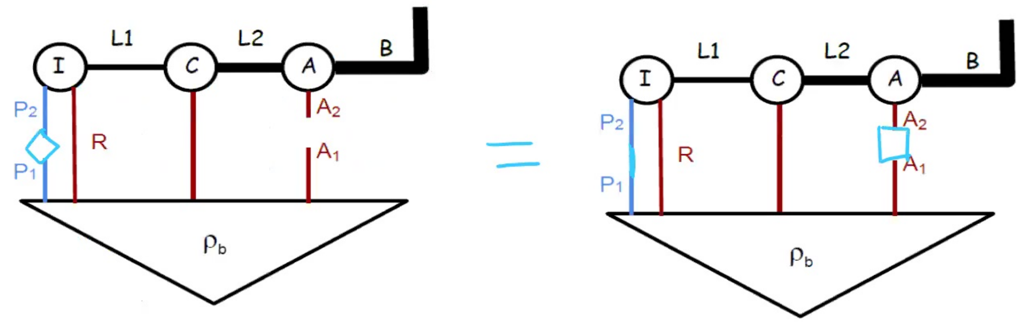




Operator reconstruction

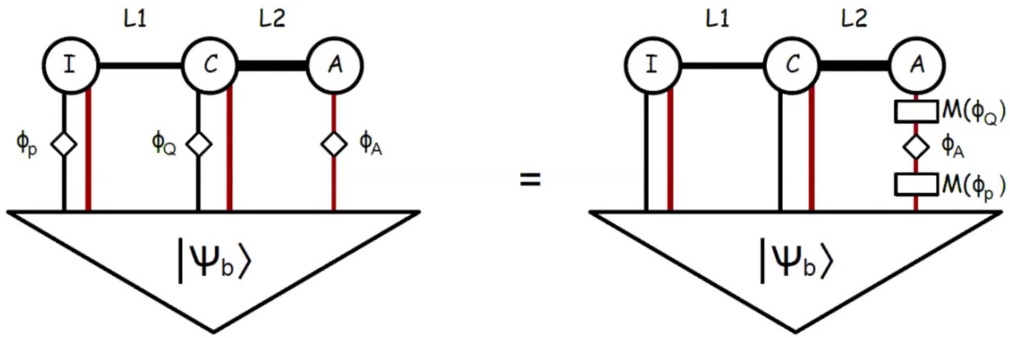
- In AdS/CFT, operators in the bulk can be mapped to boundary.
- For a boundary region A, only bulk operators in the entanglement wedge Σ_A can be reconstructed in A.
- Island I is the analog of entanglement wedge.
- Not all operators in the island region can be reconstructed in A. A small perturbation in I can be reconstructed.





Closed universe

- For closed universe with pure state matter, small operators outside the entanglement island ϕ_Q can also be mapped to A.



- Different operator ordering

$$\mathcal{M}(\phi_P \eta_P) = \mathcal{M}(\phi_P)\mathcal{M}(\eta_P)$$
$$\mathcal{M}(\phi_Q \eta_Q) = \mathcal{M}(\eta_Q)\mathcal{M}(\phi_Q)$$

- Physical interpretation: small perturbation in I is part of A. Small perturbation outside IA is maximally entangled with A.

Conclusion and open questions

- Effective entropy can be defined, at least for weak gravity.
- Quantum extremal surface and entanglement island appears in general geometries beyond AdS.
- Is there a unique way to determine the location of region A?
- Is this entropy well-defined beyond low energy effective theory?
- Is quantum gravity from post-selected quantum mechanics?
- How to develop a more covariant understanding to the tensor network state? Maybe related to the space-time random tensor networks (XLQ and Zhao Yang https://arxiv.org/abs/1801.05289)