Title: Topological Metals

Speakers: Anton Burkov

Series: Colloquium

Date: September 30, 2020 - 2:00 PM

URL: http://pirsa.org/20090008

Abstract: One of the major themes of the modern condensed matter physics is the study of materials with nontrivial electronic structure topology. Particularly significant progress in this field has happened within the last decade, due to the discovery of topologically nontrivial states of matter, that have a gap in their energy spectrum, namely Topological Insulators and Topological Superconductors. In this talk I will describe the most recent work, partly my own, extending the notions of the nontrivial electronic structure topology to gapless states of matter as well, namely to semimetals and even metals. I will discuss both the theoretical concepts, and the recent experimental work, realizing these novel states of condensed matter.

Topological metals

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Perimeter Institute, September 30, 2020

Outline

- Introduction: what is a topological metal?
- Transport in weakly-interacting topological metals.
- Strongly-interacting topological metals.

Where do gapless excitations come from?

- Liquid of weakly-interacting bosons, e.g. liquid helium.

$$\epsilon(\mathbf{k}) = \frac{\hbar^2\mathbf{k}^2}{2m} - \mu$$

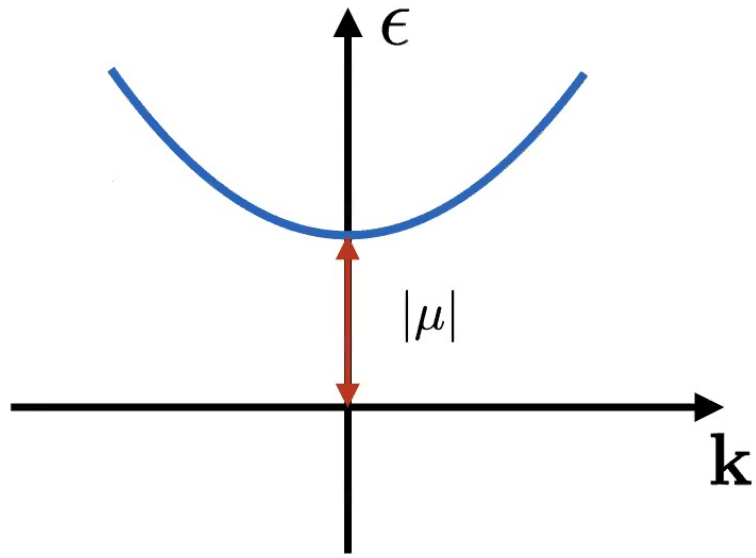
Where do gapless excitations come from?

- Chemical potential is large and negative at high T, thus all bosons have high energy relative to it, no low energy excitations.

$$\epsilon(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu$$

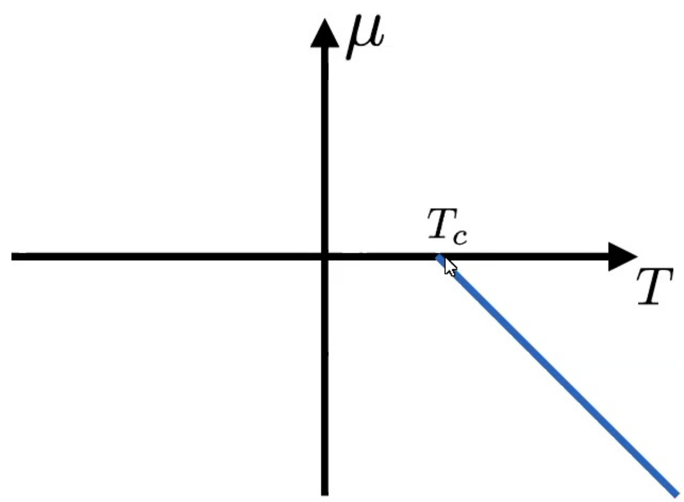
$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

$$F = E - TS$$



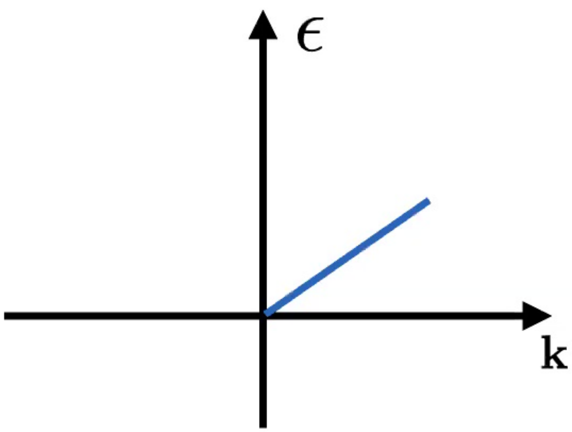
Where do gapless excitations come from?

- Transition to superfluid at low T.



Where do gapless excitations come from?

- Excitations are phonons with a gapless linear dispersion.

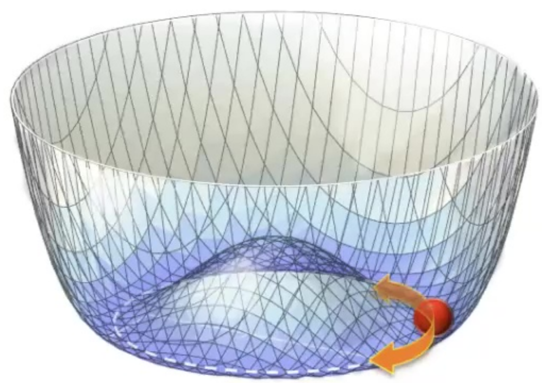


$$\epsilon(\mathbf{k}) = vk$$

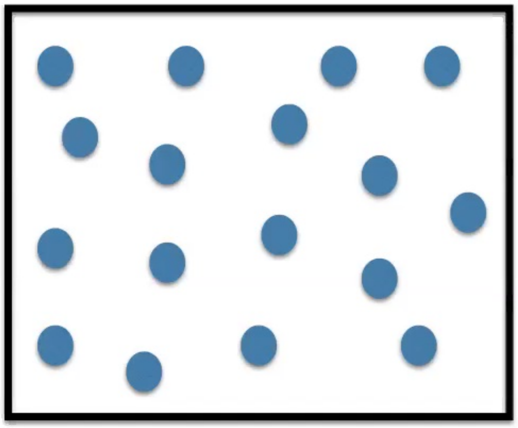
Where do gapless excitations come from?

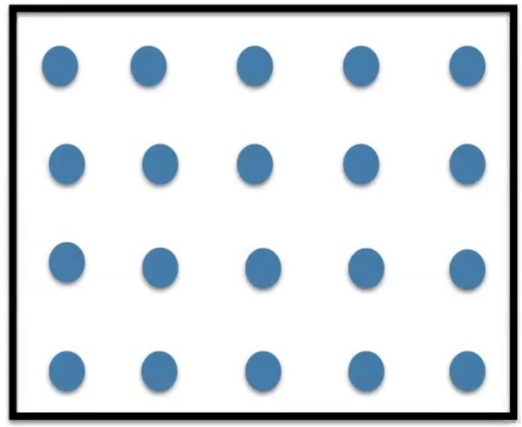
- The origin of phonons is spontaneous symmetry breaking.

$$\langle \phi(\mathbf{r},t)\rangle = \langle \sqrt{\rho}e^{i\theta(\mathbf{r},t)}\rangle \neq 0$$



Gapless excitations in crystals

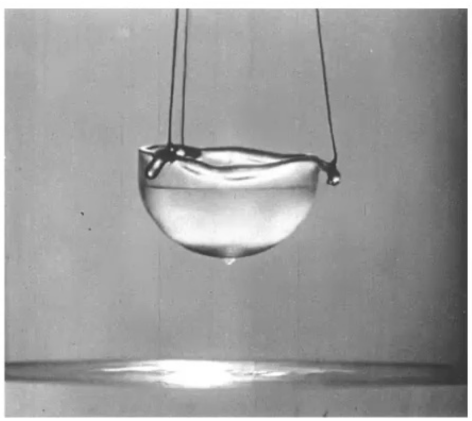




- Crystal breaks translational symmetry of space, has gapless acoustic phonons.

Where do gapless excitations come from?

- Lesson 1: gapless excitations do not arise accidentally, there must be a reason.
- Lesson 2: their existence is associated with interesting physics, such as superfluidity, crystallinity, etc.



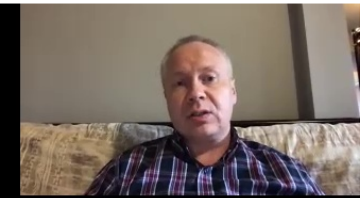
Gapless fermions: metals vs insulators

- When a material is a metal or an insulator?

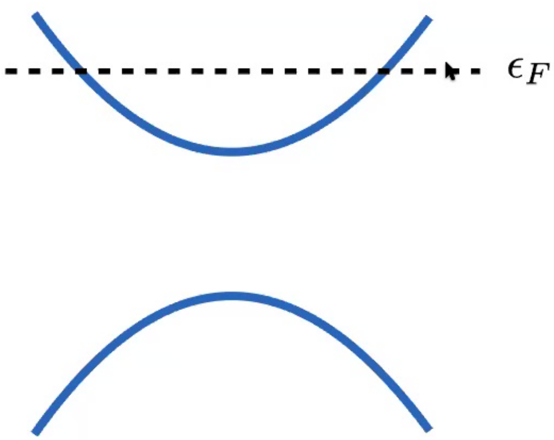
Gapless fermions: metals vs insulators

- Energy spectrum of electrons in solids has form of bands separated by bandgaps.
- Electrons obey Pauli principle.
- Number of states in a band is always 2 times the number of unit cells (momentum space in a crystal is compact: first Brillouin zone).

Metal



- Fractional (not even integer) number of electrons per unit cell: metal, Fermi surface of gapless excitations.

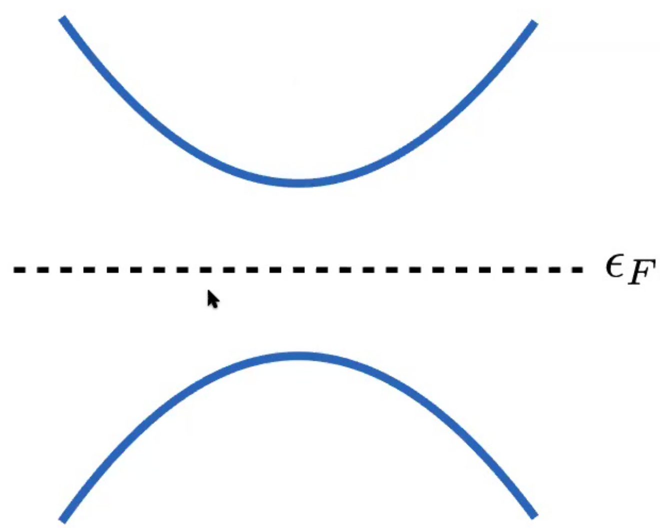


$$\frac{2V_F}{(2\pi)^d} = \frac{\nu - 2n}{v_c}$$

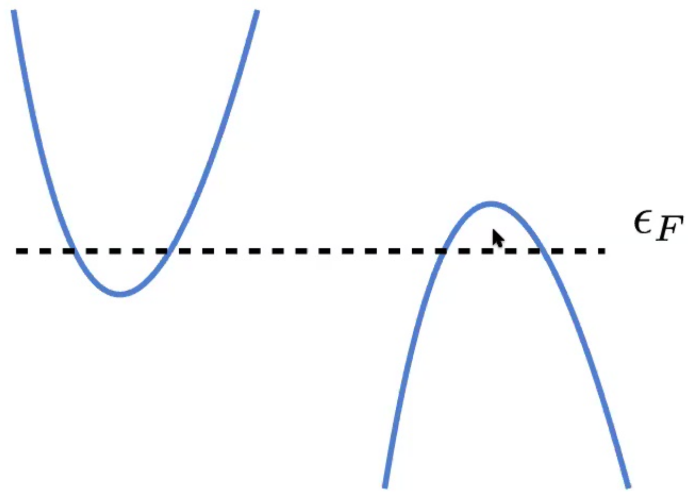
Luttinger

Insulator

- Even integer number of electrons per unit cell: insulator, no gapless excitations.



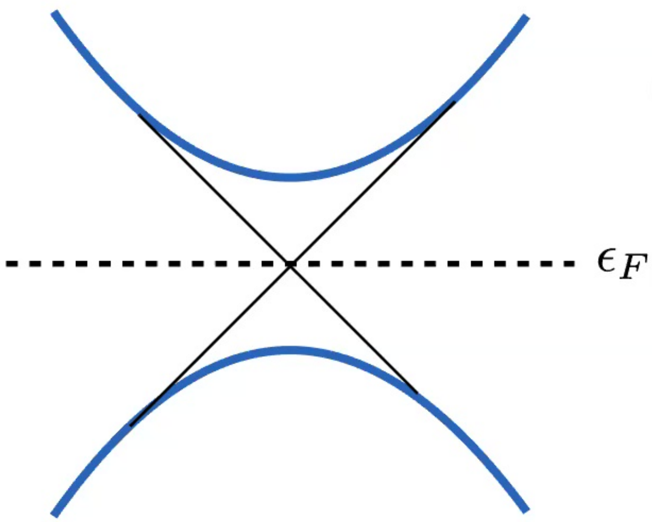
Accidental compensated semimetal

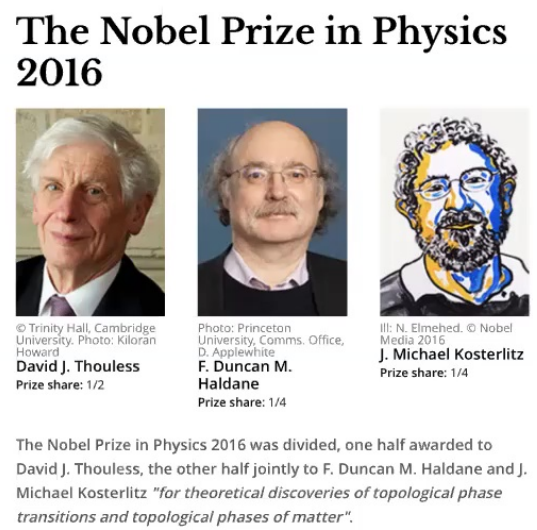


- Bands can overlap: materials with even number of electrons per unit cell often fail to be insulators, they may be compensated semimetals with zero net Luttinger volume.

Topological insulators

- Metal can exist on the surface of an insulator for topological reasons.





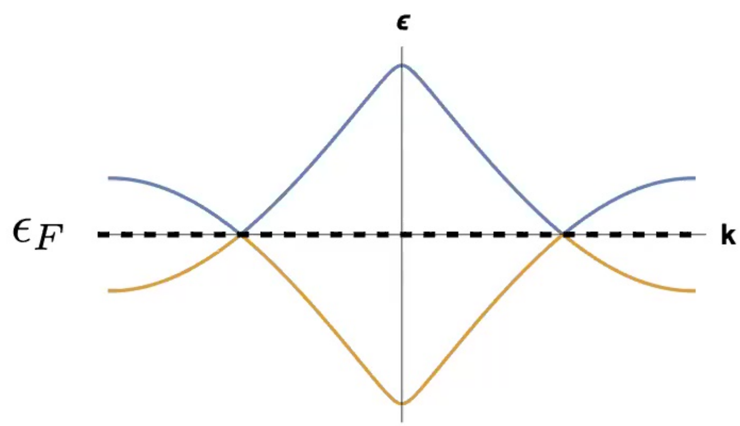
Bulk topological metals

- Can bulk 3D metals exist for topological reasons?

- Can we have a bulk 3D topologically-protected metal when the material should be an insulator by band filling?

Weyl semimetal

- Weyl semimetal: gapless topological phase which arises in 3D materials lacking time-reversal or inversion symmetries.



Murakami, 2007

Wan et al., 2011

AAB & Balents, 2011

- Exists unavoidably as an intermediate phase between a topological and ordinary insulator in 3D.

Weyl fermions

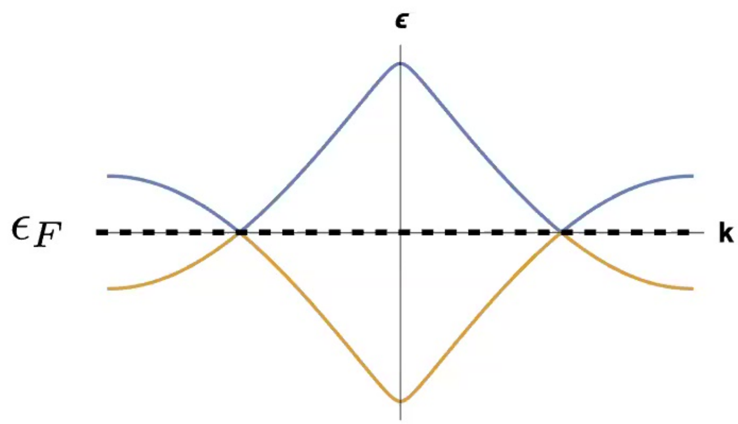
- The band Hamiltonian in the vicinity of a band-degeneracy point has a universal from:

$$H = \pm \boldsymbol{\sigma} \cdot \mathbf{k}$$

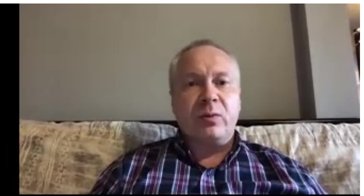
- This coincides with the Hamiltonian for relativistic massless chiral fermions, first proposed by Hermann Weyl in 1929.

3D Integer Quantum Hall transition

- "Hydrogen atom" of Weyl semimetals: intermediate phase between an ordinary 3D insulator and an integer quantum Hall insulator.

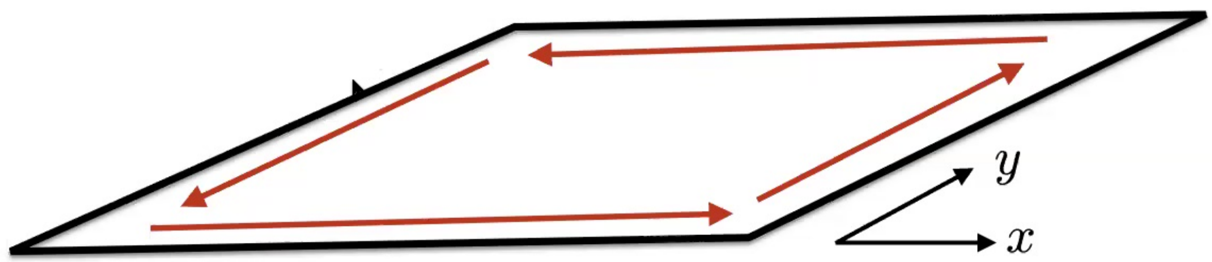


AAB & Balents, 2011



2D Integer Quantum Hall Effect

- 2D insulator with broken time-reversal symmetry.

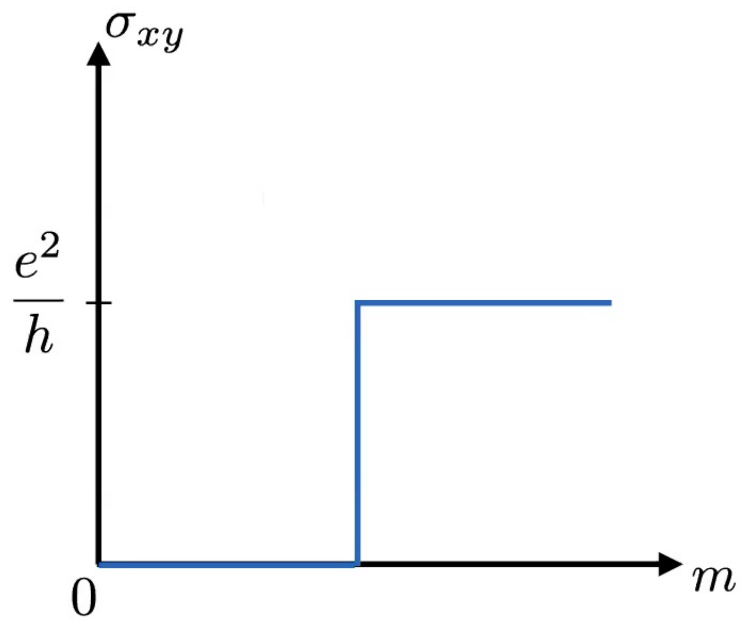


$$\sigma_{xx} = 0 \qquad \sigma_{xy} = \frac{e^2}{h} C$$

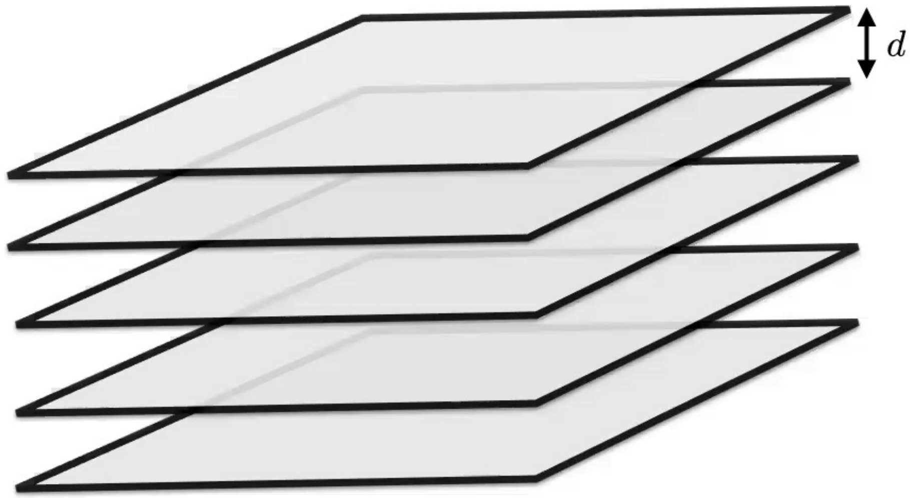
C is the number of chiral edge modes.

Plateau transition in 2D

- Transition between insulators with C=0 and C=1 must be sharp.



3D Integer Quantum Hall Effect



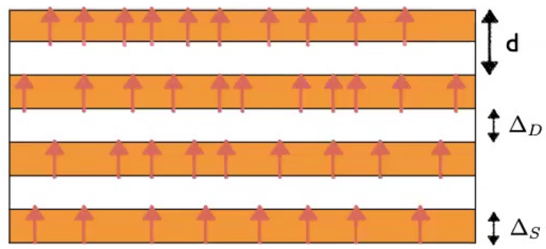
$$\sigma_{xy} = \frac{e^2}{h}\frac{G}{2\pi}$$

$$G = \frac{2\pi}{d}$$

Kohmoto, Halperin, Wu

- Hall conductivity involves a wavevector, transition to zero must happen smoothly.

3D Integer Quantum Hall Effect



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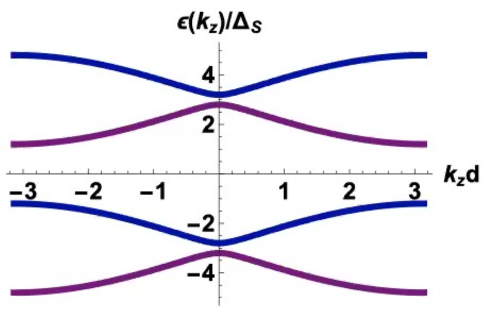

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AAB & Balents

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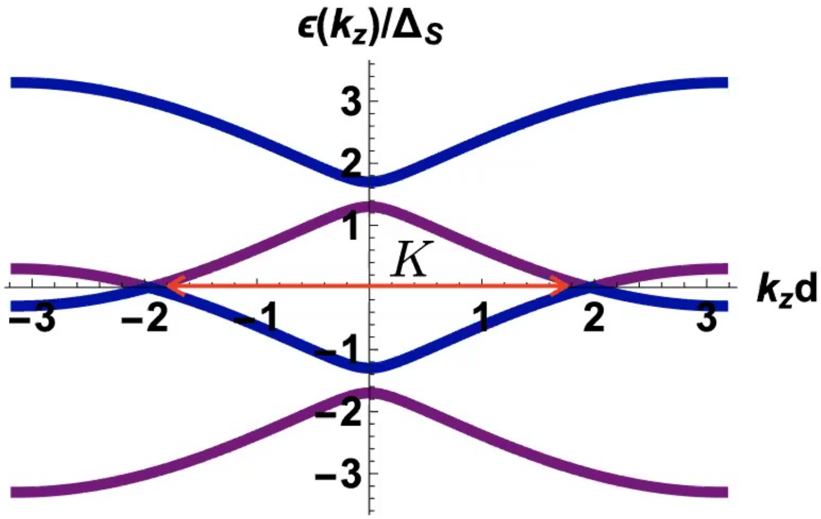
3D Integer Quantum Hall Effect



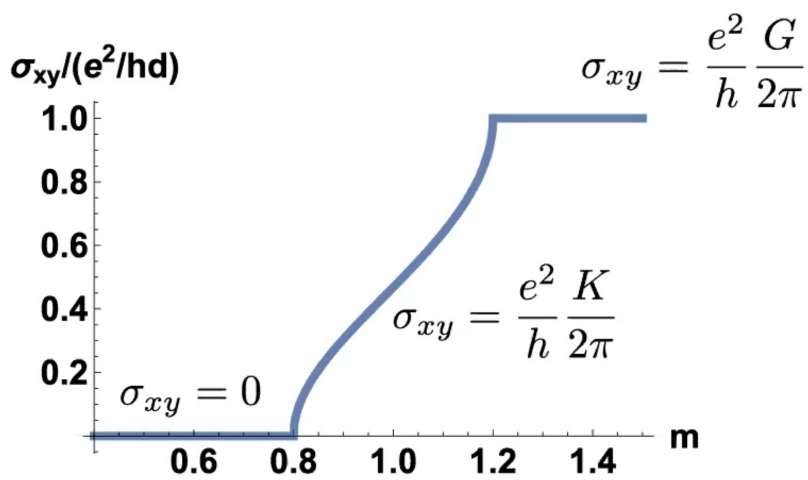
$$\sigma_{xy} = \frac{e^2}{h}\frac{K}{2\pi}$$

AAB & Balents

- Hall conductivity is proportional to the distance between the nodes and varies smoothly.

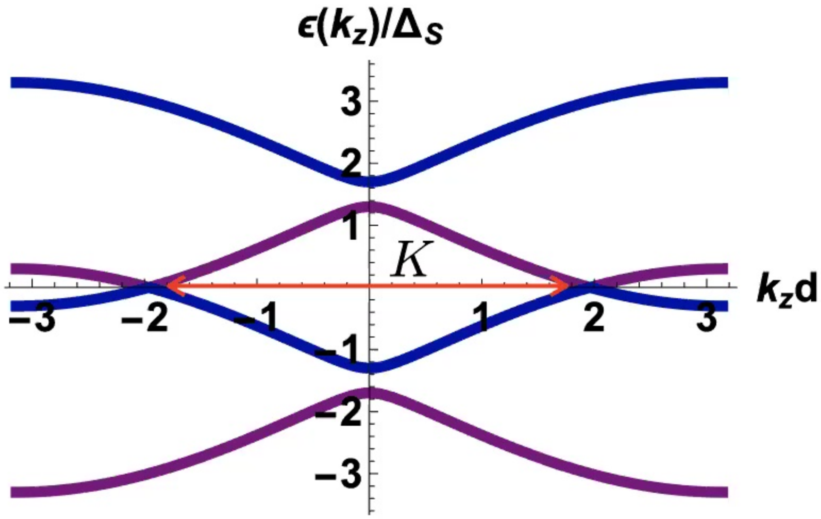
"Plateau transition" in 3D

$$\sigma_{xy} = \frac{e^2}{h}\frac{K}{2\pi}$$



- Plateau transition is sharp in 2D, but broadens into Weyl semimetal phase in 3D.

3D Integer Quantum Hall Effect



$$\sigma_{xy} = \frac{e^2}{h}\frac{K}{2\pi}$$

AAB & Balents

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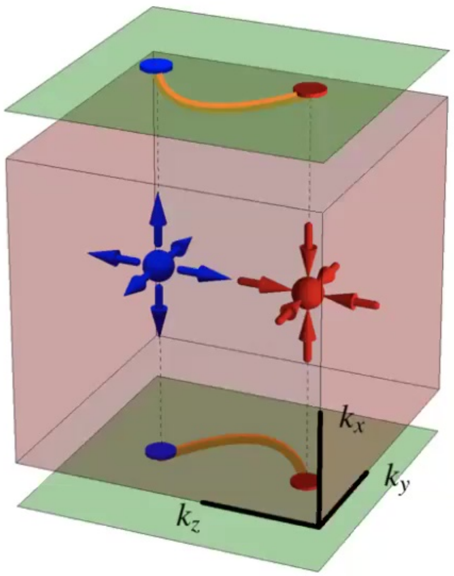
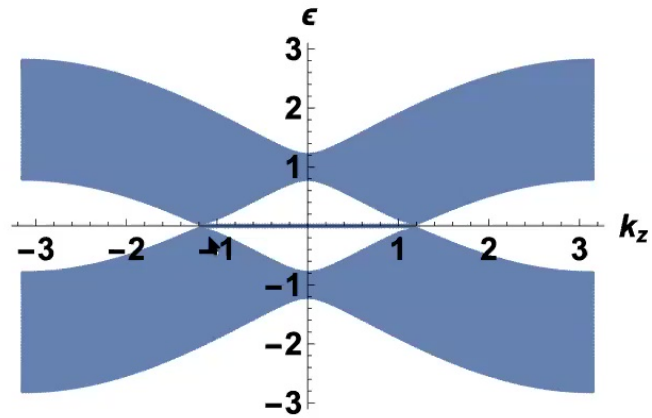
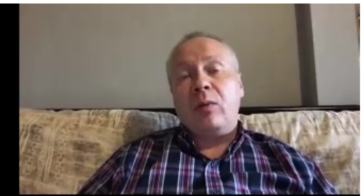
Fermi arcs

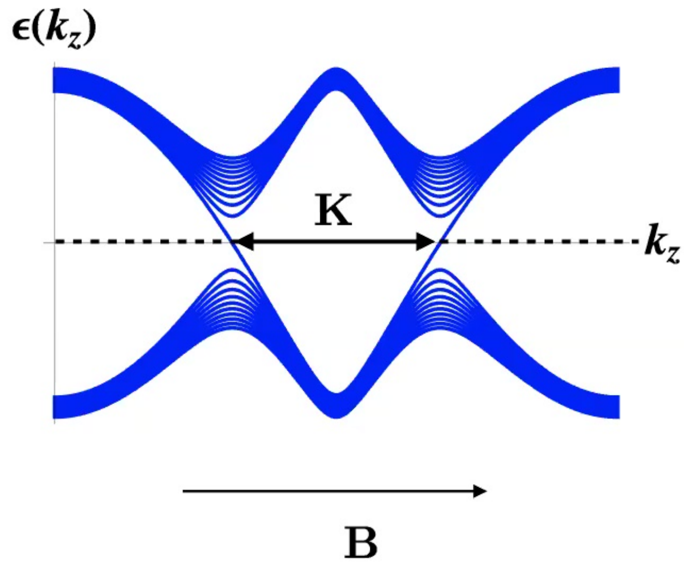




Chiral anomaly



- Nonzero Luttinger volume due to the lowest Landau level.



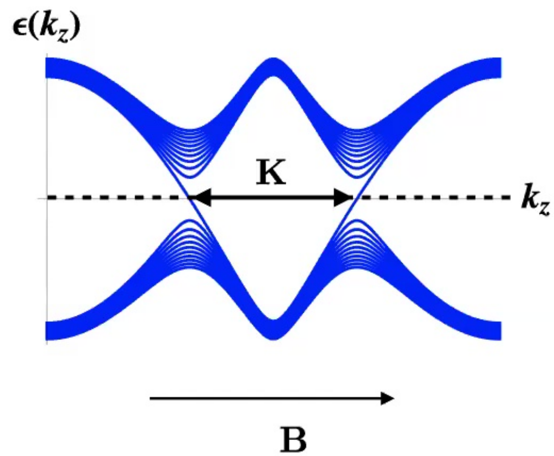
$$\sigma_{xy} = e\frac{\partial n}{\partial B} = e\frac{K}{2\pi\hbar}\frac{\partial}{\partial B}\frac{1}{2\pi\ell_B^2} = \frac{e^2}{h}\frac{K}{2\pi}$$

- Hall conductivity is a derivative of the Luttinger volume with respect to the magnetic field.

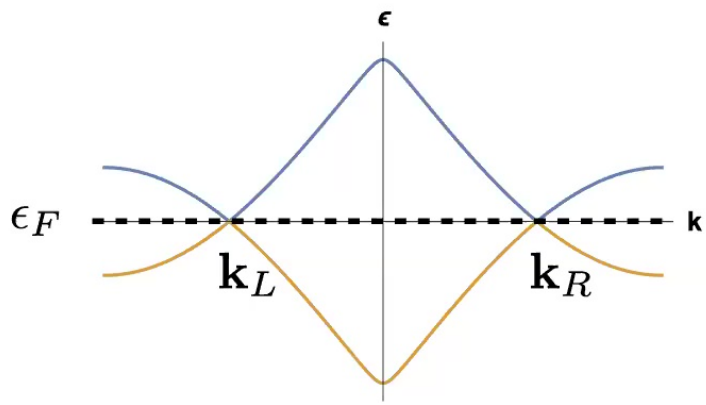
Chiral anomaly

- "Fractional" Hall conductivity in the absence of a Fermi surface inevitably implies Weyl nodes.

$$\sigma_{xy} = e\frac{\partial n}{\partial B} = e\frac{K}{2\pi\hbar}\frac{\partial}{\partial B}\frac{1}{2\pi\ell_B^2} = \frac{e^2}{h}\frac{K}{2\pi}$$



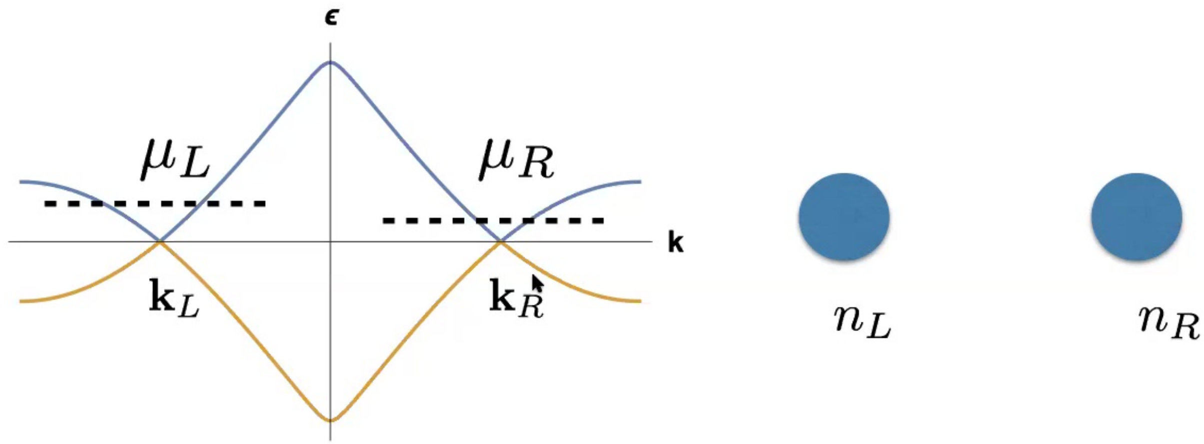
Chiral anomaly



$$\langle u(\mathbf{k}_L)|u(\mathbf{k}_R)\rangle = 0$$

- New conserved quantity: chiral charge.

Chiral anomaly



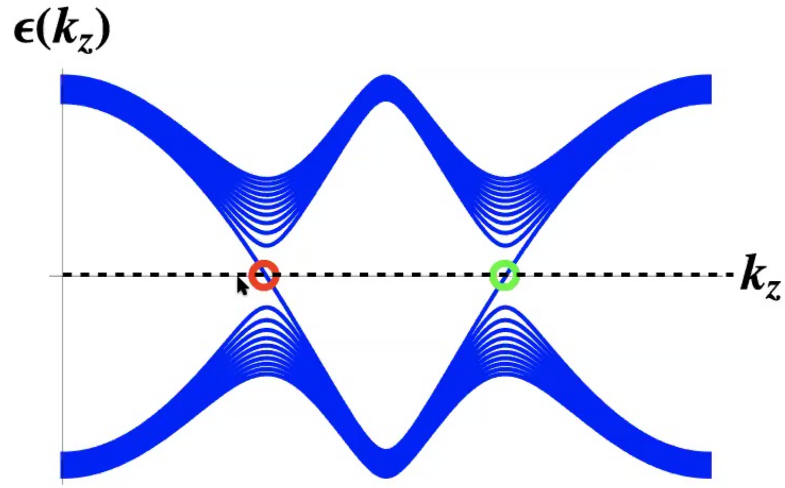
- Left-handed and right-handed charges should be separately conserved.

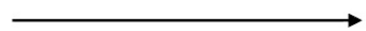
Chiral anomaly

- Conservation is violated in the presence of collinear electric and magnetic fields.

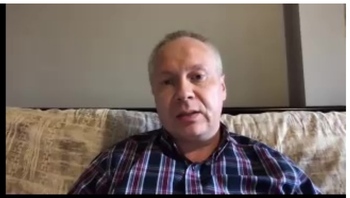
$$\frac{\partial n_R}{\partial t} = \frac{1}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$

$$\frac{\partial n_L}{\partial t} = -\frac{1}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$





$$\mathbf{E} \parallel \mathbf{B}$$



Density response in a Weyl metal

Total charge: $n = n_R + n_L$

Chiral charge: $n_c = n_R - n_L$

- If both were conserved, both would obey independent continuity (diffusion) equations:

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2 n \qquad \frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2 n_c - \frac{n_c}{\tau_c}$$

Density response in topological metal

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2(n + gV) + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n_c + gV_c)$$

$$\frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2(n_c + gV_c) - \frac{n_c + gV_c}{\tau_c} + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n + gV)$$

- New transport coefficients:

$$n = n_R + n_L$$

$$n_c = n_R - n_L$$

$$\boldsymbol{\Gamma} = \frac{e\mathbf{B}}{2\pi^2 g}$$

Chiral charge relaxation time: $\tau_c \gg \tau$

Density response in topological metal

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$$\frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2(n_c + gV_c) - \frac{n_c + gV_c}{\tau_c} + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n + gV)$$

- First derivatives will dominate at long length scales. This leads to propagating density modes and quasiballistic conductance.

Diffusion eigenmodes

$$i\omega_\pm = Dq^2 + \frac{1}{2\tau_c} \pm \sqrt{\frac{1}{4\tau_c^2} - \Gamma^2 q^2}$$

$$q < \frac{1}{2\Gamma\tau_c}$$

- Ordinary diffusion of conserved electric and almost conserved chiral charges:

$$i\omega_+ = Dq^2 + \frac{1}{\tau_c} \qquad i\omega_- = Dq^2$$

Diffusion eigenmodes

$$i\omega_{\pm} = Dq^2 + \frac{1}{2\tau_c} \pm \sqrt{\frac{1}{4\tau_c^2} - \Gamma^2 q^2}$$

$$q > \frac{1}{2\Gamma\tau_c}$$

- Get a propagating mode:

$$\omega \approx \Gamma q - iDq^2$$

Propagating mode

- Get a propagating mode:

$$\omega \approx \Gamma q - iDq^2$$

- This mode is weakly damped as long as:

$$q < \frac{\Gamma}{D} = \frac{1}{L_a}$$

$$L_a = \frac{D}{\Gamma} \sim \ell(k_F \ell_B)^2$$

$$\ell_B = \sqrt{\hbar c/eB}$$

Propagating mode

- Linearly-dispersing propagating mode:

$$\omega = \Gamma q$$

- Exists as long as: $\dfrac{1}{L_*} < q < \dfrac{1}{L_a}$

$$L_* = \frac{L_c^2}{L_a} \qquad L_c = \sqrt{D\tau_c}$$ chiral charge diffusion length

- The existence of such a propagating mode in the diffusive transport regime in weak magnetic fields is a qualitatively new feature of topological metals.

Propagating mode

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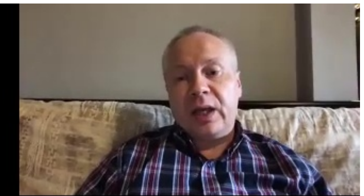
Propagating mode



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Chiral Magnetic Effect

$$\mathbf{j} = \frac{\sigma}{e}\boldsymbol{\nabla}\mu + eg\mu_c\boldsymbol{\Gamma}$$

Kharzeev et al.

$$\sigma = \frac{ne^2\tau}{m}$$

involves irreversible randomization of momentum, dissipative.

- Second term is nondissipative.

Density response in topological metal

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2(n + gV) + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n_c + gV_c)$$

$$\frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2(n_c + gV_c) - \frac{n_c + gV_c}{\tau_c} + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n + gV)$$

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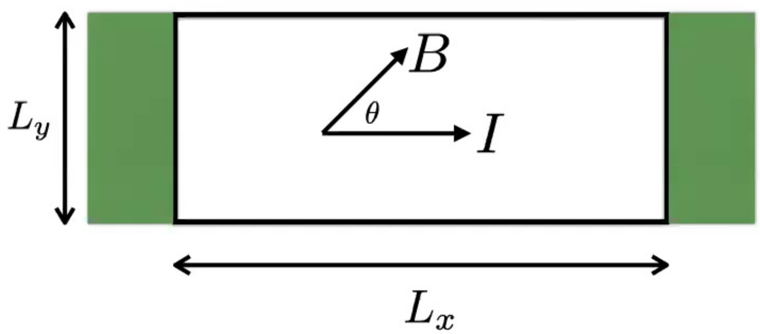
Anisotropic MR



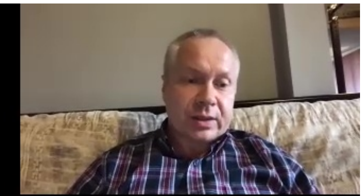
Kharzeev et al.

- Chiral magnetic effect: $\mathbf{j} = \frac{\sigma}{e}\boldsymbol{\nabla}\mu + eg\mu_c\boldsymbol{\Gamma}$

$$\rho_{xx} = \rho_\perp - \Delta\rho\cos^2\theta,$$
$$\rho_{yx} = -\Delta\rho\sin\theta\cos\theta,$$

$$\Delta\rho = \rho_\perp - \rho_{||}$$

$$\rho_\perp = 1/\sigma$$



Anisotropic MR

Negative LMR: $\rho_{xx} = \rho_\perp - \Delta\rho \cos^2\theta$

Son & Spivak AAB

Planar Hall Effect: $\rho_{yx} = -\Delta\rho \sin\theta\cos\theta$

AAB $\Delta\rho = \rho_\perp - \rho_\parallel = \dfrac{1}{\sigma}\dfrac{(L_c/L_a)^2}{1+(L_c/L_a)^2}$

$$L_a = \frac{D}{\Gamma} \sim \ell(k_F\ell_B)^2$$

$$\ell_B = \sqrt{\hbar c/eB}$$

purely quantum phenomena!

Anisotropic MR

$$\Delta\rho = \rho_{\perp} - \rho_{\parallel} = \frac{1}{\sigma}\frac{(L_c/L_a)^2}{1+(L_c/L_a)^2}$$

- AMR has opposite sign to what is typically seen in ferromagnets and much larger magnitude.

$$\frac{\Delta\rho}{\rho_{\perp}} \approx 50\%$$ largest AMR in a FM metal in U_3As_4

- Several hundred percent in Na_3Bi, N.P. Ong et al.

$$\frac{\Delta\rho}{\rho_{\parallel}} = \frac{(L_c/L_a)^4}{1+(L_c/L_a)^2}$$

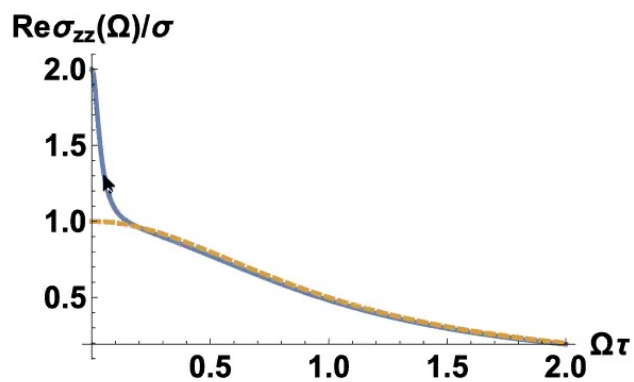
Optical conductivity

- From charge continuity equation:

$$\sigma_{zz}(\omega) = \lim_{q \to 0} \frac{ie^2\omega}{q^2} \chi_{00}(q, \omega)$$

$$\mathrm{Re}\sigma_{zz}(\omega) = \frac{\sigma}{1+\omega^2\tau^2}\left[1 + \left(\frac{L_c}{L_a}\right)^2 \frac{1-\omega^2\tau\tau_c}{1+\omega^2\tau_c^2}\right]$$

AAB



Chiral anomaly and interactions

- Chiral anomaly inevitably implies Weyl nodes in case of weak interactions.
- Does this remain true when the interactions are not weak?
- In other words, can we gap out the Weyl nodes while preserving the chiral anomaly and while not breaking any symmetries?

Anisotropic MR

Negative LMR: $\rho_{xx} = \rho_\perp - \Delta\rho\cos^2\theta$

Son & Spivak   AAB

Planar Hall Effect: $\rho_{yx} = -\Delta\rho\sin\theta\cos\theta$

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$$L_a = \frac{D}{\Gamma} \sim \ell(k_F\ell_B)^2$$

$$\ell_B = \sqrt{\hbar c/eB}$$

purely quantum phenomena!

Density response in topological metal

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2(n + gV) + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n_c + gV_c)$$

$$\frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2(n_c + gV_c) - \frac{n_c + gV_c}{\tau_c} + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n + gV)$$

- First derivatives will dominate at long length scales. This leads to propagating density modes and quasiballistic conductance.

Propagating mode

- Get a propagating mode:

$$\omega \approx \Gamma q - iDq^2$$

- This mode is weakly damped as long as:

$$q < \frac{\Gamma}{D} = \frac{1}{L_a}$$

$$L_a = \frac{D}{\Gamma} \sim \ell (k_F \ell_B)^2$$

$$\ell_B = \sqrt{\hbar c / eB}$$

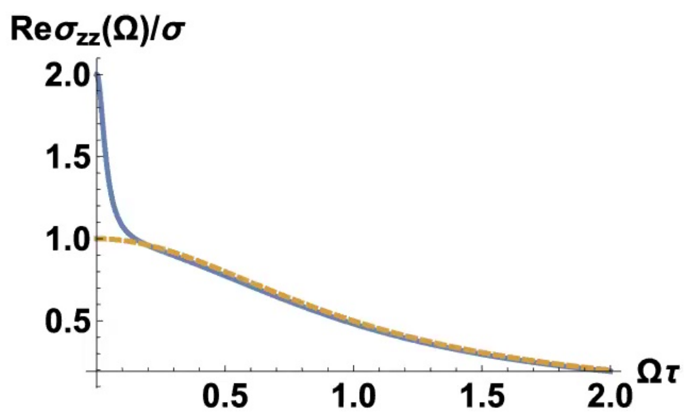
Optical conductivity

- Transfer of spectral weight to a narrow low-frequency peak.

$$\mathrm{Re}\sigma_{zz}(\omega) = \frac{\sigma}{1+\omega^2\tau^2}\left[1+\left(\frac{L_c}{L_a}\right)^2 \frac{1-\omega^2\tau\tau_c}{1+\omega^2\tau_c^2}\right]$$

- Drude weight is preserved.

$$\int_0^\infty d\omega\, \mathrm{Re}\sigma(\omega) = \frac{\pi\sigma}{2\tau}$$



Chiral anomaly and interactions

- Chiral anomaly inevitably implies Weyl nodes in case of weak interactions.
- Does this remain true when the interactions are not weak?
- In other words, can we gap out the Weyl nodes while preserving the chiral anomaly and while not breaking any symmetries?

3D Fractional Quantum Hall Effect

PHYSICAL REVIEW LETTERS **124**, 096603 (2020)

Fractional Quantum Hall Effect in Weyl Semimetals

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(Received 8 July 2019; accepted 11 February 2020; published 6 March 2020)

- We can “defeat” the anomaly, but at the cost of fractionalizing electrons.

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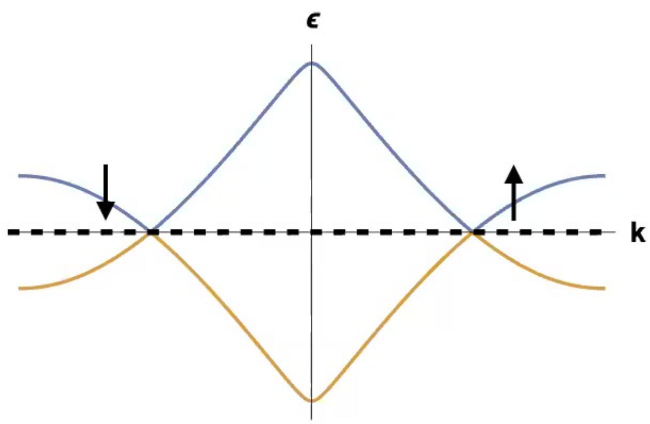
- We can "defeat" the anomaly, but at the cost of fractionalizing electrons.
- This is analogous to asking if we can have a gapped Mott insulator not breaking any symmetries at odd integer electron filling per unit cell.

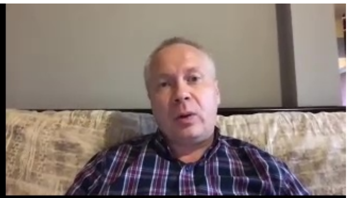
Vortex condensation

- Induce fully gapped superconductivity in Weyl semimetal.
- Destroy SC coherence by condensing vortices while keeping the pairing gap: this produces an insulator (superconductor to insulator transition).
- Chiral anomaly places strong restrictions on the procedure and prohibits a simple insulator, has to have topological order.

Weyl superconductor

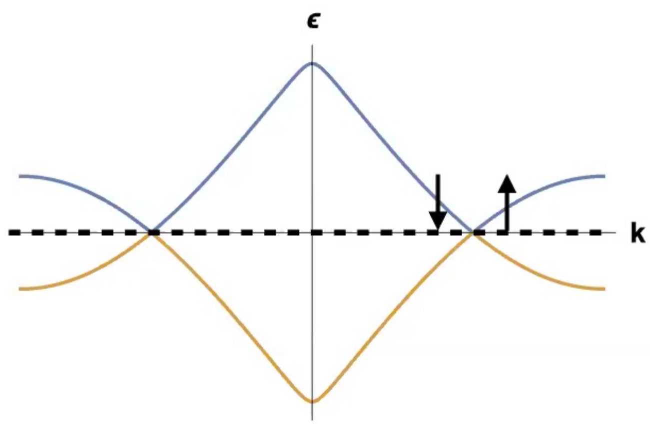
- BCS: pairing k and -k states, i.e. internodal pairing.





Weyl superconductor

- FFLO (Fulde-Ferrell-Larkin-Ovchinnikov): pairing states on the opposite side of each Weyl point, i.e. intranodal pairing.



BCS pairing

- Weak BCS pairing can not open a gap, since the two chiralities are not mixed by the pairing term:

$$H = v_F \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \tau^z \boldsymbol{\sigma} \cdot \mathbf{k} c_{\mathbf{k}} + \Delta \sum_{\mathbf{k}} (c^\dagger_{\mathbf{k}R} i \sigma^y c^\dagger_{-\mathbf{k}L} + h.c.)$$

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}R\uparrow}, c_{\mathbf{k}R\downarrow}, c^\dagger_{-\mathbf{k}L\downarrow}, -c^\dagger_{-\mathbf{k}L\uparrow})$$

$$H = \sum_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} (v_F \boldsymbol{\sigma} \cdot \mathbf{k} + \Delta \tau^x) \psi_{\mathbf{k}}$$

Meng & Balents   Bednik, Zyuzin, AAB

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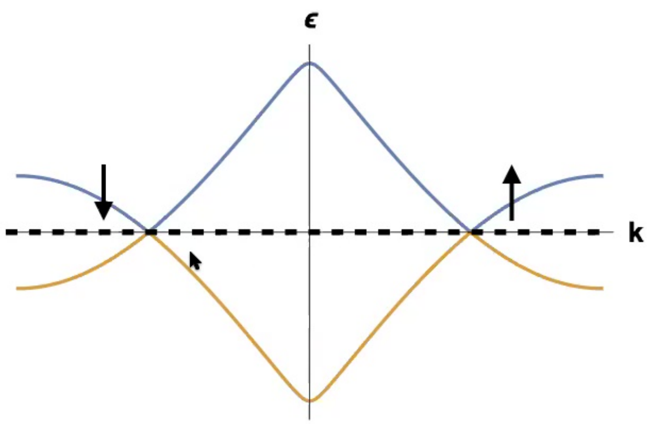

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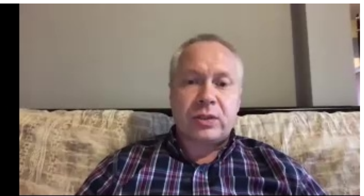

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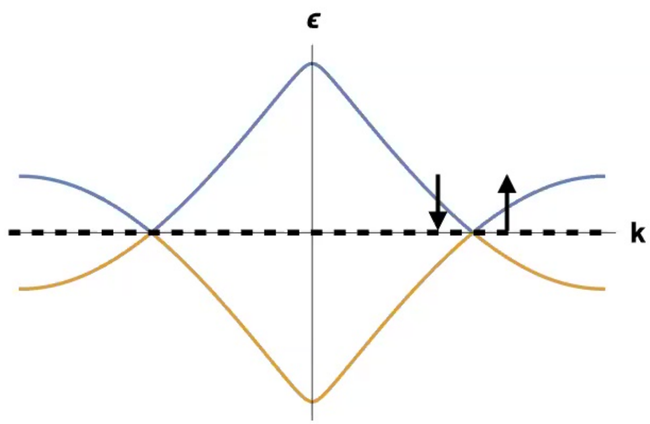
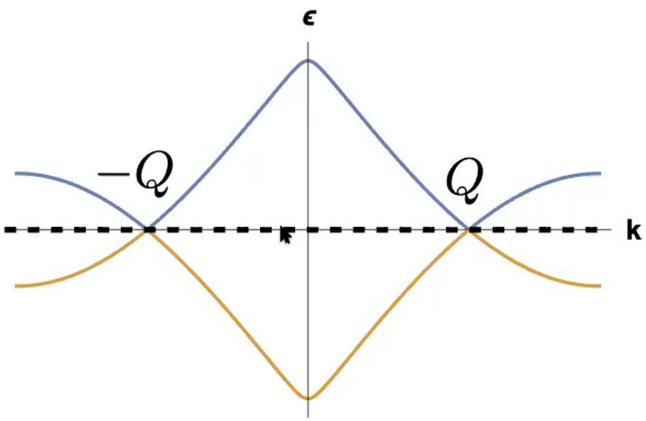


FFLO pairing

- FFLO does open a gap, but breaks translational symmetry:



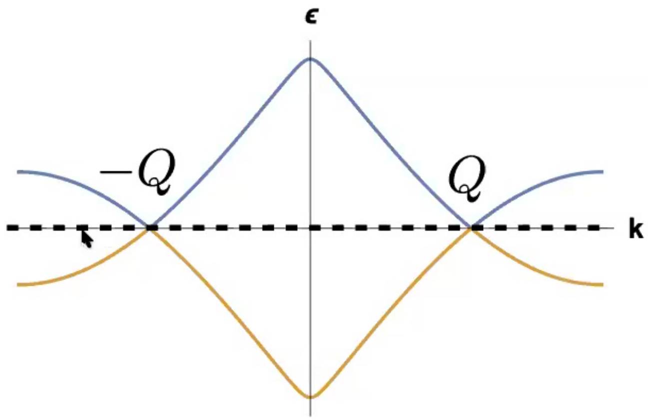
$$\Delta(\mathbf{Q}) \sim \sum_{\mathbf{k}} \langle c^{\dagger}_{\mathbf{Q}+\mathbf{k}} c^{\dagger}_{\mathbf{Q}-\mathbf{k}} \rangle$$

carries momentum 2Q.

$$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$$

carries momentum 4Q.

FFLO pairing



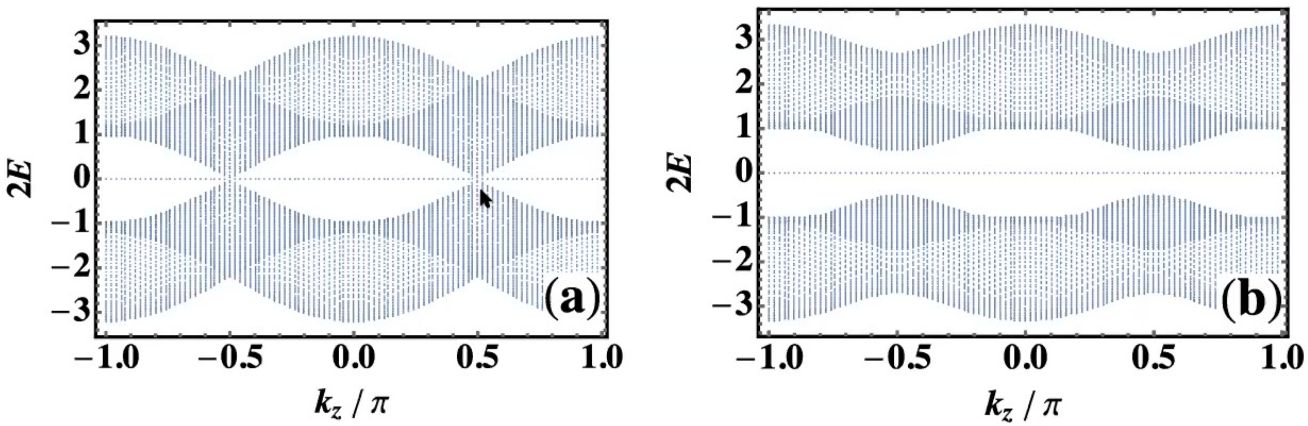
$$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$$

carries momentum 4Q.

- This breaks translational symmetry, unless $\mathbf{Q} = \mathbf{G}/4$
- In other words, FFLO does not break translational symmetry when Weyl node separation is exactly half the reciprocal lattice vector.

Majorana arc

- Fermi arc becomes Majorana arc, which occupies twice the momentum interval of the Fermi arc, i.e. 4Q.

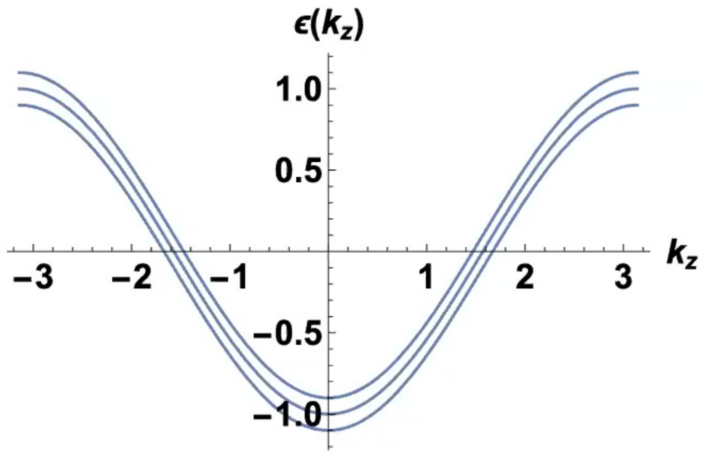


$$\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Vortex condensation in FFLO state

- n-fold vortex (Φ=nhc/2e) in FFLO paired state: get n chiral Majorana modes in the vortex core.

$$\epsilon_p(k_z) = \epsilon_F\left(1 - \frac{2p}{n+1}\right) + v_F k_z. \qquad p = 1, \ldots, n.$$

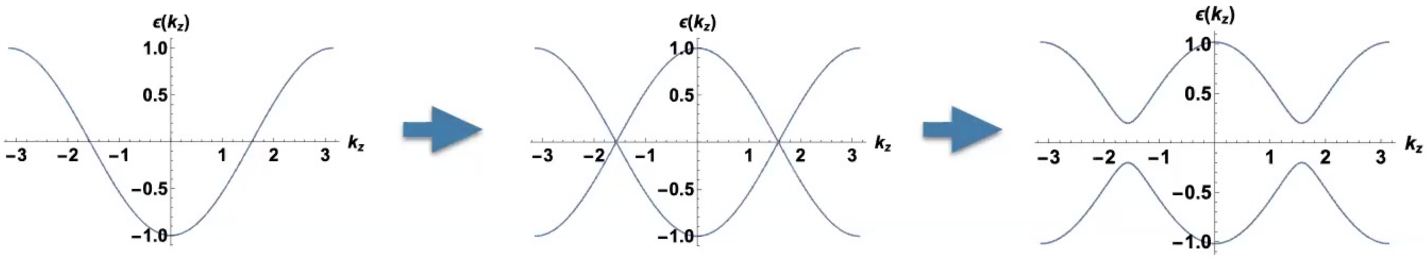


Callan & Harvey

Jackiw & Rossi

Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:



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$$H = v_F \sum_{k_z} [k_z c^\dagger_{k_z} \tau^z c_{k_z} + \Delta(c^\dagger_{k_z} i\tau^y c^\dagger_{-k_z} + \text{h.c.})/2]$$

- An odd number of Majorana modes can not be eliminated without breaking translational symmetry, thus a fundamental SC vortex may not be condensed.

$$\Phi = \frac{hc}{2e} = \pi \qquad \hbar = c = e = 1$$

Vortex condensation in FFLO state

- A double vortex does not have Majorana modes, but may still not be condensed.

- This follows from the fact that the insulating state we want to obtain must preserve the chiral anomaly, i.e. must have a Hall conductivity of half conductivity quantum per atomic plane:

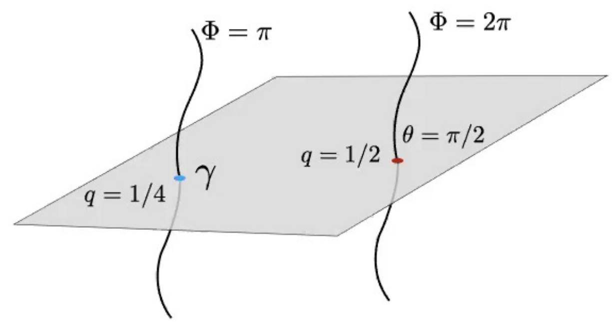
$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi}$$

Vortex condensation in FFLO state

- A vortex will induce a charge when intersecting an atomic plane:

$$\mathcal{L} = \frac{\sigma_{xy}}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

$$q = \frac{\Phi}{4\pi} = \frac{1}{2}$$



- A pair of such charges will have semion exchange statistics.

$$\theta = 2\pi^2\sigma_{xy} = \frac{\pi}{2}$$

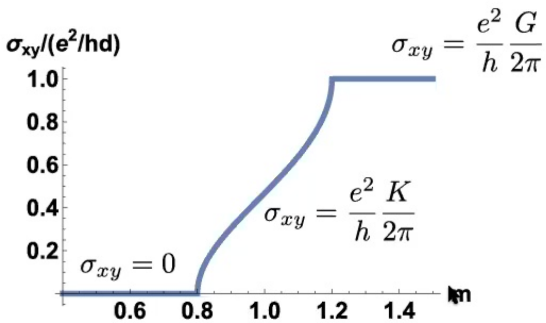
Vortex condensation in FFLO state

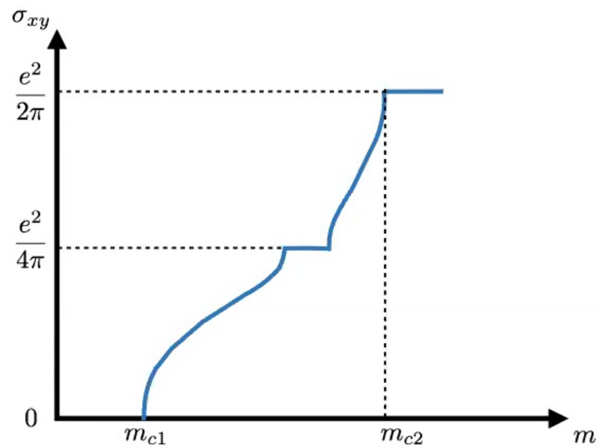
- Following the same logic, quadruple vortices have bosonic statistics and thus may be condensed without breaking any symmetries.

- This is an insulating state that preserves the chiral anomaly and does not break any symmetries.

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi} \qquad \kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Nontrivial generalization of FQHE to 3D





- In the presence of interactions, smooth evolution of the Hall conductivity with the magnetization in a Weyl semimetal may be interrupted by a half-quantized plateau.

BF theory of the 3D FQHE

- 2D FQHE: Chern-Simons theory.

Odd-denominator Laughlin state $\nu = \dfrac{1}{2q+1}$

$$\mathcal{L} = i\frac{2q+1}{4\pi}\epsilon_{\mu\nu\lambda}b_\mu\partial_\nu b_\lambda + \frac{ie}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu b_\lambda + ib_\mu j_\mu$$

- Excitations are quasiparticles (vortices), which carry fractional charge and fractional statistics:

$$Q = \frac{e}{2q+1} \qquad \theta = \frac{\pi}{2q+1}$$

BF theory of the 3D FQHE

$$\mathcal{L} = \mathcal{L}_f(-a_\mu) + \frac{i}{2\pi}(A_\mu + a_\mu + 2c_\mu)\epsilon_{\mu\nu\lambda\rho}\partial_\nu b_{\lambda\rho} - \frac{2i}{4\pi}\epsilon_{z\mu\nu\lambda}c_\mu\partial_\nu c_\lambda + ib_{\mu\nu}j_{\mu\nu} + ic_\mu j_\mu$$

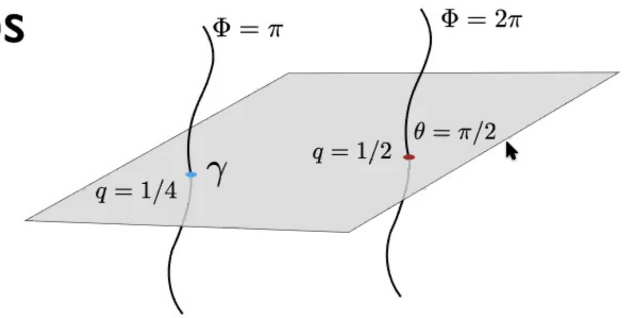
- Neutral fermions (couple to a_μ).
- Charged bosons (couple to c_μ).
- Vortex loops (couple to $b_{\mu\nu}$).
- a_μ is a Z_2 gauge field, while c_μ is a Z_4 gauge field.

Thakurathi & AAB

BF theory of the 3D FQHE

$$\mathcal{L} = \mathcal{L}_f(-a_\mu) + \frac{i}{2\pi}(A_\mu + a_\mu + 2c_\mu)\epsilon_{\mu\nu\lambda\rho}\partial_\nu b_{\lambda\rho} - \frac{2i}{4\pi}\epsilon_{z\mu\nu\lambda}c_\mu\partial_\nu c_\lambda + ib_{\mu\nu}j_{\mu\nu} + ic_\mu j_\mu$$

- Intersections of vortex loops with atomic planes are anyons.



Conclusions

- Topological metal is a bulk metal which exists for topological rather than electron filling reasons.
- In Weyl semimetals bands at the Fermi energy touch at an even number of nodes, which behave as chiral Weyl fermions.
- These nodes are topological objects and lead to observable phenomena: Fermi arc surface states, giant anisotropic magnetoresistance and dissipationless transport, non-Drude optical conductivity, 3D fractional quantum Hall effect, etc.

Thanks

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Area of Research:
Condensed Matter

CHONG WANG

Faculty
Faculty

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BF theory of the 3D FQHE

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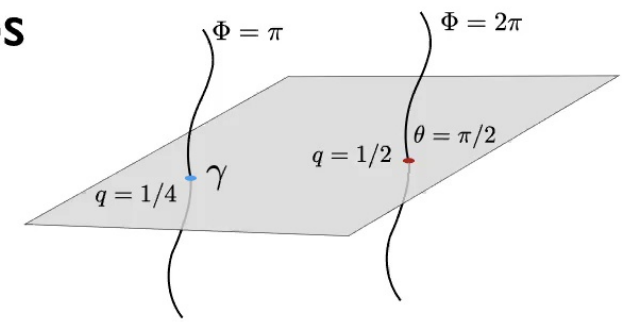

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