Title: Holography of information from semiclassical gravity

Speakers: Suvrat Raju

Series: Quantum Gravity

Date: September 17, 2020 - 11:30 AM

URL: http://pirsa.org/20090001

Abstract: We will argue that even with semiclassical gravity, it can be shown that a copy of all the information on a Cauchy slice resides near the boundary of the slice. We will first demonstrate this in asymptotically global AdS, and then in four-dimensional asymptotically flat space. We will then describe a physical protocol that can be used to verify this property at low-energies and within perturbation theory. This property of gravity implies that information about the black-hole interior is always present "outside" the black hole, which leads to a fresh perspective on the information paradox.

References:
1) https://arxiv.org/abs/2002.02448
2) https://arxiv.org/abs/2008.01740

Holography of information from semiclassical gravity

Suvrat Raju

International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

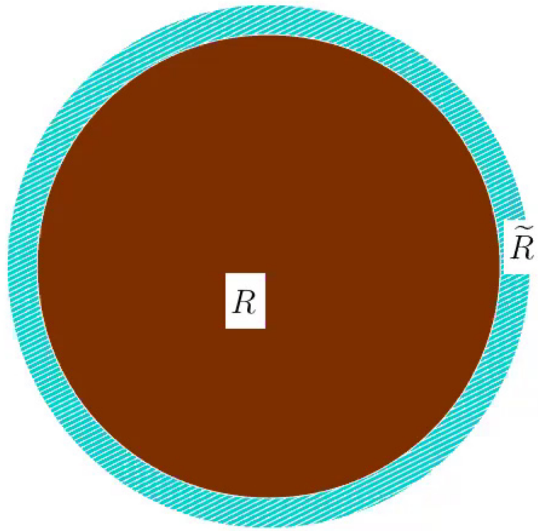
Perimeter Institute
17 September 2020

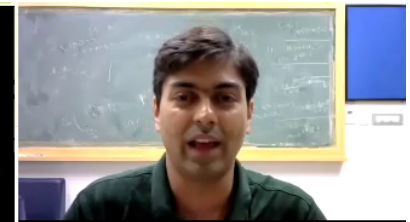
Collaborators and References

- 2008.01740, Chandramouli Chowdhury, Olga Papadoulaki, S.R.
- 2002.02448, Alok Laddha, Siddharth Prabhu, Pushkal Shrivastava, S.R.
- arXiv:1903.11073, S.R.
- arXiv:1809.10154, S.R.
- arXiv:1603.02812, Souvik Banerjee, Jan-Willem Bryan, Kyriakos Papadodimas, S.R.

Holography of quantum information

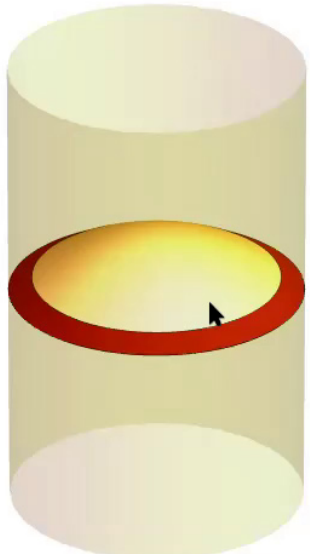
In quantum gravity, a copy of all the information on a Cauchy slice is available near the boundary of the slice.





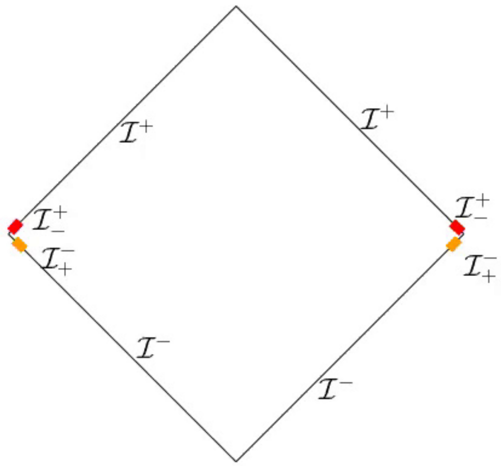
Holography of information in AdS

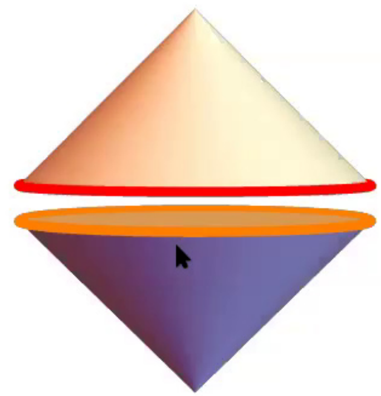
We will establish this in global AdS.



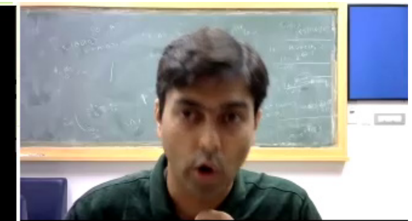
Also describe physical protocol for observers in brown "annular region" to obtain complete information about the state near center of AdS.

Holography of information in flat space

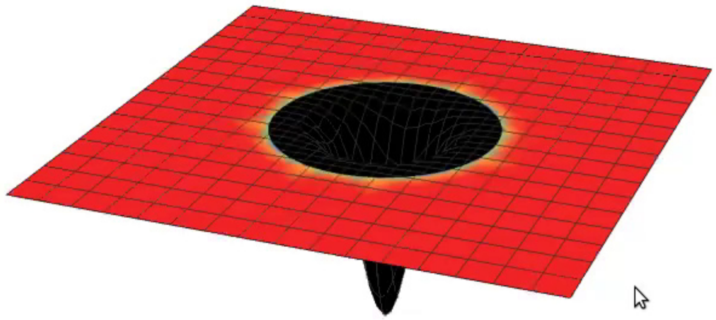




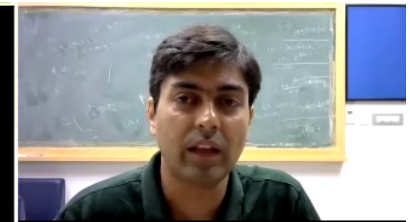
In 4D asymptotically flat spacetime, all information on null infinity is present on the past boundary of future null infinity (or future boundary of past null infinity).



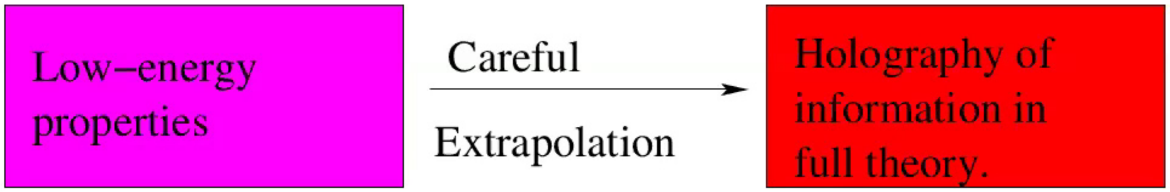
Implications for black holes



When applied to black holes, suggests that the exterior region always contains a complete copy of the information in the interior.



Philosophy



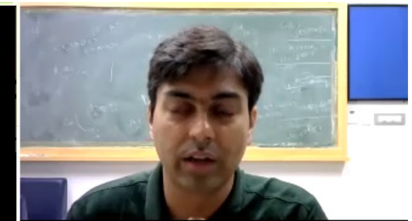
Draws on previous work but detailed conclusions and arguments are different.

[Marolf, 2006–13]

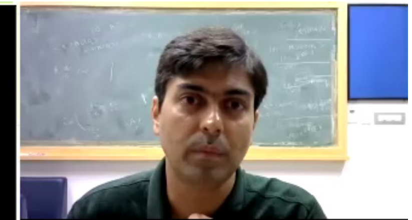
[de Boer, Solodukhin, 2003]

[Bagchi, Grumiller, Pasterski, Shu-Heng Shao, Strominger, 2016–19]

1. Introduction
2. AdS
3. Asymptotically flat spacetimes
4. Perturbative verification
5. Black Holes



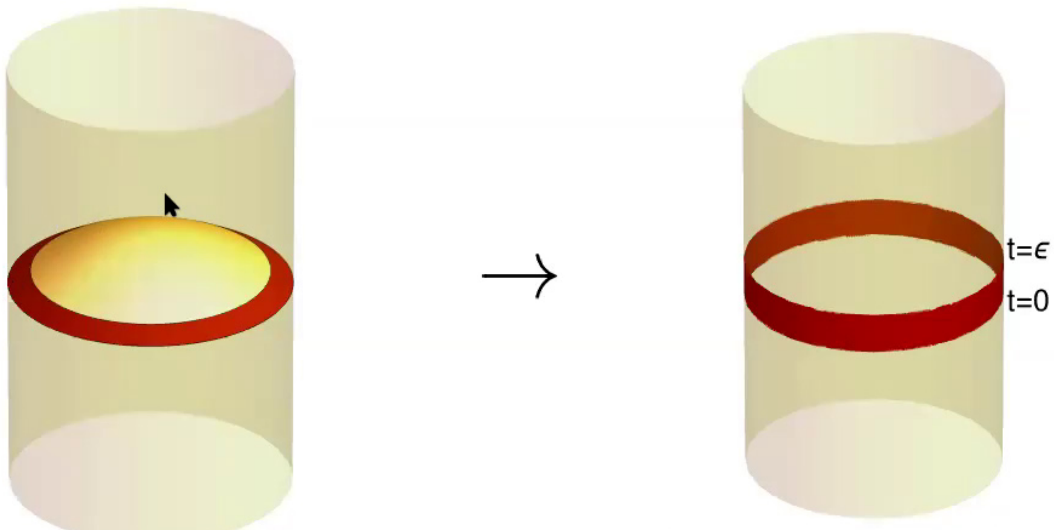
Outline



Setting

It is more precise to consider asymptotic observables for $t \in [0, \epsilon]$ rather than observables in $r \in [r_0, \infty]$.

$$r_0 \sim \cot(\frac{\epsilon}{2})$$



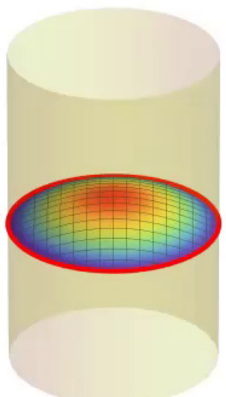
Holography of Information in AdS

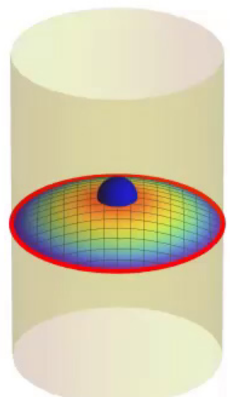
If two bulk states are distinct, they can be distinguished by observables in the time band $[0, \epsilon]$ near the boundary.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[S.R., 2019]

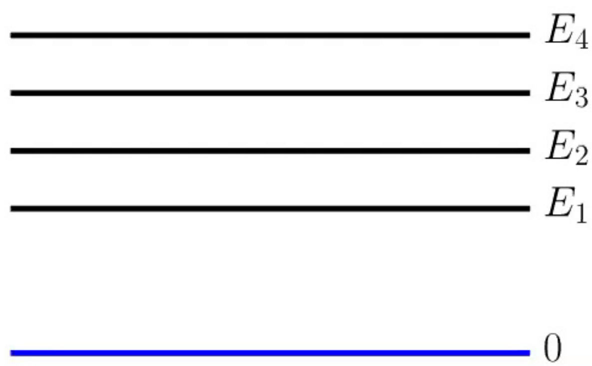
[Marolf, 2006–13]





Low-energy states

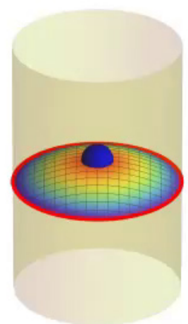
Effective field theory tells us the spectrum at low energies: unique vacuum and discrete excitations.



Boundary Hamiltonian

In semi-classical gravity, energy is a boundary term.

$$H = (d/16\pi G_N) \lim_{r\to\infty} r^{d-2} \int d^{d-1}\Omega h_{tt}$$



Assumption: The vacuum of H coincides with the vacuum of the full theory and H remains a positive operator in boundary algebra.

This is not to assume that H is the true Hamiltonian.

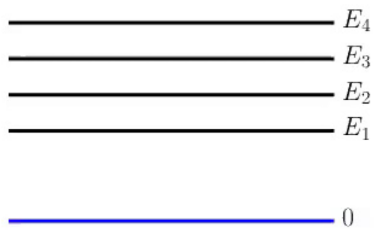
Projector on the vacuum

We apply standard rules of quantum mechanics to this observable.

$$H = \sum_E E P_E$$

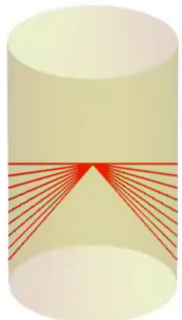
The probability of getting 0 upon measuring H is

$$\langle g | \mathcal{P}_0 | g \rangle$$



So $\mathcal{P}_0 = |0\rangle\langle 0|$ is a boundary observable in a theory of gravity.

Assumption 2: Hilbert space

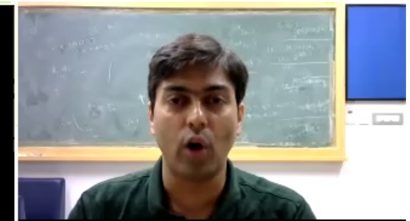


We can formulate the theory in the space given by exciting the vacuum by boundary operators at arbitrary times.

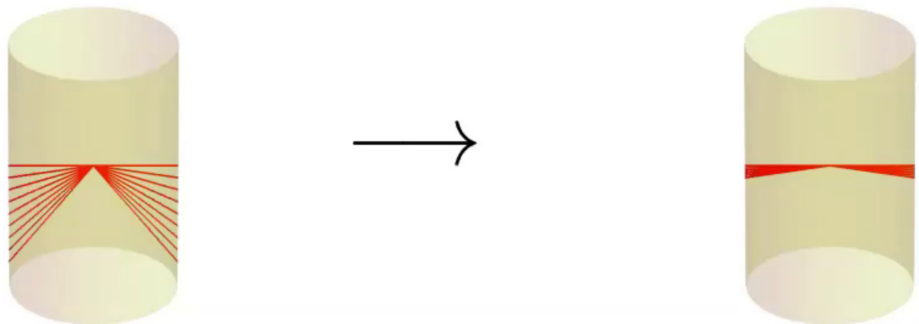
$$\mathcal{H} = A(t)|0\rangle$$

for $t \in [-\infty, \infty]$

- $\mathcal{H}$ is manifestly closed under time-evolution. So unitarity cannot force us to include additional states.
- $\mathcal{H}$ is quite large. Accommodates all black holes formed from collapse.



Squeezing the Hilbert space



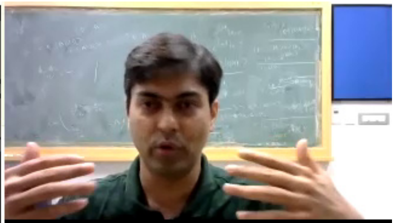
Since

$$\langle n|A(t)|0\rangle$$

is analytic in upper-half t-plane, any state in this space can be generated from

$$|n\rangle = X_n|0\rangle$$

where X_n is a boundary operator from time band $[0, \epsilon]$.



Basis using entanglement: Idea

This is not a deep result.

$$|\Psi\rangle = a|0\rangle|0\rangle + b|1\rangle|1\rangle.$$

Operators on the first qubit generate a basis for the Hilbert space.

$$\frac{1}{2a}(1+\sigma_z^1)|\Psi\rangle = |0\rangle|0\rangle$$
$$\frac{1}{2b}(1-\sigma_z^1)|\Psi\rangle = |1\rangle|1\rangle$$
$$\sigma_x^1|\Psi\rangle = a|1\rangle|0\rangle + b|0\rangle|1\rangle$$
$$\sigma_y^1|\Psi\rangle = ia|1\rangle|0\rangle - ib|0\rangle|1\rangle$$

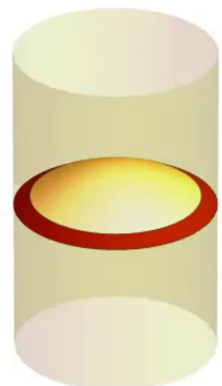
- Doesn't violate locality. Physically, can only act with unitary operators.
- Doesn't give information about second qubit. All unitaries on second qubit commute with observations on first.

Completing the argument

Applying bdry operators to $|0\rangle$ generates the H-space. (Also true in LQFT and gives no bulk info by itself.)

Vacuum can be identified from the boundary in gravity.

Boundary operators contain all info in gravity.



Canonical argument for holography of information

Let O be any operator in the theory.

$$O = \sum_{nm} c_{nm} |n\rangle\langle m|$$

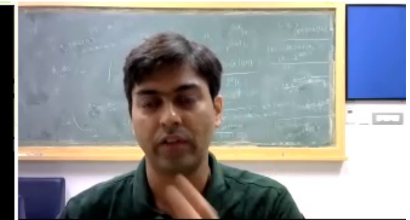
We have

$$|n\rangle = X_n|0\rangle; \qquad |m\rangle = X_m|0\rangle$$

where X_n, X_m are boundary operators. So

$$O = \sum c_{nm} X_n |0\rangle\langle 0| X_m^\dagger$$
$$= \sum c_{nm} X_n \mathcal{P}_0 X_m^\dagger$$

which is a sum of products of boundary operators, and so a boundary operator.



Importance of gravity

Non-gravitational theories, including gauge theories, contain exactly local gauge-invariant bulk operators which commute with all elements of $\mathcal{A}_{\text{bdry}}$.

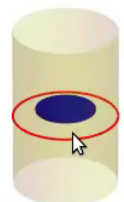
$$|g\rangle \quad \text{and} \quad e^{i\text{Tr}(F^2)(0)}|g\rangle$$

cannot be distinguished by any boundary measurement without gravity.

Importance of quantum mechanics

In the classical theory, one can specify initial data in the interior of a ball independently of data outside.

[Corvino, Schoen, 2003]



$$|g_1\rangle = \sum a_i |E_i\rangle \qquad\qquad |g_2\rangle = \sum a'_i |E_i\rangle$$

Classically, we can only measure

$$\langle H \rangle = \sum |a_i|^2 E_i$$

But in QM we get access to

$$\langle H^m \rangle = \sum |a_i|^2 E_i^m \neq \langle H \rangle^m$$

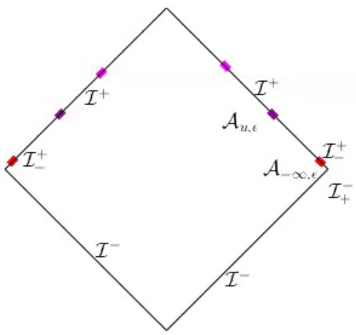
Outline

1. Introduction
2. AdS
3. Asymptotically flat spacetimes
4. Perturbative verification
5. Black Holes

Asymptotic boundary conditions

We consider four dimensional asymptotically flat spacetimes

$$ds^2 = -du^2 - 2dudr + r^2\gamma_{AB}d\Omega^A d\Omega^B + rC_{AB}d\Omega^A d\Omega^B + \frac{2m_{\rm B}}{r}du^2 + \gamma^{DA}D_D C_{AB}dud\Omega^B + \ldots$$

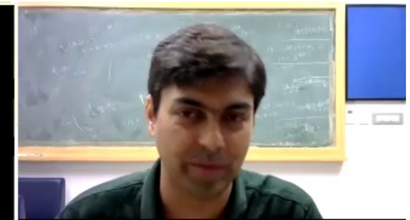


The algebra of observables between the cuts $[u, u+\epsilon]$ is $\mathcal{A}_{u,\epsilon}$. Includes the Bondi news, Bondi mass

$$N_{AB} = \partial_u C_{AB}; \qquad M(u) = \int \sqrt{\gamma}\, m_{\rm B}(u,\Omega) d^2\Omega,$$

and all possible products and linear combinations.

$\mathcal{A}_{-\infty,\epsilon}$ is the algebra near $u \to -\infty$.



Hilbert space

The vacuum is infinitely degenerate

$$\mathcal{Q}_{\ell,m}|\{s\}\rangle = s_{\ell,m}|\{s\}\rangle.$$

and on top of each vacuum we can build a Fock space

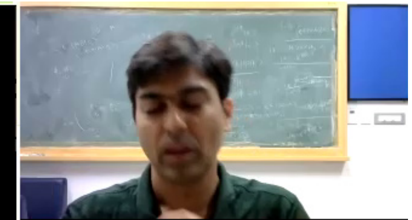
$$\mathcal{H}_{\{s\}} = \text{span of}\{N(f_1)N(f_2)\ldots N(f_n)\}|\{s\}\rangle,$$

[Ashtekar, Faddeev, Kulish, Strominger, He, Lysov, Mitra, Pasterski, Compere, Laddha, Campiglia . . . , 1981–2020]

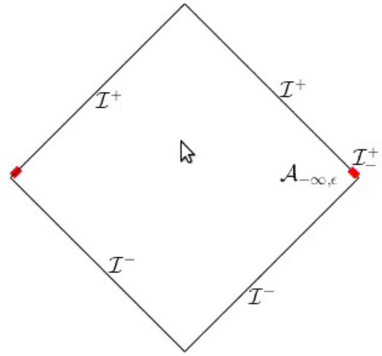
The full Hilbert space of massless particles is

$$\mathcal{H} = \bigoplus_{\{s\}} \mathcal{H}_{\{s\}},$$

Our statements will be confined to this Hilbert space (which excludes massive excitations)



Result : Information at $\mathcal{I}^+_-$



We will now describe.

Any two distinct states in $\mathcal{H}$ can be distinguished just by observables in $\mathcal{A}_{-\infty,\epsilon}$

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Marolf, 2006–13]

[de Boer, Solodukhin, 2003]

[Bagchi, Grumiller, Pasterski, Shu-Heng Shao, Strominger, 2016–19]

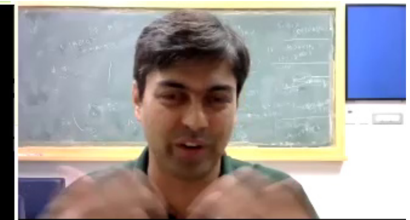
Step 1: Squeezing the Hilbert space

The positivity of the full Hamiltonian guarantees that any state $|n\rangle \in \mathcal{H}_{\{s\}}$ can be approximated arbitrarily well as

$$|n\rangle \doteq X_n|\{s\}\rangle$$

where $X_n \in \mathcal{A}_{-\infty,\epsilon}$.

The argument is precisely as in AdS.



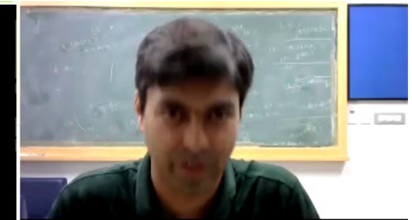
Step 2: Projector onto all vacua

- Both the Hamiltonian and supertranslation charges are in $\mathcal{A}_{-\infty,\epsilon}$.
- Upon measurement of charge, probability of getting answer between $s + \Delta s$ is

$$\langle g|\mathcal{P}_{\ell,m}[s]|g\rangle$$

- So $\mathcal{P}_{\ell,m}[s] \in \mathcal{A}_{-\infty,\epsilon}$, and also the projector on the manifold of vacua

$$\mathcal{P}_0 = \int \left(\prod_{\ell,m} ds_{\ell,m}\right) |\{s\}\rangle\langle\{s\}| \in \mathcal{A}_{-\infty,\epsilon}.$$



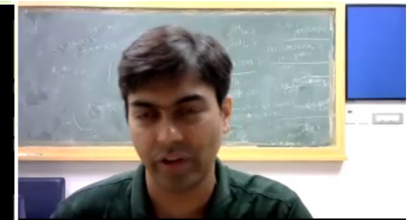
Projector onto a specific vacuum

$$\mathcal{P}_0 \prod_{\ell,m} \mathcal{P}_{\ell,m}[s_{\ell,m}] = |\{s\}\rangle\langle\{s\}| \in \mathcal{A}_{-\infty,\epsilon}.$$

In flat space as in AdS, one can select a specific vacuum using observables from the boundary of the spacetime. Unique feature of gravity.

Using projectors and simple operators, we can construct

$$T_{\{s\},\{s'\}} = |\{s\}\rangle\langle\{s'\}| \in \mathcal{A}_{-\infty,\epsilon}.$$



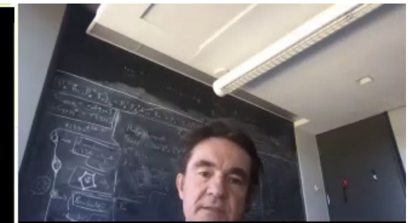
Argument for result

- Any operator, $\mathcal{H} \to \mathcal{H}$, can be written as

$$A = \sum_{s,s',n,m} c(n,m,s,s') |n_{\{s\}}\rangle\langle m_{\{s'\}}|$$
$$\doteq \sum c(n,m,s,s') X_n |\{s\}\rangle\langle\{s'\}| X_m^\dagger$$
$$= \sum c(n,m,s,s') X_n T_{\{s\},\{s'\}} X_m^\dagger.$$

The RHS is manifestly in $\mathcal{A}_{-\infty,\epsilon}$

- So any operator, $\mathcal{H} \to \mathcal{H}$, can be approximated arbitrarily well by an operator near $\mathcal{I}^+_-$.



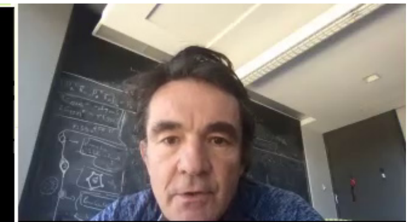
Argument for result

- Any operator, $\mathcal{H} \to \mathcal{H}$, can be written as

$$
\begin{aligned}
A &= \sum_{s,s',n,m} c(n,m,s,s') |n_{\{s\}}\rangle \langle m_{\{s'\}}| \\
&\doteq \sum c(n,m,s,s') X_n |\{s\}\rangle \langle \{s'\}| X_m^\dagger \\
&= \sum c(n,m,s,s') X_n T_{\{s\},\{s'\}} X_m^\dagger.
\end{aligned}
$$

  The RHS is manifestly in $\mathcal{A}_{-\infty,\epsilon}$

- So any operator, $\mathcal{H} \to \mathcal{H}$, can be approximated arbitrarily well by an operator near $\mathcal{I}^+_-$.



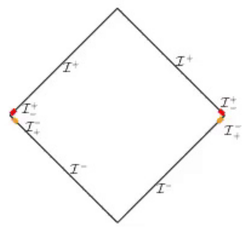
Assumptions for the full theory

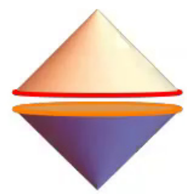
This result extends to the full theory of quantum gravity, if we assume

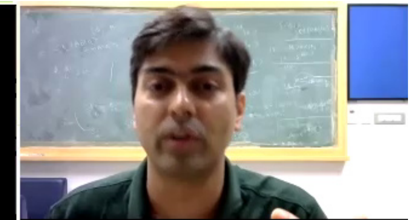
1. Vacua in the full theory are still labelled by operators near $\mathcal{I}^+_-$.

2. Operators that map the space of vacua back to itself are contained in $\mathcal{A}_{-\infty,\epsilon}$.

So, if the full theory shares low-energy properties of the semi-classical theory, all information about massless particles is available near $\mathcal{I}^+_-$ (or near $\mathcal{I}^-_+$).





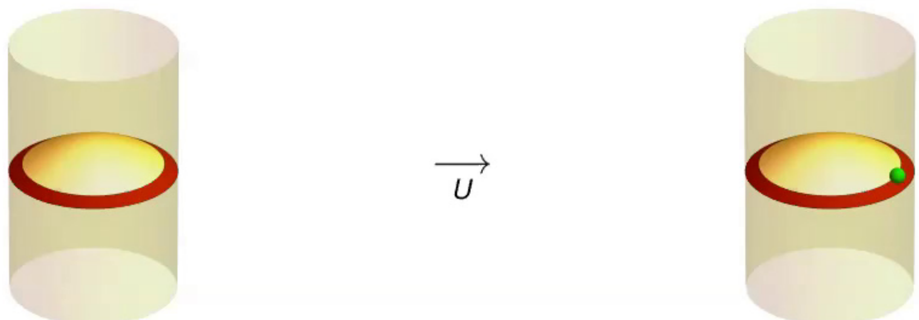


Protocol for verification

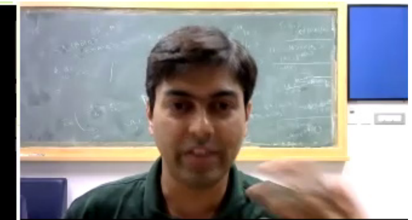
Consider near-boundary observers allowed to act with simple low energy unitaries from the time-band.

$$\text{for example } U = e^{i\int_0^{\epsilon} O(t,\Omega)f(t,\Omega)}$$

and make projective measurements of the energy.

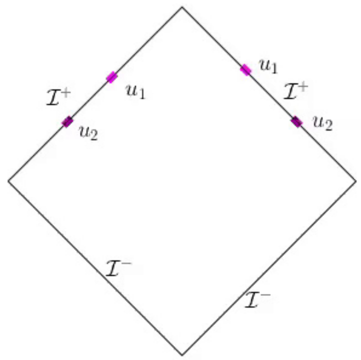


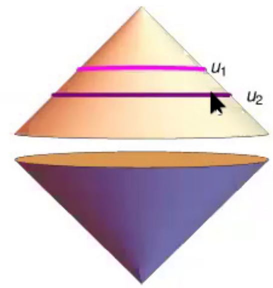
Observers have access to identically prepared systems, so they can reliably determine probabilities.



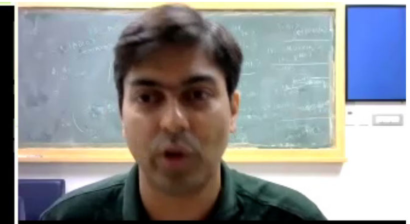
A stronger result

All information accessible through $\mathcal{A}_{u_1,\epsilon}$ is also available through $\mathcal{A}_{u_2,\epsilon}$ for any $u_2 < u_1$.





But this requires stronger assumptions about the UV-theory.



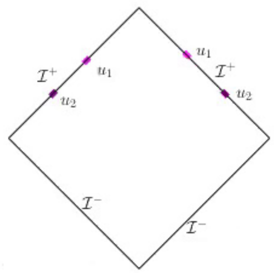
Assumptions for result 2

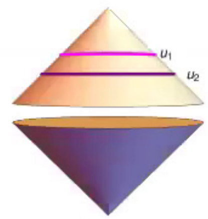
- In the semiclassical theory

$$[M(u), C_{AB}(u', \Omega)] = -4\pi G i \partial_{u'} C_{AB}(u', \Omega)\theta(u' - u),$$
$$[M(u), O(u', \Omega)] = -4\pi G i \partial_{u'} O(u', \Omega)\theta(u' - u).$$

- These depend only on the weak field limit. So could assume they carry over to the full UV theory.

Argument for result 2



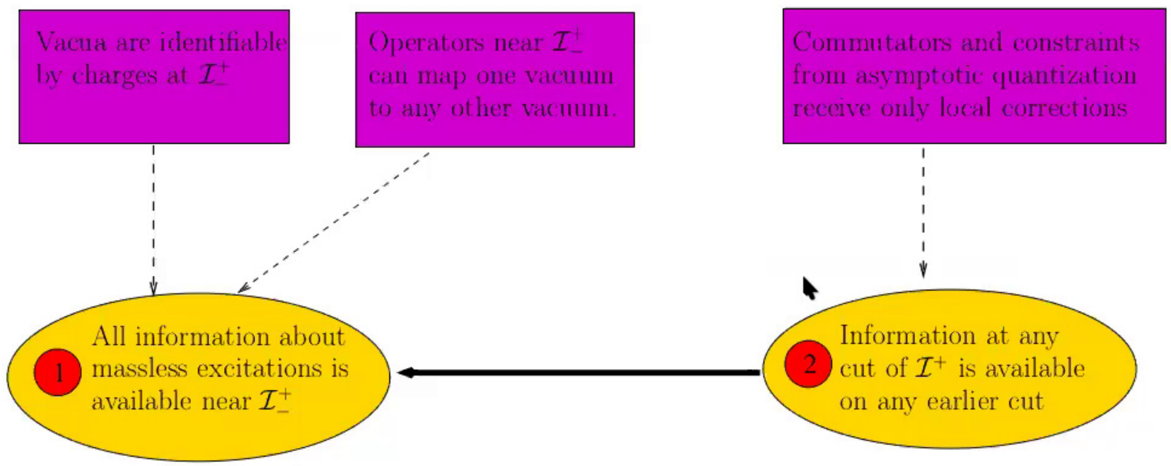


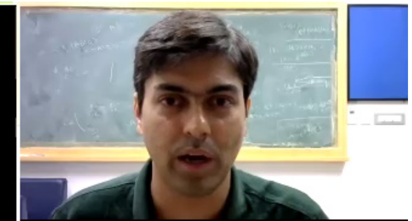
- For $u_1 > u_2$

$$C_{AB}(u_1, \Omega) = e^{\frac{iM(u_2)}{4\pi G}(u_1 - u_2)} C_{AB}(u_2, \Omega) e^{\frac{-iM(u_2)}{4\pi G}(u_1 - u_2)};$$

- Unlike AdS, we seem to lose information as we move up on $\mathcal{I}^+$.

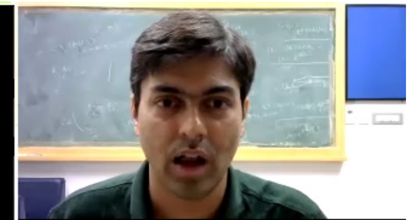
Recap of assumption and result





Outline

1. Introduction
2. AdS
3. Asymptotically flat spacetimes
4. **Perturbative verification**
5. Black Holes

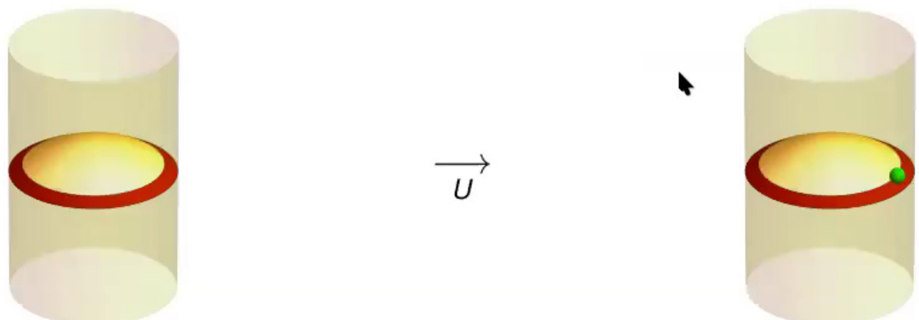


Protocol for verification

Consider near-boundary observers allowed to act with simple low energy unitaries from the time-band.

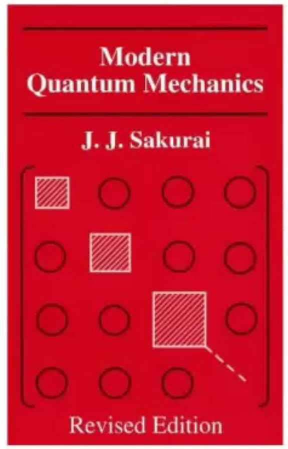
$$\text{for example } U = e^{i \int_0^{\epsilon} O(t,\Omega) f(t,\Omega)}$$

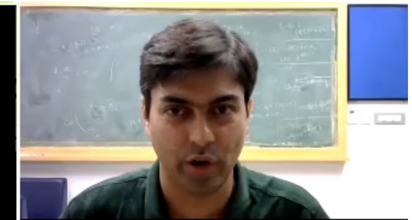
and make projective measurements of the energy.



Observers have access to identically prepared systems, so they can reliably determine probabilities.

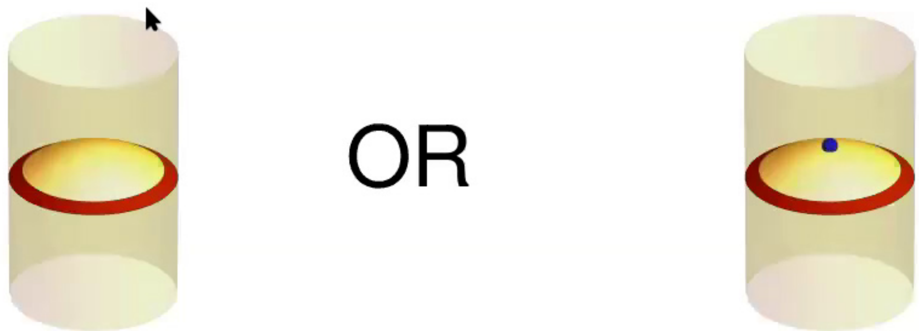
Textbook framework for QM

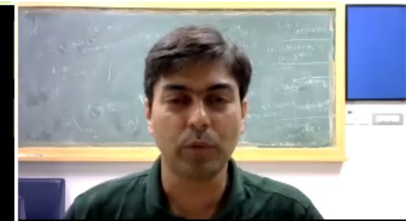




Simple task 1: Identifying the vacuum

- The system is in some state $|g\rangle$.
- The near boundary observers are asked to determine if $|g\rangle = |0\rangle$ or not.

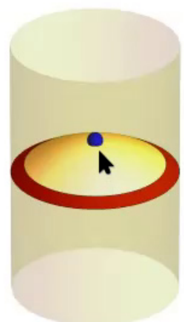


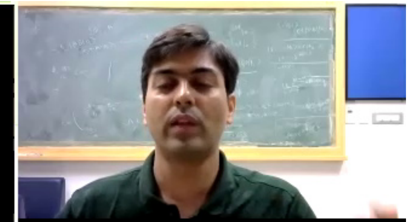


Contrast with LQFT

- In a local QFT (including non-gravitational gauge theories), the task is impossible.
- Near-boundary observers cannot distinguish $|0\rangle$ from an orthogonal state $U_{\text{bulk}}|0\rangle$.

indistinguishable
(in LQFT)
from



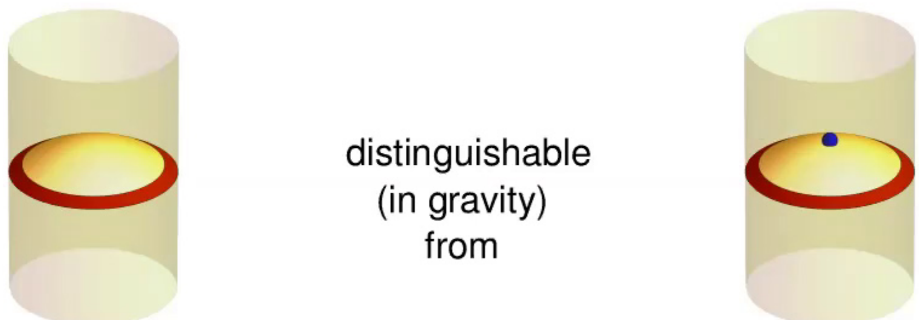


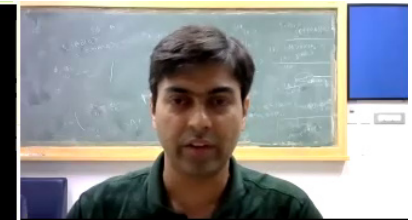
Success in gravity

- In gravity, the observers just measure H!
- Probability of getting 0 directly yields

$$|\langle g|0\rangle|^2$$

- Successfully distinguishes between $|0\rangle$ and any other state!



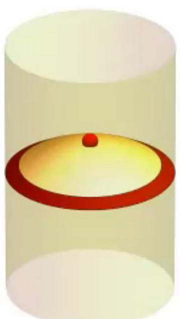


Simple task 2: another comparison

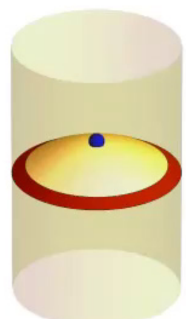
- The observers are in a state, $|g\rangle$ with $\langle g|0\rangle = 0$.
- Let X be a simple Hermitian operator from the time-band $[0, \epsilon]$ (Not necessarily unitary).

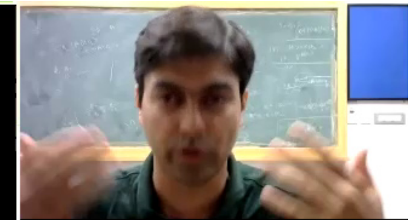
$$|X\rangle = X|0\rangle.$$

- The observers are asked to determine if the state of the system, $|g\rangle = |X\rangle$ or not.



OR





Success in gravity

This task is hopeless in a LQFT but, in gravity, the observers complete it through a two-step process.

1. Act with the unitary, $U = e^{iJX}$ for small J. This takes

$$|g\rangle \to U|g\rangle = (1 + iJX - \frac{J^2X^2}{2})|g\rangle + \mathrm{O}\left(J^3\right)$$

2. Subsequently, measure the energy to second order in J. The probability of getting 0 is

$$\langle g|U^\dagger \mathcal{P}_0 U|g\rangle = \langle g|(1 - iJX - \frac{1}{2}J^2X^2)\mathcal{P}_0(1 + iJX - \frac{1}{2}J^2X^2)|g\rangle$$

Since $\mathcal{P}_0 = |0\rangle\langle 0|$, and $\langle 0|g\rangle = 0$, and $X|0\rangle = |X\rangle$ So

$$\langle g|U^\dagger \mathcal{P}_0 U|g\rangle = J^2|\langle g|X\rangle|^2 + \mathrm{O}\left(J^3\right)$$

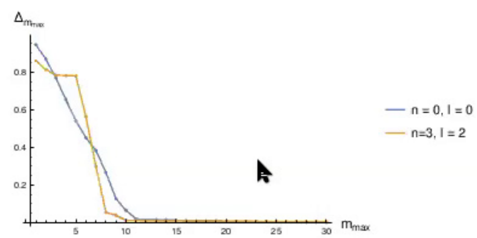
Check: basis from time-band

Can numerically check $X|0\rangle$ forms a basis.

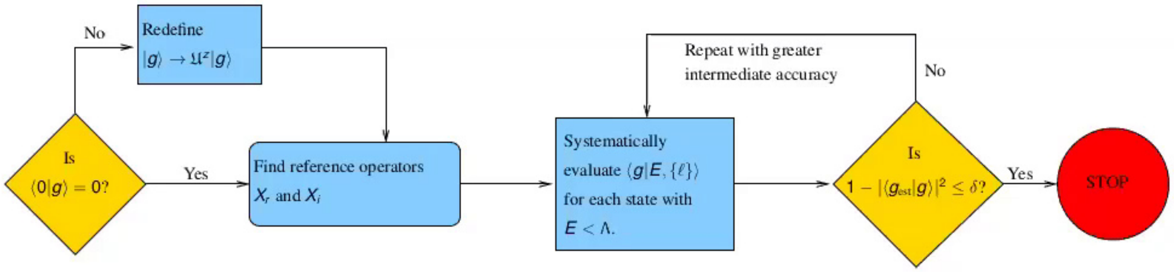
$$|h_m\rangle = \int_0^{\epsilon} \left[O(t,\Omega) Y_\ell^*(\Omega) e^{imt} w(t) d^{d-1}\Omega dt \right] |0\rangle$$

Minimize

$$\Delta_{m_{\max}} = \left| |n,\ell\rangle - \sum_{m=-m_{\max}}^{m_{\max}} c_m |h_m\rangle \right|^2 .$$

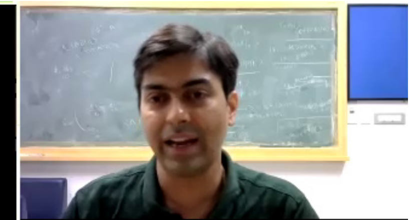


Complete protocol



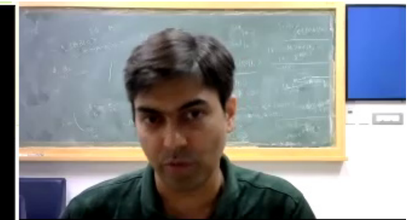
By extending this simple protocol, we showed that any low-energy state could be uniquely identified within perturbation theory.

[Papadoulaki, Chowdhury, S.R, 2020]

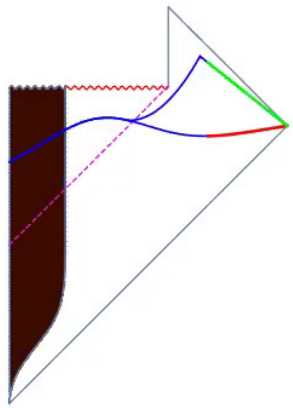


Outline

1. Introduction
2. AdS
3. Asymptotically flat spacetimes
4. Perturbative verification
5. **Black Holes**

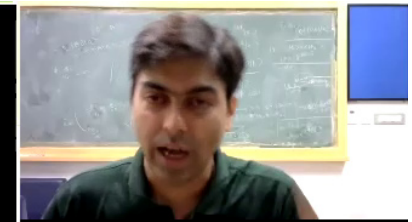


Black hole information

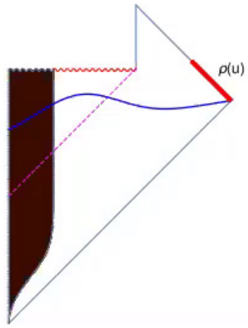


One often asks “how does information come out of the black hole as it evaporates?” (“how can it be obtained from the green slice?”)

Our results suggest information is always outside! With the right measurements, even before evaporation, one obtains complete information about the state from outside. (Info available also on the red slice.)



von Neumann entropy on $\mathcal{I}^+$



Common to define a state on $\mathcal{I}^+$ satisfying,

$$\mathrm{Tr}(\rho(u)b) = \langle b \rangle.$$

where b and $\rho(u)$ are both in the algebra on $[-\infty, u]$.

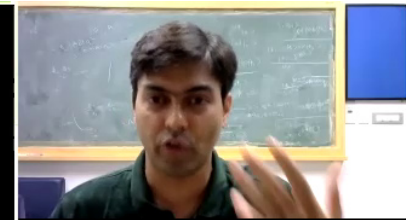
But, we can always choose

$$\rho(u) = \sigma \in \mathcal{A}_{-\infty,\epsilon},$$

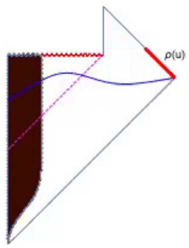
So

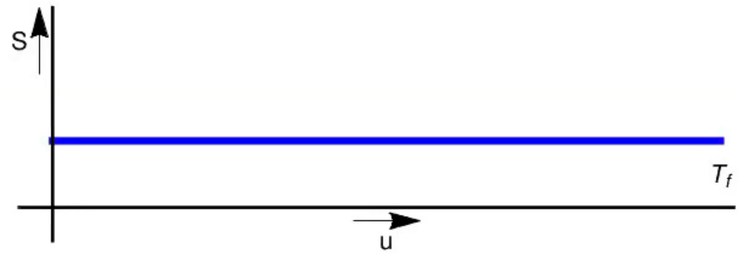
$$S = -\mathrm{Tr}(\rho(u)\log(\rho(u)))$$

is independent of u!



von Neumann entropy on $\mathcal{I}^+$



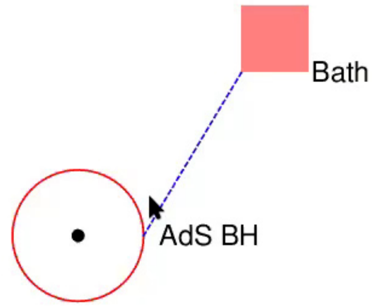


- The conventional Page curve fails since it is based on known incorrect assumptions about factorization of the Hilbert space.
- Perhaps, appropriate restriction of algebra on $\mathcal{I}^+$ will yield Page curve.

Physical point is that information is always available outside a black hole in flat space even before the Page time.

Page curve in AdS/CFT

- Recent work considers gravity coupled to a non-gravitational bath.



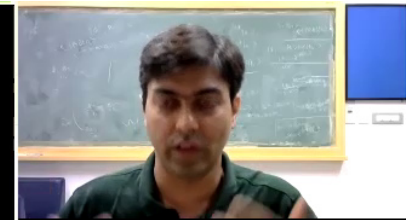
[Pennington, Almheiri, Mahajan, Maldacena, Hartman,]

[Shagoulian, Tajdini, Stanford, Shenker, Yang..., 2019]

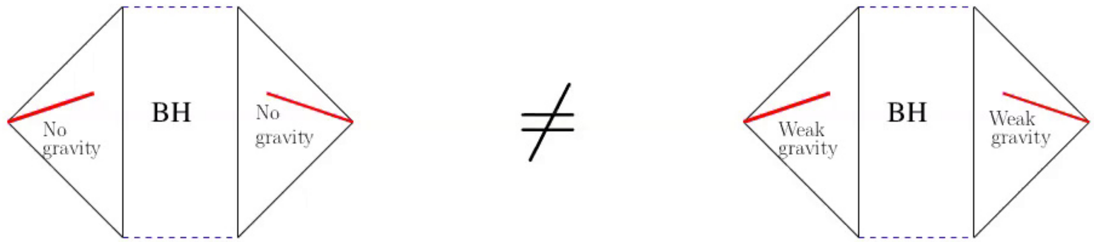
- The Hilbert space factorizes

$$\mathcal{H} = \mathcal{H}_{\text{bh}} \otimes \mathcal{H}_{\text{bath}}$$

- And both S_{bh} and S_{bath} follow a conventional Page curve.



Real black holes?



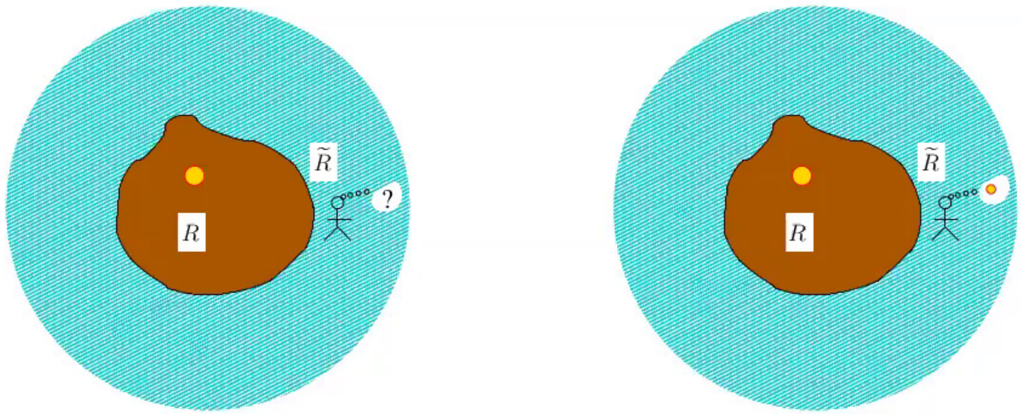
For fine-grained quantum-information questions

weak effects in gravity can conspire to give radically different answers from local theories

- So in systems where gravity is non-dynamical beyond some region, information may emerge only after Page time.
- For more-realistic black holes, information is always outside.

Conclusions

In gravity, copy of information inside a region is also available outside.



Not an abstract subtlety. Concrete physical protocol to extract information at low-energy in AdS.

Provides a new perspective on black hole information.