Title: Composite fermi liquids and non-commutative field theory

Speakers: Senthil Todadri
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Abstract: The interplay between topology, strong correlations, and kinetic energy presents a new challenge for the theory of quantum matter. In this talk I will describe some recent progress on understanding a simple class of problems where these effects can all be analytically handled. I will first present results on a microscopic lowest Landau theory of the composite fermi liquid state of bosons at filling 1. Building on work from the 1990s I will derive an effective field theory for this system that takes the form of a non-commutative field theory. I will show that an approximate mapping of this theory to a commutative field theory yield the familiar Halperin-Lee-Read action but with parameters correctly described by the interaction strength. I will describe the effect of a finite bandwidth introduced to the Landau Level and describe the evolution between the composite Fermi liquid and a boson superfluid. Time permitting, I will describe some generalizations that will include the evolution between a Quantum Anomalous Hall state and a Landau Fermi liquid that may be experimentally accessible in moire graphene systems.

Composite Fermi Liquids and non-commutative field theory

T. Senthil (MIT)

Zhihuan Dong, grad student@ MIT

https://arxiv.org/abs/2006.01282, and forthcoming

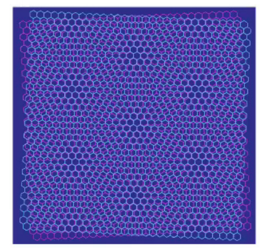
Ongoing collaborations: Hart Goldman, Adrian Po (MIT postdocs)

A new frontier in quantum matter science:

Band topology meets strong correlation

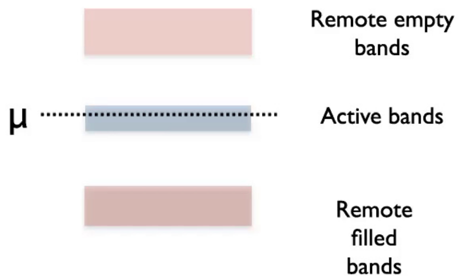
Physics of interacting electrons in a partially filled topological band??

The main conceptually new theoretical problem posed by moire graphene materials.



Other contexts - ongoing development (Checkelsky, Comin,…)
of Kagome flat band materials (eg CoSn).

Microscopic modeling of a typical correlated solid



Typical interaction strength << gap between active and remote bands.

Project interaction to active band to obtain effective model in the Bloch (k-space) basis.

$$H = \int_k \epsilon_{\vec{k}} \, c^\dagger_{\vec{k}} c_{\vec{k}} + \int_{\vec{k}_1,\vec{k}_2,\vec{k}_3} U(\vec{k}_1,\vec{k}_2,\vec{k}_3) c^\dagger_{\vec{k}_1} c^\dagger_{\vec{k}_2} c_{\vec{k}_3} c_{-(\vec{k}_1+\vec{k}_2+\vec{k}_3)}$$

(suppressing spin/other internal indices)
Additional: Phonons, impurities,….

Challenge to this paradigm

Band topology meets strong correlation

The story of the previous 2 slides becomes complicated if the bands in question are topological.

Chern bands in 2d

Thouless et al 1983; Haldane 1988

In $2d$, for an isolated band, total momentum space magnetic flux $\int_{T^2} d^2k\, \mathcal{B}_k = 2\pi C_n$ is quantized with an integer C_n ("Chern number")

C_n counts the number of Berry magnetic monopoles inside the k-space torus, and is a topological invariant.

Bands with different C_n are topologically distinct.

Physical consequences of Chern number

Chern insulator = material with a filled Chern band.

$j_x = \sigma_{xy} E_y$ where $\sigma_{xy} = \frac{C_n e^2}{h}$ is quantized and measures the Chern number

Integer quantum Hall effect

Other related phenomena: chiral edge states

Requires absence of time reversal symmetry.

Chern bands and Wannier representability

Theorem (Brouder et al 2007): Exponentially localized Wannier functions cannot be constructed for Chern bands.

Intuition: A Chern band encloses a magnetic monopole in k-space

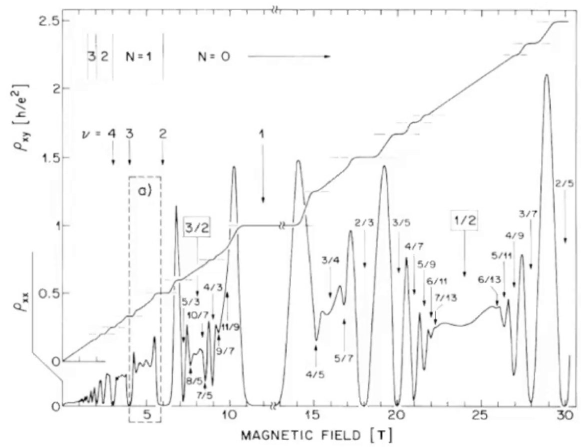
=> Phase of Bloch wave-function cannot be globally well-defined

The integrand in the k-space integral that defines the Wannier function is not smooth => no well localized Wannier functions.

Corollary: A correlated partially filled Chern band cannot be simply described by a lattice effective model (without introducing extra auxiliary bands to ``cancel'' the Chern number).

Strong correlations in a special topological flat band: The quantum Hall effect

A single Landau level: a special case of a Chern band
(with flat energy dispersion, flat Berry curvature,…)



Strong correlations at partial filling give rich physics of fractional quantum Hall systems.

Do not understand in terms of lattice tightbinding models but in other ways (eg, wavefunctions, effective field theories,……).

How to deal with strong correlations ***away*** from this special (Landau level) limit of a Chern band, or for ***other topological bands***, is largely an open question in many body physics.

This is precisely the situation presented to us by many moire graphene materials.

Can we base the theory on our understanding of correlated Landau levels?

May be but there are some problems…..

?? Generalizing from Landau levels: Problem I ??

Differences between a Landau Level and moire topological bands

1. Time-reversal symmetric (so roughly like a system with two sets of Landau levels corresponding to equal and opposite fields, coupled together by interactions).

2. When there are Chern bands, the Berry curvature is not uniform within the Brillouin zone.

3. Bandwidth is not zero (and can be tuned in some systems)

Nevertheless, intuition from quantum Hall is less likely to misguide, and could be a viable starting point.

?? Generalizing from Landau levels: Problem II ??

More basic problem:

Dealing analytically with a microscopic model restricted to a single Landau level is hard and not satisfactorily understood even for classic quantum Hall phenomena.

A program and plan of talk

Identify simple models to study the interplay of
band topology, strong correlations, and kinetic energy.

Part I: Revisit old unsettled questions about quantum Hall physics within a single Landau level (usually the Lowest level)

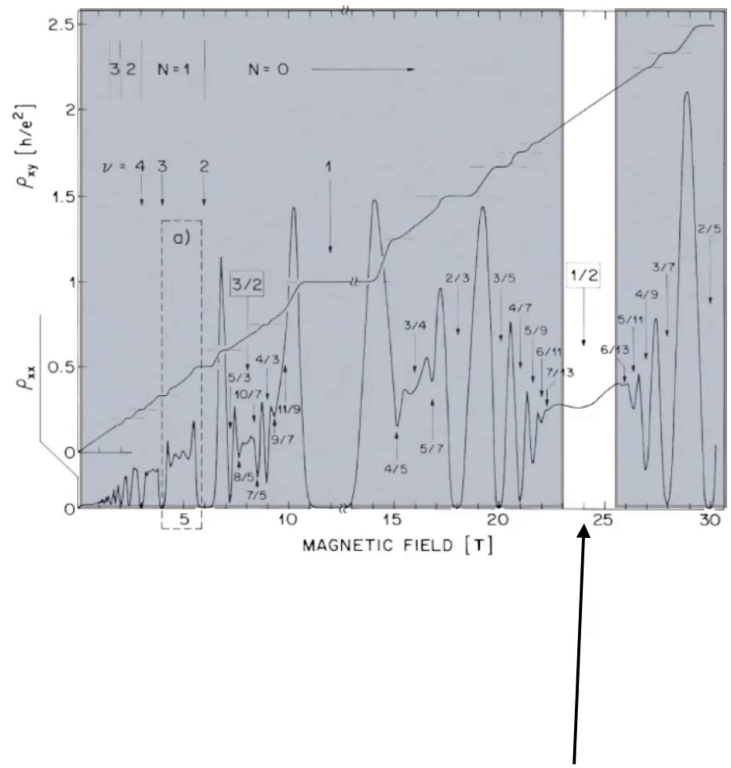
Part II: Broaden Landau level into a Chern band to study competition with kinetic energy.

2d electrons in the ``quantum Hall" regime

Filling factor ν = 1,2., 3, ... (IQHE)

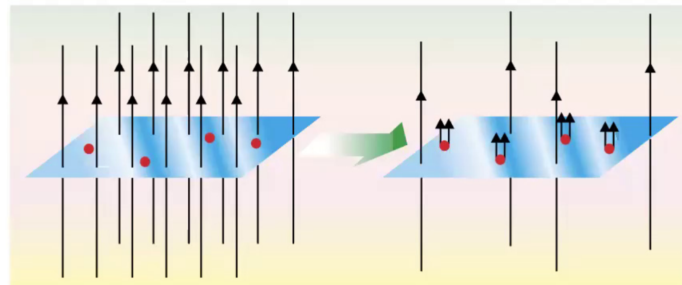
ν = 1/3, 1/5, ... (FQHE)

ν = 1/2, 1/4,…: Metal with $\rho_{xx} \neq 0$, $\rho_{xy} \neq 0$, but $\rho_{xx} \ll \rho_{xy}$. (Composite Fermi Liquids)



Composite fermi liquid theory
(Halperin, Lee, Read (HLR) 1993)

Assume (Jain 89) each electron captures two flux quanta to form a new fermion
("Composite fermions")



See reduced effective field
$B^* = B - (2h/e)\rho$

At $\nu = 1/2$, $B^* = 0$

=> form Fermi surface of composite fermions

Effective theory:

$$\mathcal{L} = \overline{\psi}_{CF}\left(i\partial_t - a_0 - iA_0^{ext} + \frac{(\vec{\nabla} - i(\vec{a} + \vec{A}))^2}{2m}\right)\psi_{CF} + \frac{1}{8\pi}a_\mu\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda \qquad (1)$$

(Short-hand for better version with properly quantized Chern-Simons terms)

Other composite fermi liquids

1. Electrons at $\nu = 1/4$,…..

2. Bosons at filling $\nu = 1$ (numerics: ground state is a version gapped by pairing but the metallic state is still worth studying)

In these cases too, the HLR construction admits a composite fermi liquid ground state

Effective theory: Fermi surface + U(1) gauge field with Chern-Simons coupling

Composite Fermi Liquids (CFLs) are central to our overall understanding of quantum Hall phenomena.

The many successes of HLR

Explanation/succesfull prediction of many aspects of the observed metallic state

Parent state of nearby Jain sequence of prominent quantum Hall states

Parent state of paired non-abelian quantum Hall state at same filling

Successful backing from numerics

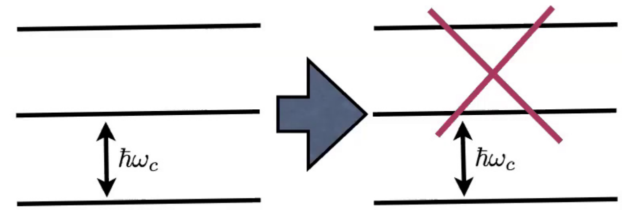
Despite these successes there were some vexing open questions that were extensively thought about in the 1990s without resolution.

Unsatisfactory aspects of the HLR theory-I

Theory should make sense within the Lowest Landau Level (LLL) but HLR not suited to projecting to LLL.

Mean field effective mass = bare electron mass in HLR

LLL limit: take m to zero; what happens??



$V_{int} \ll \hbar\omega_c$: project to LLL $\quad$ $V_{int} \ll \hbar\omega_c$: project to LLL

Much discussed in the late 90s (Shankar, Murthy; Read; Halperin, Stern, Simon, van Oppen; D.-H. Lee, Pasquier, Haldane,.......) but dust never settled.

Experience of learning to work within LLL will be useful in other contexts (as LLL is simplest example of a topological band)

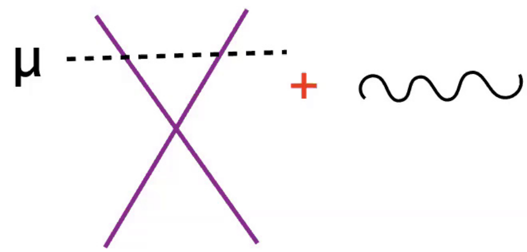
Unsatisfactory aspects of the HLR theory-II

At ν = 1/2 LLL theory has an extra particle-hole symmetry that HLR is blind to.

Issue identified in the 90s (Grotov, Gan, Lee, Kivelson, 96; Lee 98) but a possible resolution only in last few years

``Dirac composite fermions”

Son + many others (2015 - present)



This progress sidestepped issue of LLL projection;

I will not discuss p/h symmetry today.

Dirac fermions at finite density
+ U(1) gauge fields

The basic problem

Particles in lowest Landau level with 2-body repulsive interactions.

$|m\rangle$: Some basis for single particle states in LLL

Many body states spanned by $|m_1, m_2, m_3, ..., m_N\rangle$ (anti)symmetrized for (fermions) bosons.

Hamiltonian has only interaction energy

$$\mathcal{H} = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2} U(\mathbf{q})\rho_L(\mathbf{q})\rho_L(-\mathbf{q})$$

Projected "density" operators $\rho_L(\mathbf{q})$ satisfy the "GMP algebra" (a.k.a W_∞ algebra)

Girvin-MacDonald-Platzman

$$[\rho_L(\mathbf{q}), \rho_L(\mathbf{q}')] = 2i\sin\left(\frac{(\mathbf{q}\times\mathbf{q}')l_B^2}{2}\right)\rho_L(\mathbf{q}+\mathbf{q}')$$

$U(\mathbf{q}) = e^{-\frac{q^2 l_B^2}{2}} U_0(\mathbf{q})$ with U_0 the Fourier transform of the microscopic 2-body interaction.

Composite Fermi liquid of bosons at $\nu = 1$: Important progress in the 1990s

Pasquier-Haldane (1998): Redundant representation of density operator satisfying GMP algebra in terms of fermionic partons (= LLL version of composite fermions)

Impose constraints to recover physical Hilbert space.
Constraint operators themselves satisfy GMP algebra (with opposite sign).

Read(1998): ``W_∞ gauge structure" broken to U(1) by mean field composite fermi liquid solution.

Low energy theory: Fermi surface + U(1) gauge field

Fluctuation effects treated diagrammatically in a conserving approximation;

Physically sensible final results.

Not much further development since…..

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Not much further development since…..

Open questions about Pasquier-Haldane-Read

1. What is the low energy effective field theory?

2. How related to HLR?

3. How to treat competition with paired bosonic Pfaffian at same filling? Jain states at proximate filling?

4. Can these methods be generalized to other problems?

A suggested effective field theory

Composite fermion as a fermionic vortex

$$\mathcal{L}_{vcfl} = \bar{\psi}_v(\partial_\tau + ia_0)\psi_v + \frac{1}{2m^*}|\,(\partial_i + ia_i)\,\psi_v|^2 - \frac{i}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda - \frac{i}{4\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

Suggested in Read's original 1998 paper but not explicitly derived.

No Chern-Simons term in action; contrast with usual HLR (with renormalized parameters).

$$\mathcal{L}_{HLR} = \bar{\psi}(\partial_\tau + i(a_0 + A_0))\psi_v + \frac{1}{2m^*}|\,(\partial_i + i(a_i + A_i))\,\psi_v|^2 - \frac{i}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

Do they describe the same IR physics, or are they distinct theories?

Which of these is actually obtained in the Pasquier-Haldane-Read microscopic theory?

See also: Alicea, Hermele, Motrunich, Fisher(2005) - flux attachment to fermionize vortices (not in LLL)
Wang, TS (2016) - standard HLR + Fermi surface Berry phase

Our answers

1. This bosonic CFL is described by a ***non-commutative field theory*** of fermions + U(1) gauge field with no Chern-Simons term

2. Approximate map(*) to commutative field theory for long wavelength, low amplitude gauge fluctuations: HLR (+ suleading corrections) with parameters determined by interactions

(*) Using Seiberg, Witten (1999)

Remarks on non-commutative field theory

Field theories defined in a space-time where spatial coordinates do not commute.

Long history in physics/math literature; much studied in high energy physics 1998-2003

Of course the best example of such a space is the Landau Level - so perhaps not surprising that LLL effective theories are non-commutative.

Incompressible states: Proposals for a non-commutative Chern-Simons description (Susskind 2001, Polychronakos 2002,…….)

Connection to microscopics? Added value over standard commutative TQFT description?

Here I will focus on the metallic state of bosons at $\nu = 1$.

Pasquier-Haldane-Read parton construction

Represent states in many body boson Hilbert space by

$$|m_1, ..., m_N\rangle = \epsilon^{n_1 n_2 .....n_N} c^\dagger_{n_1 m_1} c^\dagger_{n_2 m_2} ......c^\dagger_{n_N m_N} |0\rangle$$

Symmetric

Anti-symmetric Tensor

c_{mn}: usual fermion anticommutation relations; destroy fermionic partons.

Physical states: singlets under $SU(N)$ rotations on the right:

$$c_{mn} \rightarrow c_{mn'} U^R_{n'n}$$

Generators $\rho^R_{nn'} = c^\dagger_{nm} c_{mn'}$ of these $SU(N)_R$ satisfy constraint:

$$\rho^R_{nn'} |\psi_{phys}\rangle = \delta_{nn'} |\psi_{phys}\rangle$$

'Left' $SU(N)$ rotations generated by $\rho^L_{mm'} = c^\dagger_{nm'} c_{mn}$ are physical operators.

Take $N \rightarrow \infty$, and $|m\rangle, |n\rangle$ as single particle basis states for a Landau level.

Momentum space formulation

Fourier transform c_{mn} using magnetic translation operator $\tau_{\mathbf{k}} = e^{i\mathbf{k}\cdot\mathbf{R}}.$

$$c_{mn} = \int \frac{d^2k}{(2\pi)^2} \langle m|\tau_{\mathbf{k}}|n\rangle c_{\mathbf{k}}$$

Guiding center coordinate

Usual fermion operator

Fourier-transformed density operators (= fermion bilinears) satisfy:

$$[\rho^L(\mathbf{q}), \rho^L(\mathbf{q}')] = 2i \sin\left(\frac{(\mathbf{q}\times\mathbf{q}')l_B^2}{2}\right)\rho^L(\mathbf{q}+\mathbf{q}')$$

GMP algebra

$$[\rho^R(\mathbf{q}), \rho^R(\mathbf{q}')] = -2i \sin\left(\frac{(\mathbf{q}\times\mathbf{q}')l_B^2}{2}\right)\rho^R(\mathbf{q}+\mathbf{q}')$$

anti-GMP algebra

$$[\rho^L(\mathbf{q}), \rho^R(\mathbf{q}')] = 0$$

The Hamiltonian $\mathcal{H} = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2} U(\mathbf{q})\rho_L(\mathbf{q})\rho_L(-\mathbf{q})$

Constraint $\rho^R(\mathbf{q})|phys\rangle = 0$ for all non-zero $\mathbf{q}$.

Hartree-Fock solution

$\mathcal{H}$ is quartic in fermions - first study within Hartree-Fock.

A simple (global) symmetry preserving state:

$$\langle c^\dagger_{\mathbf{k}} c_{\mathbf{k}'} \rangle = n_{\mathbf{k}} \delta^{(2)}(\mathbf{k} - \mathbf{k}')$$

Fermions acquire a dispersion (*) and form a Fermi sea with area set by their density.

Mean field composite fermi liquid

(*) Approximate as parabolic - what matters is near Femi surface anyway.

Mean field action:

$$\mathcal{S}_{HF} = \int d\tau \frac{d^2\mathbf{k}}{(2\pi)^2} \ \bar{c}_{\mathbf{k}} \frac{dc_{\mathbf{k}}}{d\tau} - \left(\frac{\mathbf{k}^2}{2m^*}\right) c^\dagger_{\mathbf{k}} c_{\mathbf{k}}$$

Beyond mean field

Read (1998): Diagrammatic conserving approximation but here we seek an effective Lagrangian.

Mean field breaks infinite gauge symmetry generated by ρ^R - the breaking is weak as $\mathbf{q} \to 0$.

=> Include small-q gauge fluctuations on top of mean field

Difficulty: In k-space, mean field state is simple but ρ_R transformations mix states at different momenta.

In m, n (Landau orbital) basis, gauge transformations look simple but mean field state is complicated.

Convenient formulation: Fields in non-commutative space and time

Define abstract fields as a function of the non-commuting guiding center coordinate

$$c(\mathbf{R},\tau) = \int \frac{d^2\mathbf{k}}{(2\pi)^{\frac{1}{2}}} e^{i\mathbf{k}\cdot\mathbf{R}} c_{\mathbf{k},\tau}$$

Here $[R_i, R_j] = i\theta_{ij} = i\Theta\epsilon_{ij}$.
$\Theta = -l_B^2 =$ "non-commutativity" parameter.

$\mathbf{R}$ is an operator in the space of single particle states of the LLL.

Therefore $c(\mathbf{R},\tau)$ is also an operator in this space.

Product of two such 'non-commutative" fields $f(\mathbf{R})$ and $g(\mathbf{R})$ has a Fourier transform

$$f(\mathbf{R})g(\mathbf{R}) = \int \frac{d^2k d^2k'}{(2\pi)^3} \left(e^{-i\frac{\Theta \mathbf{k}\times\mathbf{k}'}{2}} \tilde{f}(\mathbf{k})\tilde{g}(\mathbf{k}') \right) e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{R}}$$

Convenient formulation:
Fields in non-commutative space and time (cont'd)

To any $f(\mathbf{R})$ associate an ordinary field $f(x)$ defined in commutative space through the standard Fourier transform

$$f(\mathbf{x}) = \int \frac{d^2k}{(2\pi)^{\frac{1}{2}}} e^{i\mathbf{k}\cdot\mathbf{x}} f_{\mathbf{k}}$$

The product $f(\mathbf{R})g(\mathbf{R})$ is then associated with a 'star product'

$$f(\mathbf{x}) * g(\mathbf{x}) = e^{i\frac{\theta_{ij}}{2}\partial_{x_i}\partial_{x'_j}} f(\mathbf{x})g(\mathbf{x}')|_{\mathbf{x}'=\mathbf{x}}$$

The star product is associative but not commutative:.

Can define derivatives, and integrals

How does this help?

Simple action of transformations generated by $\rho_{L,R}$:

$$c(\mathbf{x},\tau) \to U^L(\mathbf{x},\tau) * c(\mathbf{x},\tau) * U^R(\mathbf{x},\tau)$$

Here take $U^{L,R} = e^{i\theta_{L,R}(\mathbf{x},\tau)}$ with exponential defined through power series with all products being star products.

Mean field action also looks simple:

$$\mathcal{S}_{HF} = \int d\tau d^2\mathbf{x} \quad \left(\bar{c}(\mathbf{x},\tau) * \frac{dc(\mathbf{x},\tau)}{d\tau} + \frac{1}{2m}\nabla\bar{c}(\mathbf{x},\tau) * \nabla c(\mathbf{x},\tau)\right)$$

Now require that action be invariant under long wavelength `right' gauge transformations: Replace all derivatives by covariant derivatives

Non-commutative action for composite fermi liquid

$$\mathcal{S} = \int d^2\mathbf{x} d\tau \;\; \bar{c} * D_0 c + i a_0 \underline{\rho} + \frac{1}{2m^*} \overline{D_i c} D_i c$$

The covariant derivatives

$$D_\mu c = \partial_\mu c - i c * a_\mu - i A_\mu * c$$

a_μ: dynamical non-commutative $U(1)$ gauge field coupling to 'right' currents.

A_μ: background non-commutative $U(1)$ gauge field coupling to 'left' currents

Fermions at finite density, form a Fermi surface; gauge fields vary slowly on scale l_B.

Gauge invariance:

$$\begin{aligned} c &\rightarrow c + i c * \theta_R + i\theta_L * c \\ a_\mu &\rightarrow a_\mu + \partial_\mu \theta_R + i(a_\mu * \theta_R - \theta_R * a_\mu) \\ A_\mu &\rightarrow A_\mu + \partial_\mu \theta_L + i(\theta_L * A_\mu - A_\mu * \theta_L) \end{aligned}$$

Comments

1. Theory has Fermi surface + dynamical U(1) gauge field without any Chern-Simons term but is formulated in non-commutative space-time.

2. Hartree-Fock theory gives estimate of bare effective mass m* in terms of interaction strength.

How to compare with previous proposals?

Previous proposed field theories (HLR, fermionic vortex liquid) are all ordinary commutative field theories.

However, the non-commutativity occurs at scale of magnetic length, we can hope to approximate long wavelength gauge fluctuations in terms of a `coarge-grained' commutative theory.

Key tool: The Seiberg-Witten map

Seiberg, Witten (1999): Map pure non-commutative gauge theory with gauge fields a_μ to a commutative gauge theory with gauge fields $\hat{a}_\mu$ in a systematic expansion in powers of Θ.

Expansion is local (coefficients only involve fields and derivatives at same space-time point).

Gauge transformation parameters are mapped as functions of the gauge field configurations themselves.

Here we will need a small generalization that includes the fermion fields.

Pure $U(1)$ gauge theory: Exact non-perturbative Seiberg-Witten map exists (Liu, Michelson, Okawa, Ooguri 01, Mukhi, Suryanarayana 01).

Appealing physical interpretation: Relation to map between Lagrangian and Eulerian descriptions of a fluid (Jackiw, Pi, Polychronakos, 02; also Susskind 01)

Seiberg-Witten map to linear order

$$
\begin{aligned}
A(\hat{A}) &= \hat{A} + \Delta A(\hat{A}) & a(\hat{a}) &= \hat{a} + \Delta a(\hat{a}) \\
\theta_L(\hat{\theta}_L, \hat{A}) &= \hat{\theta}_L + \Delta\theta_L(\hat{\theta}_L, \hat{A}) & \theta_R(\hat{\theta}_R, \hat{a}) &= \hat{\theta}_R + \Delta\theta_R(\hat{\theta}_R, \hat{a}) \qquad (1) \\
c(\psi, \hat{A}, \hat{a}) &= \psi + \Delta\psi(\psi, \hat{A}, \hat{a})
\end{aligned}
$$

Here ΔA, Δa, $\Delta\theta_R$, $\Delta\theta_L$, and Δc are all of $o(\Theta)$.

The map can be found explicitly by requiring that the hatted fields satisfy ordinary U(1) gauge invariance for both dynamical and background gauge fields.

Strategy: Plug in the map to get an effective commutative gauge theory

Emergence of HLR

Effective commutative Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{HLR} + \mathcal{L}_{corr} \\
\mathcal{L}_{HLR} &= \bar{\psi}\partial_0\psi - i(\hat{a}_0 + \hat{A}_0)\bar{\psi}\psi + i\hat{a}_0\underline{\rho} + \frac{1}{2m^*}\left|\left(\partial_i - i(\hat{a}_i + \hat{A}_i)\right)\psi\right|^2 - i\frac{1}{4\pi}\epsilon^{\alpha\beta\gamma}\hat{a}_\alpha\partial_\beta\hat{a}_\gamma \\
\mathcal{L}_{corr} &= -\frac{\Theta}{2}\epsilon^{\alpha\beta}\left((\hat{f}_{0\beta} - \hat{F}_{0\beta})\partial_\alpha(\bar{\psi}\psi) - \partial_\alpha(\hat{a}_\beta - \hat{A}_\beta)(\bar{\psi}D_0\psi - \frac{1}{2m^*}|\hat{D}_i\psi|^2)\right)
\end{aligned}
$$

First term is just the standard HLR theory!
(but with correct effective mass)

Second term is a correction that is small for long wavelength, low amplitude gauge fluctuations

Comments

1. The commutative theory has a Chern-Simons term (as expected in HLR) which is absent in the non-commutative theory.

This is traced to the non-trivial form factors in the density operator in the non-commutative theory (roughly correspond to a non-trivial Berry phase)

2. Can use the commutative effective theory to, say, study Jain states near $\nu = 1$ (technically a bit hard within the microscopic Pasquier-Haldane-Read approach) The correction terms may need to be included for some purposes.

3. We do not find the fermionic vortex field theory.

A program and plan of talk

Identify simple models to study the interplay of
band topology, strong correlations, and kinetic energy.

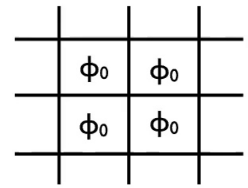
Part I: Revisit old unsettled questions about quantum Hall physics within a single Landau level (usually the Lowest level)

Part II: Broaden Landau level into a Chern band to study competition with kinetic energy.

Introducing bandwidth to a Landau level

Add a periodic potential whose unit cell encloses one flux quantum .

Concrete choice: Square unit cell with lattice constant l.



Flux through unit cell $l^2/l_B^2 = 2\pi$.

Discrete magnetic translations by $l\hat{x}, l\hat{y}$ commute => Can define crystal momentum inside first Brillouin zone.

$\mathcal{H}_V = V(-\mathbf{q})\rho_L(\mathbf{q})$

$V(\mathbf{q}) = \int d^2x V_0(\cos(\frac{2\pi x}{l}) + \cos(\frac{2\pi y}{l})$

This periodic potential leads to a dispersion of single particle states.

LLL $\xrightarrow{\text{Add V}}$ Chern band with non-zero bandwidth

Topological bands, correlations, and kinetic energy: a simple model

LLL + periodic potential + interactions:

$$\mathcal{H} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{1}{2} U(\mathbf{q}) \rho_L(\mathbf{q}) \rho_L(-\mathbf{q}) + V(-\mathbf{q}) \rho_L(\mathbf{q})$$

Correlations

Projected "density" operators $\rho_L(\mathbf{q})$ satisfy the "GMP algebra" (a.k.a W_∞ algebra)

$$[\rho_L(\mathbf{q}), \rho_L(\mathbf{q}')] = 2i \sin\left(\frac{(\mathbf{q} \times \mathbf{q}') l_B^2}{2}\right) \rho_L(\mathbf{q} + \mathbf{q}')$$

LLL, band topology

$U(\mathbf{q}) = e^{-\frac{q^2 l_B^2}{2}} U_0(\mathbf{q})$ with U_0 the Fourier transform of the microscopic 2-body interaction.

Topological bands, correlations, and kinetic energy: a simple model

LLL + periodic potential + interactions:

$$\mathcal{H} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{1}{2} U(\mathbf{q})\rho_L(\mathbf{q})\rho_L(-\mathbf{q}) + V(-\mathbf{q})\rho_L(\mathbf{q})$$

Bandwidth

Correlations

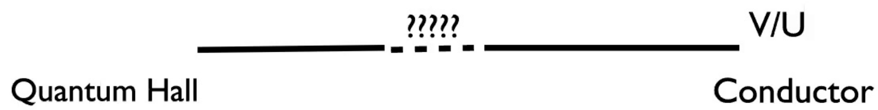
Projected "density" operators $\rho_L(\mathbf{q})$ satisfy the "GMP algebra" (a.k.a W_∞ algebra)

$$[\rho_L(\mathbf{q}), \rho_L(\mathbf{q}')] = 2i \sin\left(\frac{(\mathbf{q}\times\mathbf{q}')l_B^2}{2}\right)\rho_L(\mathbf{q}+\mathbf{q}')$$

LLL, band topology

$U(\mathbf{q}) = e^{-\frac{q^2 l_B^2}{2}} U_0(\mathbf{q})$ with U_0 the Fourier transform of the microscopic 2-body interaction.

Limiting cases



V = 0: Classic Quantum Hall
- Incompressible quantum Hall ground states/compressible composite fermi liquids depending on filling and particle statistics.

U = 0: Free particles in a dispersive band
- Bose condensate/Fermi gas depending on statistics

Small U: Superfluid/Fermi liquid depending on statistics.

Change V/U: Evolution from quantum Hall ground state to superfluid/fermi liquid.

Examples

Laughlin state ——— ????? ——— V/U Fermi liquid

Electrons at $\nu = 1/3$

Composite Fermi Liquid ——— ????? ——— V/U Fermi liquid

Electrons at $\nu = 1/2$

Laughlin state ——— ????? ——— V/U Superfluid

Bosons at $\nu = 1/2$

I will briefly describe progress in analyzing this evolution for bosons at $\nu = 1$.

Bosons at ν = 1 in a periodic potential: Mean field

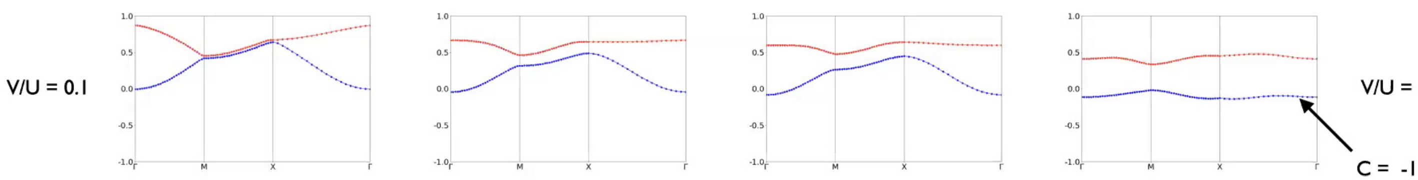
Study evolution from composite fermi liquid to superfluid using the formalism described in first part of talk.

Weak V: Composite fermion dispersion gets a band gap at Brillouin zone boundary.

Fermi surface reconstructs into a `particle'-like sheet and `hole'-like sheet of equal area.

Increase V: Particle and hole sheets shrink and eventually disappear.

Composite fermions completely fill a single band which however itself has a Chern number -1



Evolution of mean field composite fermion band structure

Bosons at $\nu = 1$ in a periodic potential: Fluctuations beyond mean field

Weak V: Fermi surface reconstructs into a \`particle'-like sheet and \`hole'-like sheet of equal area

These couple to a dynamical U(1) gauge field - HLR-like action in commutative approximation.

Increase V: Composite fermions completely fill a single band which however itself has a Chern number -1

Bosons at ν = 1 in a periodic potential: Fluctuations beyond mean field

Weak V: Fermi surface reconstructs into a `particle'-like sheet and `hole'-like sheet of equal area.

These couple to a dynamical U(1) gauge field - HLR-like action in commutative approximation.

Increase V: Composite fermions completely fill a single band which however itself has a Chern number -1

Integrate out composite fermions: effective action (in commutative approximation) is

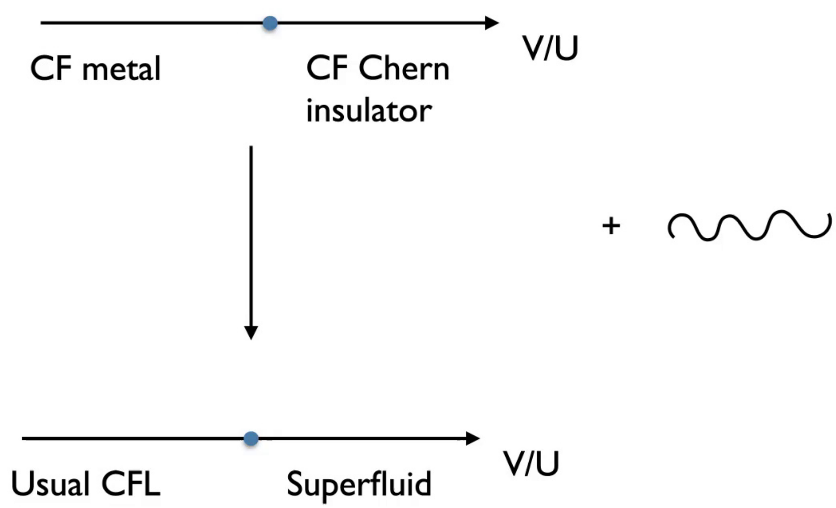
$$\int d^3x - \frac{1}{4\pi}(\hat{a}+\hat{A})d(\hat{a}+\hat{A}) + \frac{1}{4\pi}\hat{a}d\hat{a} = -\frac{\hat{A}d\hat{a}}{2\pi} - \frac{1}{4\pi}\hat{A}d\hat{A}$$

Integrating out $\hat{a}$ sets $\hat{A} = 0$. Response of a superfluid!

Pictorial sketch

Mean field:

Fluctuations:



Comments/generalizations (cont'd)

1. This parton framework has enabled analytic understanding both the composite fermi liquid and its evolution into the superfluid in a periodic potential.

Can also include composite fermion pairing to access the bosons Pfaffian state.

2. Generalizations - multicomponent bosons/fermions at total integer filling in LLL + periodic potential

Example: Spinful electrons at total filling 1.

Small V - Quantum Hall ferromagnet

Large V - Fermi liquid

In progress: Address using a composite boson theory formulated within the LLL
(and may be accessible experimentally in some moire graphene systems)

Outlook

Revisiting quantum Hall theory:

- Other composite fermi liquids?

Some simple generalizations easy (eg spinfull bosons at total filling 1)

Fermions at 1/2-filling?

In all cases it is natural that a LLL theory is non-commutative .

- Coupling to geometry?

General correlated topological bands:

- Correlated Chern bands with non-uniform Berry curvature?
- Other kinds of band topology?