Title: Temperature enhancement of thermal Hall conductance quantization

Speakers: David Mross

Series: Quantum Matter

Date: August 04, 2020 - 1:30 PM

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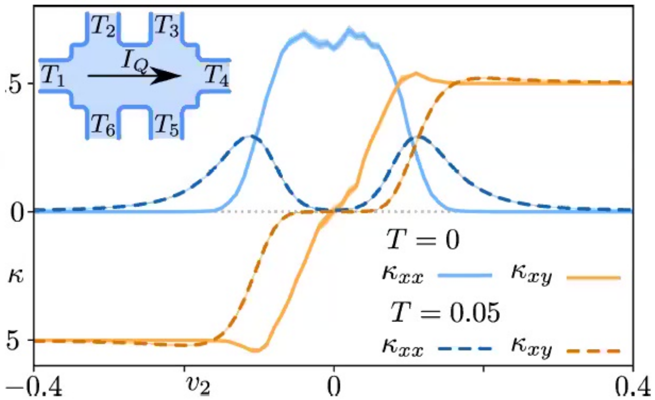
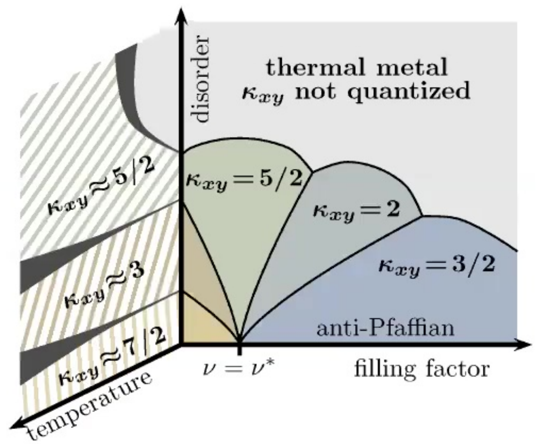
Abstract: The quest for non-Abelian quasiparticles has inspired decades of experimental and theoretical efforts. Among their clearest signatures is a thermal Hall conductance with quantized half-integer value. Such a value was indeed recently observed in a quantum-Hall system at Î½=5/2 and in the magnetic insulator Î±-RuCl3. I will explain that a non-topological "thermal metal" phase that forms due to quenched disorder may disguise as a non-Abelian phase by well approximating the trademark quantized thermal Hall response. Remarkably, the quantization here improves with temperature, in contrast to fully gapped systems. In my talk, I will provide analytical and numerical evidence for this effect and discuss its possible implications for the aforementioned experiments.

Temperature enhancement of thermal Hall conductance quantization

David F. Mross





I. C. Fulga, Y. Oreg, A. D. Mirlin, A. Stern, DFM, cond-mat/2006.09392

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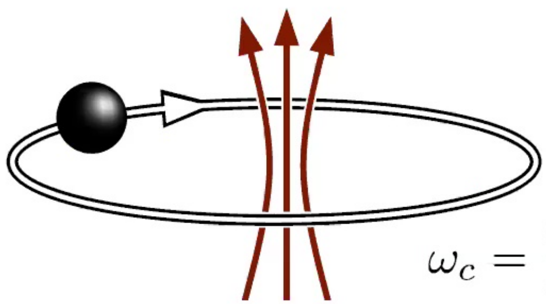
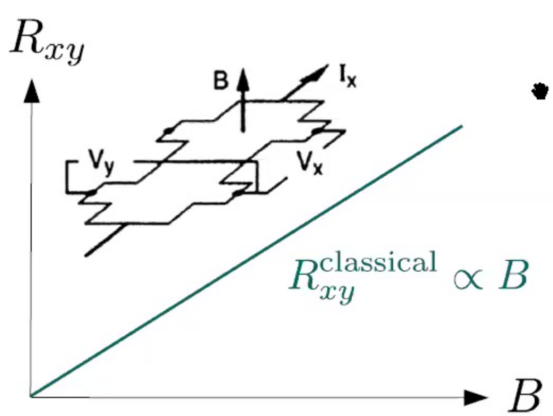
Classical Hall effect

Classical: cyclotron orbits

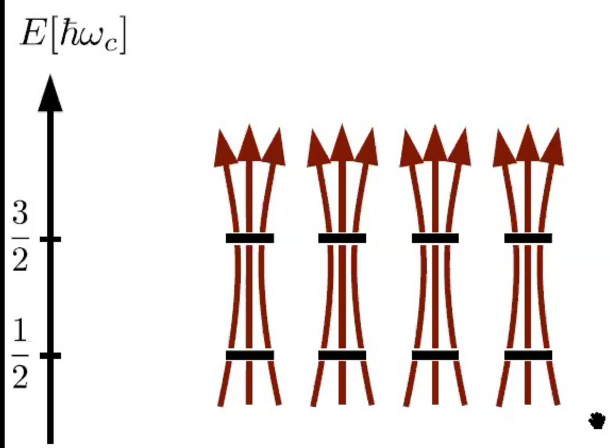


$$\omega_c = \frac{qB}{mc}$$



Quantum Hall effect I: clean case

Quantum mechanical: energy levels



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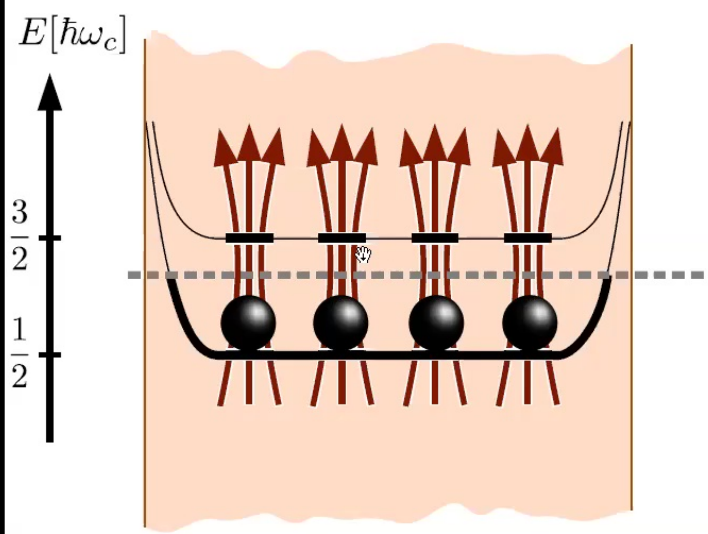
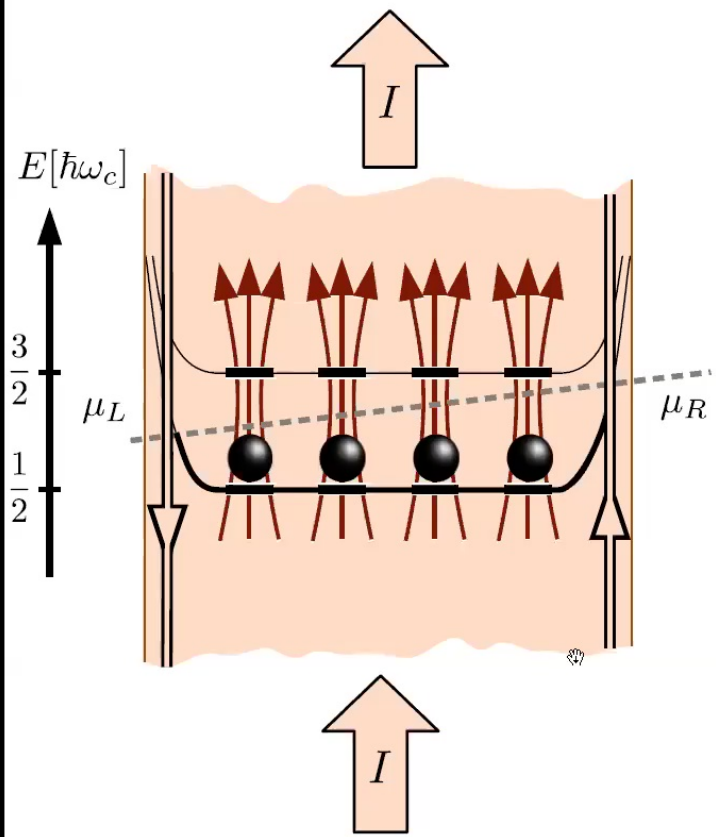
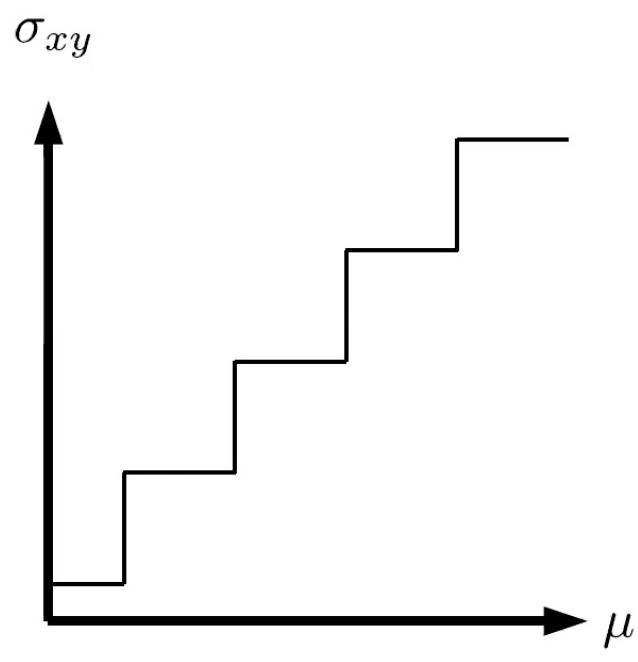
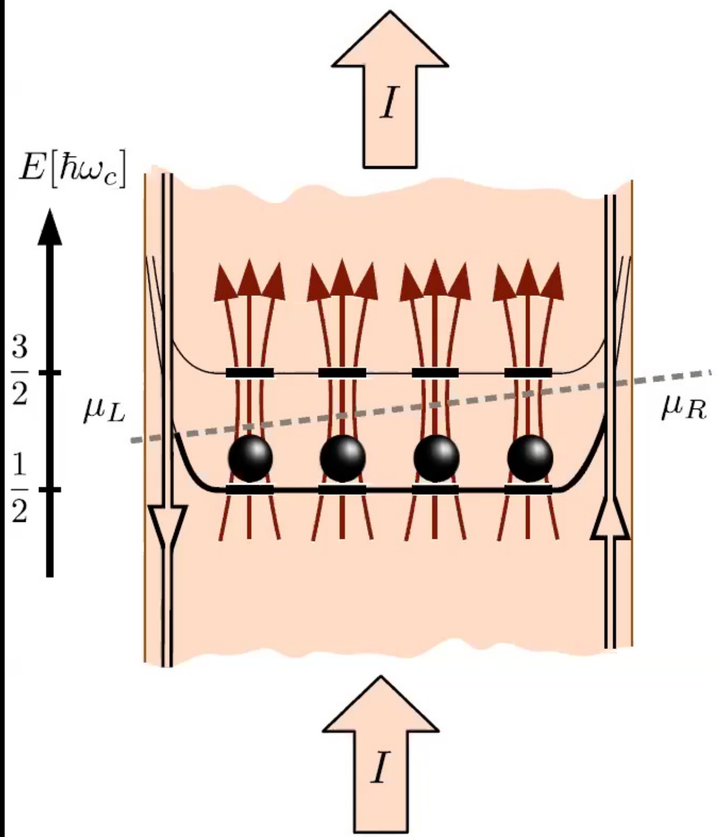


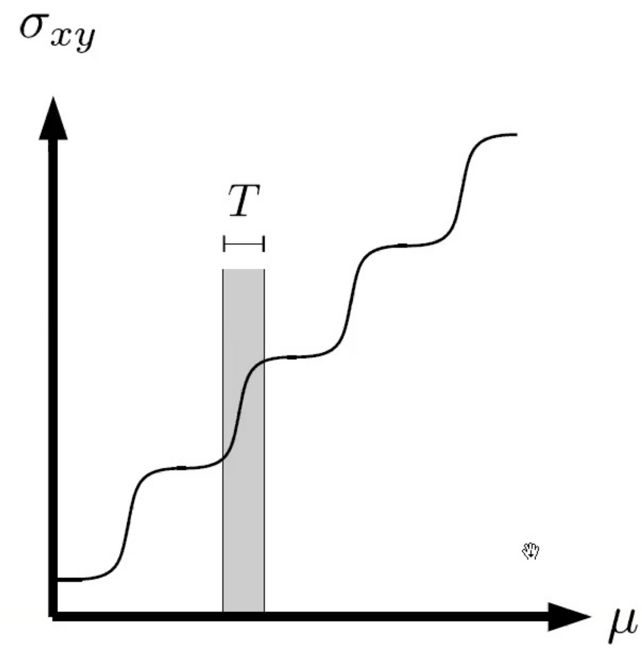
Quantum Hall effect I: clean case





Quantum Hall effect I: clean case





Non-zero temperature

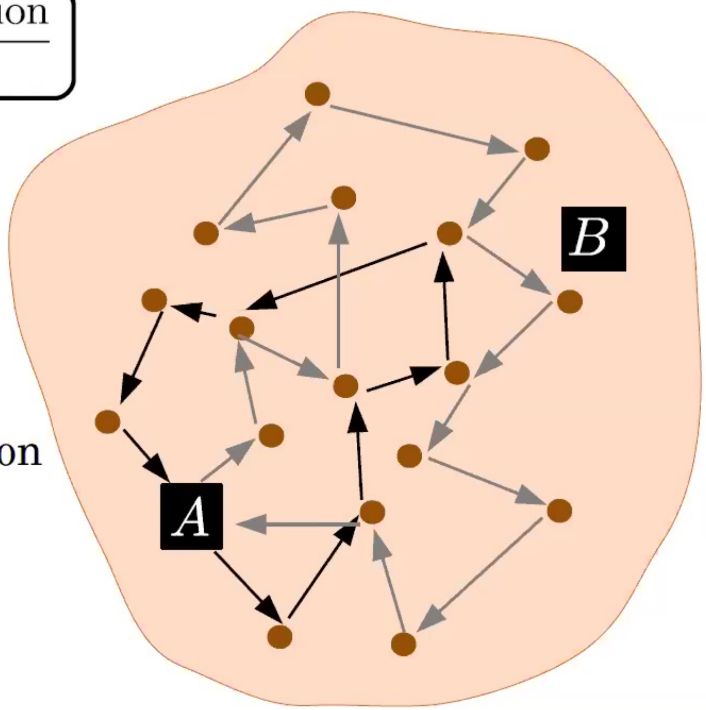
$$\delta\sigma_{xy} \sim e^{-|\mu-\mu_n|/T}$$

Quantum Hall effect II: disorder

$$\text{conductance} = \text{conductivity}\,\frac{\text{cross section}}{\text{length}}$$

$$\beta(g) = \frac{dg}{d\log L} = (d-2)g - \ldots$$

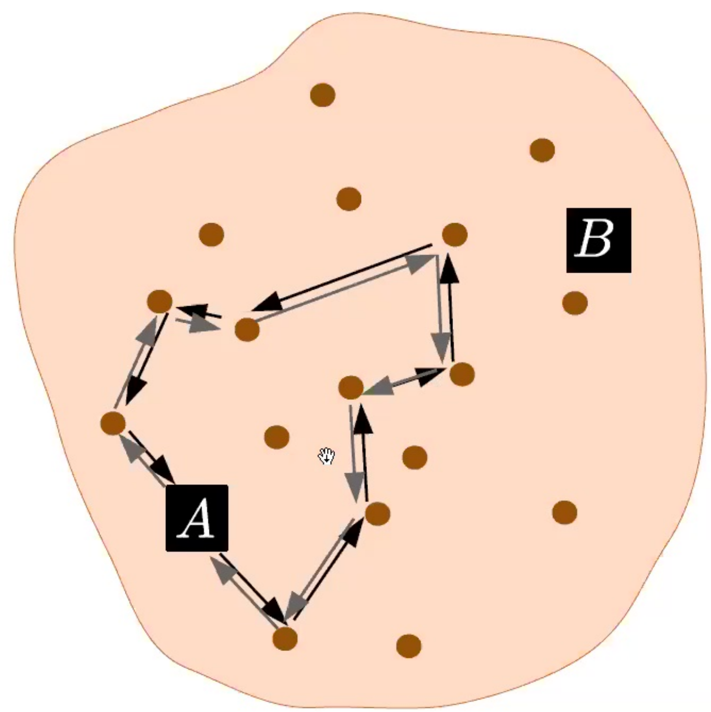
Quantum correction
(interference)



Quantum Hall effect II: disorder

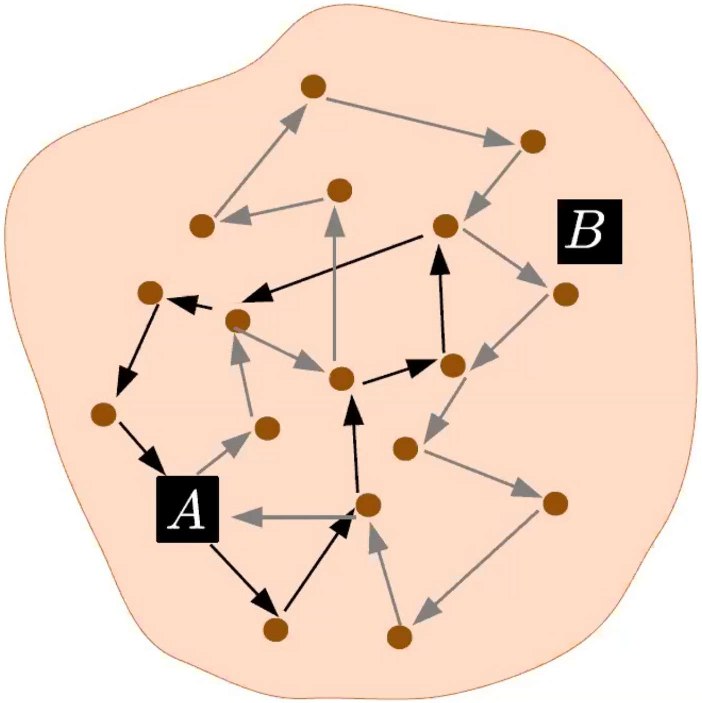
$$\beta(g) = \frac{dg}{d\log L} = (d-2)g - \ldots$$

With time-reversal symmetry, return is favored



Quantum Hall effect II: disorder

$$\beta(g) = \frac{dg}{d\log L} = (d-2)g - \dots$$



Without time-reversal symmetry, return is slightly favored

$$\beta_A^{2d}(g) = -\frac{1}{2\pi^2 g} \quad \Rightarrow \quad g(L) = \sqrt{g_0^2 - \frac{1}{\pi^2}\log\frac{L}{\ell_0}}$$

(very) weak localization

Quantum Hall effect II: disorder

Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
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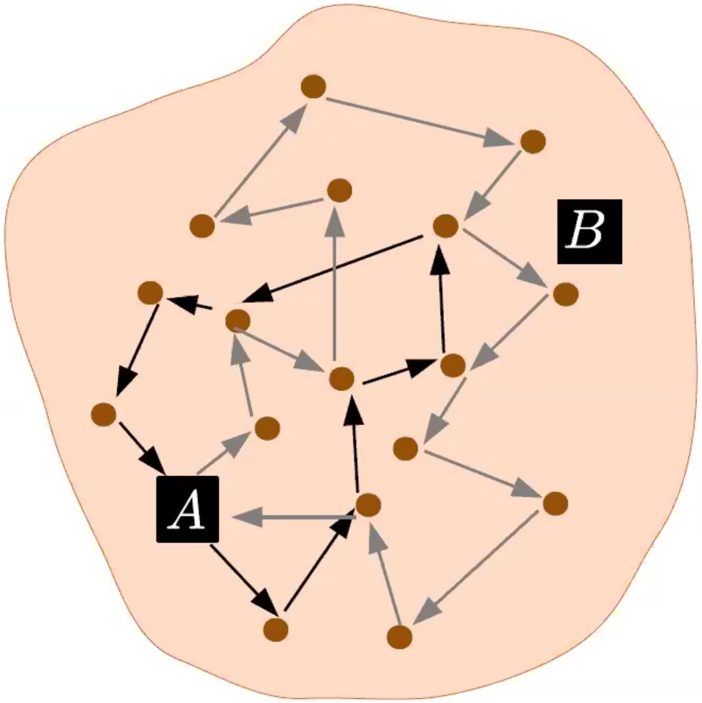

$$g(L) = \sqrt{g_0^2 - \frac{1}{\pi^2} \log \frac{L}{\ell_0}}$$

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Quantum Hall effect II: disorder

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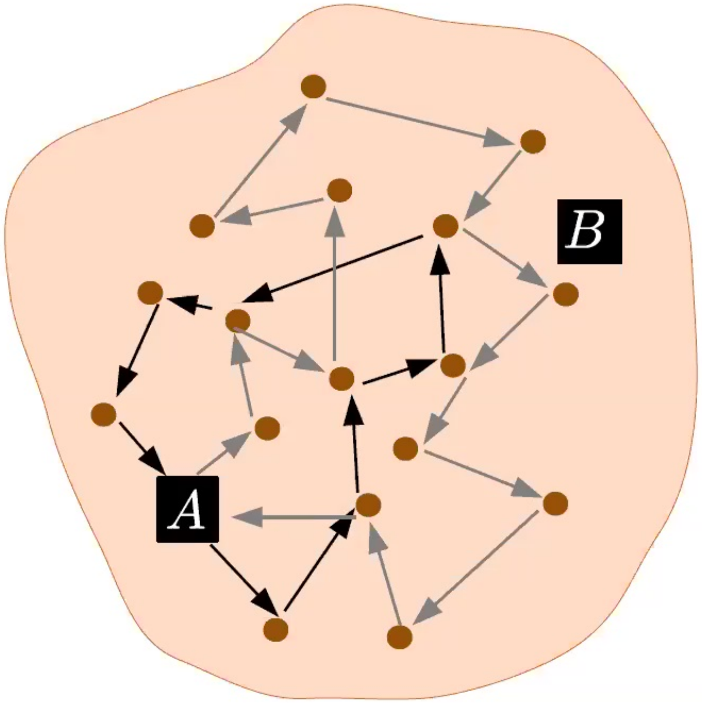

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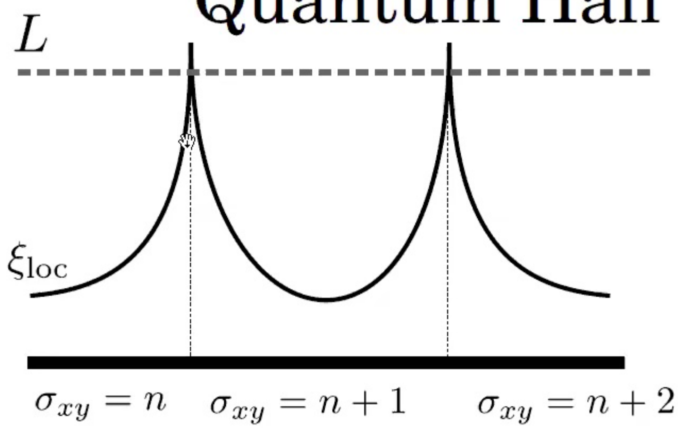
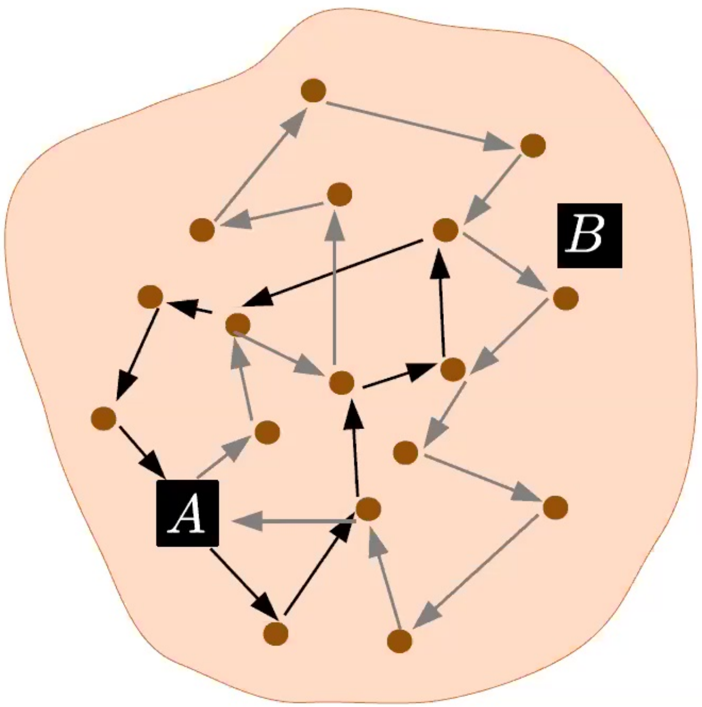
Quantum Hall effect II: disorder



- ξ_{loc} weakly energy dependent
- ξ_{loc} diverges at topological phase transitions
- Quantized σ_{xy} for $\xi_{loc} \ll L$



$$g(L) = \sqrt{g_0^2 - \frac{1}{\pi^2}\log\frac{L}{\ell_0}}$$

(very) weak localization

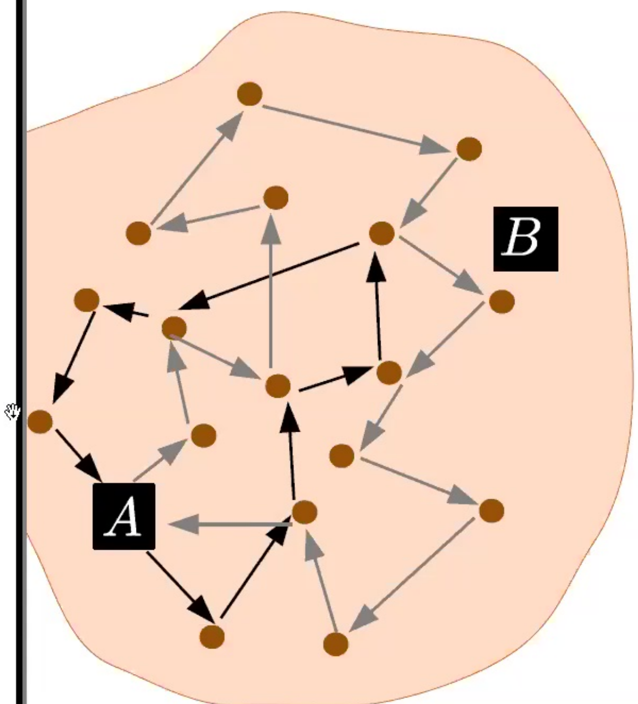
II: disorder

Dephasing

Quasiparticles lose their phase information through interactions with phonons or other quasiparticle

→ Can no longer interfere

$$g(L > \ell_\phi) \approx g(\ell_\phi)$$



$$(L) = \sqrt{g_0^2 - \frac{1}{\pi^2} \log \frac{L}{\ell_0}}$$

very) weak localization

II: disorder

Dephasing

Quasiparticles lose their phase information through interactions with phonons or other quasiparticle

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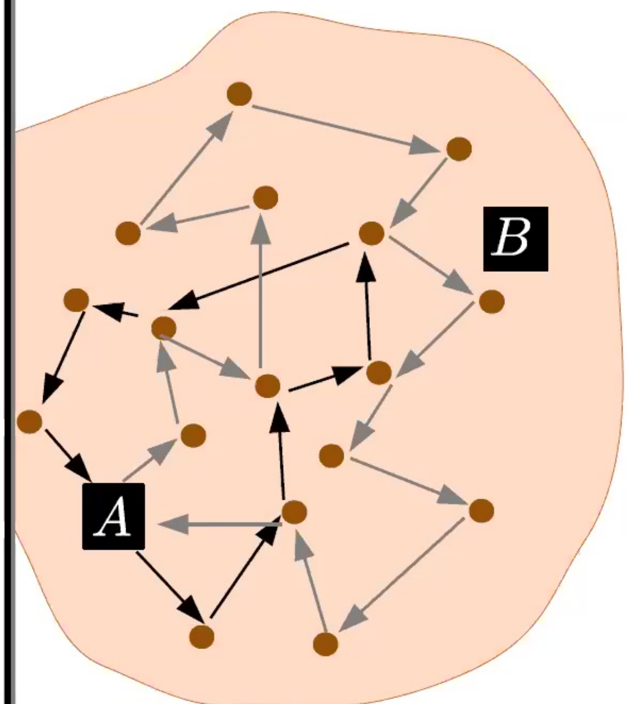
Interactions are sensitive to temperature, typically yield power laws

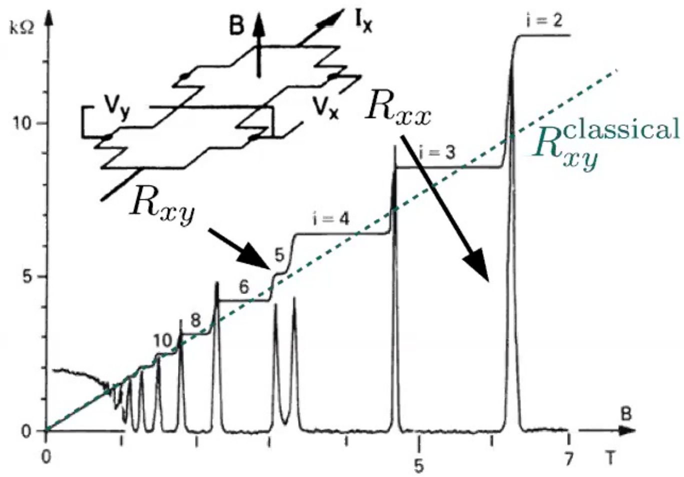
$$\ell_\phi(T) \sim T^{-\alpha}$$



$$(L) = \sqrt{g_0^2 - \frac{1}{\pi^2}\log\frac{L}{\ell_0}}$$

(very) weak localization

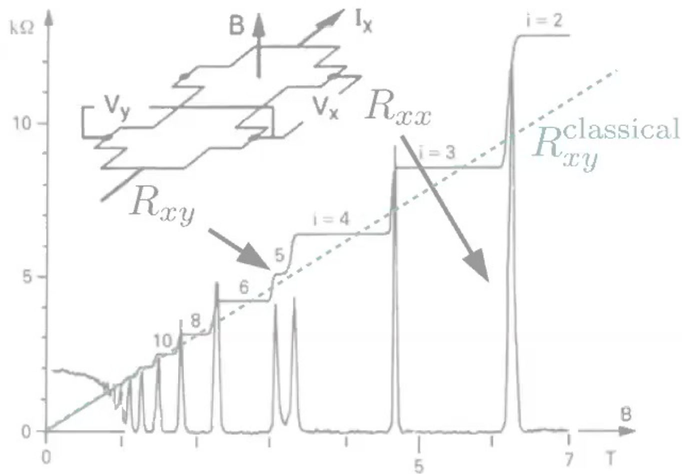
Quantum Hall and thermal Hall effect

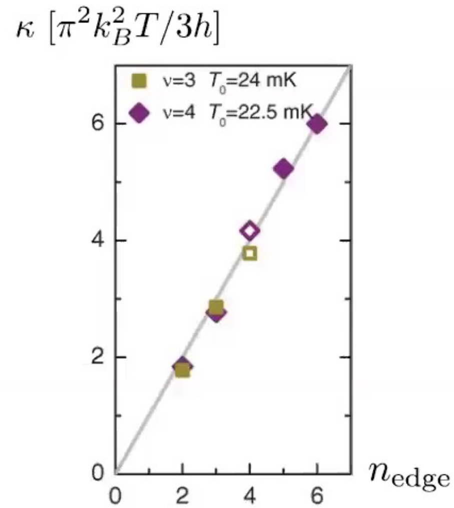


Hall conductance

$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$

Quantum Hall and thermal Hall effect





Jezouin *et al.* (2013)

Hall conductance

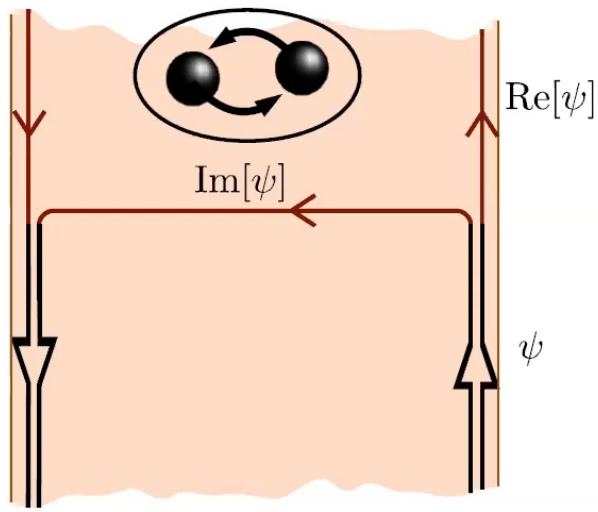
$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$

Thermal Hall conductance

$$\kappa_{xy} = n_{\text{edge}} \frac{\pi^2 k_B^2}{3h} T$$

Any charge carrying edge state, fractional or integer, carries an integer thermal conductance κ_0

Theory: Kane and Fisher (1997)
Experiment: Banerjee *et al.* (2017)



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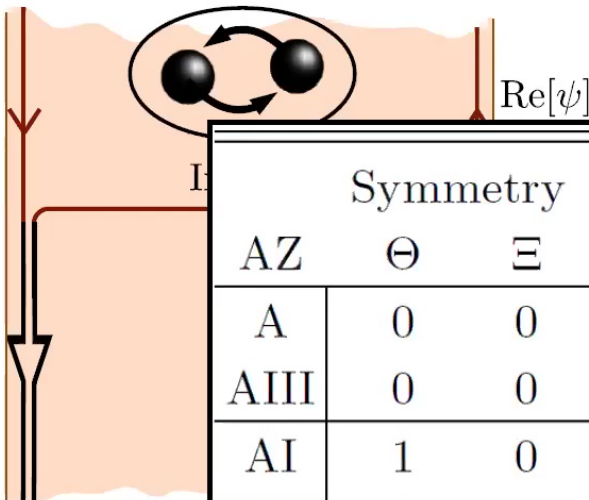

Symmetry				d		
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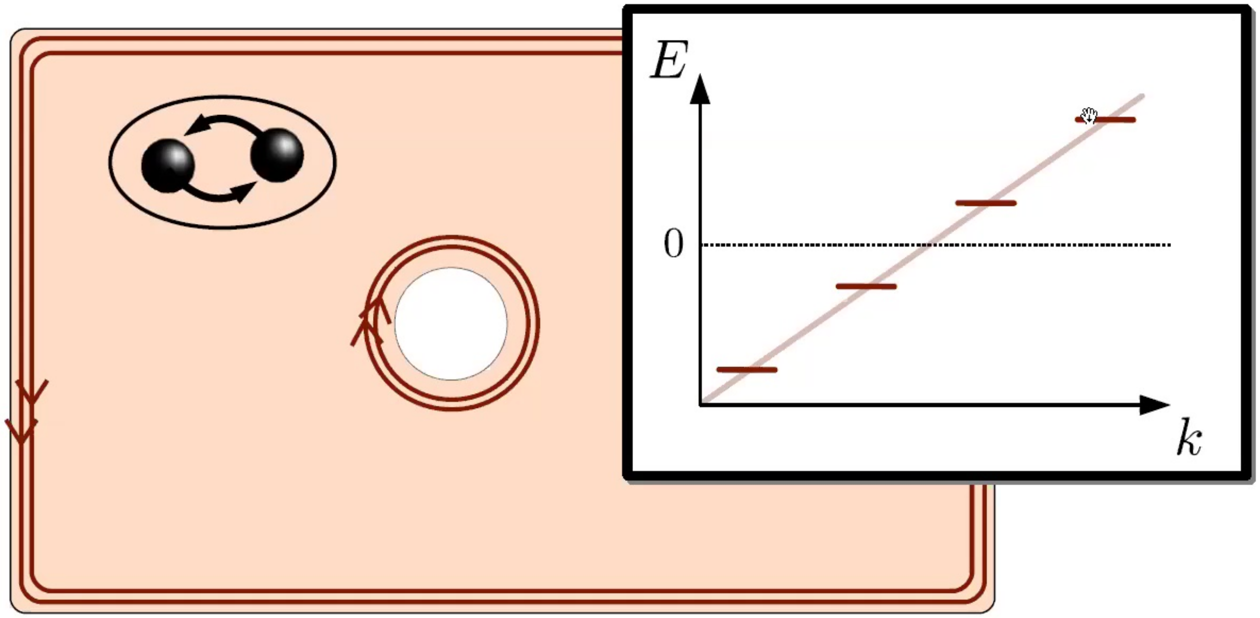
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

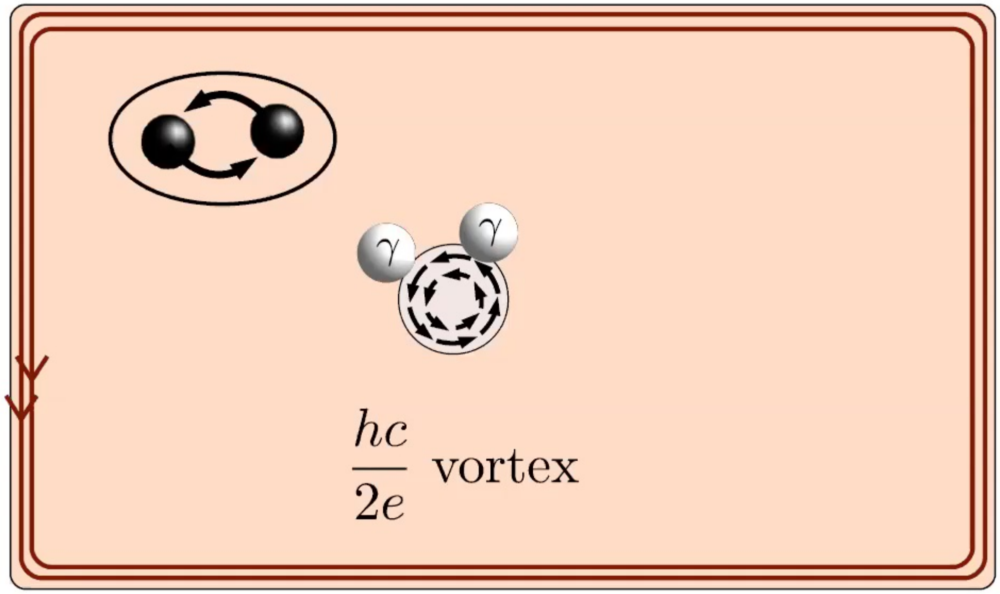
Topological superconductors



n_{Majorana} chiral Majoranas propagating at the edge (absolutely stable)

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2}\kappa_0$$

Topological superconductors

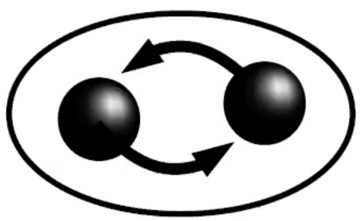


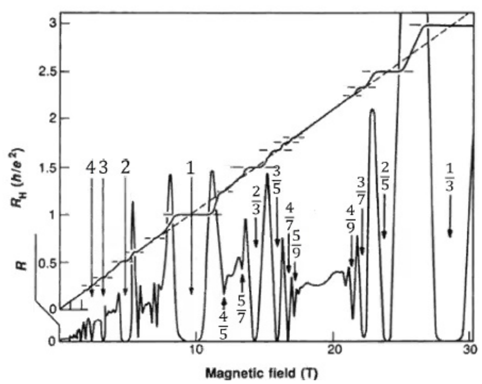
n_{Majorana} chiral Majoranas propagating at the edge (absolutely stable)

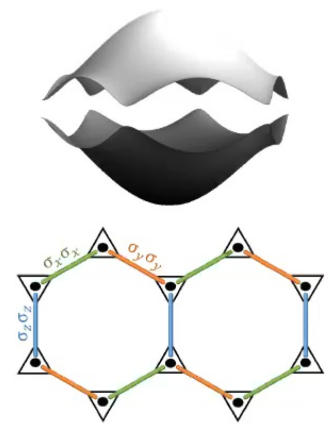
n_{Majorana} Majorana zero modes localized at a vortex (stable mod 2)

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2}\kappa_0$$

Physical systems





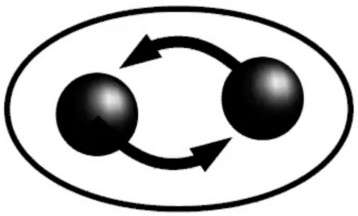


Topological superconductors

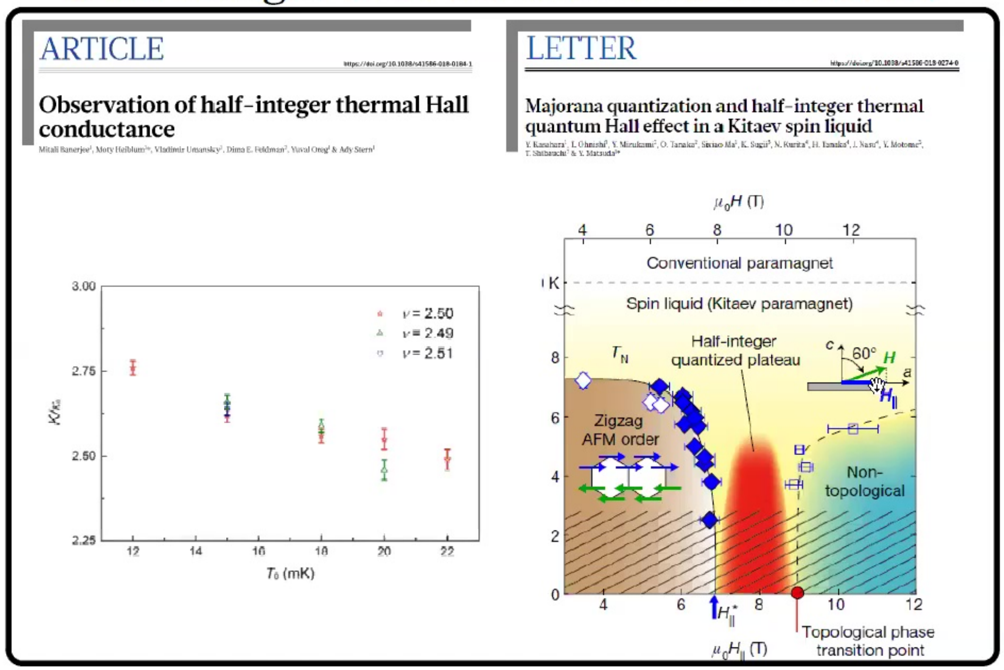
Fractional quantum Hall states

Quantum spin liquids

Physical systems



Half-integer Hall conductance measured

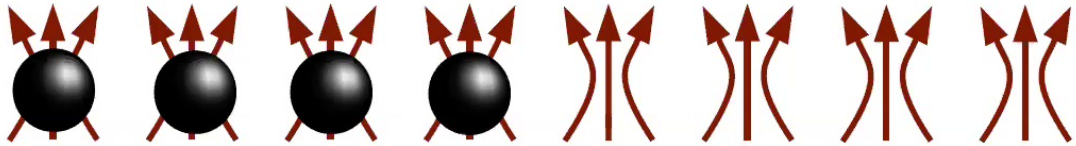


Topological superconductors

Fractional quantum Hall states

Quantum spin liquids

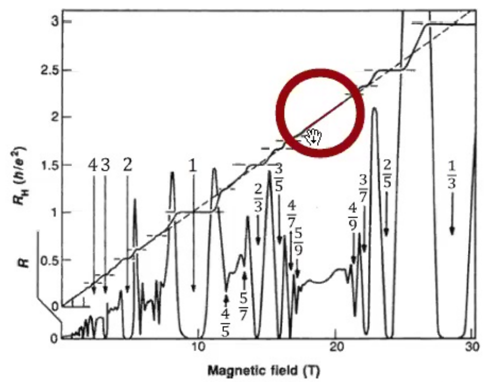
Fractional quantum Hall effect

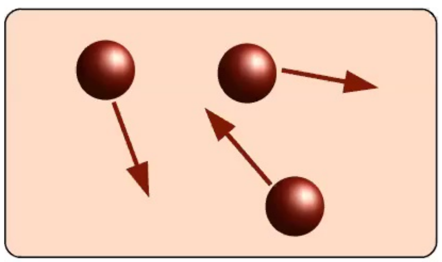


Two flux quanta per electron → many degenerate states

Fractional quantum Hall effect

Two flux quanta per electron → zero flux for composite fermions





Composite-fermion metal

Fractional quantum Hall effect

→ zero flux for composite fermions

Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
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AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
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CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
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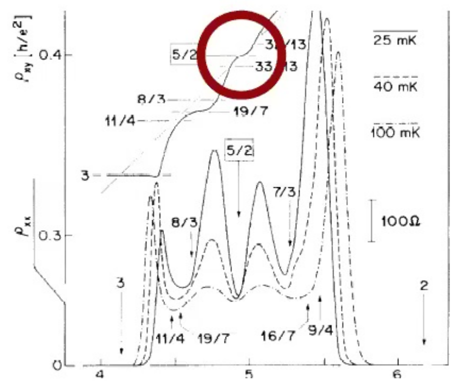
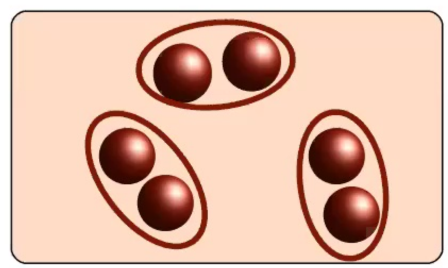
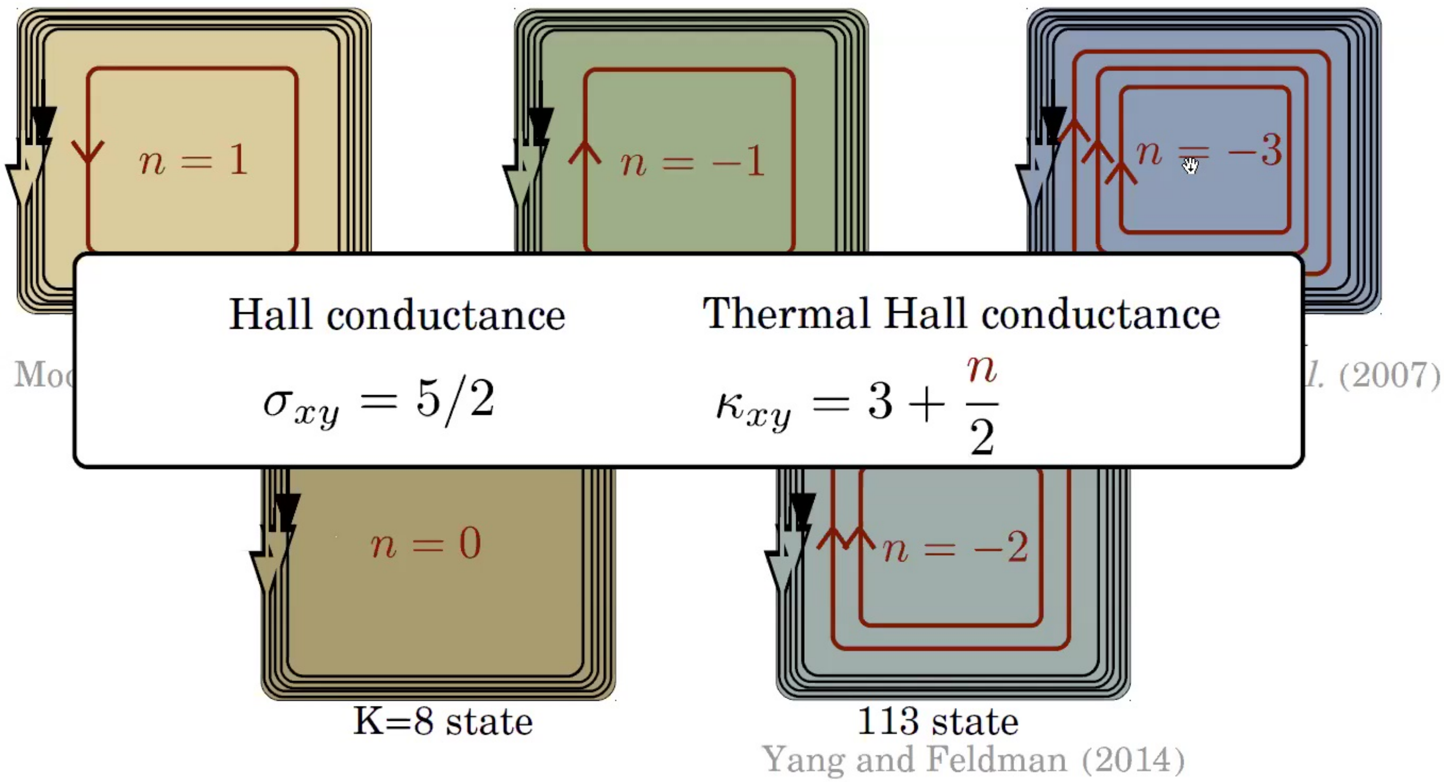




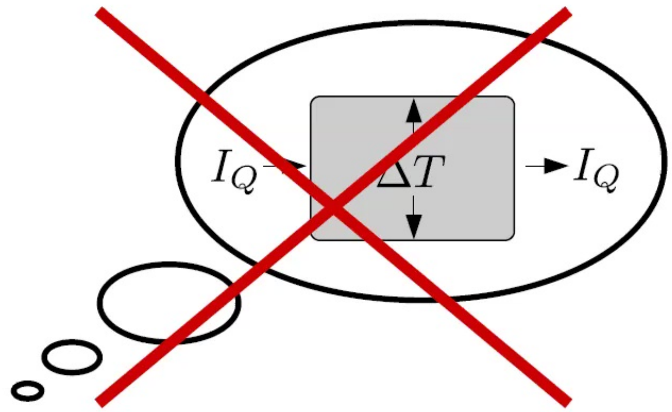
Composite-fermion superconductor

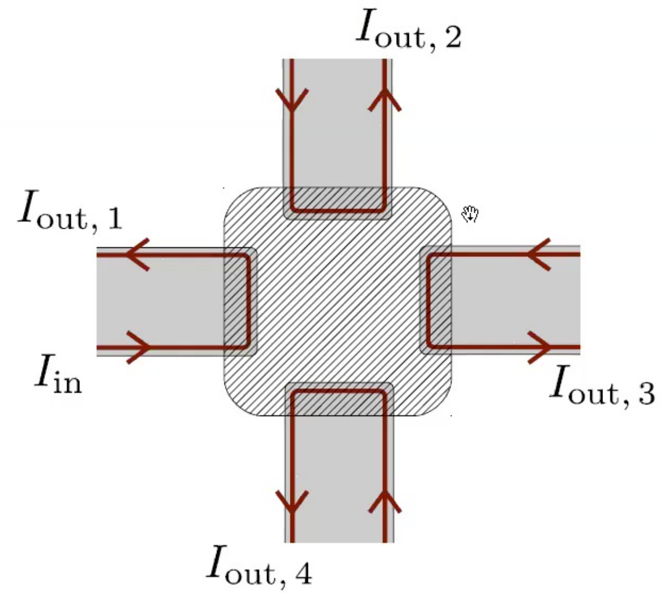
Fractional quantum Hall effect at ν=5/2



Hall conductance	Thermal Hall conductance
$\sigma_{xy} = 5/2$	$\kappa_{xy} = 3 + \frac{n}{2}$

Thermal Hall effect at ν=5/2

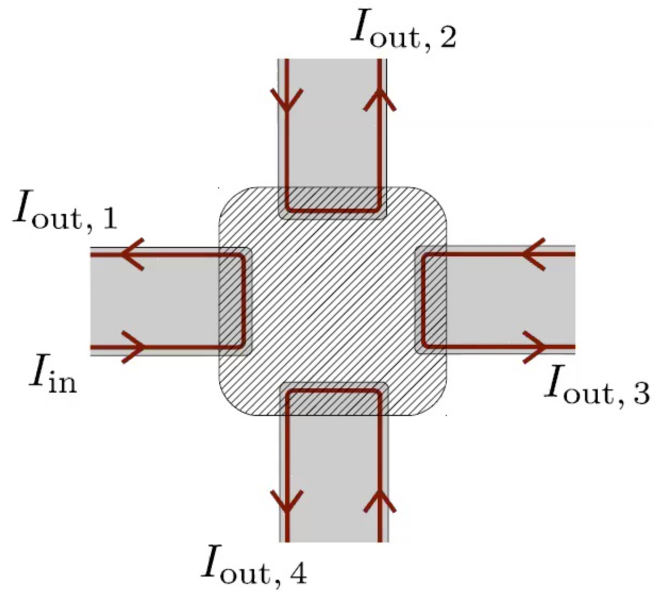




Thermal Hall effect at ν=5/2

Charge conservation

$$I_{\text{in}} = \sum_i I_{\text{out},\,i}$$



Power balance

$$\Delta P = P_{\text{in}} - \sum_i P_{\text{out},\,i}$$

Thermal Hall effect at ν=5/2

Charge conservation

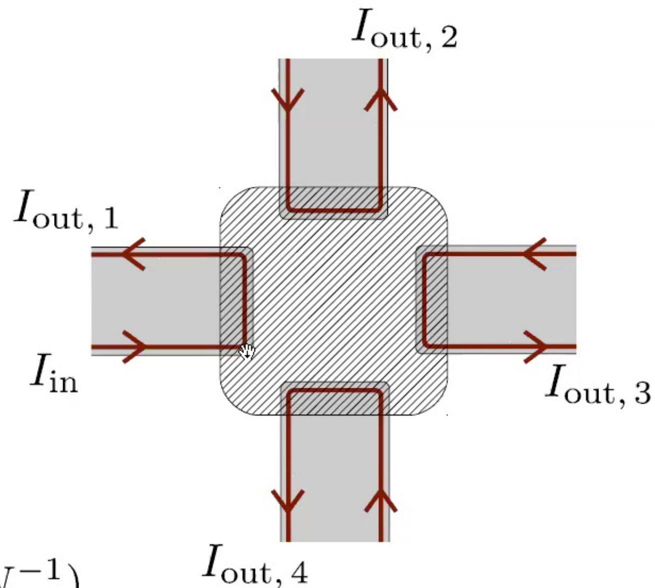
$$I_{\text{in}} = \sum_i I_{\text{out},\,i}$$

Joule's Law

$$P_i = I_i^2/2G$$

Power balance

$$\Delta P = P_{\text{in}} - \sum_i P_{\text{out},\,i} = P_{\text{in}}(1 - N^{-1})$$



Thermal Hall effect at ν=5/2

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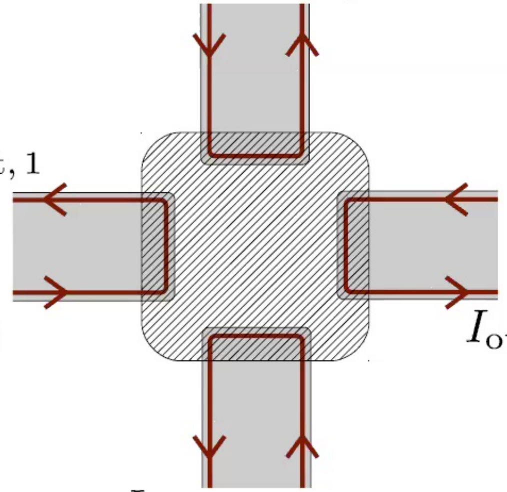
$$\Delta P = P_{\text{in}} - \sum_i P_{\text{out},\,i} = P_{\text{in}}(1 - N^{-1})$$

Heat flow out of metal

$$\Delta P = \frac{1}{2} K N T_{\text{metal}}^2$$



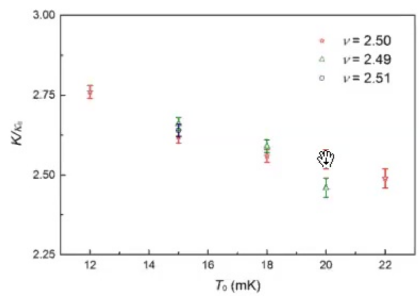
Thermal Hall effect at ν=5/2

ARTICLE

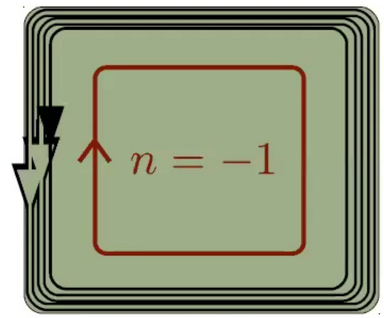
https://doi.org/10.1038/s41586-018-0184-1

Observation of half-integer thermal Hall conductance

Mitali Banerjee[1], Moty Heiblum[1*], Vladimir Umansky[1], Dima E. Feldman[2], Yuval Oreg[1] & Ady Stern[1]

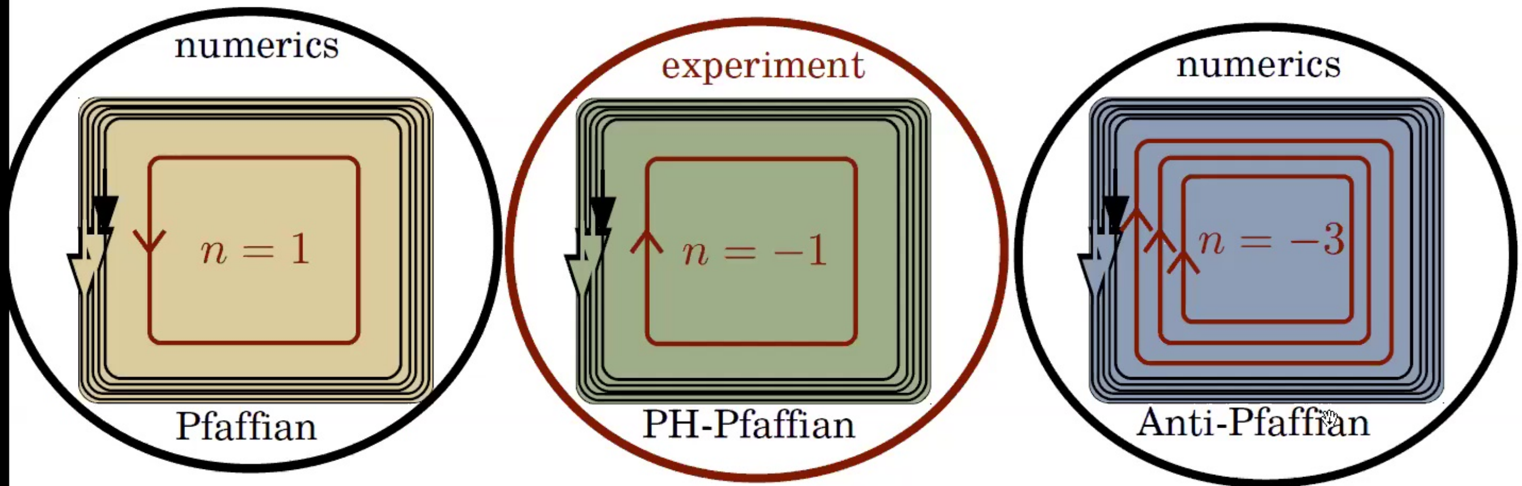


$$K/\kappa_0 = 3 + \frac{n_{\text{Majorana}}}{2}$$



PH-Pfaffian

Numerics at ν=5/2



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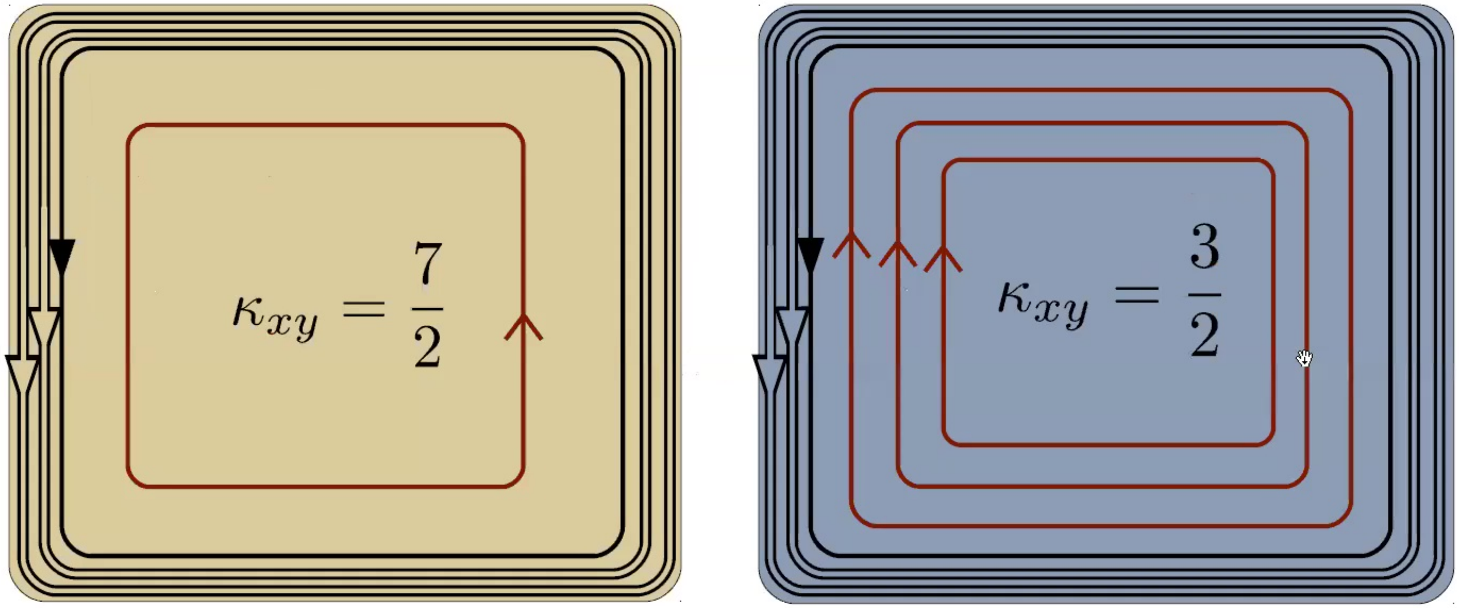
Fractional quantum Hall effect at ν=5/2

Numerics: In **clean** system, Pfaffian or Antipfaffian



degenerate when PH-symmetric ($\nu = 5/2$)

Thermal Hall effect at ν=5/2

Charge conservation

$$I_{\text{in}} = \sum_i I_{\text{out},\,i}$$

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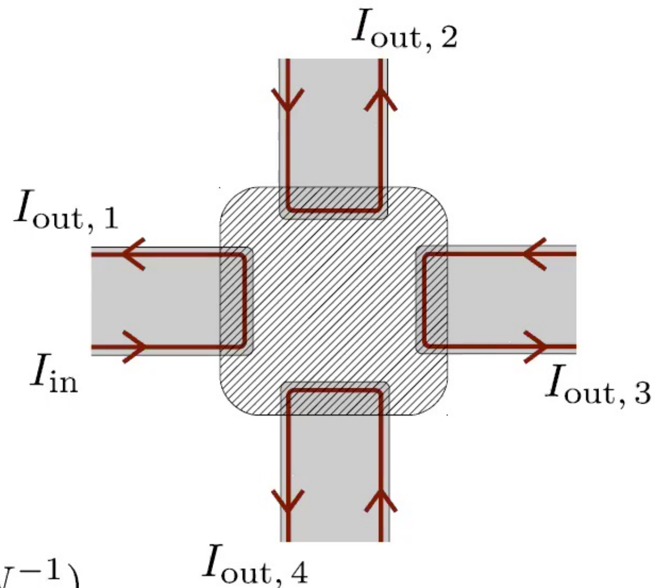
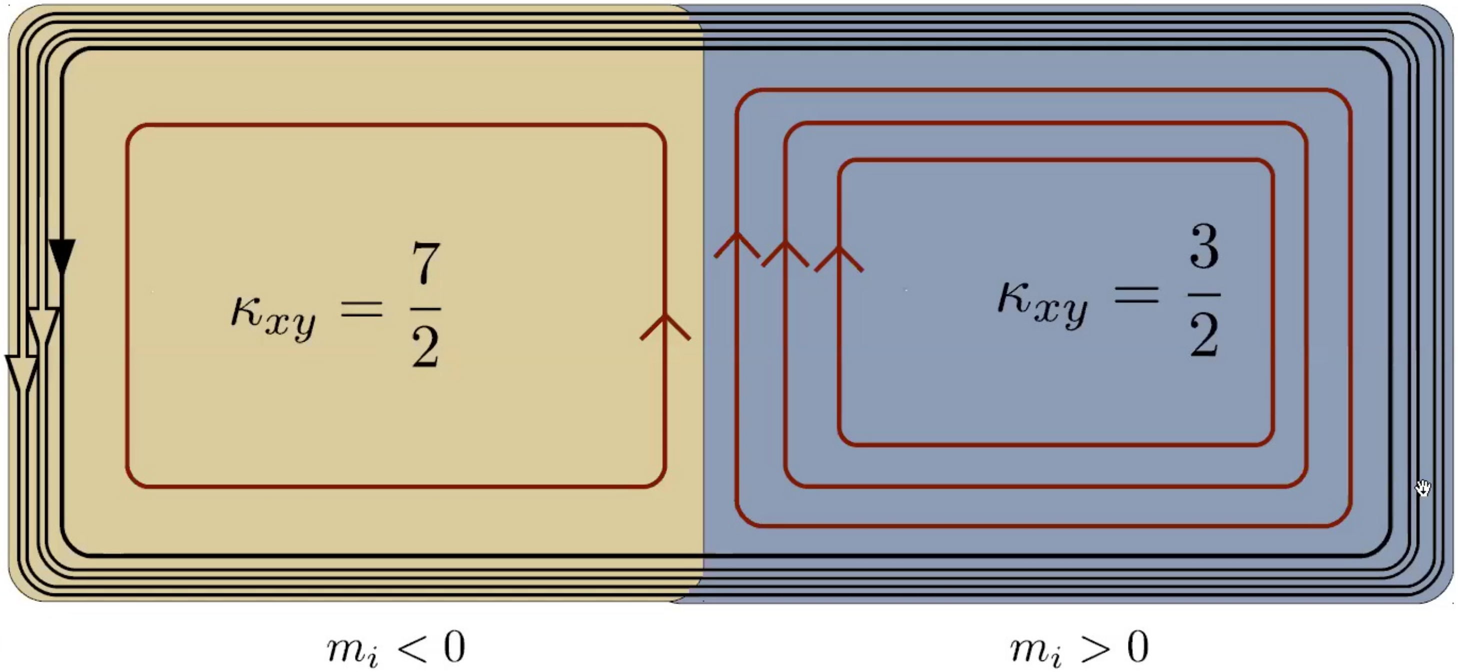


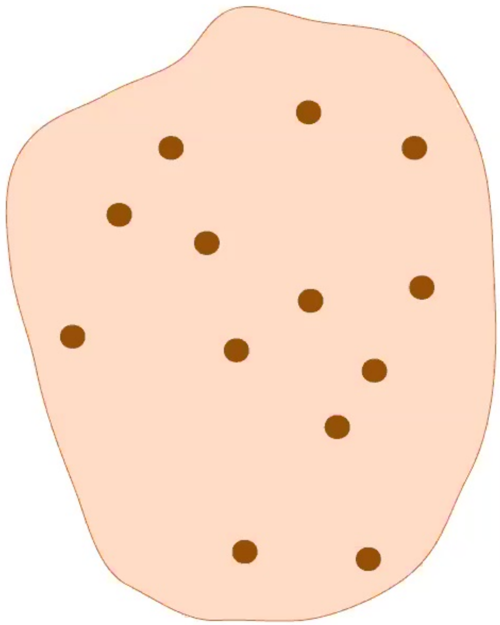
Fractional quantum Hall effect at ν=5/2

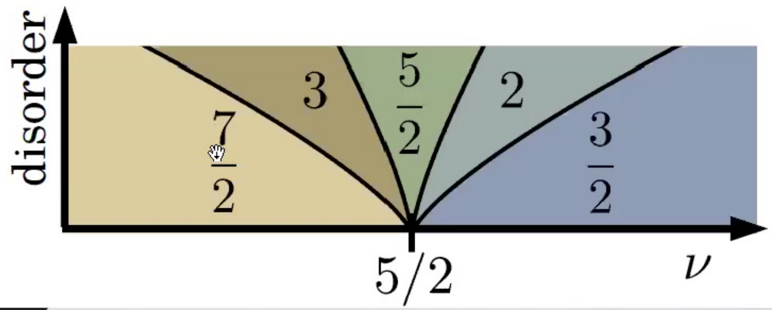
Numerics: In **clean** system, Pfaffian or Antipfaffian



$$H = \sum_{i=1}^{4} \eta_i^T \left[i\tau_x \partial_x + i\tau_z \partial_z + m_i \tau_y \right] \eta_i$$

Fractional quantum Hall effect at ν=5/2





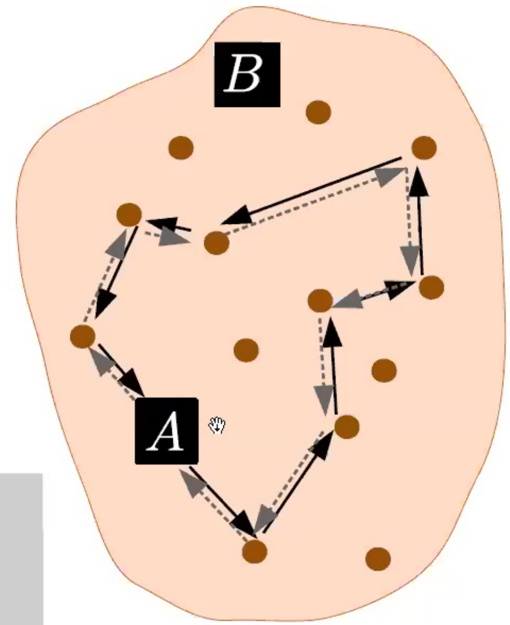
Disorder splits multi-critical point into sequence of phase transitions

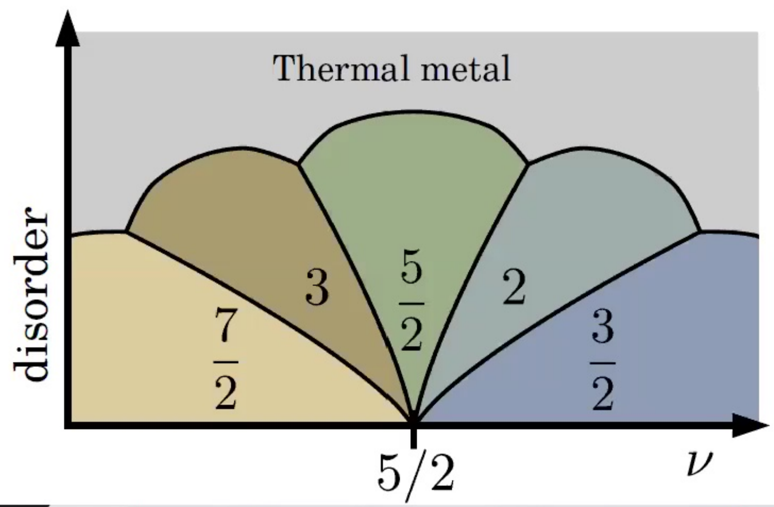
Fractional quantum Hall effect at ν=5/2

Interference between particles and holes

$$\beta_D^{2d}(g) = \frac{1}{\pi} \qquad \text{(weak anti-localization)}$$

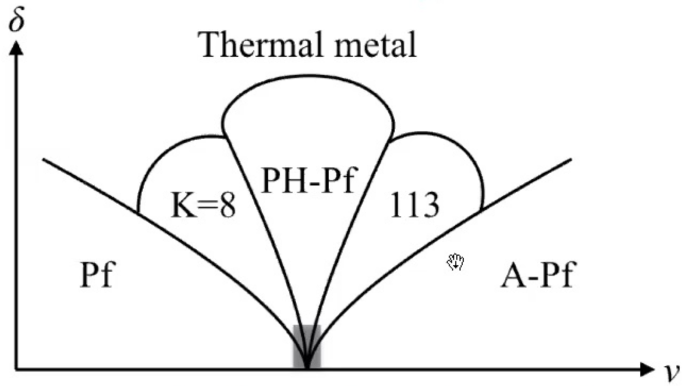
$$g(L) = g_0 + \frac{1}{\pi} \log \frac{L}{\ell_0}$$



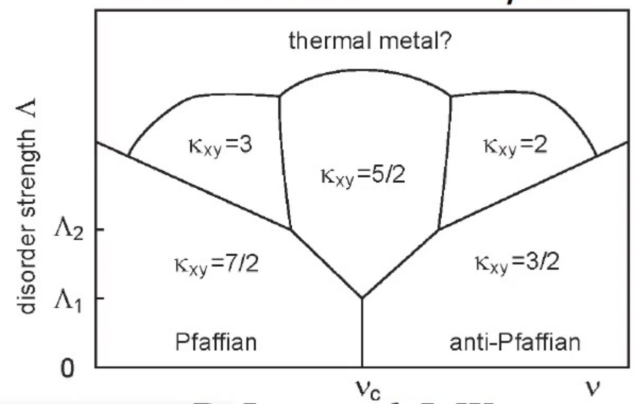


Disorder splits multi-critical point into sequence of phase transitions

Fractional quantum Hall effect at ν=5/2

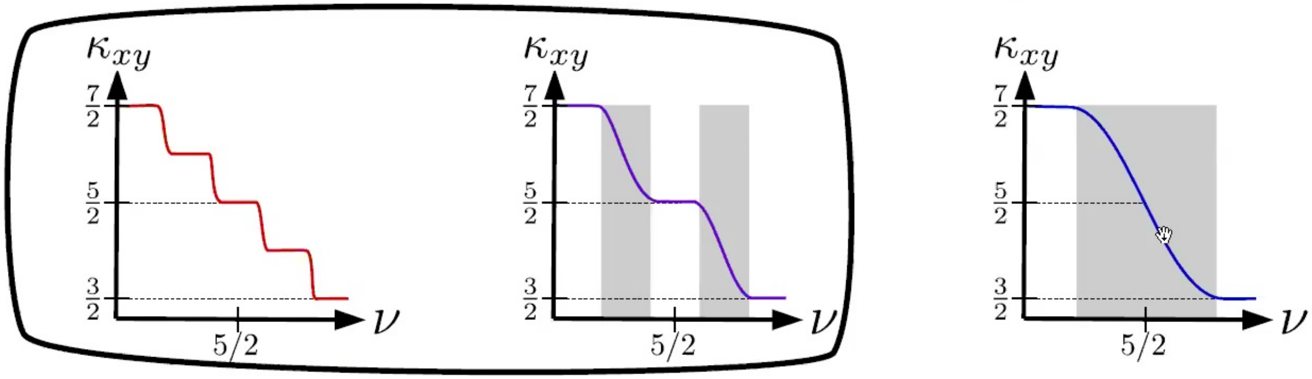


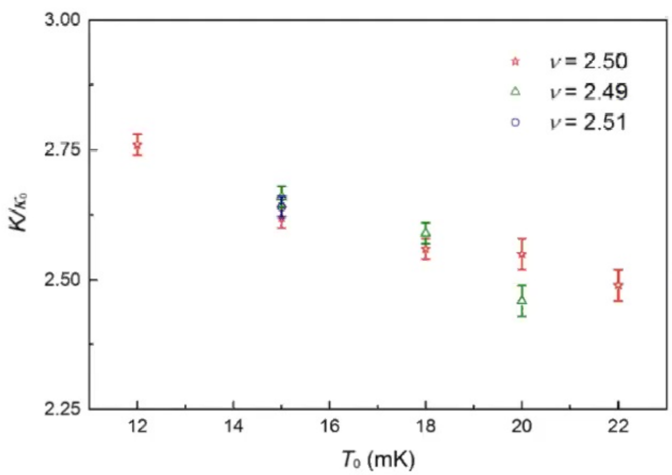
C. Wang, A. Vishwanath, B. Halperin, PRB 98, 045112 (2018)



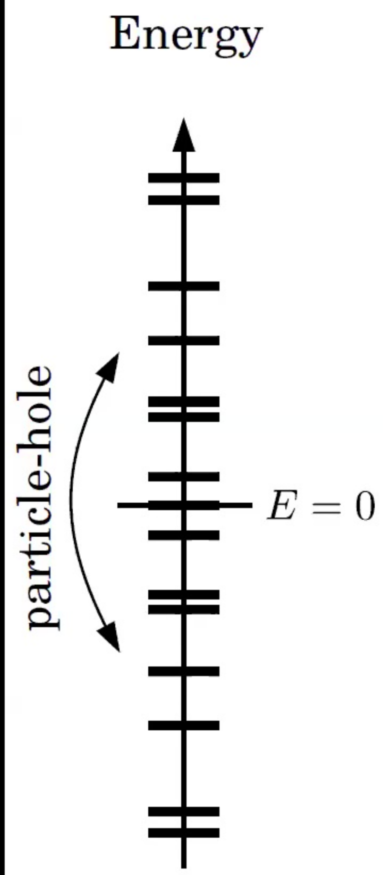
B. Lian and J. Wang PRB 97 165124 (2018)

Thermal metal at ν=5/2?



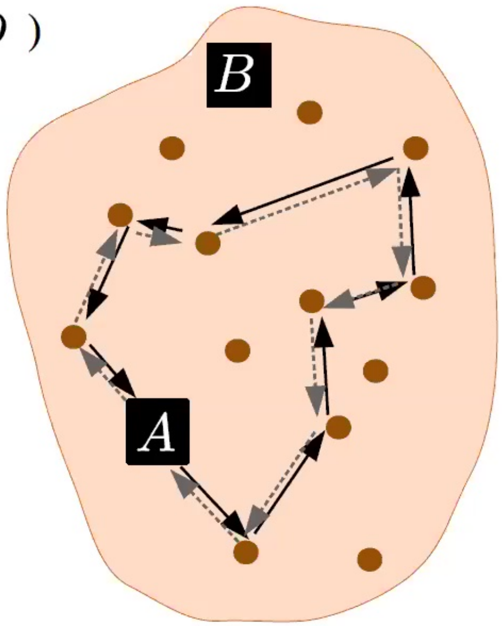


Particle-hole symmetry

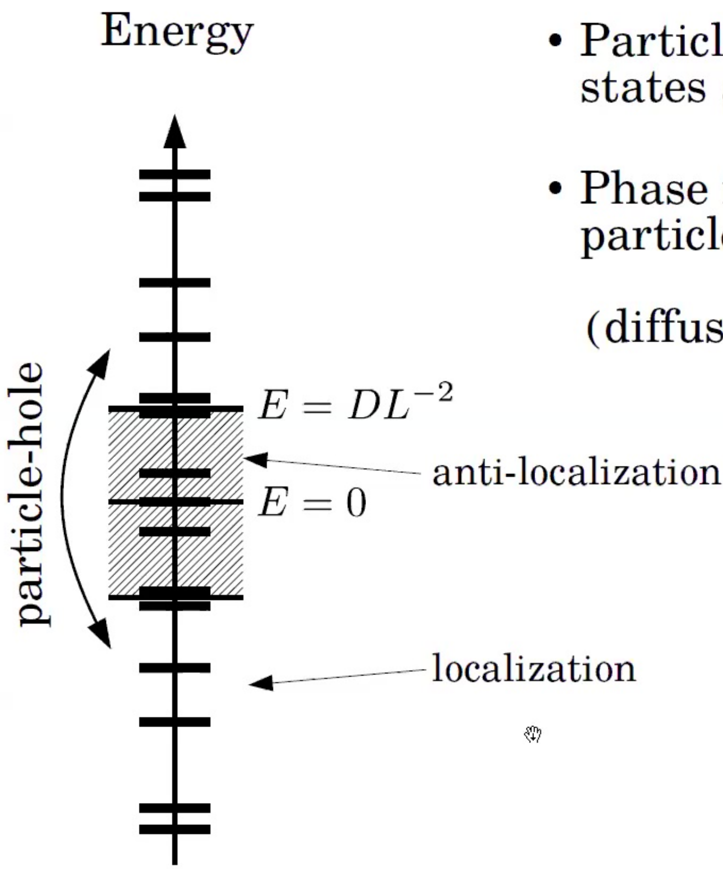


- Particle-hole symmetry relates states at energies $+E$ and $-E$.
- Phase factor $e^{iE\tau}$ between particle and hole trajectories

(diffusion: $\tau = \ell^2/D$)



Particle-hole symmetry



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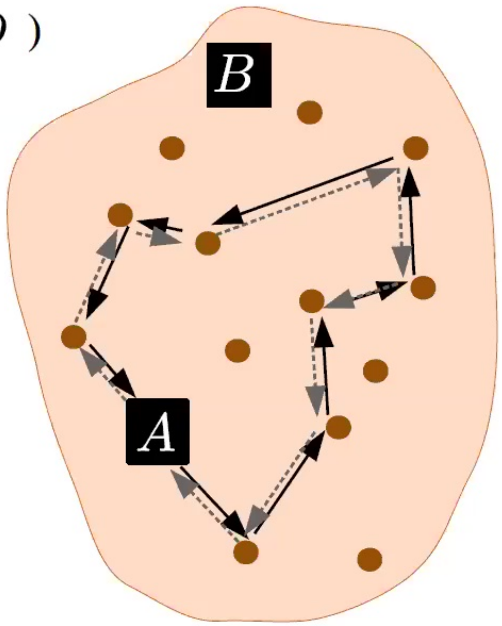
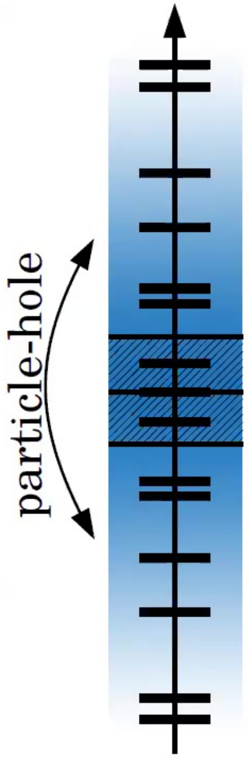


Particle-hole symmetry

- Thermal conductance involves states between energies $+T$ and $-T$.



Most participating states are localized

$\kappa_{xx} \to 0$

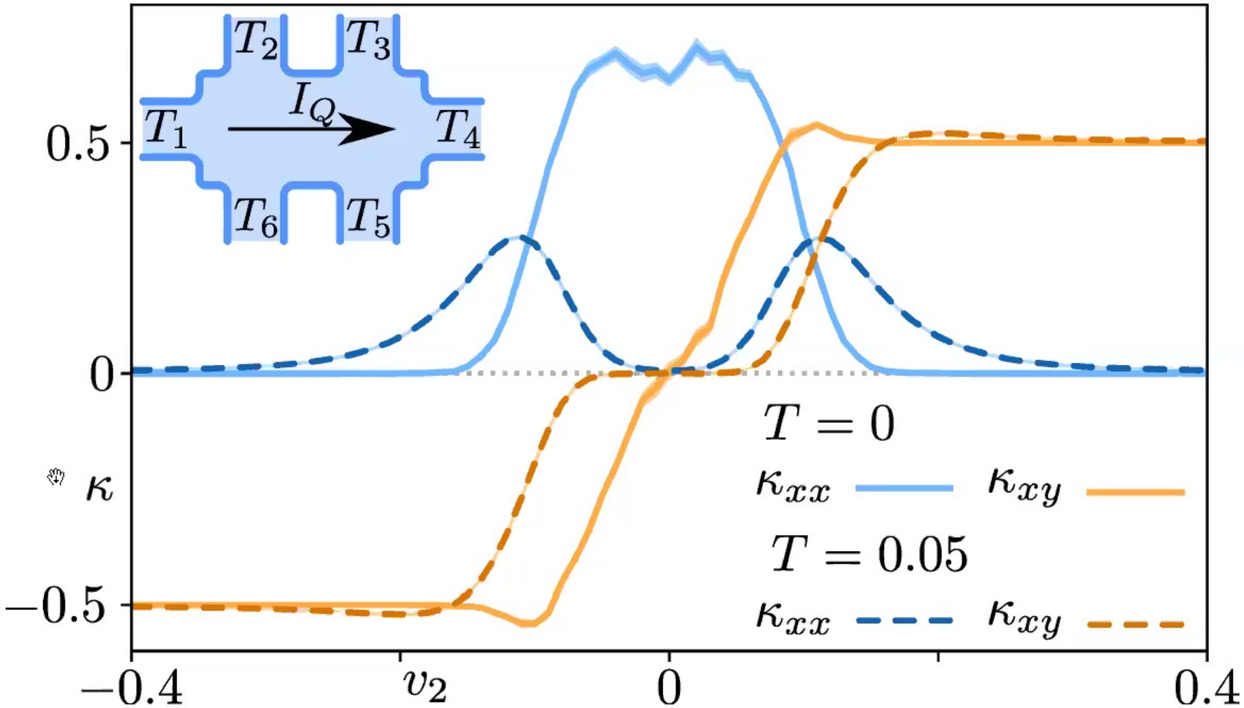
$L \to \infty$ at finite T

Most participating states are delocalized

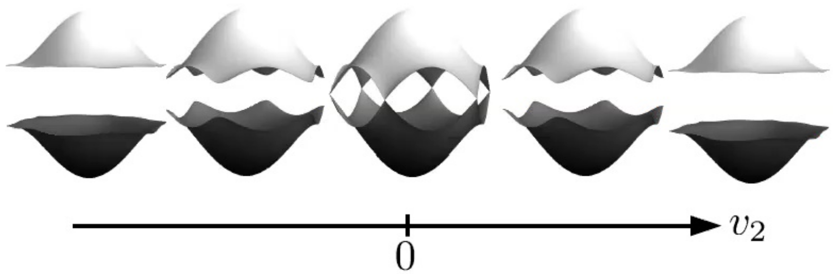
$\kappa_{xx}/T \to \infty$

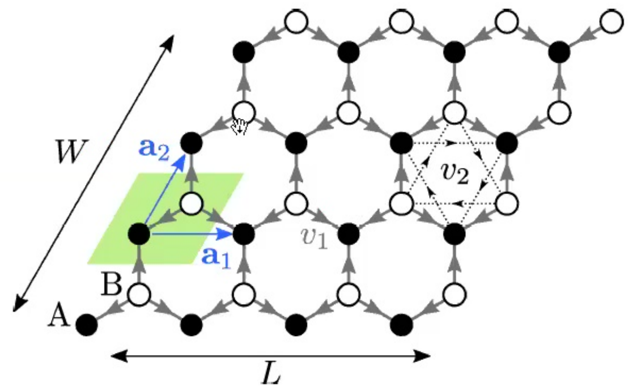
$T \to 0$ at finite L

Particle-hole symmetry



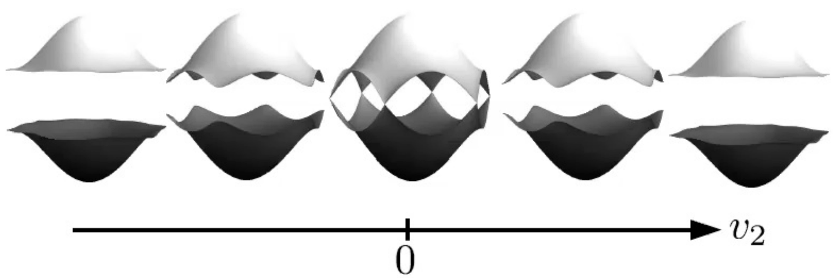
Model system: topol. phases in clean limit

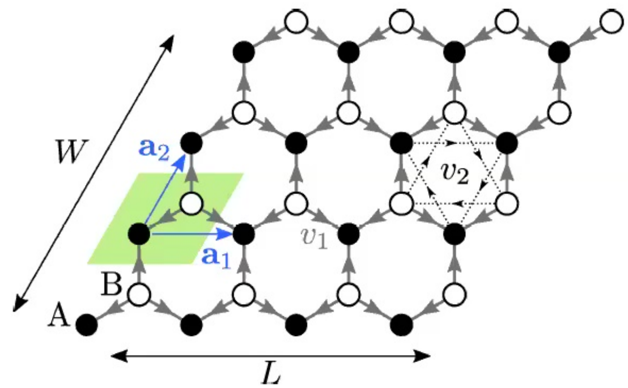


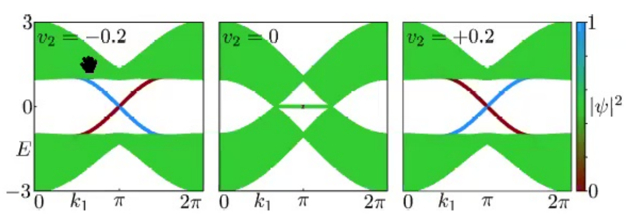


$$H = iv_1 \sum_{\langle jk \rangle} \gamma_j \gamma_k + iv_2 \sum_{\langle\langle jk \rangle\rangle} \gamma_j \gamma_k$$

Model system: topol. phases in clean limit



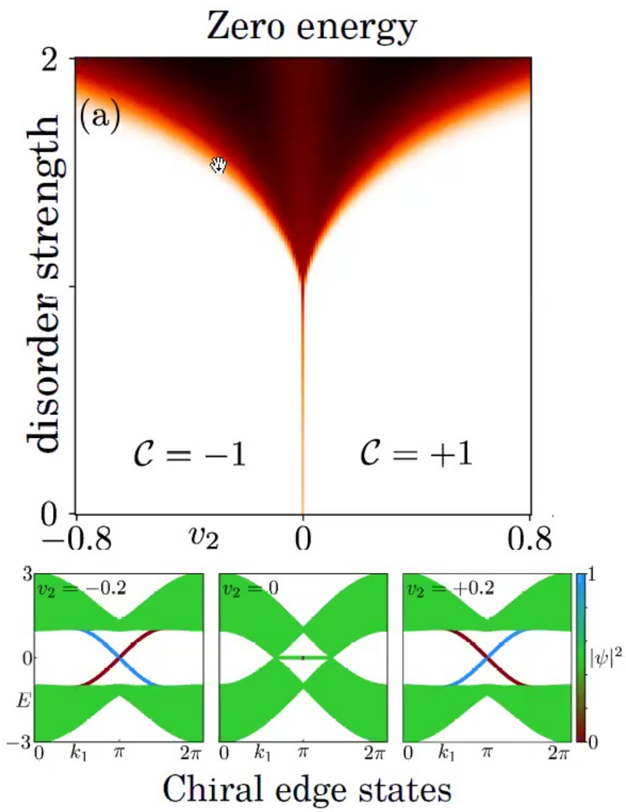


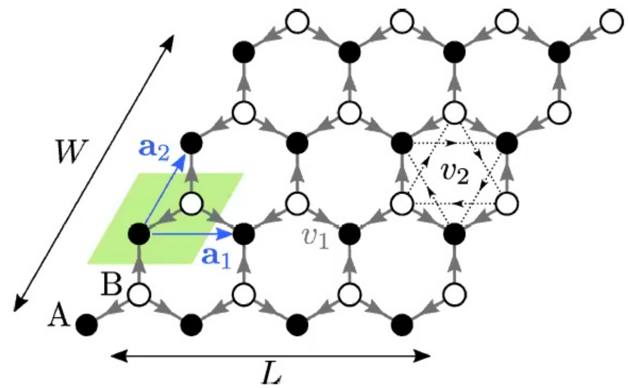


Chiral edge states

$$H = iv_1 \sum_{\langle jk \rangle} \gamma_j \gamma_k + iv_2 \sum_{\langle\langle jk \rangle\rangle} \gamma_j \gamma_k$$

Model system with disorder: thermal metal

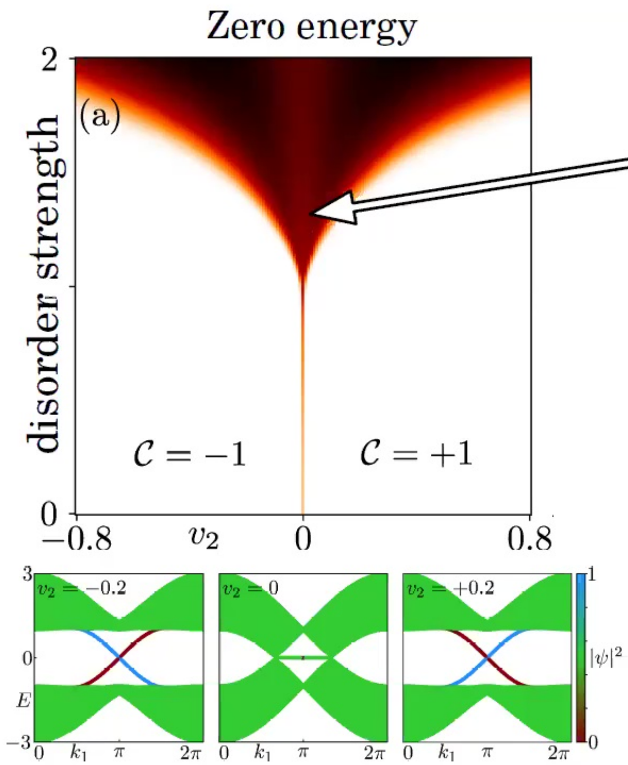




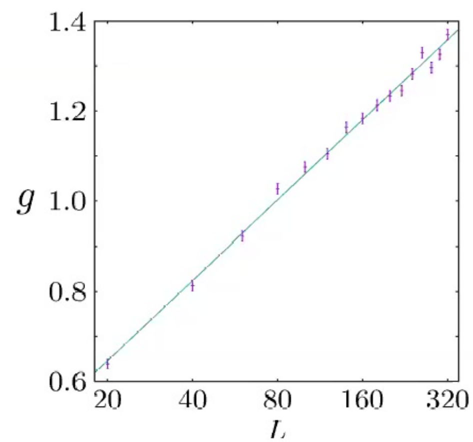
$$H = iv_1 \sum_{\langle jk \rangle} \gamma_j \gamma_k + iv_2 \sum_{\langle\langle jk \rangle\rangle} \gamma_j \gamma_k$$

add randomness

Model system with disorder: thermal metal



Chiral edge states



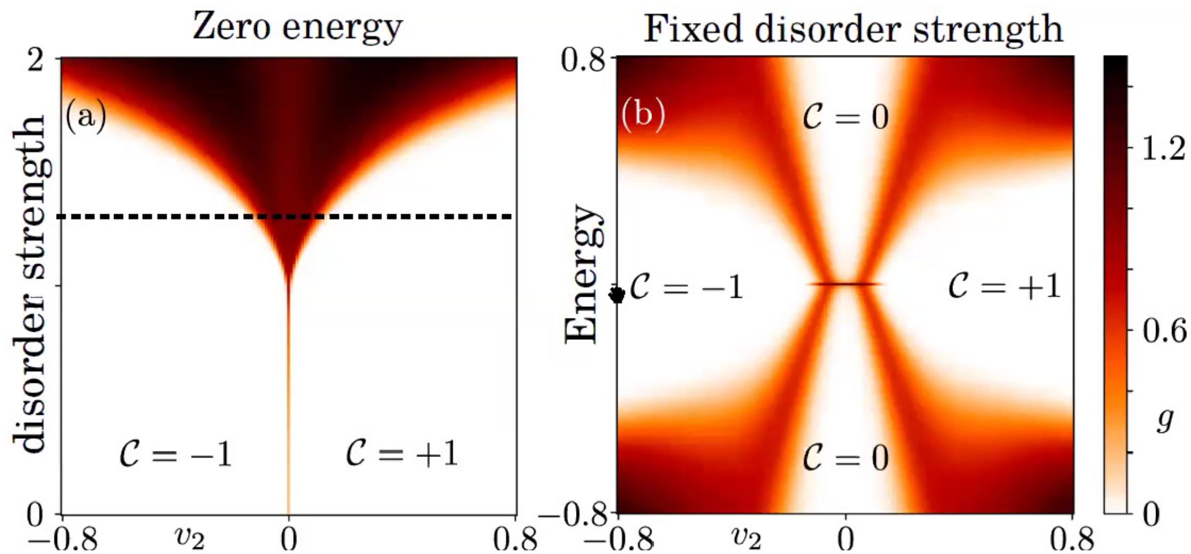
$$g[L] = g_0 + 0.26 \log L/\ell_0$$

(RG predicts slope $\pi^{-1} \approx 0.32$)

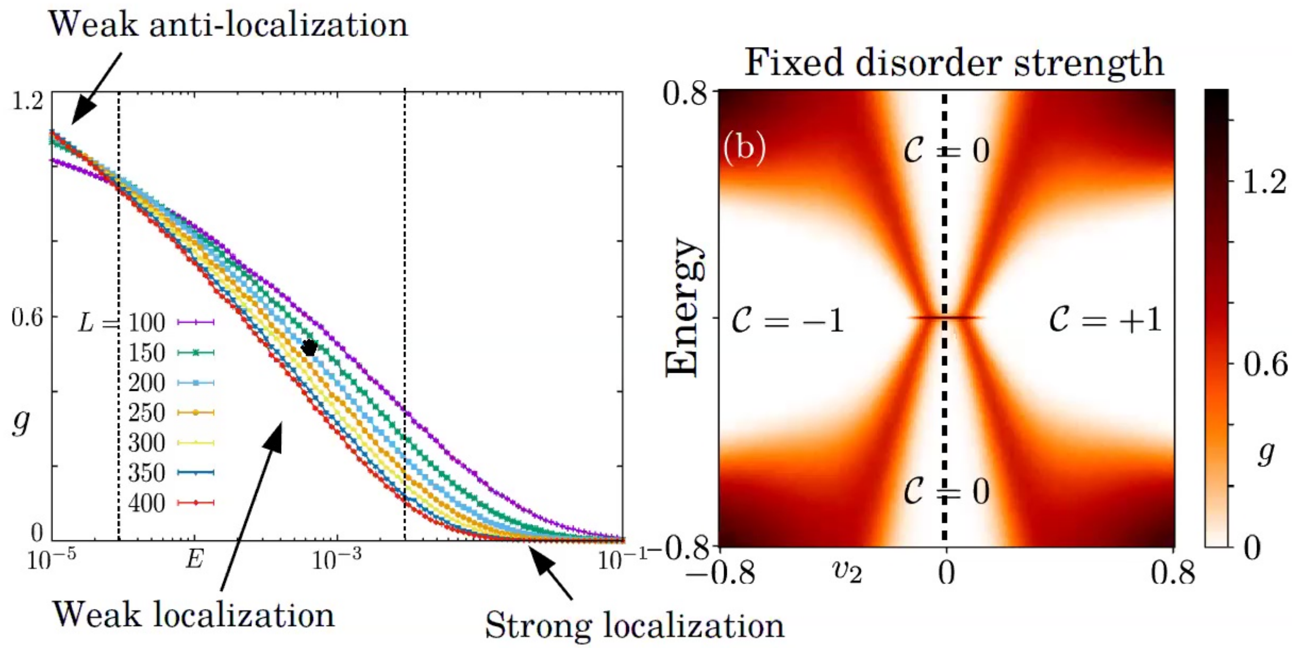
$$H = iv_1 \sum_{\langle jk \rangle} \gamma_j \gamma_k + iv_2 \sum_{\langle\langle jk \rangle\rangle} \gamma_j \gamma_k$$

add randomness

Model system with disorder: thermal metal



Localization–anti-localization crossover

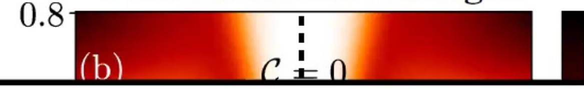


Localization–anti-localization crossover

$$g(L) = g_{0,1} + \frac{1}{\pi}\log\frac{L}{\ell_0}$$

Weak anti-localization

Fixed disorder strength



Cross-over when $E/E_0 = (\ell_0/L_c)^2$

$$g_{2,0} = g_1(L_c) = \frac{1}{2\pi}\log\frac{E_0}{E}$$

Weak localization

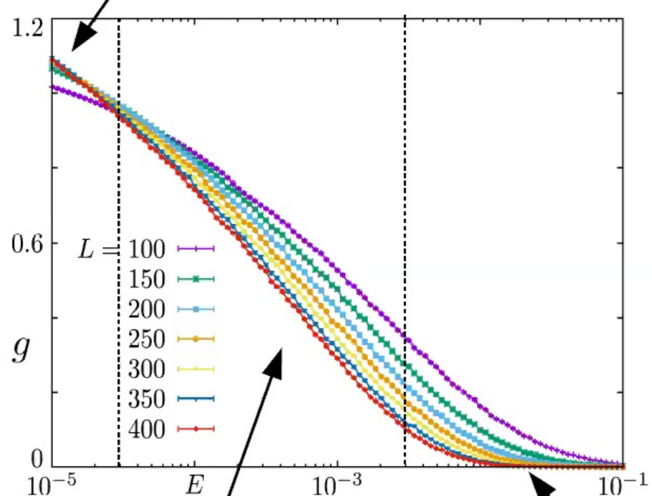
Strong localization

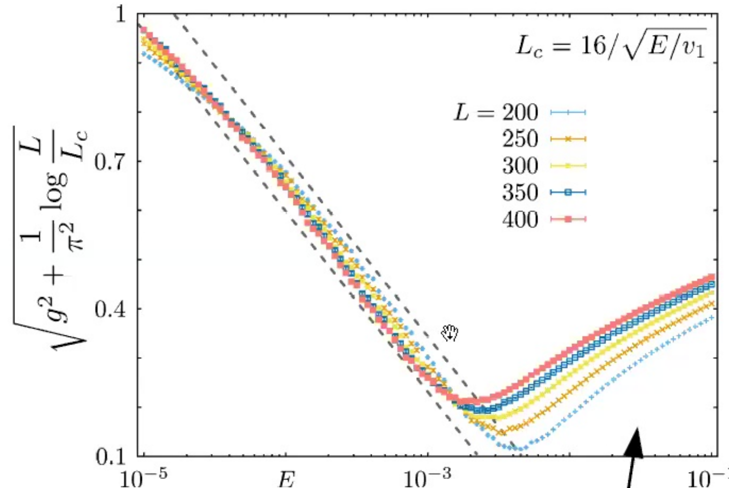
$$g(L) = \sqrt{g_{0,2}^2 - \frac{1}{\pi^2}\log\frac{L}{\ell_0}}$$

Localization–anti-localization crossover

$$g(L) = g_{0,1} + \frac{1}{\pi}\log\frac{L}{\ell_0}$$

Weak anti-localization





Weak localization

Strong localization

Strong localization

$$g(L) = \sqrt{g_{2,0}^2 - \frac{1}{\pi^2}\log\frac{L}{L_c}}$$

$$g_{2,0} = \frac{1}{2\pi}\log\frac{E_0}{E}$$

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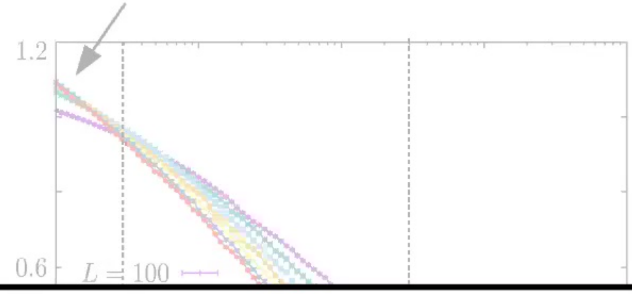
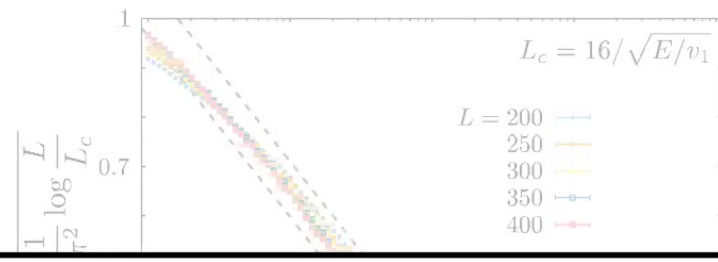
$$g(L) = g_{0,1} + \frac{1}{\pi}\log\frac{L}{\ell_0}$$

Weak anti-localization





Localization length

$$g(L=\xi) = 1 \quad \Rightarrow \quad \xi(E) = \frac{\ell_0}{\sqrt{E/E_0}} \exp\frac{1}{4}\log^2\frac{E_0}{E}$$

Weak localization

Strong localization

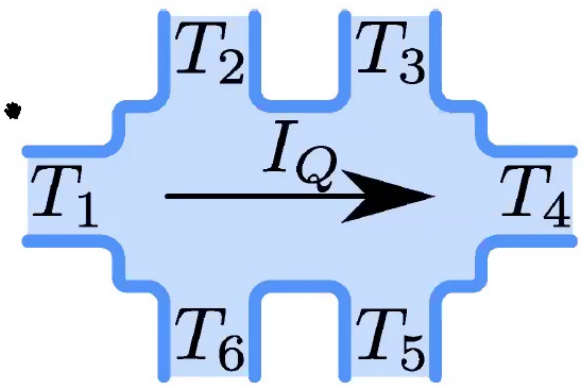
Strong localization

$$g(L) = \sqrt{g_{2,0}^2 - \frac{1}{\pi^2}\log\frac{L}{L_c}}$$

$$g_{2,0} = \frac{1}{2\pi}\log\frac{E_0}{E}$$

Landauer formula in six-terminal geometry

$$\begin{pmatrix} I_Q \\ 0 \\ 0 \\ -I_Q \\ 0 \\ 0 \end{pmatrix} = \int_0^\infty dE\ E \frac{\partial f(E,T)}{\partial T} \begin{pmatrix} N_1 - \mathcal{R}_1 & \cdots & -\mathcal{T}_{16} \\ \vdots & \ddots & \vdots \\ -\mathcal{T}_{61} & \cdots & N_6 - \mathcal{R}_6 \end{pmatrix} \begin{pmatrix} \Delta T/2 \\ T_2 \\ T_3 \\ -\Delta T/2 \\ T_5 \\ T_6 \end{pmatrix}$$



Landauer formula in six-terminal geometry

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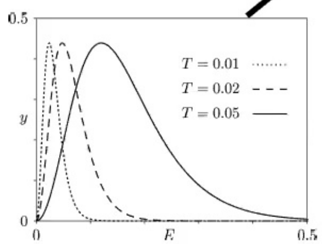
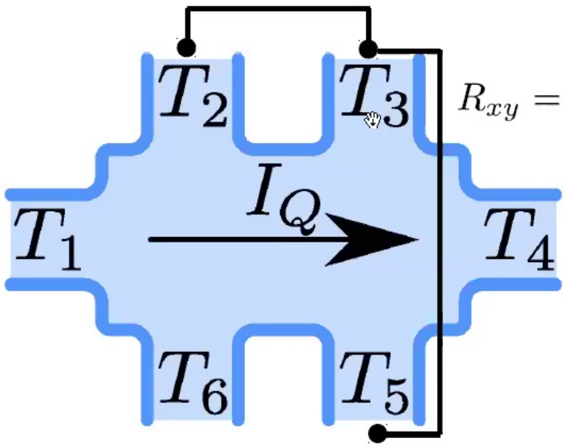


$$R_{xx} = \frac{T_2 - T_3}{I_Q}$$

$$R_{xy} = \frac{T_2 - T_6}{I_Q}$$



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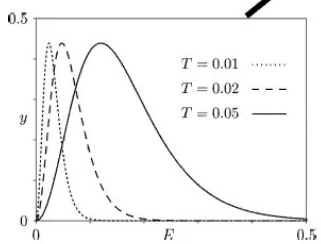
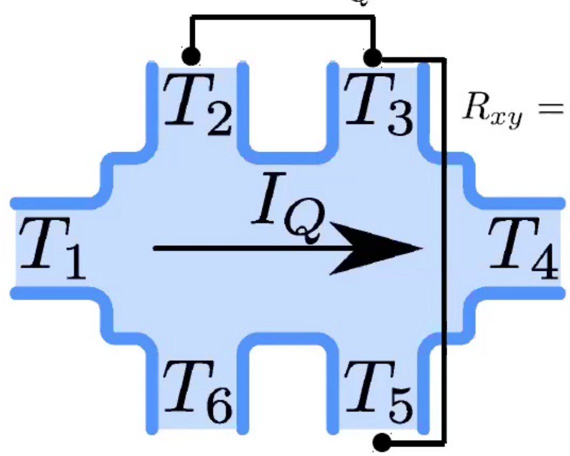


$$R_{xx} = \frac{T_2 - T_3}{I_Q}$$

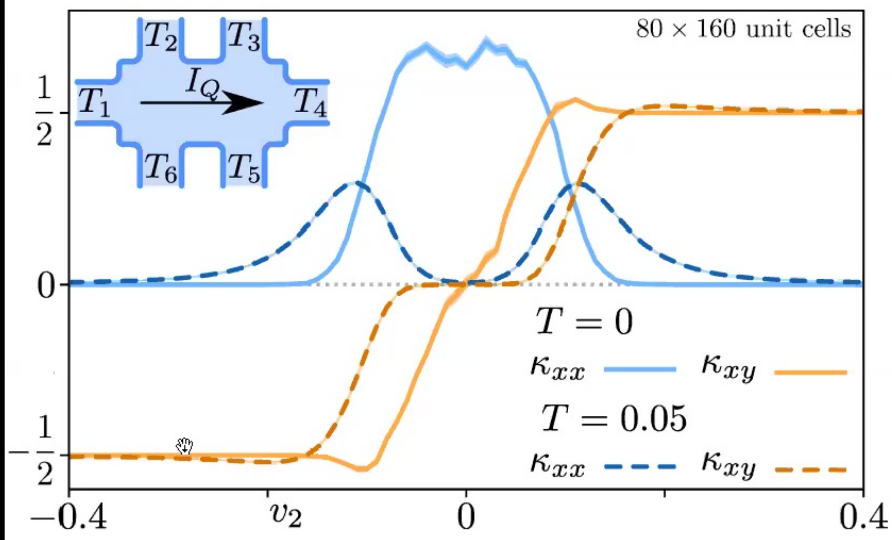


$$R_{xy} = \frac{T_2 - T_6}{I_Q}$$

$$\kappa = \left\langle \frac{3h}{\pi k_B^2 T} \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{xx} \end{pmatrix}^{-1} \right\rangle$$

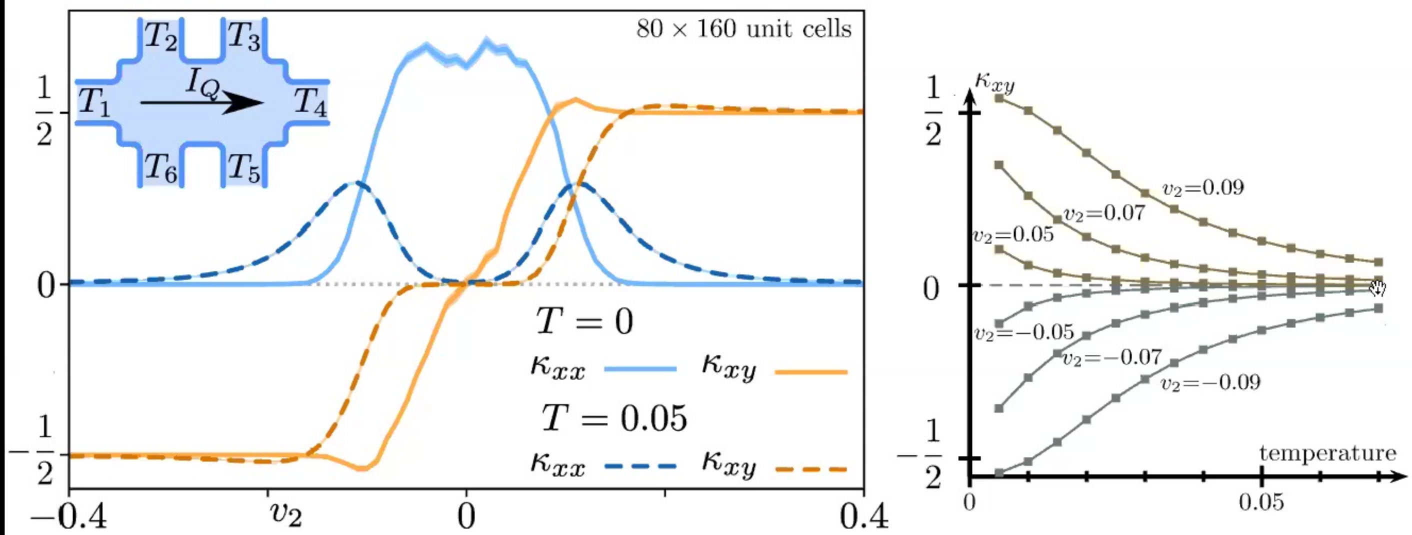
Numerics based on Kwant
(http://kwant-project.org)

Thermal Hall conductance



- Raising temperature **decreases** thermal conductance
- Quantization of thermal Hall conductance **improves**

Thermal Hall conductance



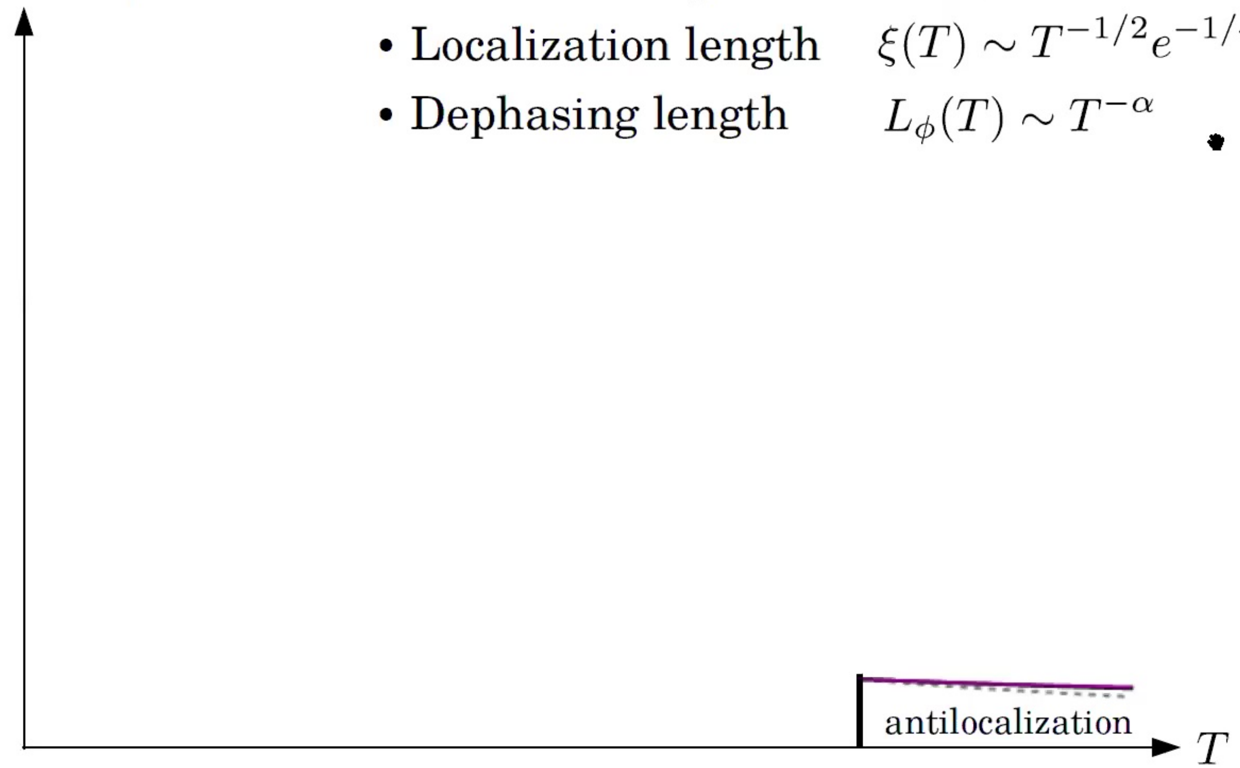
- Raising temperature **decreases** thermal conductance
- Quantization of thermal Hall conductance **improves**

Interaction effects

Dephasing due to phonons $\tau_\phi^{-1} \sim T^{2\alpha}$ (in metals, $\alpha \approx 1-2$)

Three length scales:

- Diffusion length $L_c(T) \sim T^{-1/2}$
- Localization length $\xi(T) \sim T^{-1/2} e^{-1/4 \log^2 T}$
- Dephasing length $L_\phi(T) \sim T^{-\alpha}$



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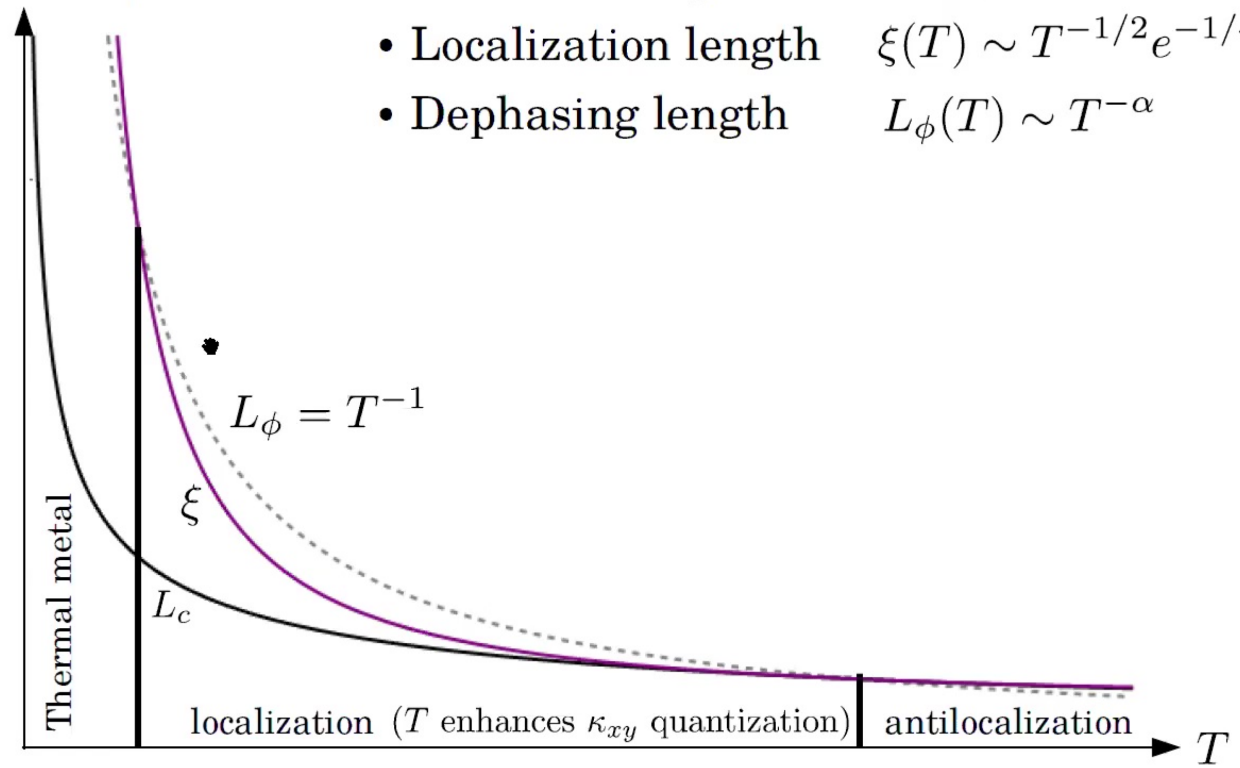
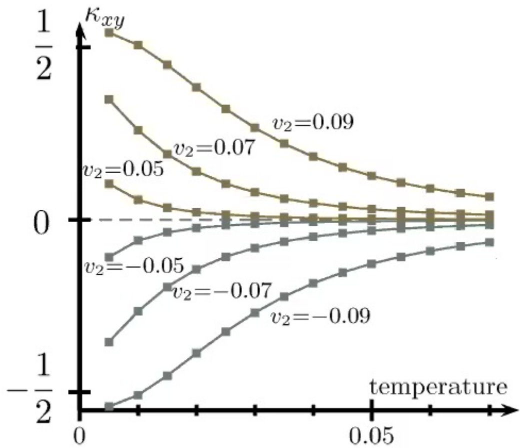
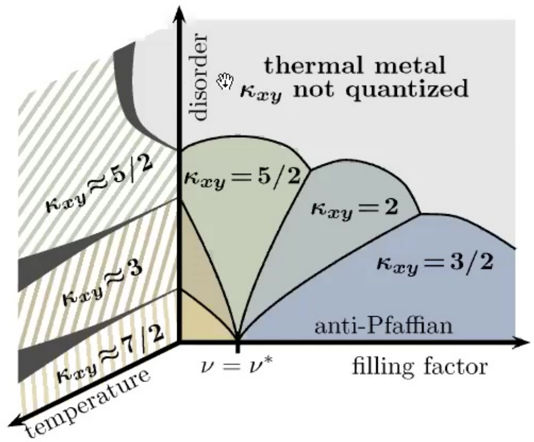


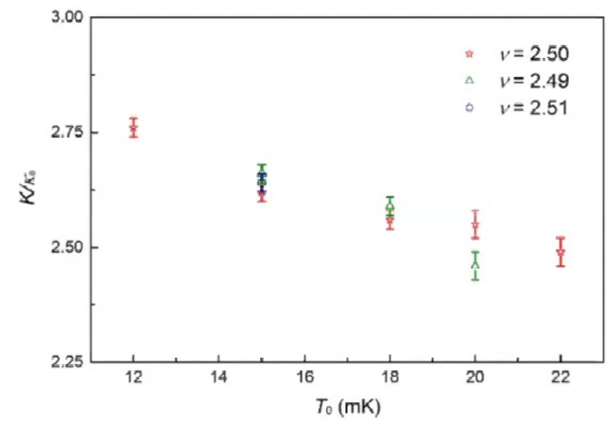
Implications



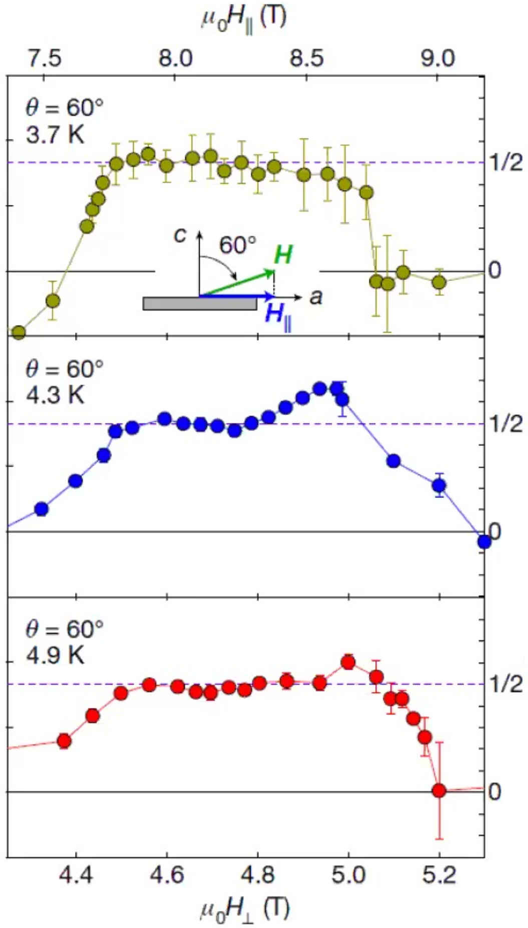


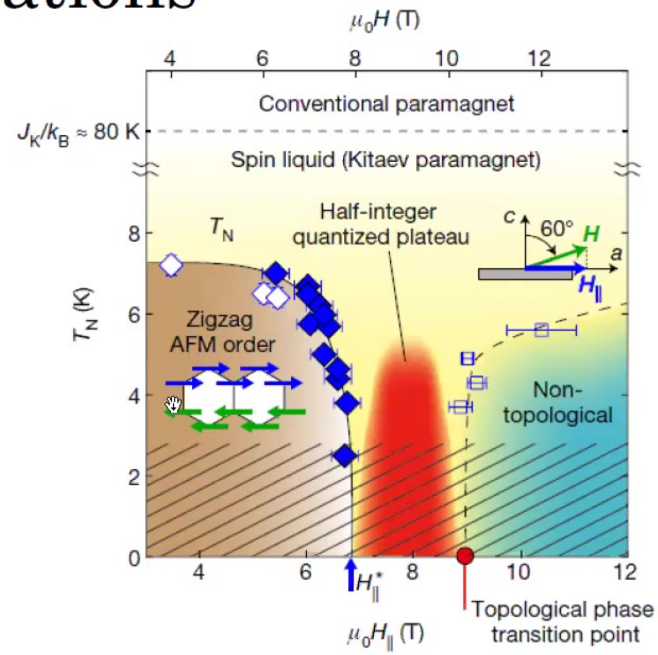
(finite size in numerics $\simeq$ dephasing in real systems)

- Consistent with observations at $\nu = 5/2$
- Edge-equilibration predicts similar trend



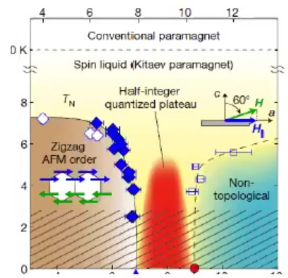
Implications

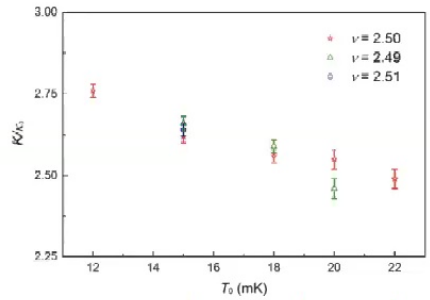




- Good understanding of non-Abelian phase in clean limit
- Data only at relatively high temperatures and large κ_{xx}

Conclusions/Outlook





- Thermal metals can masquerade as topological phases → need systematic temperature dependence

- Key role played by dephasing of emergent quasiparticles → requires theoretical study

- Can thermal metals exhibit other properties of topological phases such as interference or even braiding at finite frequencies?