Title: Non-Fermi liquid theories

Speakers: Sung-Sik Lee

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Non-Fermi liquid theories

Sung-Sik Lee

McMaster University
Perimeter Institute

Plan

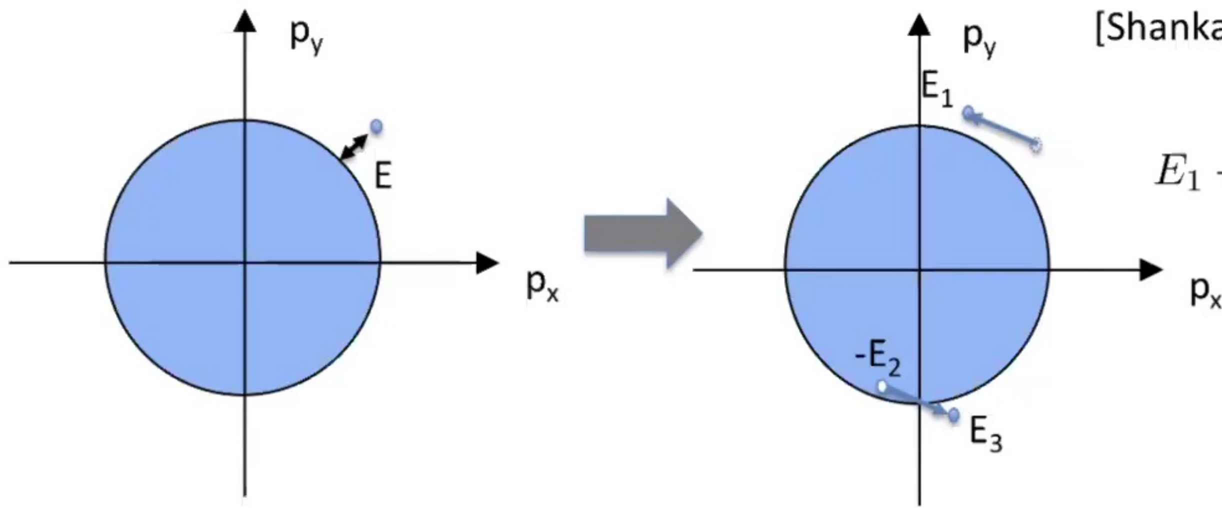
- Introduction
- NFL with hot Fermi surface and hot spots

Theory / Method	Hot Fermi surface	Hot spot
Perturbative	Part I	
Non-perturbative	Part II	

Fermi Liquids

[Landau]

[Shankar, Polchinski]

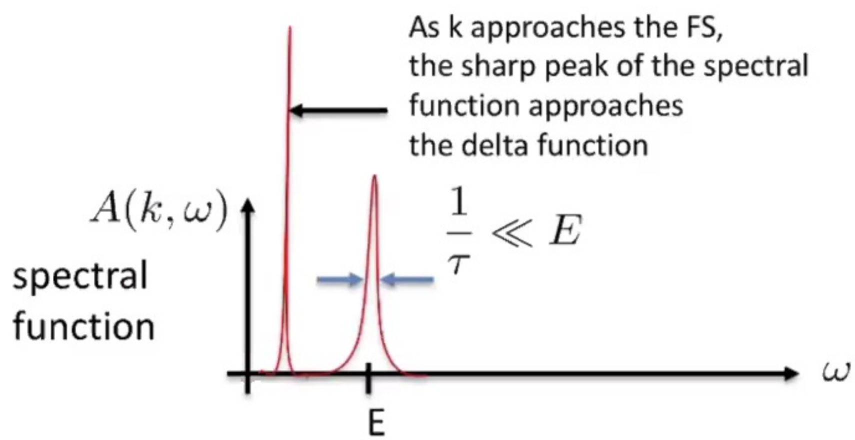


$$E_1 + E_2 + E_3 = E$$
$$E_1, E_2, E_3 \geq 0$$

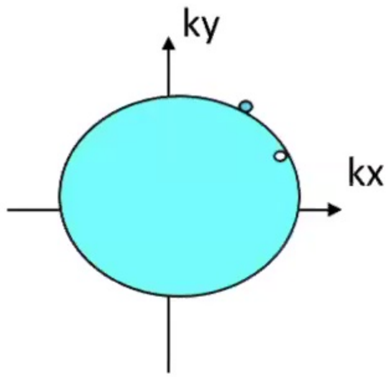
Particles close to the Fermi surface have long life-time

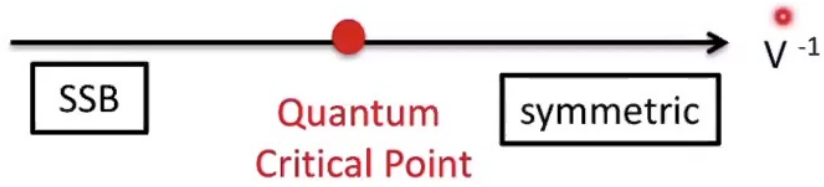
$$\frac{1}{\tau} = \alpha V^2 E^2$$

V : microscopic interaction
α : kinematic constants



NFL @ QCP





- At QCP, gapless order parameter fluctuations mediate long-range interactions between electrons
- Single-particle excitations no longer have long lifetime if the interaction is singular enough to generate strong non-forward scatterings

$$\frac{1}{\tau} = \alpha V(E)^2 E^2 > E$$

NFL's are described by interacting field theories that are not diagonalizable in single-particle basis

Two important factors :

- space dimension
- wavevector of gapless collective mode

Space dimension

- 3d : quantum fluctuations are relatively weak
- 1d : strong quantum effect but no extended Fermi surface (described by relativistic QFT)
- **2d** : challenging & interesting :
  
  – Extended Fermi surface
  – Strong quantum fluctuations at low energies

※ We will focus on NFLs in d=2.

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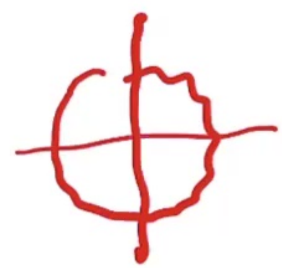
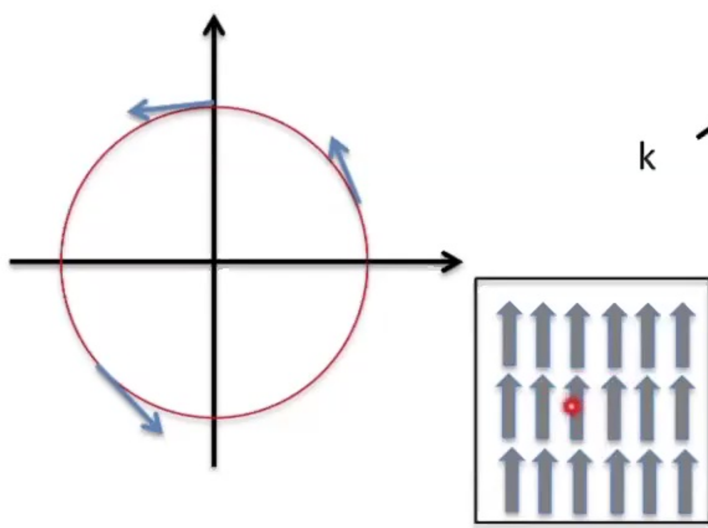
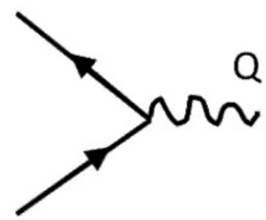


❊ We will focus on NFLs in d=2.

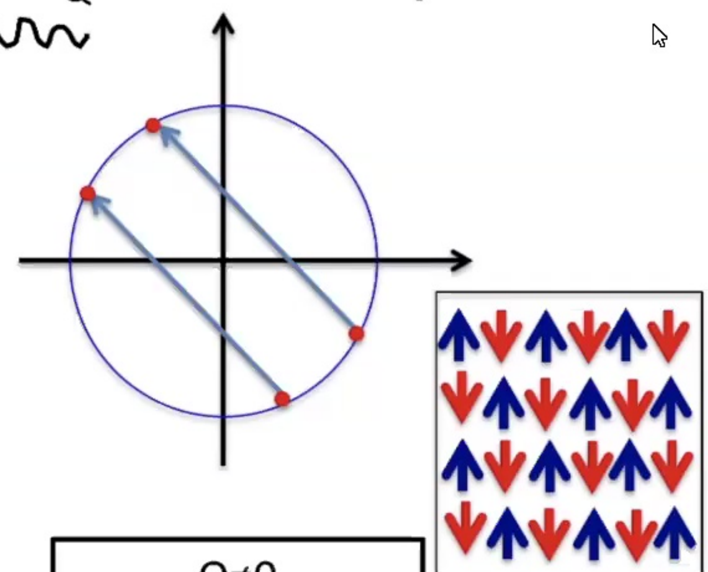
Wavevector of critical mode (Q)

Hot Fermi surface





Hot spot

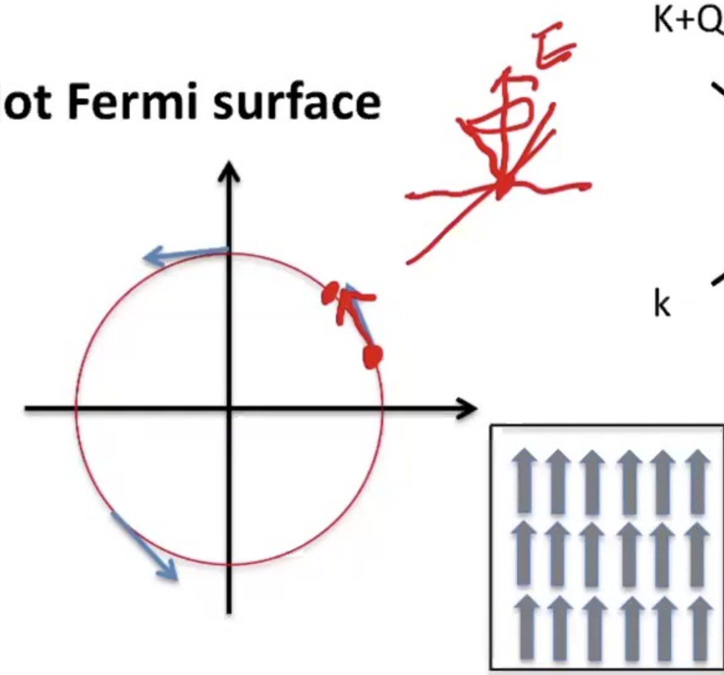


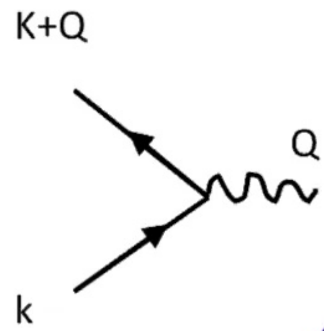
Q=0
Nematic, ferromagnetic QCP,
Spin liquids with emergent gauge boson,..

Q≠0
SDW, CDW,.. QCP

Wavevector of critical mode (Q)

Hot Fermi surface





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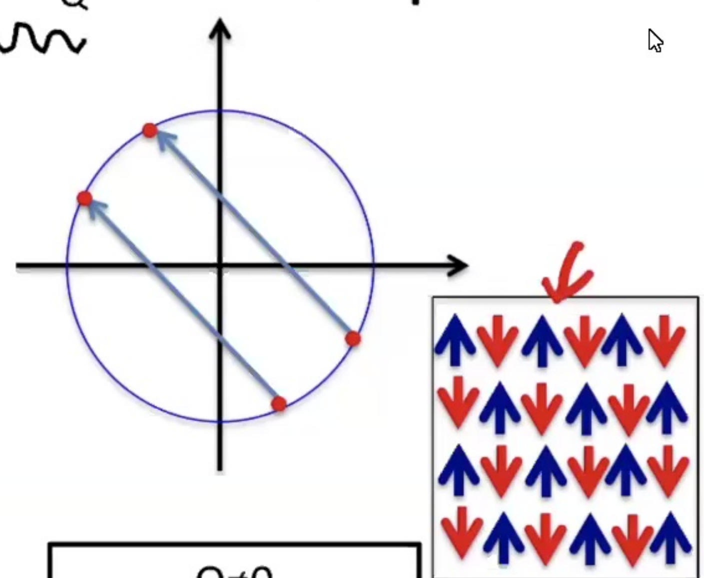

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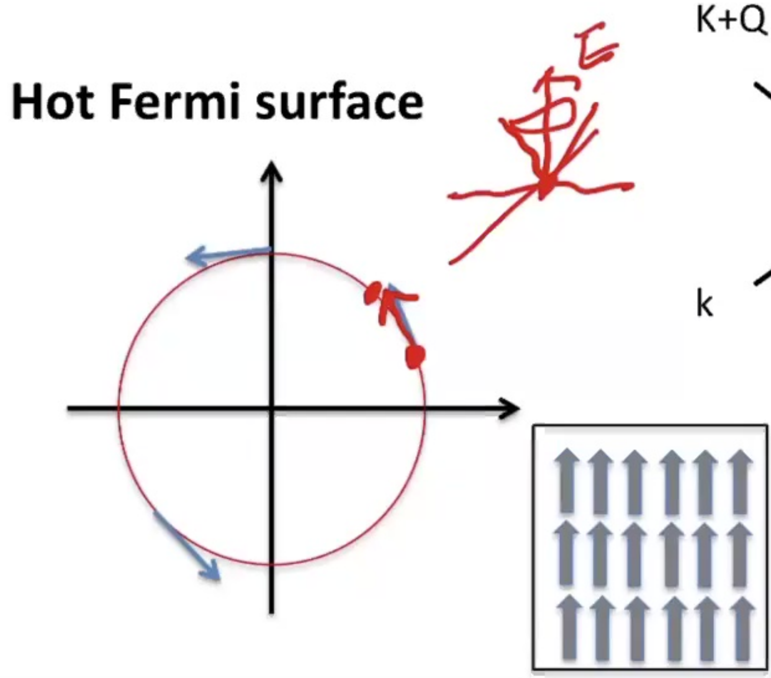

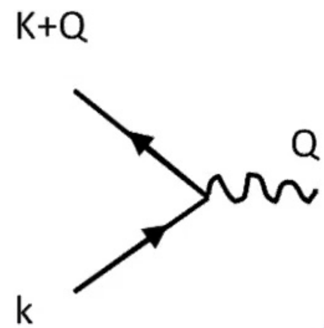

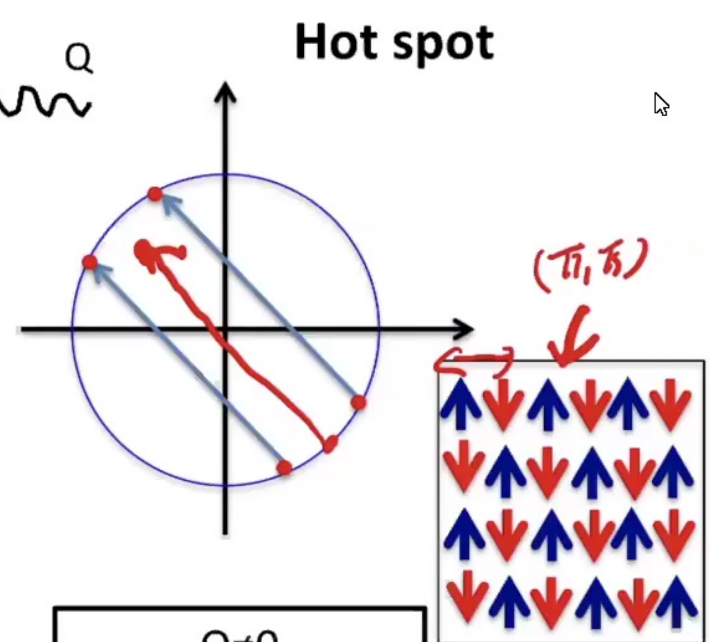

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Nematic, ferromagnetic QCP,
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SDW, CDW,.. QCP

Hot Fermi Surface

Ising-nematic quantum critical metal

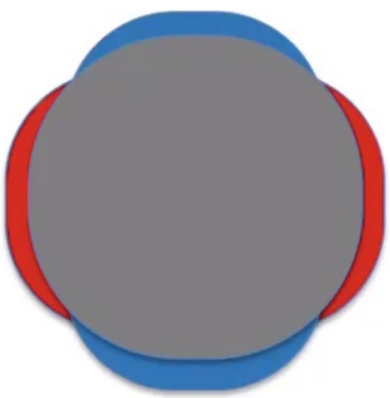
Ising-Nematic QCP

We start with a metal with the 4-fold rotational symmetry that favors a deformation of the Fermi surface in the l=2 channel

$$\hat{H} = \int dK_x dK_y \; \epsilon(\vec{K}) c_j^\dagger(\vec{K}) c_j(\vec{K}) - V \int dq_x dq_y \; \Delta_{\vec{q}} \Delta_{-\vec{q}}$$

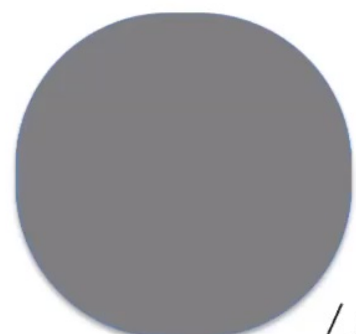
$$\Delta_{\vec{q}} = \int dK_x dK_y \; (\cos K_x - \cos K_y) c_j^\dagger(\vec{K} + \vec{q}) c_j(\vec{K})$$

Δ describes describes the l=2 deformation of FS



large V

$\langle \Delta_{q=0} \rangle \neq 0$



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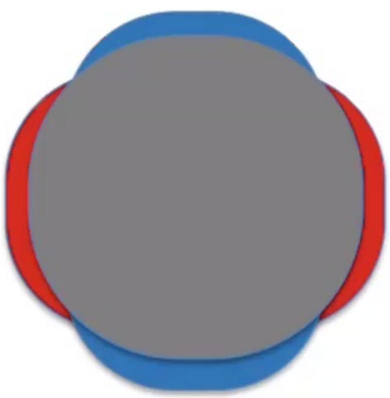

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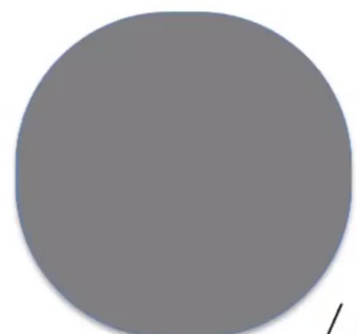

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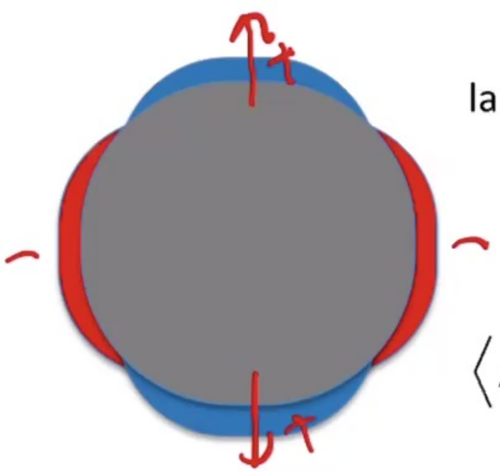

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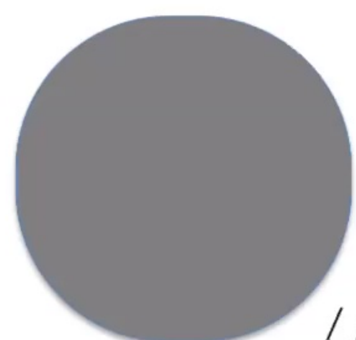

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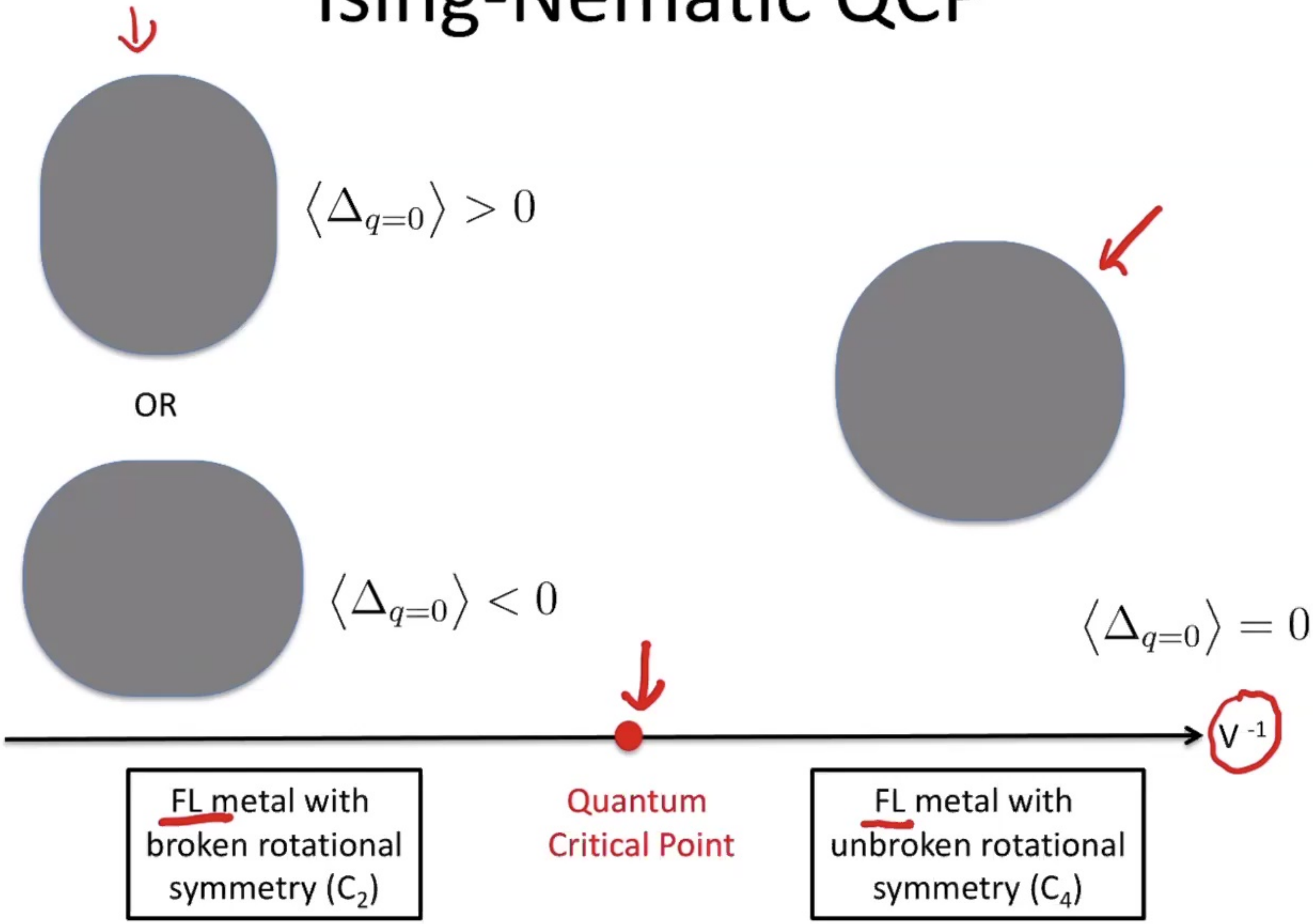
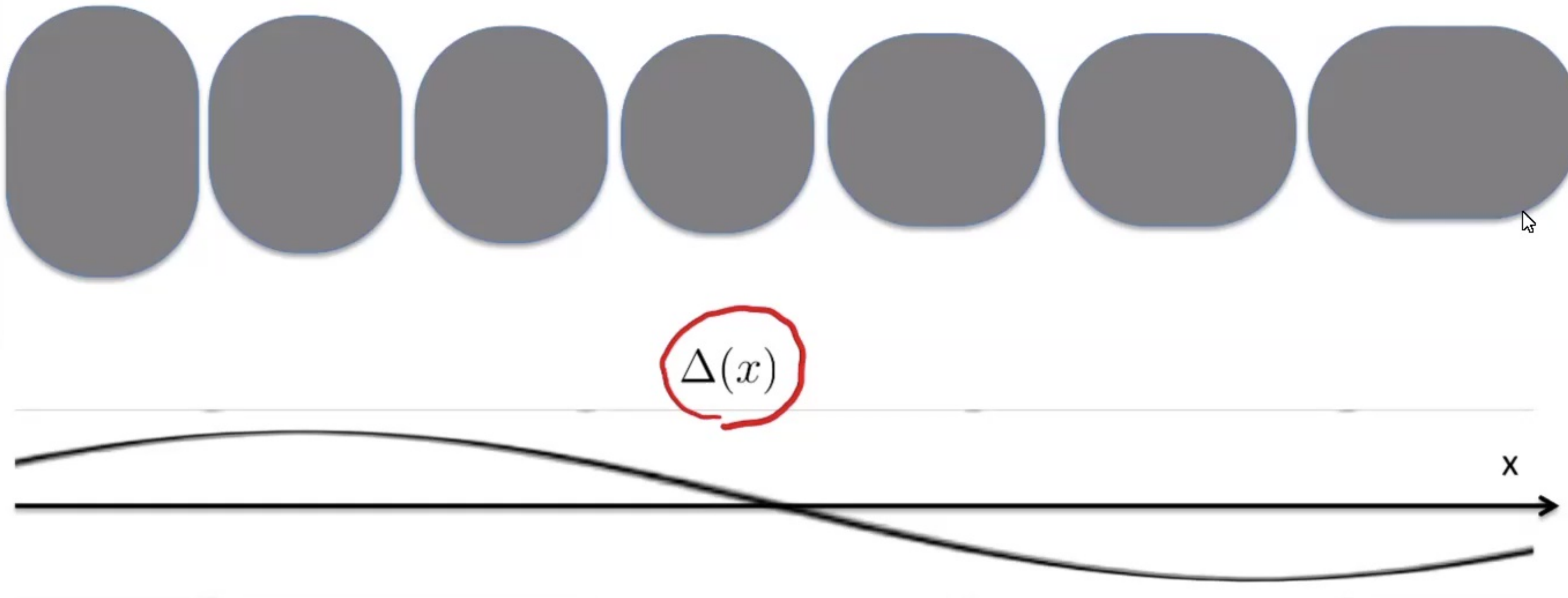


At quantum critical point



Long wave fluctuations of the nematic order parameter emerges as a gapless bosonic excitation

Low-energy theory at QCP

- The form of the low-energy field theory is fixed by
  - **Locality & symmetry**

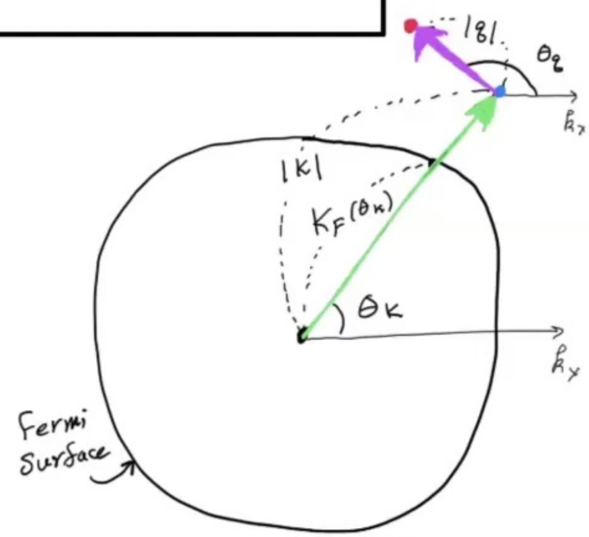
$$
\begin{aligned}
S = & \int dK\ c_j^\dagger(K)\Big[iK_0 + v_F(\theta_K)\left\{|K| - K_F(\theta_K)\right\}\Big]c_j(K) \\
& + \frac{1}{2}\int dq\ \left[q_0^2 + c^2 q^2\right]|\phi(q)|^2 + u\int dq_1 dq_2 dq_3\ \phi(q_1)\phi(q_2)\phi(q_3)\phi(-q_1-q_2-q_3) \\
& + \int dK dq\ \ e(\theta_K)\ c_j^\dagger(K+q)c_j(K)\phi(q) \\
& + \int dK_1 dK_2 dK_3\ \lambda_{j_1 j_2 j_3 j_4}(\theta_1,\theta_2,\theta_3,\theta_4)\ c_{j_1}^\dagger(K_1)c_{j_2}^\dagger(K_2)c_{j_3}(K_3)c_{j_4}(-K_1-K_2-K_3) \\
& + \dots
\end{aligned}
$$

$K = (K_0, |K|, \theta_K)$

$q = (q_0, |q|, \theta_q)$

Infinitely many low-energy data
(coupling functions)

$$K_F(\theta),\ v_F(\theta),\ e(\theta),\ \lambda_{j_1,..,j_4}(\theta_1,..,\theta_4)$$



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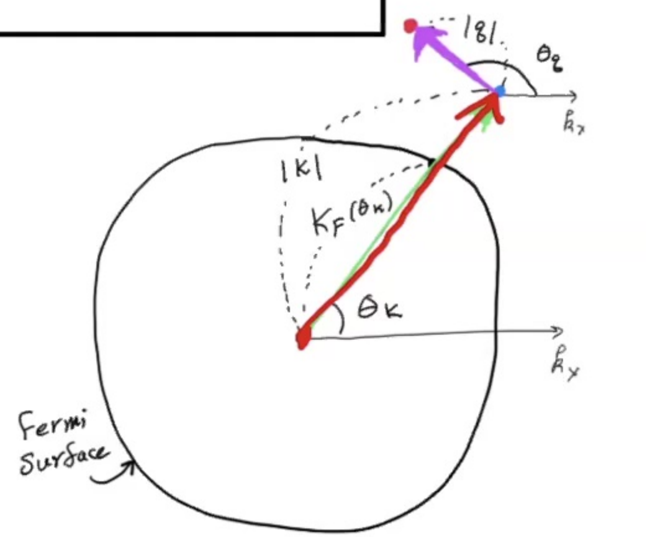

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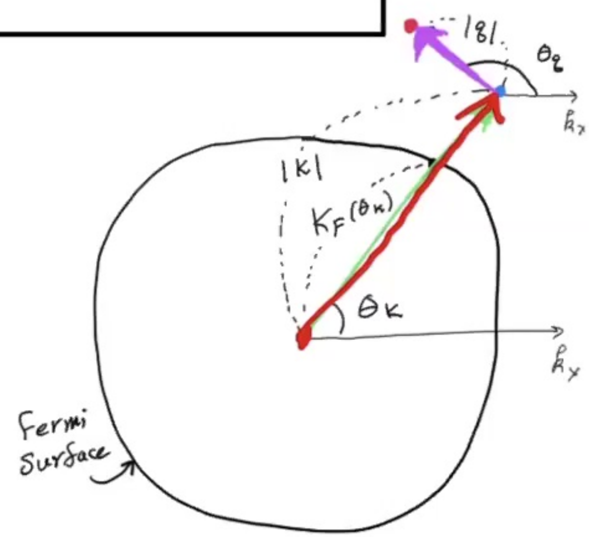
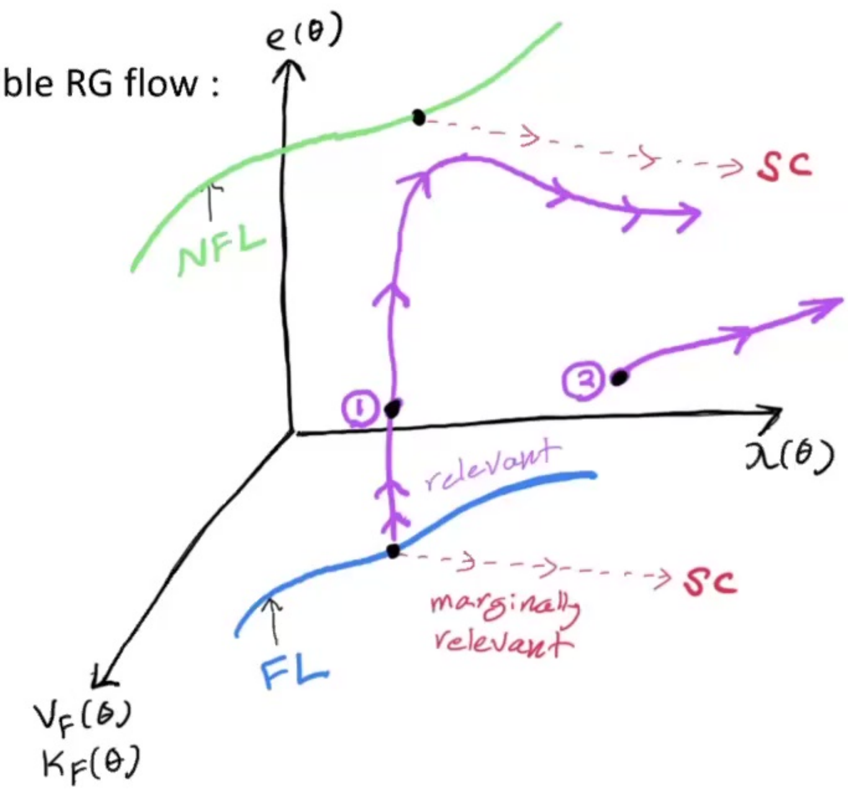
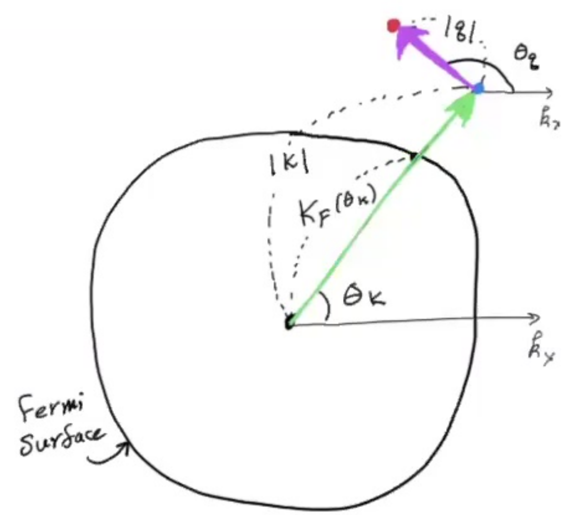


Goal

- **To characterize the manifold of low-energy fixed points in the space of the coupling functions**
  - Critical exponents are functionals of the coupling functions that parameterize the manifold, and functions of angle on the FS

A possible RG flow :

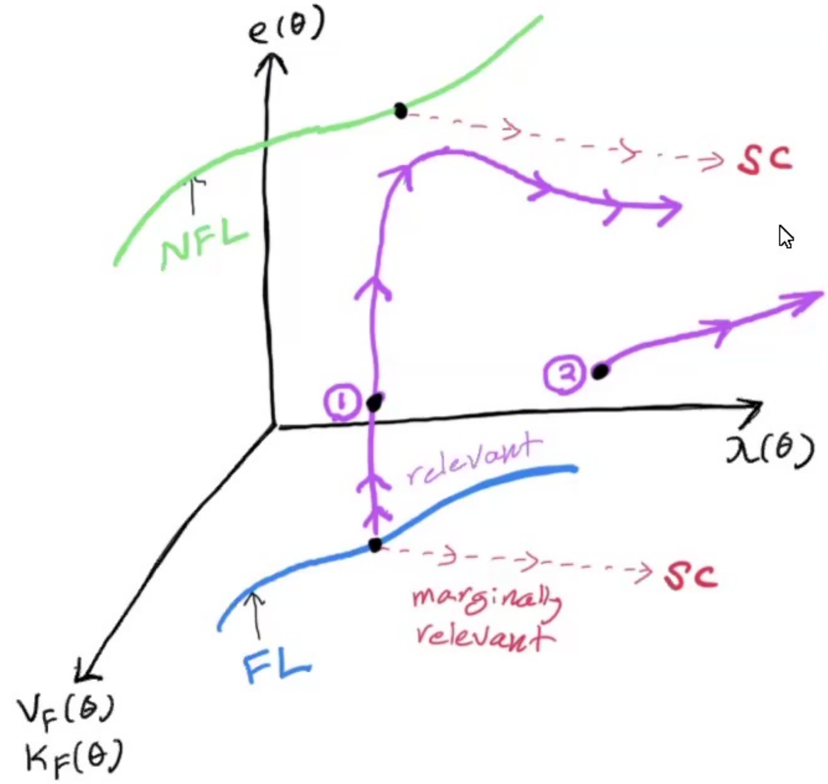




The first step

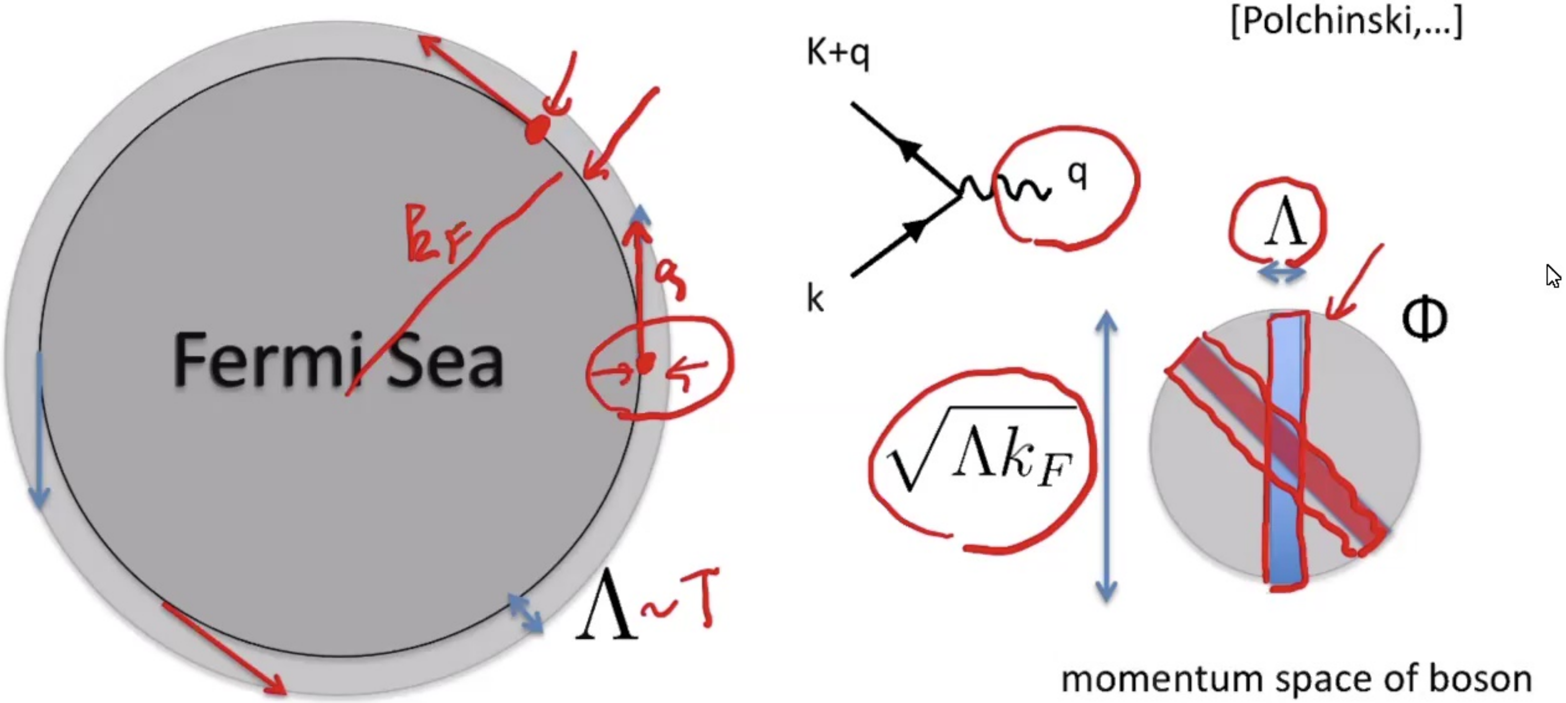
- Solving the full problem requires a **functional RG for strongly coupled field theories with infinitely many d.o.f.,** which is hard

- It is easier to understand NFL without SC instabilities
  - There are some NFLs without SC instabilities (focus of this lecture)
  - Even if SC instabilities are present, physics in an intermediate energy scales can be controlled by unstable NFL fixed points



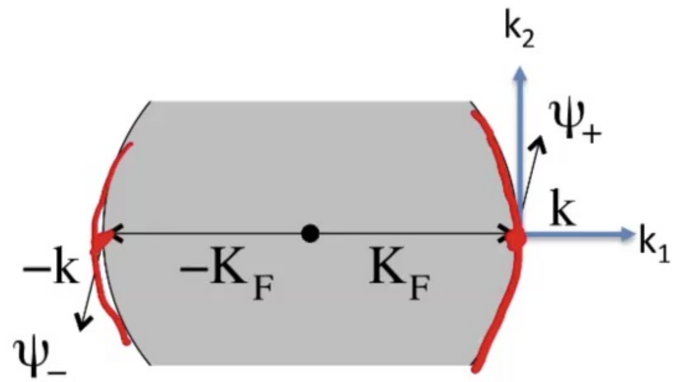
Without pairing interaction, the theory has an emergent locality in momentum space

[Polchinski,...]



- At low energies, fermions are primarily scattered along the directions tangential to FS
- Fermions with non-parallel tangential vectors are decoupled from each other in the low-energy limit

Patch theory



$$
\begin{aligned}
S \;=\; & \sum_{s=\pm}\sum_{j=\uparrow,\downarrow}\int d^3k\,\psi^\dagger_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k) \\
& + \frac{1}{2}\int d^3q\,\left[q_0^2 + c^2|\vec{q}|^2\right]\phi(-q)\phi(q) \\
& + e\sum_{s=\pm}\sum_{j=\uparrow,\downarrow}\int d^3k d^3q\;\phi(q)\;\psi^\dagger_{s,j}(k+q)\psi_{s,j}(k) \\
& + u\int dq_1dq_2dq_3\;\phi(q_1)\phi(q_2)\phi(q_3)\phi(-q_1-q_2-q_3) \\
& + \sum_{s_a,j_a}\lambda_{s_1,s_2,..,j_1,j_2,..}\int dk_1dk_2dk_3\;\psi^\dagger_{s_1,j_1}(k_1)\psi^\dagger_{s_2,j_2}(k_2)\psi_{s_3,j_3}(k_3)\psi_{s_4,j_4}(-k_1-k_2-k_3)
\end{aligned}
$$

$$
\begin{aligned}
\psi_{+,j}(k) &= c(K_0, \vec{K}_F + \vec{k}) \\
\psi_{-,j}(k) &= c(K_0, -\vec{K}_F + \vec{k})
\end{aligned}
$$

- Coupling functions are replaced with coupling constants within a patch

Patch theory

$$S = \sum_{s=\pm}\sum_{j=\uparrow,\downarrow}\int d^3k\psi^\dagger_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k)$$
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$$+ e\sum_{s=\pm}\sum_{j=\uparrow,\downarrow}\int d^3kd^3q\ \phi(q)\ \psi^\dagger_{s,j}(k+q)\psi_{s,j}(k)$$
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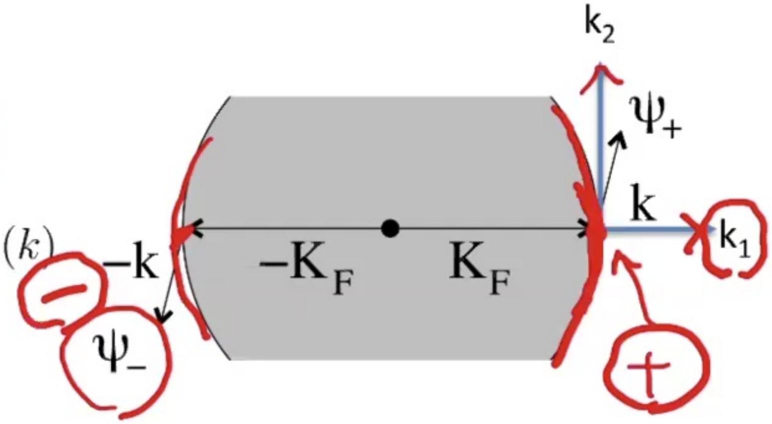

$$\psi_{+,j}(k) = c(K_0, \vec{K}_F + \vec{k})$$
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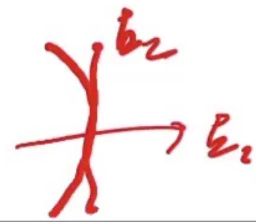
- Coupling functions are replaced with coupling constants within a patch

Gaussian scaling



b < 1

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$$k_1 \to bk_1,\quad k_2 \to b^{1/2}k_2,\quad k_0 \to bk_0$$
$$\Psi \to b^{-\frac{7}{4}}\psi,\quad \phi \to b^{-\frac{7}{4}}\phi$$
$$e \to b^{-\frac{1}{4}}e,\quad u \to b^{1/2}u,\quad \lambda \to b^{1/2}\lambda$$

- In the low-energy limit,
  - $[q_0^2 + c^2q_1^2]$ term in the boson kinetic term and u decreases (irrelevant)
  - the fermion-boson coupling grows (relevant)
  - four-fermion coupling is irrelevant by power-counting*

(*However, this itself does not exclude SC instabilities)

Gaussian scaling

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$$+ u\int dq_1 dq_2 dq_3\ \phi(q_1)\phi(q_2)\phi(q_3)\phi(-q_1-q_2-q_3)$$

$$+ \sum_{s_a,j_a}\lambda_{s_1,s_2,..,j_1,j_2,..}\int dk_1 dk_2 dk_3\ \psi^\dagger_{s_1,j_1}(k_1)\psi^\dagger_{s_2,j_2}(k_2)\psi_{s_3,j_3}(k_3)\psi_{s_4,j_4}(-k_1-k_2-k_3)$$

$b < 1$

$k_1 \to bk_1,$	$k_2 \to b^{1/2}k_2,$	$k_0 \to bk_0$
	$\Psi \to b^{-\frac{7}{4}}\psi,$	$\phi \to b^{-\frac{7}{4}}\phi$
$e \to b^{-\frac{1}{4}}e,$	$u \to b^{1/2}u,$	$\lambda \to b^{1/2}\lambda$

- In the low-energy limit,
  - $[q_0^2 + c^2 q_1^2]$ term in the boson kinetic term and u decreases (irrelevant)
  - the fermion-boson coupling grows (relevant)
  - four-fermion coupling is irrelevant by power-counting*

(*However, this itself does not exclude SC instabilities)

Gaussian scaling

$$S = \sum_{s=\pm}\sum_{j=\uparrow,\downarrow}\int d^3k\, \psi^\dagger_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k)$$
$$+ \frac{1}{2}\int d^3q\,[q_0^2 + c^2(q_x^2 + q_y^2)]\,\phi(-q)\phi(q)$$
$$+ e\sum_{s=\pm}\sum_{j=\uparrow,\downarrow}\int d^3k d^3q\;\phi(q)\;\psi^\dagger_{s,j}(k+q)\psi_{s,j}(k)$$
$$+ u\int dq_1 dq_2 dq_3\;\phi(q_1)\phi(q_2)\phi(q_3)\phi(-q_1-q_2-q_3)$$
$$+ \sum_{s_a,j_a}\lambda_{s_1,s_2,..,j_1,j_2,..}\int dk_1 dk_2 dk_3\;\psi^\dagger_{s_1,j_1}(k_1)\psi^\dagger_{s_2,j_2}(k_2)\psi_{s_3,j_3}(k_3)\psi_{s_4,j_4}(-k_1-k_2-k_3)$$

b < 1

$k_1 \to bk_1,$	$k_2 \to b^{1/2}k_2,$	$k_0 \to bk_0$
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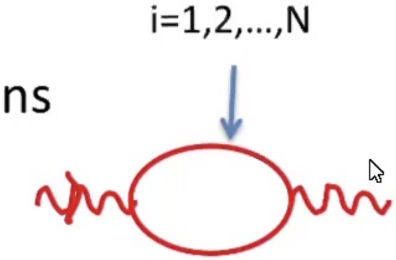
Perturbative approaches

I. 1/N - expansion

[Altshuler, Ioffe, Millis(94); Kim, Furusaki, Wen, Lee(94); Polchinski(94),...]

- In the presence of a large fermion flavors, the collective mode is heavily damped, and fluctuations of the collective mode is suppressed



- In relativistic QFT, quantum fluctuations become weak, and the 1/N expansion is organized by a perturbative series

- This is not the case in the presence of FS : the theory with large N vector flavor behaves like a large N matrix model which is not perturbatively solvable due to the proliferation of planar graphs

[SL (09)]

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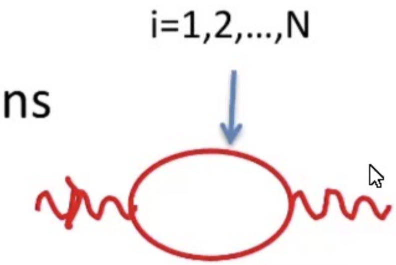

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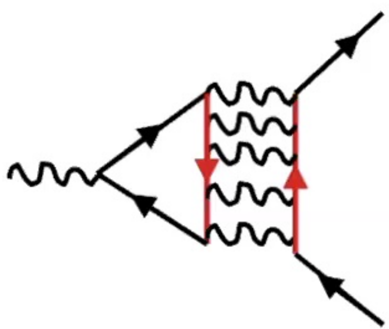
$\psi_{i=1,\dots,N}$

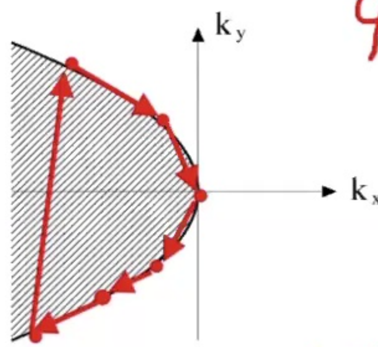
$\psi_{\alpha\beta}$

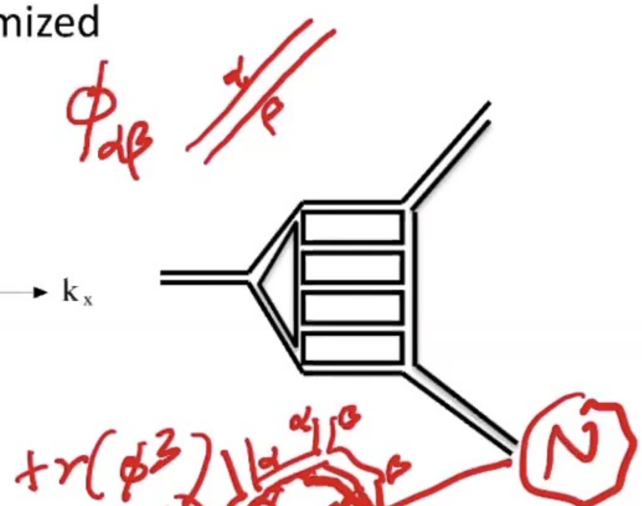
[SL (09)]

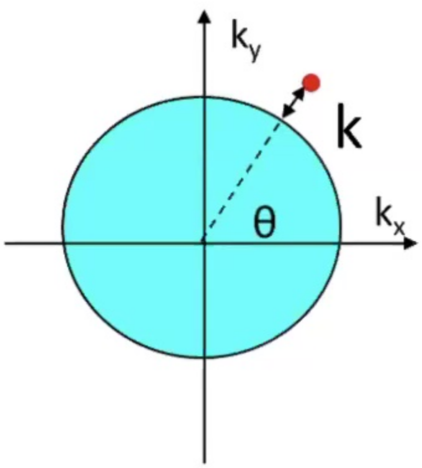
Origin of the emergent matrix structure : angle on Fermi surface becomes another flavor

- Planar diagrams represent virtual processes in which the phase space for low-energy particle-hole excitations is maximized









Vector fermion with FS in 2+1D
~ Matrix fermion in 1+1D

$$f_j(k_x, k_y) = f_{j\theta}(k)$$

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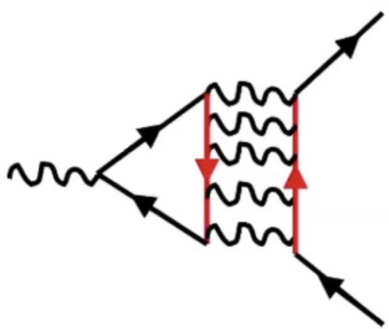
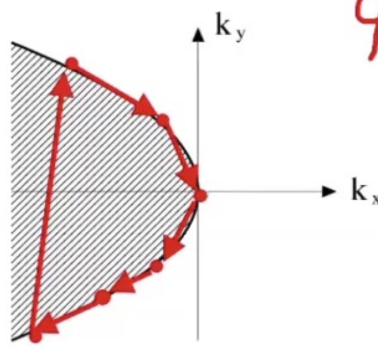

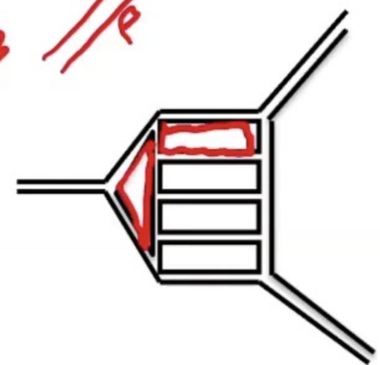
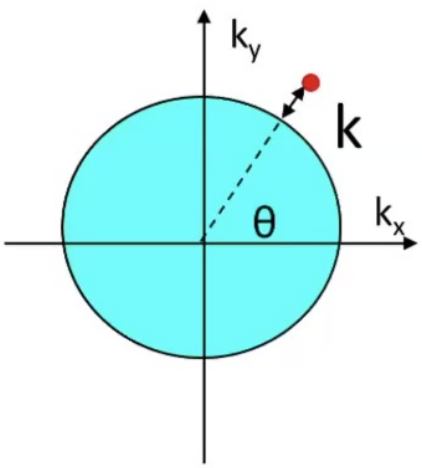

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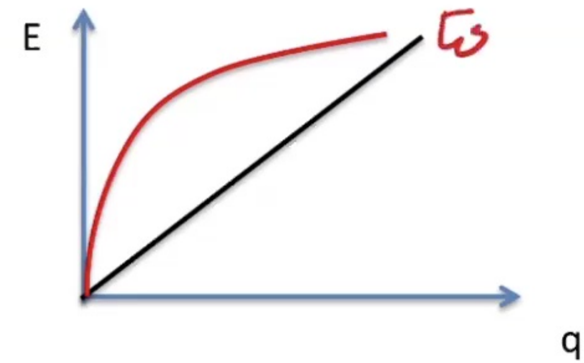
NFL in the (vector) large N limit

- For one-patch theory (chiral NFL), exact scaling exponents can be computed non-perturbatively [to be discussed in part II]
- In the two-patch theory (non-chiral NFL), even more diagrams become important at low energies [Metlitski, Sachdev (10)]
  - remains an open problem

II. Dynamical tuning

[Nayak, Wilczek(94); Mross, McGreevy, Liu, Senthil(10)]

$$|q|^2\phi^2 \rightarrow |q|^{1+\epsilon}\phi^2$$



- Tame quantum fluctuations by suppressing DOS of critical boson
- All symmetries kept
- Breaks locality of the theory : the non-local kinetic term is not renormalized perturbatively

$$\frac{|q|^{1+\epsilon-\eta}}{\Lambda^{-\eta}} = q^{1+\epsilon}\left(1-\eta\ln q/\Lambda + ..\right)$$

Quantum correction to non-local terms such as $q^{1+\epsilon}\ln\Lambda$ does not arise from short wavelength fluctuations

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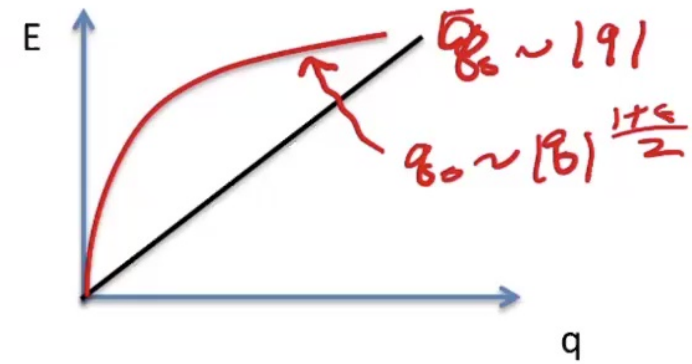

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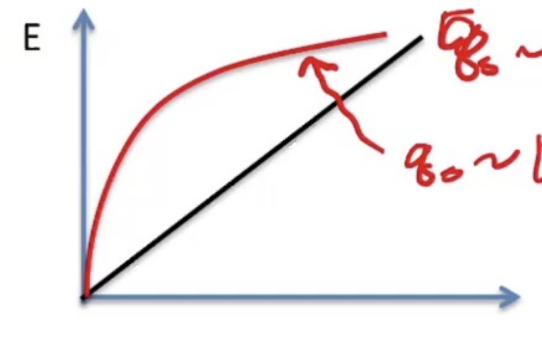

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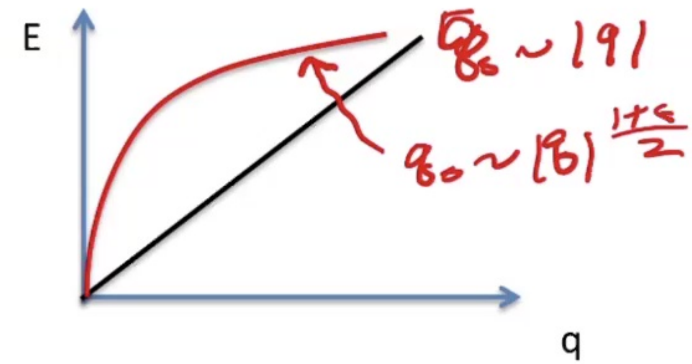

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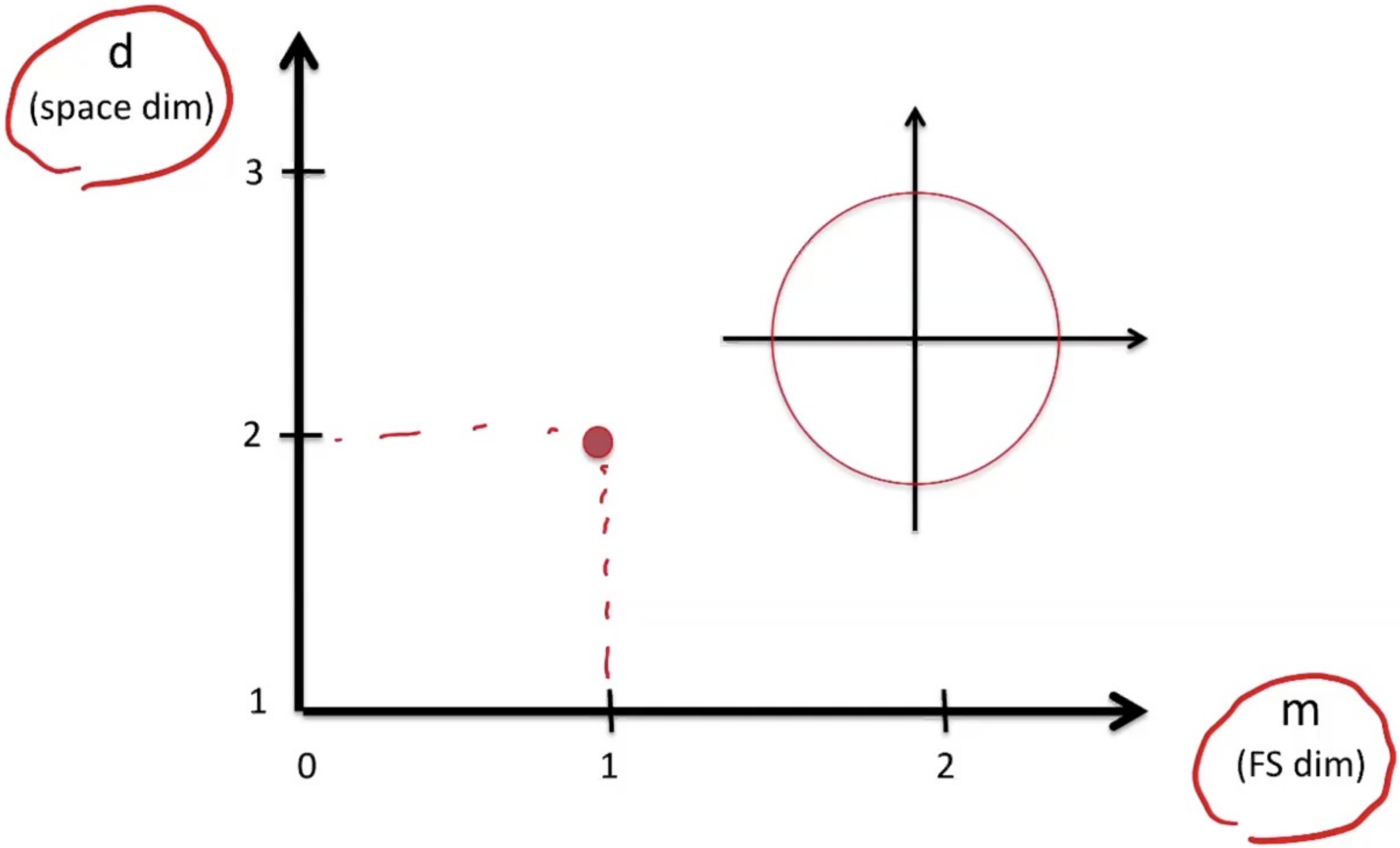
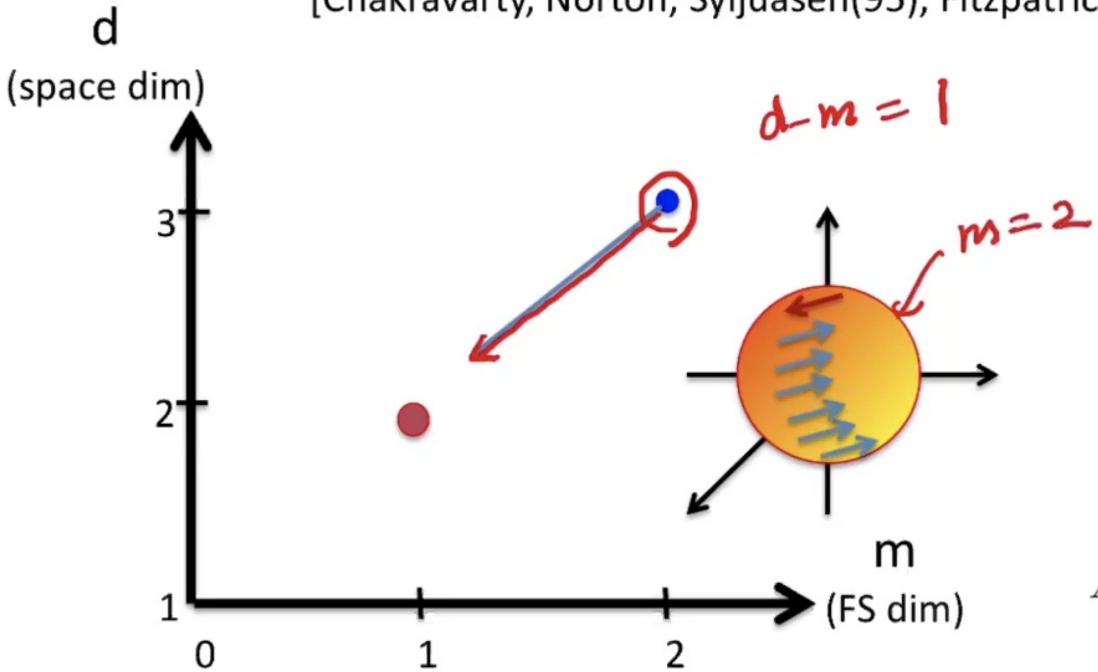
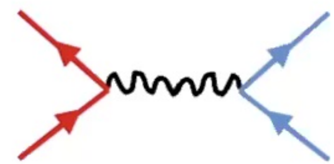
Dimensional Regularization scheme : no unique way to extend dimension



III. Tuning dim of space along with the dim of FS

[Chakravarty, Norton, Syljuasen(95), Fitzpatrick, Kachru, Kaplan, Raghu (13),..]





$$A(\omega) = \omega^{-\alpha} f\left(\left[\frac{\omega}{k_F}\right]^{m-1}\right)$$

- Most natural (symmetry, locality kept)
- Size of FS enters as a scale (UV/IR mixing) even without pairing[Mandal, SL (15)]
- Crossover function f(x) is singular in the small x limit, and m→1 limit and ω→0 limit do not commute
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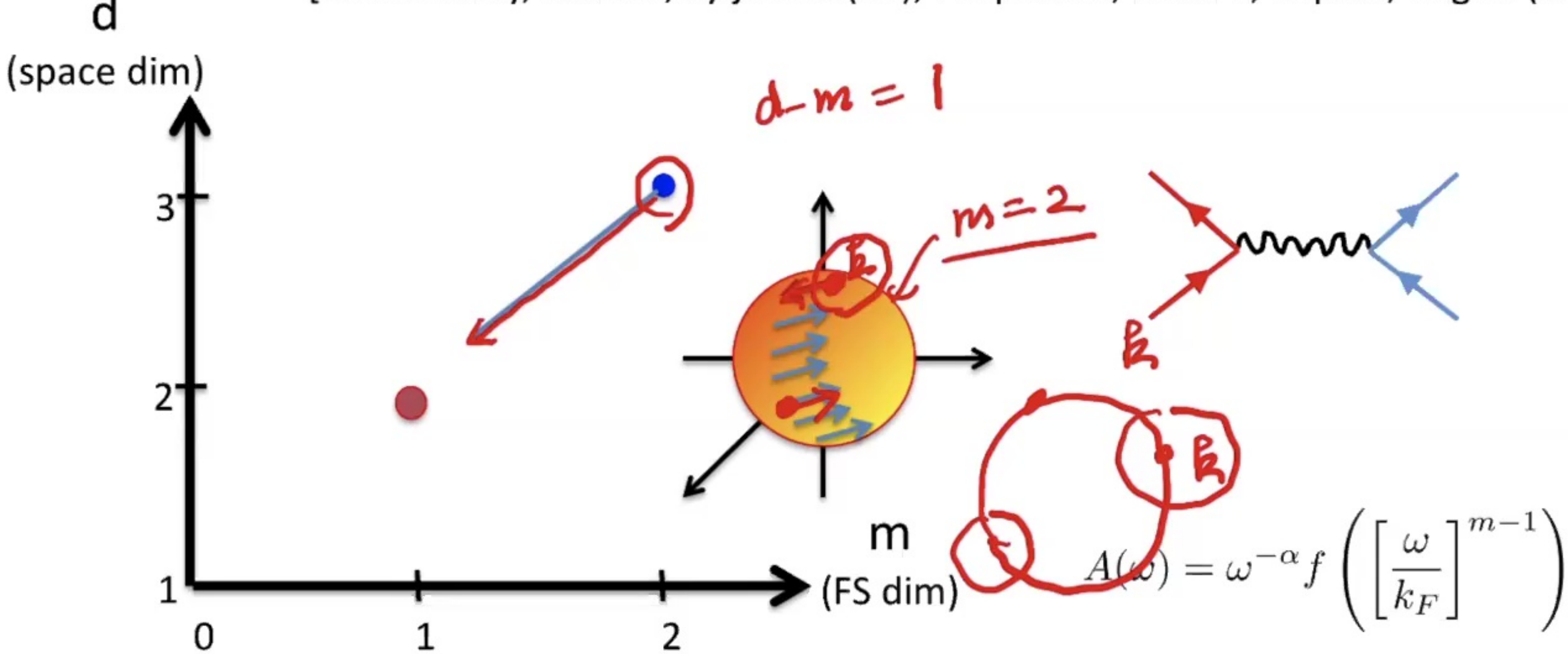

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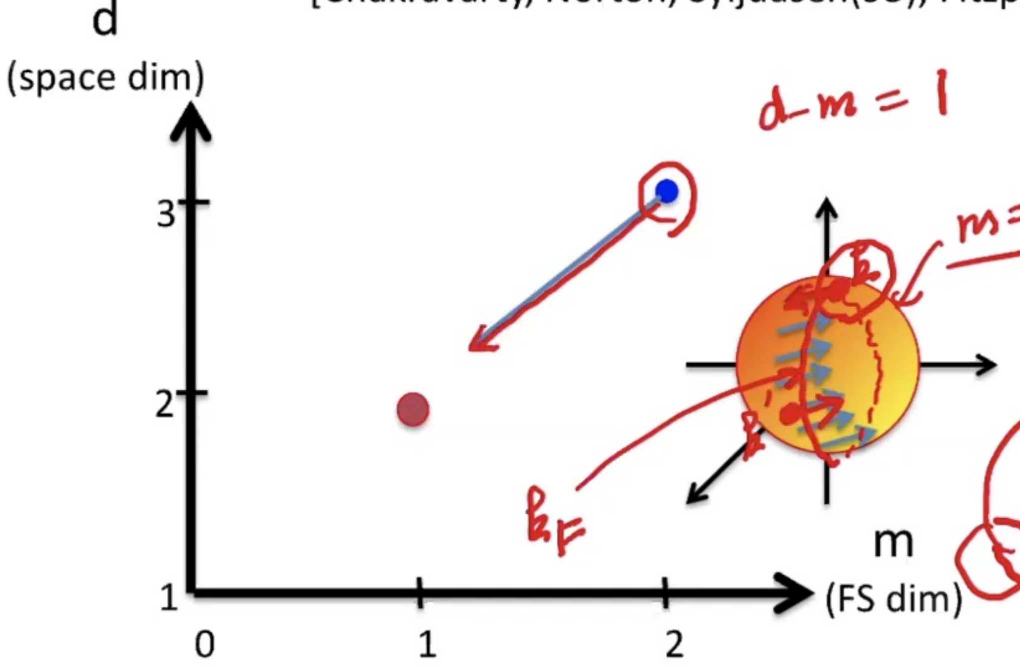

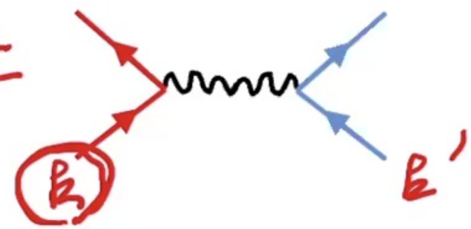

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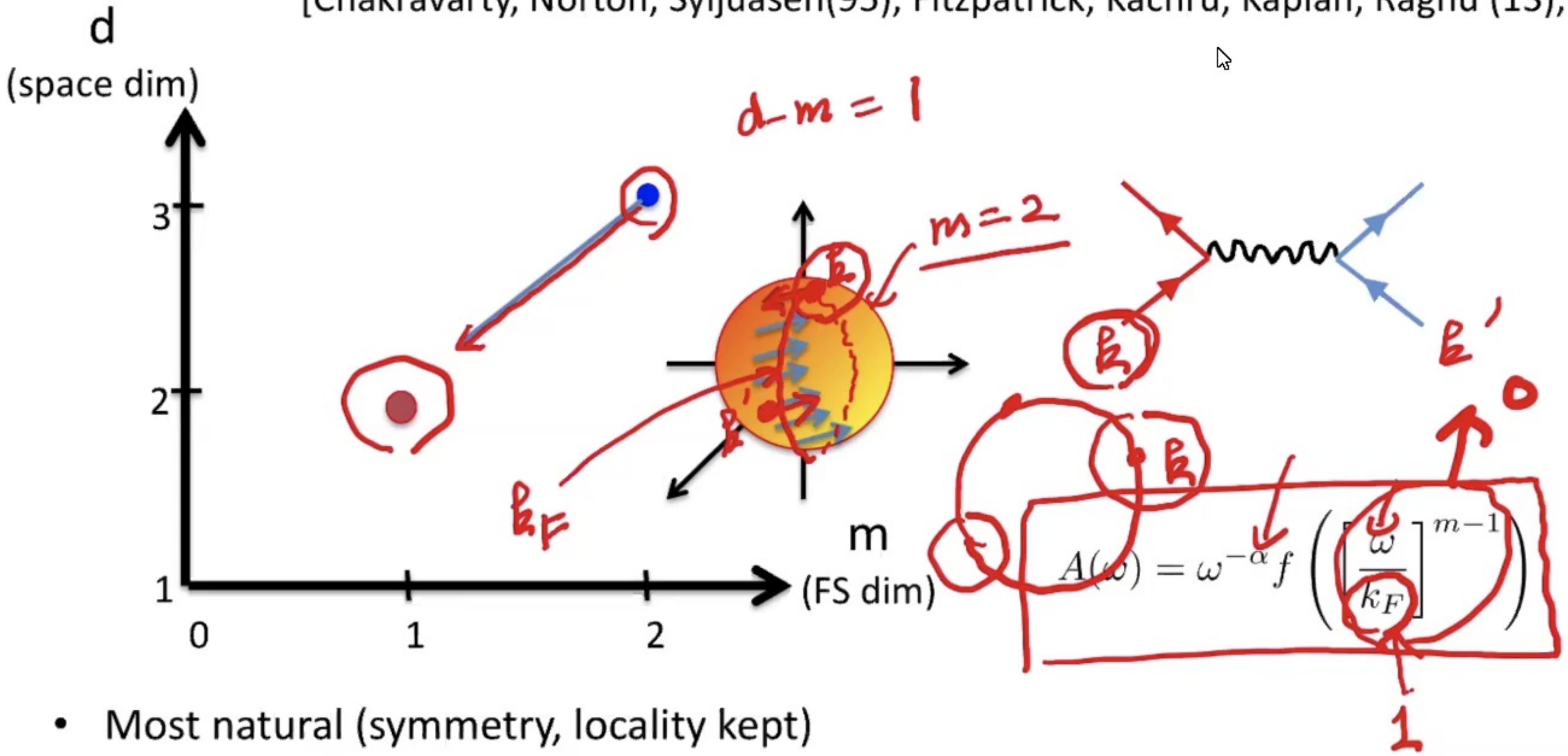

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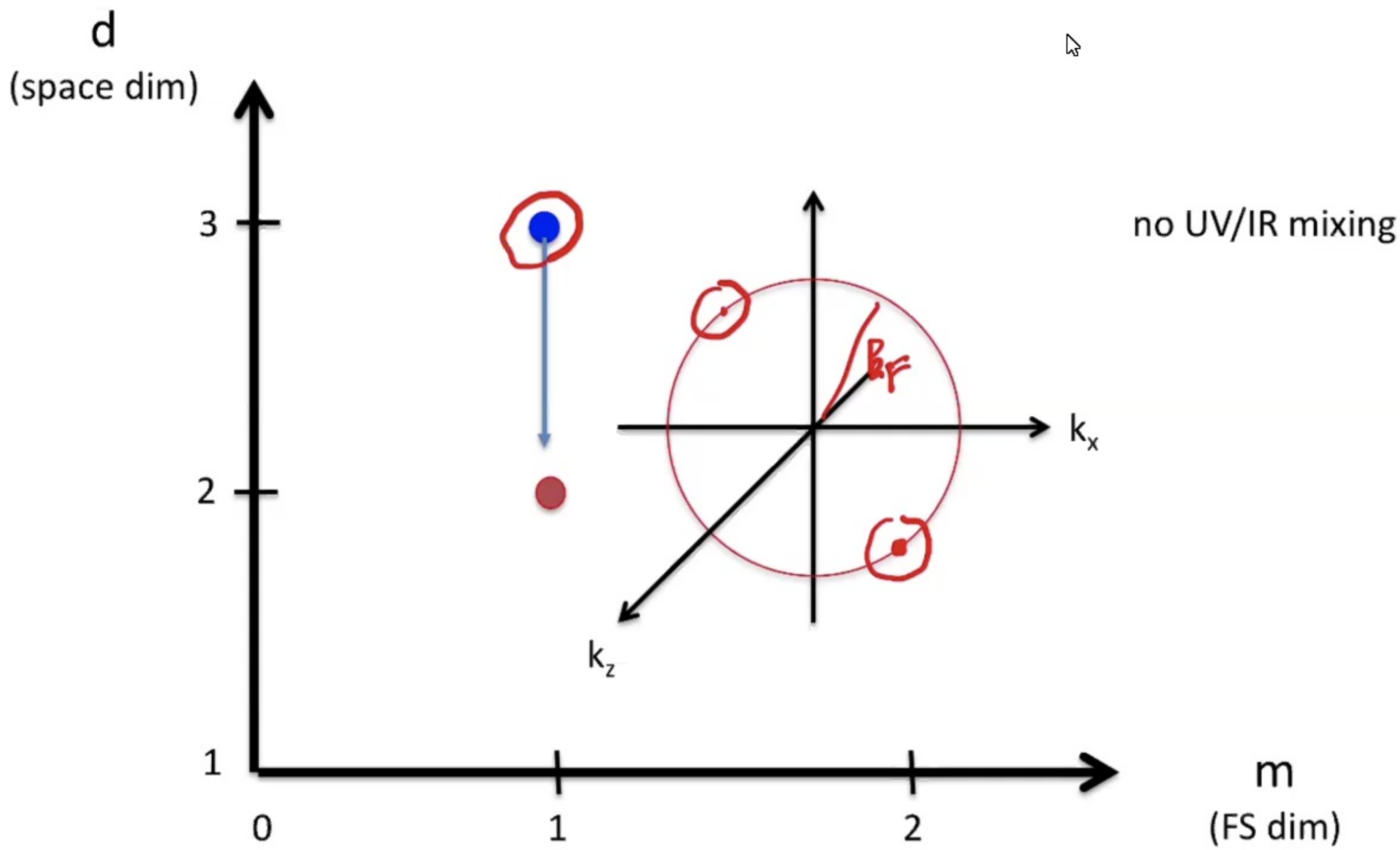
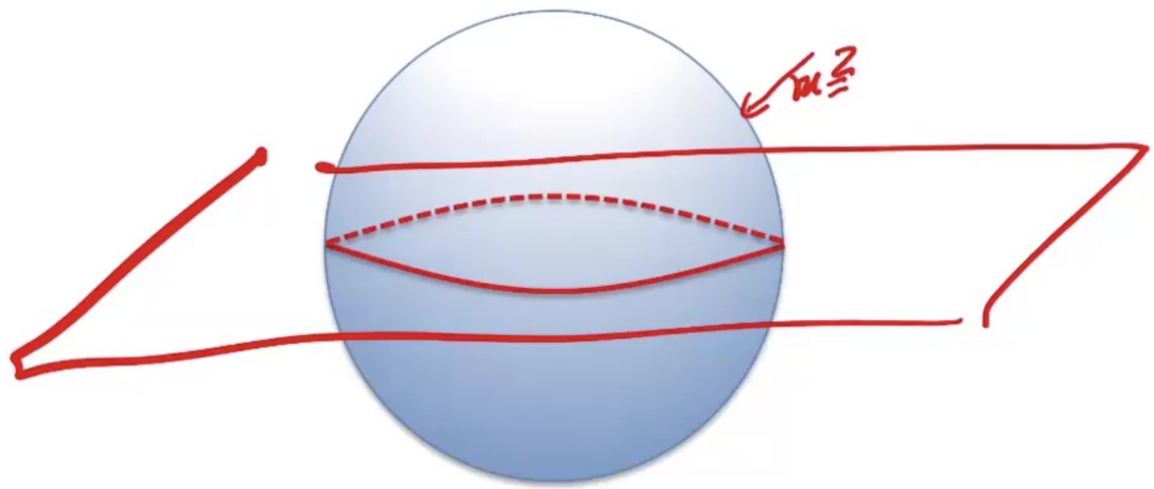
IV. Tuning co-dimension of FS



- A non-local scheme [Senthil, Shankar (09)]
- Local scheme [Dalidovich, SL (13)]

The theory at d = 3 describes a spin triplet p-wave SC

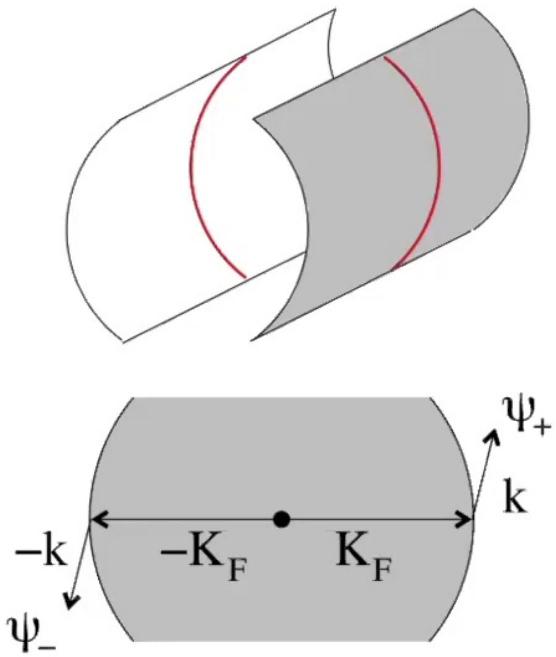


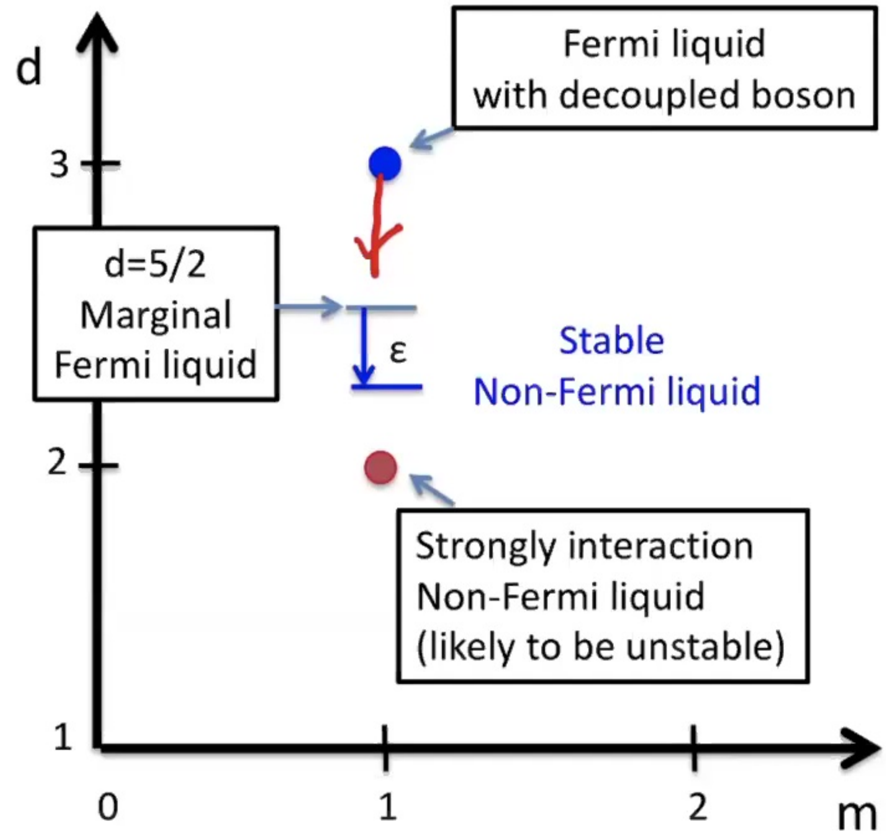
$$S \;=\; \int \frac{d^4k}{(2\pi)^3} \left\{ \sum_{s=\pm} \sum_{j=\uparrow,\downarrow} \psi^\dagger_{s,j}(k) \, (ik_0 + E_k) \, \psi_{s,j}(k) \right.$$

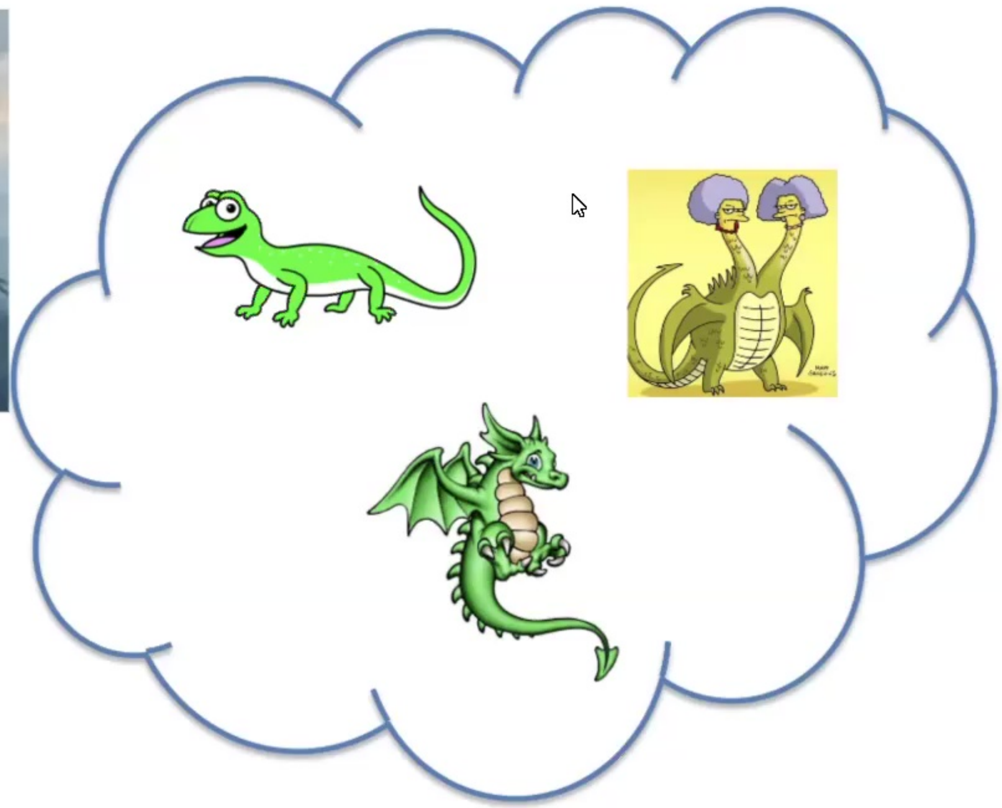
$$\left. -k_1 \left(\psi^\dagger_{+,\uparrow}(k) \psi^\dagger_{-,\uparrow}(-k) + \psi^\dagger_{+,\downarrow}(k) \psi^\dagger_{-,\downarrow}(-k) + h.c. \right) \right\}$$

- breaks the global U(1) and spin rotation

Perturbative NFL near d=5/2



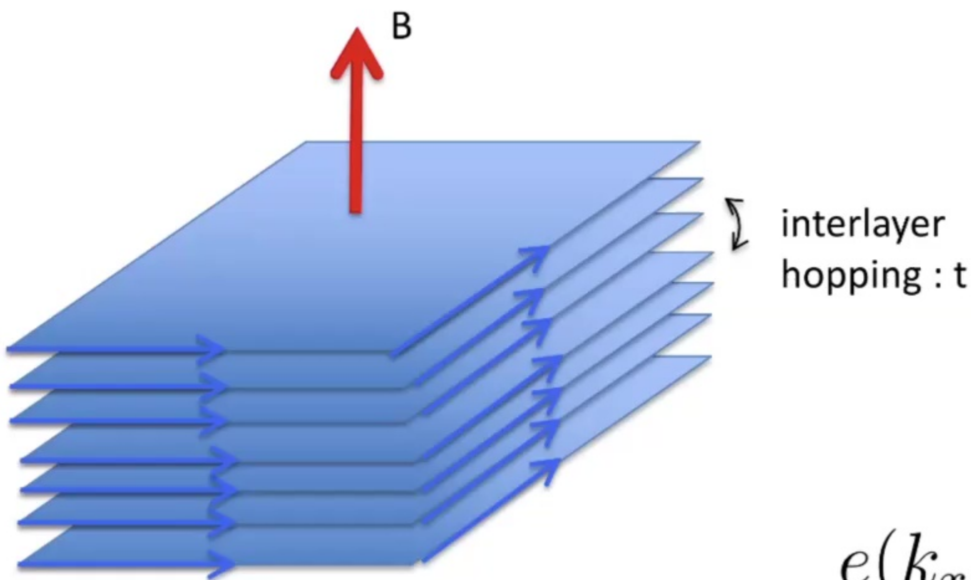


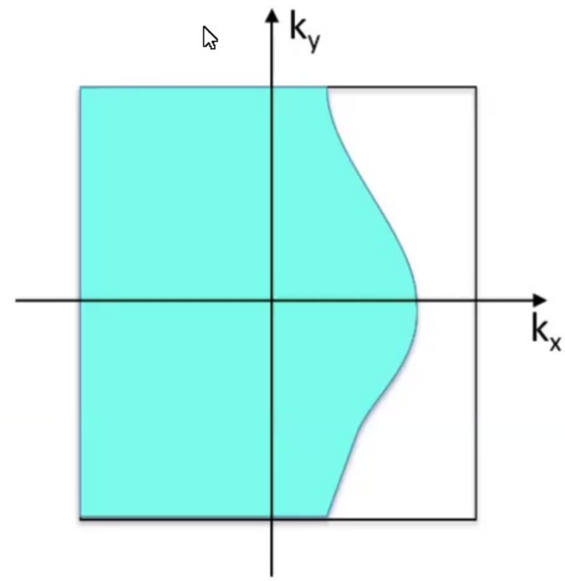


There is no perfect perturbative method.

The main purpose of perturbative methods is **to reveal emergent dynamical principles**, based on which one searches for non-perturbative ways of understanding deep quantum regime.

Chiral Metal





$$e(k_x, k_y) = k_x - t cos(k_y)$$

A stack of quantum Hall layers creates a two-dimensional chiral Fermi surface [Balents and Fisher (96)]

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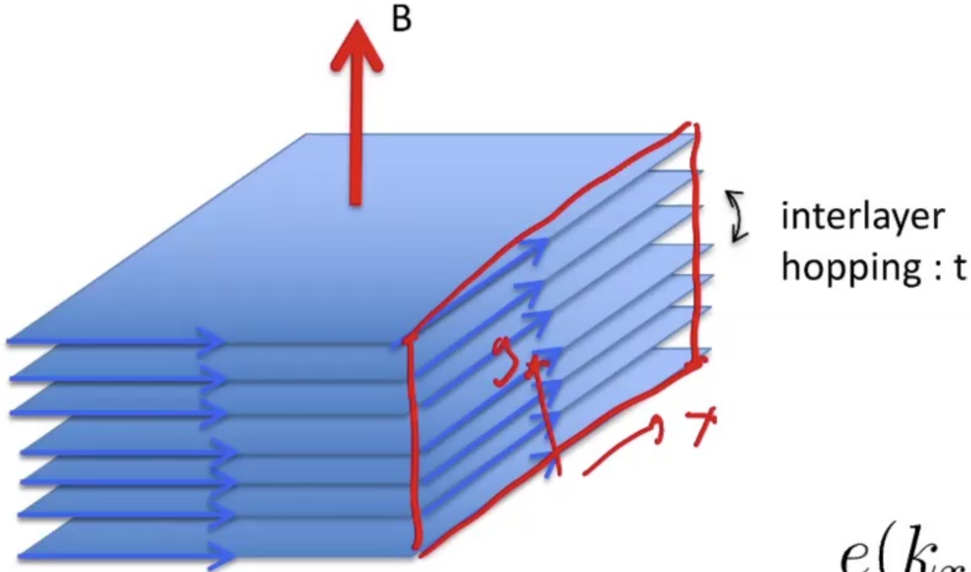

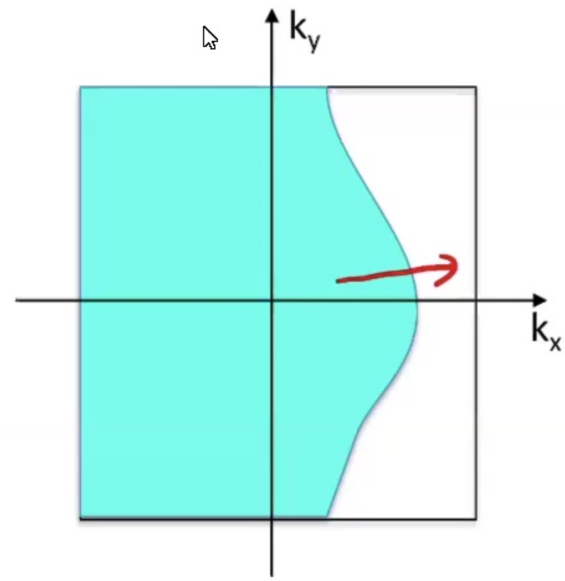

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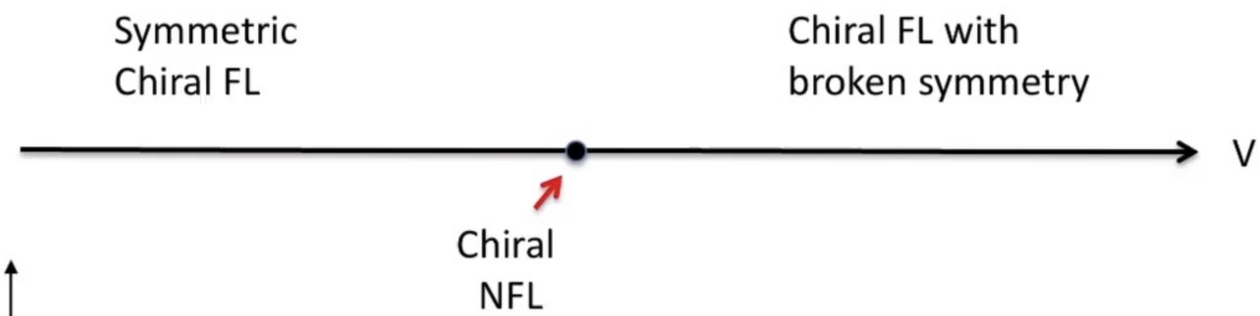
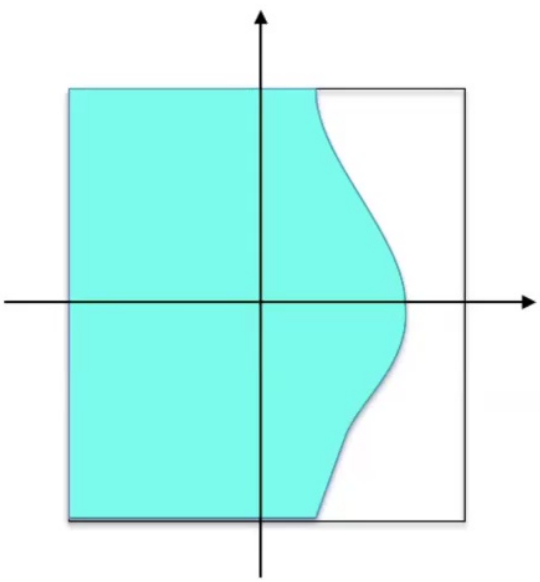
Critical chiral metal at quantum criticality





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$$+ g \int dk \; dq \; \phi_\alpha(k) \; \psi_i^*(k+q) \; T_{ij}^\alpha \; \psi_j(q)$$

+ gapless boson

under Gaussian scaling, interaction is relevant

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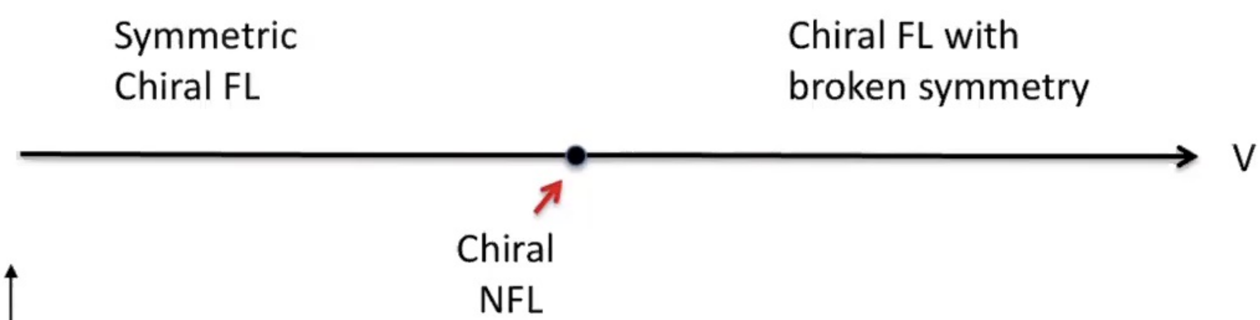

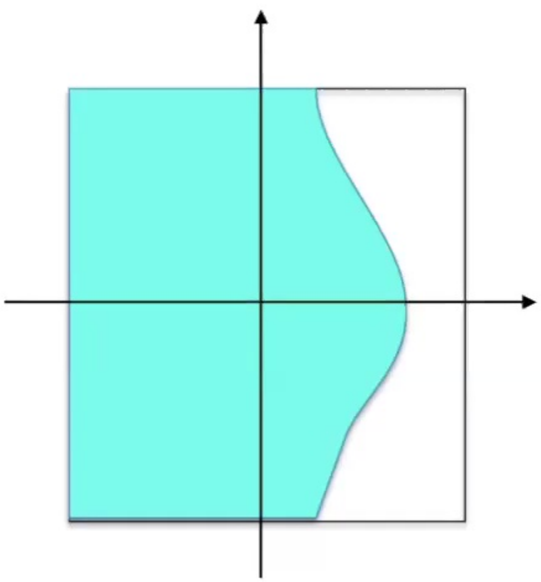

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Interaction driven scaling
(as opposed to the Gaussian scaling)

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irrelevant ($\frac{k_0}{\Lambda^{1/2}}$)

marginal (g)

$$k_x \to b k_x$$
$$k_y \to b^{1/2} k_y$$
$$k_0 \to b^z k_0$$
$$\Psi \to b^{-\frac{5+2z}{4}} \psi$$
$$\phi \to b^{-\frac{5+2z}{4}} \phi$$

$$z = \frac{3}{2}$$

- Loop corrections generate $\Sigma \sim k_0^{1/z}, \quad z > 1$
- The interaction is kept as a marginal term while one of the quadratic term is deemed irrelevant
- Irrelevant term enters as a scale

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Irrelevant

marginal

$$k_x \to b k_x$$
$$k_y \to b^{1/2} k_y$$
$$k_0 \to b^z k_0$$
$$\Psi \to b^{-\frac{5+2z}{4}}\psi$$
$$\phi \to b^{-\frac{5+2z}{4}}\phi$$

$$z = \frac{3}{2}$$

- Loop corrections generate $\Sigma \sim k_0^{1/z}$, $z > 1$
- The interaction is kept as a marginal term while one of the quadratic term is deemed irrelevant
- Irrelevant term enters as a scale

Scaling form

$$G^{-1}(k) = (k_x + k_y^2)\ g\left(\frac{|\omega|^{2/3}}{(k_x + k_y^2)}, \frac{(k_x + k_y^2)}{\Lambda}\right)$$

- In general, the UV cut-off can not be set to infinity.
  - This introduces correction of the scaling exponents with respect to the interaction driven scaling
- In this case, thanks to chirality, the theory is finite in the $\Lambda \to \infty$ limit
  - No correction to the interaction driven scaling

Interaction driven scaling (as opposed to the Gaussian scaling)

irrelevant

$$S = \int dk \left(\frac{k_0}{\Lambda^{1/2}} + k_x + \gamma k_y^2\right)\psi_j^*(k)\psi_j(k)$$

$$+ \int dk\; k_y^2\; \phi_\alpha(-k)\phi_\alpha(k)$$

$$+ g \int dk\, dq\; \phi_\alpha(k)\; \psi_i^*(k+q)\; T_{ij}^\alpha\; \psi_j(q)$$

marginal

$$k_x \rightarrow b k_x$$

$$k_y \rightarrow b^{1/2} k_y$$

$$k_0 \rightarrow b^z k_0$$

$$\Psi \rightarrow b^{-\frac{5+2z}{4}}\psi$$

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Stable fixed point

- The holomorphicity guarantees UV finiteness
- UV finiteness + absence of scale : the interaction driven scaling is exact
- Exact Scaling form of the Green's function :

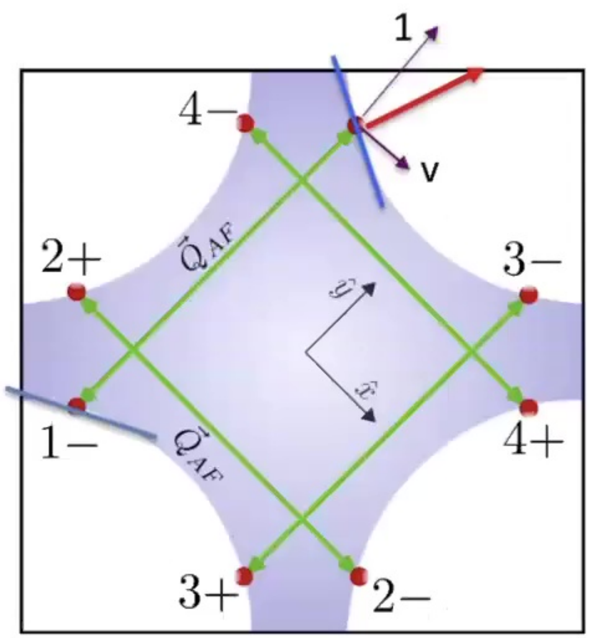
$$G^{-1}(k) = (k_x + k_y^2)\ g\left(\frac{|\omega|^{2/3}}{k_x + k_y^2}\right)$$

- The scaling exponents are one-loop exact (similar to supersymmetric QFT)
- However, this is not a perturbative fixed point
  - The full scaling function g(x) can not be computed perturbatively

[Sur, SL (2014)]

Hot spot theory

Parameters of the theory



- v : Fermi velocity perpendicular to Q_{AF}
- c : velocity of spin fluctuations
- g : coupling between electrons and spin fluctuations

- If v=0, hot spots connected by Q_{AF} are nested
- Generally, v, c, g are all non-zero

Open problems

- Beyond patch theory
  - Capturing momentum dependent universal data
  - Superconductivity
  - Disorder
- Local moments
- New non-perturbative methods