Title: Seminar: Fractons

Speakers: Meng Cheng

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Fractonic U(1) gauge theory

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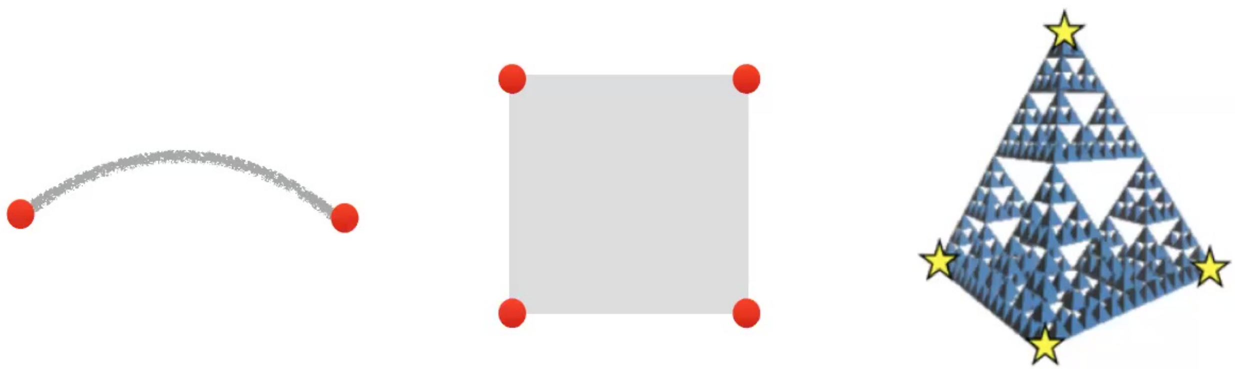
UQM Virtual Summer School
Perimeter Institute, 08/13/2020

Quantum Phases with Fractionalized Excitations

Low-energy states: well-defined quasiparticles

Quasiparticle excitations are localized, but non-local



Fractonic phases: quasiparticles with restricted mobility

Fracton Phases and Fractonic Symmetries

Both tied to rigid geometric structures

U(1) tensor gauge theory	**U(1) multipole symmetries**
Type-I fracton model	**Infinite subsystem symmetries**
Type-II fracton model	**Fractal symmetries**

Reviews:
R. Nandkishore and M. Hermele, 1803.11196
M. Pretko, X. Chen and Y. You, 2001.01722

Plan of the talk

Building some fractonic U(1) gauge theories from the ordinary Maxwell theory

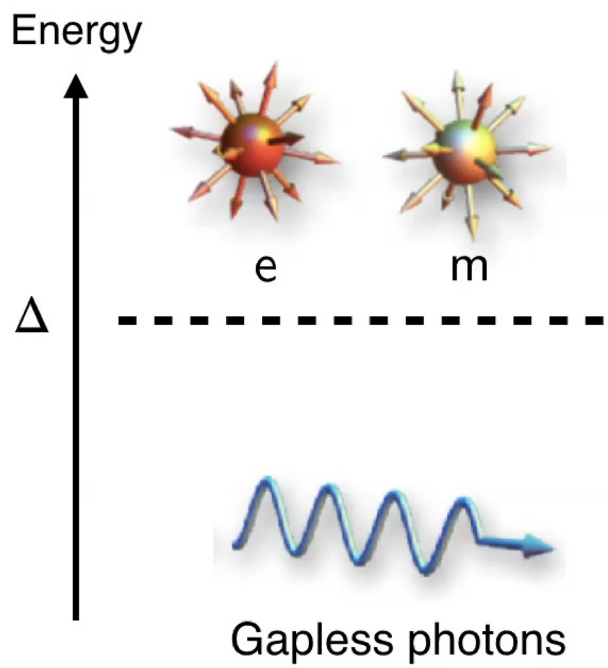
1. Gauging

2. "Defect network"

3D fractonic phases from planar U(1) Chern-Simons theories

Gapless Phases with Quasiparticles

3+1 U(1) gauge theory with electric/magnetic charges

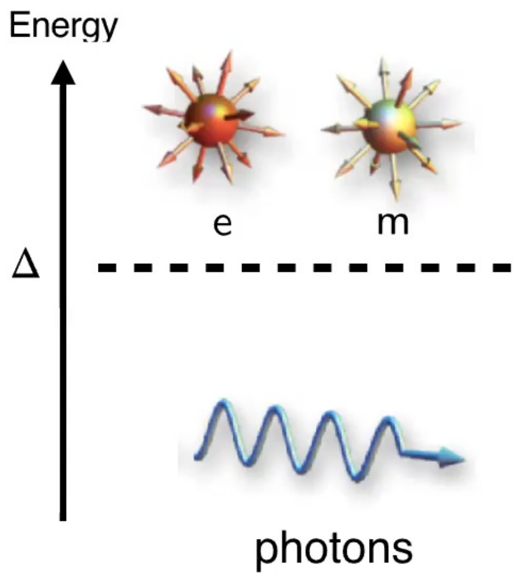


Gapped charges

$$H_{\text{eff}} = \frac{1}{2}\int d^3x \left(\epsilon \mathbf{E}^2 + \frac{1}{\mu}\mathbf{B}^2 \right)$$

Figure from Balents

U(1) Gauge Theory



(Compact) U(1) gauge field A_i

Gauge transformation $A_i \rightarrow A_i + \partial_i \chi$

Gauss's law $\nabla \cdot \mathbf{E} = \rho$

$$\int d\mathbf{x}\, \rho(\mathbf{x}) = 0$$

Gapped bosonic electric/magnetic charges

Fractonic U(1) gauge theories

Gapless photons + gapped **fractonic** charges

Ex: symmetric tensor gauge theory (with scalar charge)

$$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$$

$$H = \int d\mathbf{x}\, \frac{1}{2}(E_{ij}E^{ij} + B_{ij}B^{ij}) \qquad \omega \sim k \text{ photon}$$

Generalized Gauss's law: $\partial_i \partial_j E^{ij} = \rho$

$$\int d\mathbf{x}\, \rho(\mathbf{x}) = 0, \int d\mathbf{x}\, \mathbf{x}\rho(\mathbf{x}) = 0$$

Fractonic U(1) gauge theories

Gapless photons + gapped **fractonic** charges

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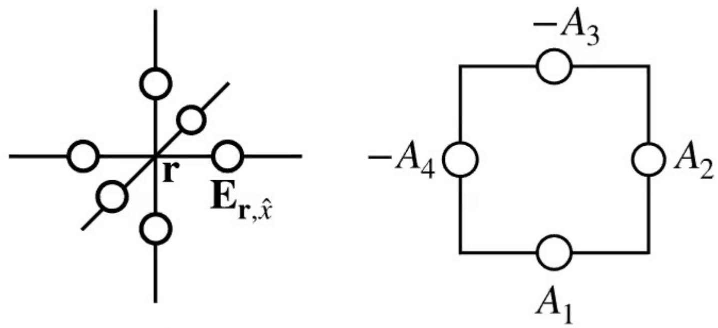
$$\int d\mathbf{x} \, \rho(\mathbf{x}) = 0, \int d\mathbf{x} \, \mathbf{x} \rho(\mathbf{x}) = 0$$

Xu, 2006; Rasmussen, You and Xu, 2016; **Pretko 2017**
Bulmash and Barkeshli 2018; Gromov 2019; …

Compact U(1) Lattice Gauge Theory

Rotors on each edge: $[A, E] = i$

$$E \in \mathbb{Z}, A \sim A + 2\pi$$



$$(\nabla \cdot \mathbf{E})_{\mathbf{r}} = q_{\mathbf{r}}$$

$$H = \Delta \sum_{\mathbf{r}} (\nabla \cdot \mathbf{E})_{\mathbf{r}}^2 - K \sum_p \cos(\nabla \times \mathbf{A})_p + \frac{U}{2} \sum_e \mathbf{E}_e^2$$

Gapless Coulomb phase for small U/K

Compact U(1) Lattice Gauge Theory

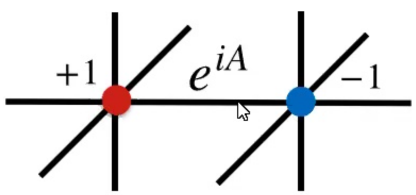
Gauss's law imposed energetically $\Delta \sum_{\mathbf{r}} (\nabla \cdot \mathbf{E})^2_{\mathbf{r}}$

Gapped charges: $(\nabla \cdot \mathbf{E})_{\mathbf{r}} = q_{\mathbf{r}} \in \mathbf{Z}$

As written, charges are completely static

= (non-relativistic) electric 1-form symmetry

Electric flux $\int_{\Sigma} \mathbf{E} \cdot \mathrm{d}\mathbf{n}$ is conserved for all closed surfaces Σ



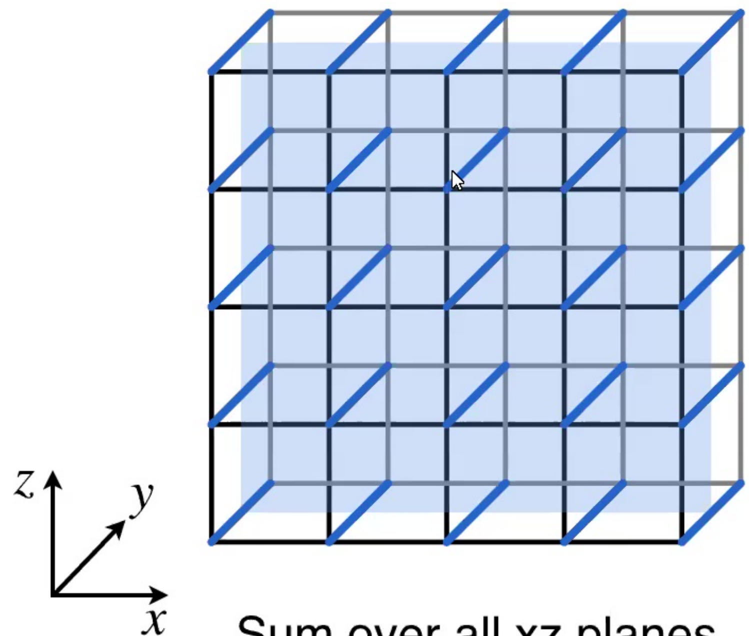
Adding "gauge-symmetry-breaking" term

Charges become dynamical

Lattice U(1) Gauge Theory

Gauging the full electric 1-form symmetry: confinement

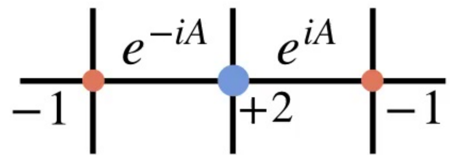
Gauging global (0-form) subgroup generated by $\sum_{e \| \mu} E_e$

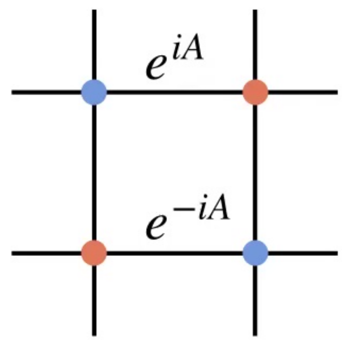


Sum over all xz planes

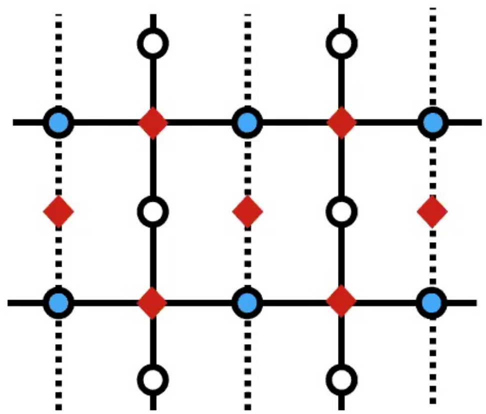
$$\rho(\mathbf{x}) = \nabla \cdot \mathbf{E} \longrightarrow \int \mathrm{d}\mathbf{x}\, E_i(\mathbf{x}) = -\int \mathrm{d}\mathbf{x}\, x_i \rho(\mathbf{x}) = -D_i$$

Conservation of electric dipole moment





Gauging dipole symmetry in 2D



Site: $\tilde{A}_{xx}, \tilde{A}_{yy}$

Plaquette: $\tilde{A}_{p,x}, \tilde{A}_{p,y}$

$$-K \sum_p \cos\left[(\nabla \times \mathbf{A})_p - \tilde{A}_{p,x} + \tilde{A}_{p,y}\right]$$

Minimal coupling

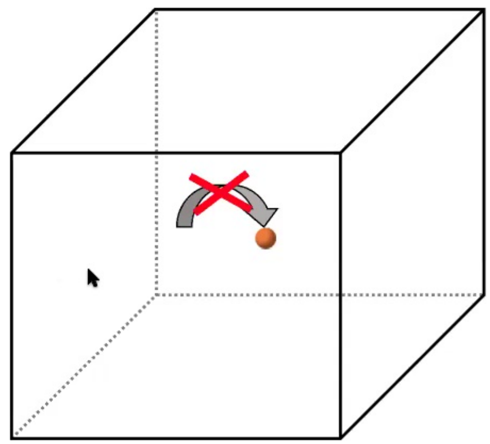
◆ Gauge fields of $\sum_e E_e^x$

"Higgsing": $\tilde{A}_{p,x} = \tilde{A}_{p,y} \equiv \tilde{A}_{xy}$

At low energy: $\underbrace{\tilde{A}_{xx}, \tilde{A}_{yy}, \tilde{A}_{xy}}$

Symmetric tensor gauge field

Fracton and translation symmetry



$|\mathrm{qp}\rangle$ and $T|\mathrm{qp}\rangle$ belong to different superselection sectors

Pai and Hermele, PRB 2019

Charge and Dipole Moment

$$Q = \int \mathrm{d}\mathbf{x}\, \rho(\mathbf{x}), \mathbf{D} = \int \mathrm{d}\mathbf{x}\, \mathbf{x}\rho(\mathbf{x})$$

Translation:

$$T(\mathbf{a}) : \rho(\mathbf{x}) \to \rho(\mathbf{x} - \mathbf{a})$$

$$T(\mathbf{a}) : Q \to Q, \mathbf{D} \to \mathbf{D} + Q\mathbf{a}$$

Three U(1) charges in 2D: Q, D_x, D_y

$$\begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix} \xrightarrow{T_x} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix}, \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix} \xrightarrow{T_y} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix}$$

Kinematics of $U(1)^N$ Gauge Theory

Suppose there are N **global** U(1) charges

$$\mathbf{Q} = (Q_1, Q_2, \cdots, Q_N)^\mathsf{T}$$

Change of variables: $\mathbf{Q}' = \mathbf{W}\mathbf{Q}, \mathbf{W} \in \mathbb{GL}(N, \mathbb{Z})$

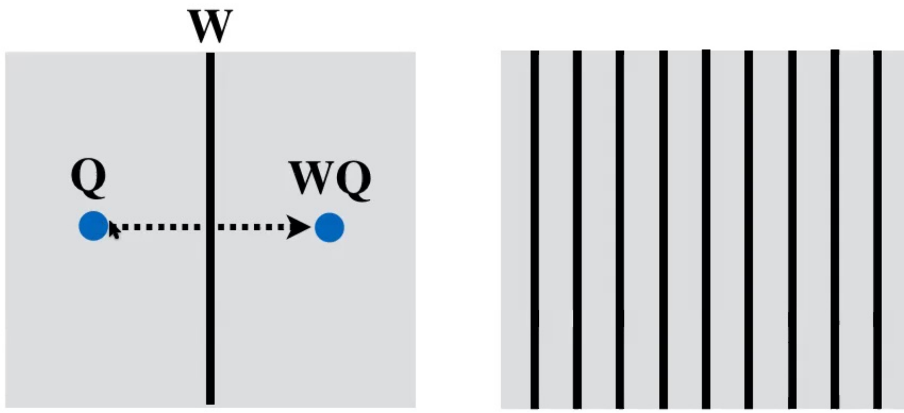
"Symmetries" of the kinematical structure

*ignore the magnetic sector now

Lattice translations $T_\mu : \mathbf{Q} \to \mathbf{W}_\mu \mathbf{Q}$

W matrices encode mobility

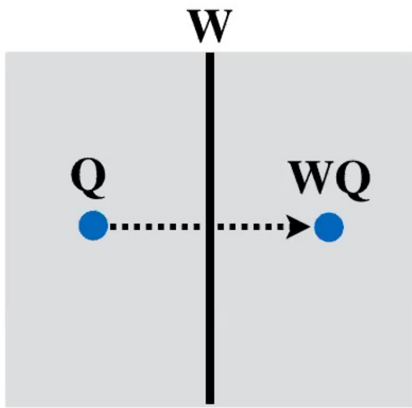
$U(1)^N$ Gauge Theory with “Defect” Network

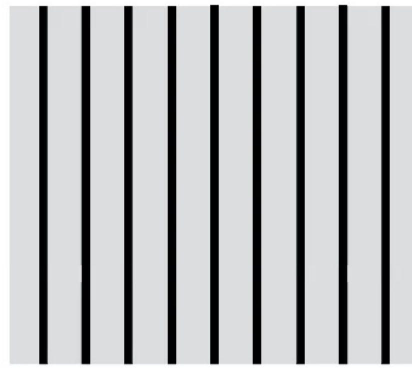


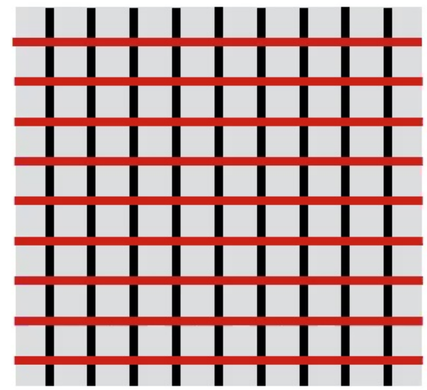
If W has infinite order, i.e. $\mathbf{W}^n \neq \mathbf{1} \ \forall n$

A general **Q** becomes immobile
unless $\boldsymbol{W^k Q = Q}$ for some nonzero k

$U(1)^N$ Gauge Theory with “Defect” Network







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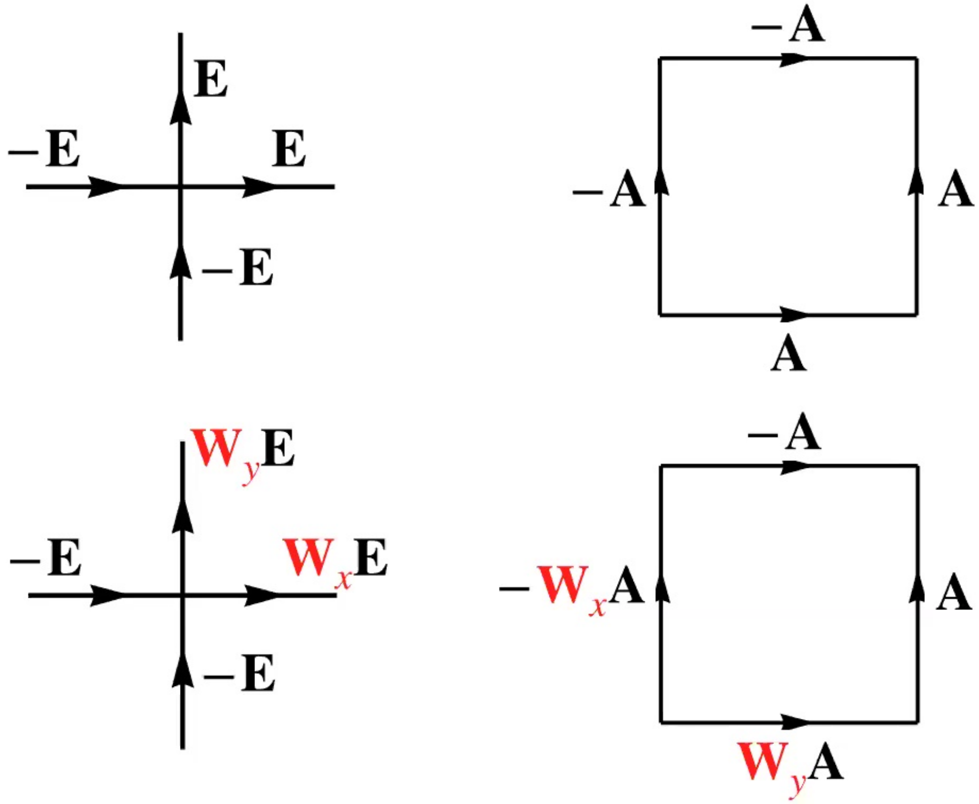
Lattice Realization

$$T_\mu : \mathbf{E} \to \mathbf{W}_\mu \mathbf{E}$$



Qing-Rui Wang and MC, in preparation

For N=3, choose W matrices corresponding to *Q* and ***D***

$$\mathbf{W}_x = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{W}_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix} \xrightarrow{T_x} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix}, \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix} \xrightarrow{T_y} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix}$$

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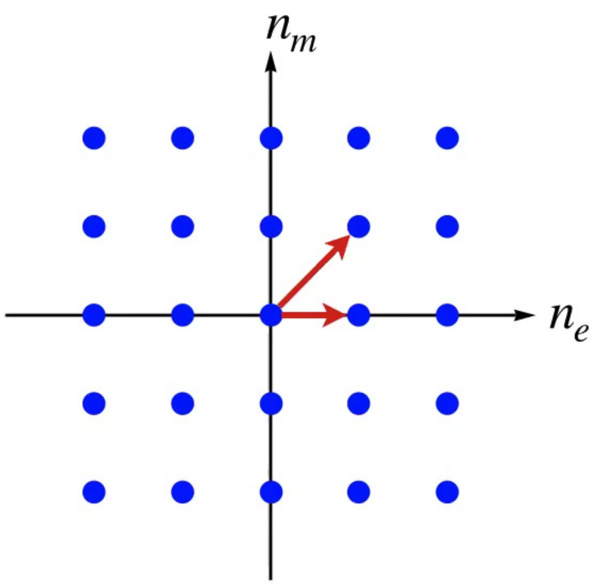
$$\begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix} \xrightarrow{T_x} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix}, \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix} \xrightarrow{T_y} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ D_x \\ D_y \end{pmatrix}$$

The model is exactly equivalent to the gauged U(1) model

For many choices of W, the photons are completely gapped

Electromagnetic Duality



Charge lattice

Basis change of the charge lattice preserves the kinematics (e.g. statistics)

$$\mathbf{S} : \begin{pmatrix} n_e \\ n_m \end{pmatrix} \to \begin{pmatrix} n_m \\ -n_e \end{pmatrix}$$

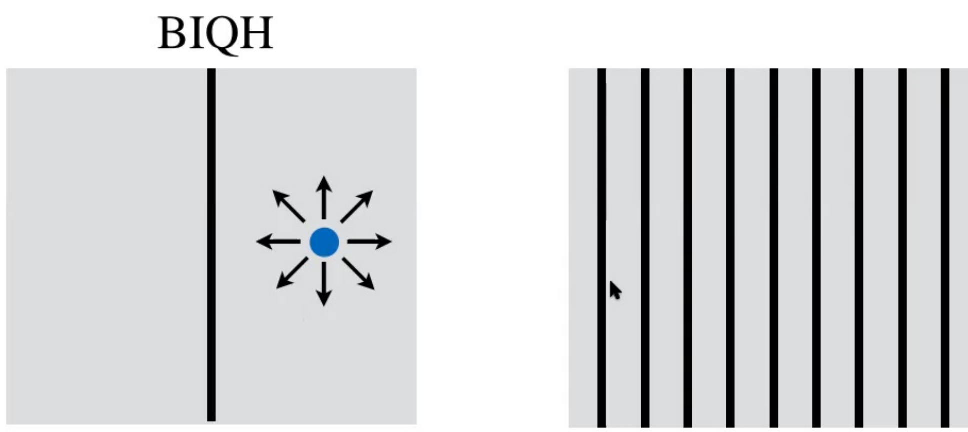
$$\mathbf{T} : \begin{pmatrix} n_e \\ n_m \end{pmatrix} \to \begin{pmatrix} n_e + n_m \\ n_m \end{pmatrix}$$

Generate SL(2, **Z**)

For $U(1)^N$ gauge theory, the duality group is Sp(2N, **Z**)

T has infinite order

$\mathbf{T}^2$ wall is a bosonic integer quantum Hall state $\sigma_H = \frac{2e^2}{h}$



Gauging 3D U(1) in a stack of BIQHs

$$\omega(\mathbf{k})^2 = v^2\mathbf{k}_{\|}^2 + v_z^2 k_{\perp}^4$$

Summary

1. Fractonic U(1) gauge theory obtained from gauging a global dipole symmetry in an ordinary U(1) gauge theory

Refs: D. Williamson, Z. Bi and MC, PRB 2019
L. Radzihovsky and M. Hermele, PRL 2020
M. Pretko, PRB 2018; N. Seiberg, 1909.10544

2. Fractonic U(1) gauge theory obtained from inserting defect network in an ordinary U(1) gauge theory

Refs: D. Williamson, Z. Bi and MC, PRB 2019
Q. R. Wang and MC, in preparation