Title: Partons 2

Speakers: Michael Levin

Collection: Online School on Ultra Quantum Matter

Date: August 13, 2020 - 11:00 AM

URL: http://pirsa.org/20080011

Properties of π-flux state

1. $|\Psi_{spin}\rangle$ invariant under spin rotation and lattice symmetries.

"spin liquid"

2. Low energy theory consists of 4 2-component Dirac fermions ("spinons") coupled to $U(1)$ gauge field.

"compact $(QED)_3$"

Physical picture for gauge field

Consider π-flux state:

$$|\Psi_{spin}\rangle = P|\Psi_0\rangle$$

$|\Psi_0\rangle$ gd. state of $H_0 = \sum_{\langle ij\rangle}\left(t_{ij} f_{i\alpha}^{+} f_{j\alpha} + h.c\right)$

One way to perturb $|\Psi_{spin}\rangle$: define

$$|\Psi_{spin}^{\theta_{ij}}\rangle = P|\Psi_0^{\theta_{ij}}\rangle$$

$|\Psi_0^{\theta_{ij}}\rangle$ gd. state of $H_0^{\theta_{ij}} = \sum_{\langle ij\rangle} t_{ij} e^{i\theta_{ij}} f_{i\alpha}^{+} f_{j\alpha}$

Claim: $|\Psi_{spin}^{\theta_{ij}+\chi_i-\chi_j}\rangle = (\text{phase})\,|\Psi_{spin}^{\theta_{ij}}\rangle$

To see this, note that

$$H_0^{\theta_{ij}+\chi_i-\chi_j} = V_\chi H_0^{\theta_{ij}} V_\chi^\dagger$$

$$V_\chi = e^{i\sum_i \chi_i f_{i\alpha}^\dagger f_{i\alpha}}$$

$$|\Psi_0^{\theta_{ij}+\chi_i-\chi_j}\rangle = V_\chi |\Psi_0^{\theta_{ij}}\rangle$$

$$\Rightarrow P|\Psi_0^{\theta_{ij}+\chi_i-\chi_j}\rangle = P V_\chi |\Psi_0^{\theta_{ij}}\rangle$$

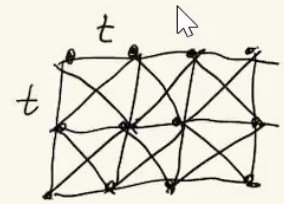
$$= e^{i\sum_i \chi_i} P|\Psi_0^{\theta_{ij}}\rangle. \boxtimes$$

$|\Psi_{spin}^{\theta_{ij}}\rangle$ describes a "gauge fluctuation."

Example 2 : A $\mathbb{Z}_2$ spin liquid

$$H_0 = \sum_{\langle ij\rangle}\left(t_{ij} f^\dagger_{i\alpha} f_{j\alpha} + \Delta_{ij} f^\dagger_{i\alpha} f^\dagger_{j\alpha} + h.c\right)$$

$$+ \sum_i \left(\Delta_3 f^\dagger_{i\uparrow} f^\dagger_{i\downarrow} + h.c\right)$$



$$t_{i,i+\hat{x}} = t_{i,i+\hat{y}} = t$$

$$\Delta_{i,i+\hat{x}+\hat{y}} = \Delta^*_{i,i-\hat{x}+\hat{y}} = \Delta_1 + i\Delta_2$$

$$t,\ \Delta_1, \Delta_2, \Delta_3 \neq 0$$

$\Delta_{i,i+\hat{x}+\hat{y}} \quad \Delta_{i,i-\hat{x}+\hat{y}} \quad \Delta_1, \Delta_2$

$t, \Delta_1, \Delta_2, \Delta_3 \neq 0$

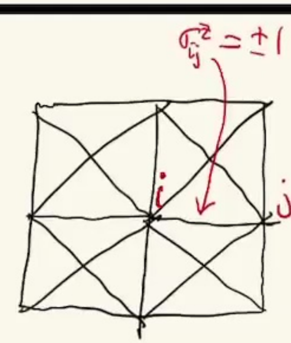
Symmetry transformations that preserve H_0 and $\vec{S}_i$:

$$f_{i\alpha} \to -f_{i\alpha}$$

$\Rightarrow G = \mathbb{Z}_2$

Introduce $\mathbb{Z}_2$ gauge fields

σ^z_{ij} on links



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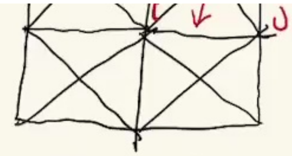


Low energy theory:

$$H_{eff} = \sum_{\langle ij\rangle}\left[t_{ij}\,\sigma_{ij}^{z} f_{i\alpha}^{\dagger} f_{j\alpha} + \Delta_{ij}\,\sigma_{ij}^{z} f_{i\alpha}^{\dagger} f_{j\alpha}^{\dagger} + h.c\right]$$

$$+ \sum_{i}\left(\Delta_3 f_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} + h.c.\right) + H_{gauge}$$

$$H_{gauge} = -h\sum_{\langle ij\rangle}\sigma_{ij}^{x} - K\sum_{\Delta}\sigma_{pq}^{z}\sigma_{qr}^{z}\sigma_{rp}^{z}$$

with $h \ll t, \Delta \ll K$

Gauge invariance constraint:

$$\prod_{*}\sigma_{ij}^{x} = (-1)^{f_{i\alpha}^{\dagger} f_{i\alpha} - 1}$$

Properties

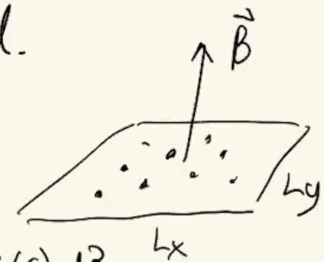
1. $|\Psi_{spin}\rangle$ invariant under rotation/lattice symmetries.

2. Low energy theory consists of gapped spin-$\frac{1}{2}$ fermions (spinons) coupled to a $\mathbb{Z}_2$ gauge field.

$\Rightarrow$ $\mathbb{Z}_2$ topological order (like toric code)

Brief review of QH states

Setting: N electrons in 2D in perp. mag. field.



$$H = \int \frac{1}{2m} c^\dagger(r)\left[i\vec{\nabla} + \vec{A}(r)\right]^2 c(r)\, d^2r$$

$$+ \text{ interactions}$$

$$\vec{\nabla} \times \vec{A} = B\hat{z}$$

Define $\nu = \dfrac{\#\text{ electrons}}{\#\text{ flux quanta}} = \dfrac{N}{B(L_x L_y)/2\pi}$

Gapped states occur at special rational values of ν.

Parton construction of QH states

Consider QH system with N electrons and $3N$ flux quanta, i.e. $\nu = \frac{1}{3}$.

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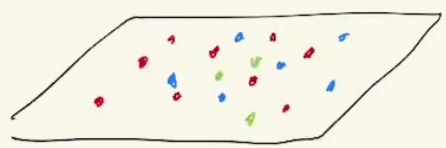
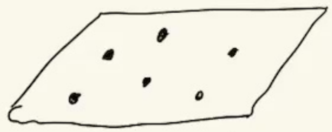
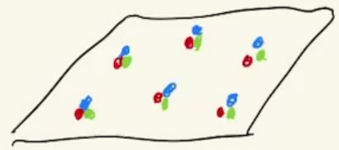
To construct a candidate state, consider a larger Hilbert space consisting of 3 species of fermions ("partons") f^1, f^2, f^3. Assume N particles from each species.



We can embed $\mathcal{H}_{\text{electron}} \subseteq \mathcal{H}_{\text{parton}}$ by thinking of electron as composite of 3 partons:



electron state



parton states

In terms of operators:

$$c = f_1 f_2 f_3$$

Next, we need to choose a parton Hamiltonian

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$$c = f_1 f_2 f_3$$

Next, we need to choose a parton Hamiltonian H_0. Need 2 properties:

1. H_0 is gapped (assuming we want to construct a gapped state)

2. Gd. state $|\Psi_0\rangle$ satisfies

$$\langle f_1^\dagger f_1 \rangle = \langle f_2^\dagger f_2 \rangle = \langle f_3^\dagger f_3 \rangle$$

Given H_0, we can construct:

1. An electron state $|\Psi_e\rangle$
2. A low energy theory.

Construction of electron state:

$$|\Psi_e\rangle = P|\Psi_0\rangle$$

projection into $\mathcal{H}_{electron}$

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Equivalently:

$$\Psi_e(\{z_i\}) = \Psi_0(\{z_i^1, z_i^2, z_i^3\})$$

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Equivalently:

$$\Psi_e(\{z_i\}) = \Psi_0(\{z_i^1, z_i^2, z_i^3\})\Big|_{z_i^1=z_i^2=z_i^3=z_i}$$

$\Psi_e(\{z_i\})$ ...
$z_i^1 = z_i^2 = z_i^3 = z_i$

$z = x + iy$

Construction of low energy theory:

similar to before.

Example

$$H_0 = \int f_p^+(r) \frac{(i\vec{\nabla} + \vec{A}(\vec{r})/3)}{2m} f_p(\vec{r}) d^3 r$$

Partons moving in mag. field $\frac{B}{3}$.

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$\Rightarrow$ each parton species is in $\nu=1$
IQH state
(one filled Landau level)

$$\Rightarrow \Psi_0(\{z_i^1, z_i^2, z_i^3\}) = \Psi^{\nu=1}(\{z_i^1\})\Psi^{\nu=1}(\{z_i^2\}) \cdot \Psi^{\nu=1}(\{z_i^3\})$$

where

$$\Psi^{\nu=1}(z_1, \dots, z_N)$$

3.

$\Rightarrow$ each parton species is in $\nu=1$ IQH state (one filled Landau level)

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where

$$\Psi^{\nu=1}(z_1, \ldots, z_N) = \prod_{i<j}(z_i - z_j)\, e^{-\sum_i \frac{|z_i|^2}{4\cdot 3\ell^2}}$$

factor of 3 because $\vec{B}/3$

$$\ell = \frac{1}{\sqrt{|B|}}$$

Electron wave-function

$$\Psi_e(\{z_i\}) = \left[\Psi^{\nu=1}(\{z_i\})\right]^3$$

$$= \prod_{i<j}(z_i - z_j)^3 \, e^{-\sum_i \frac{|z_i|^2}{4\ell^2}}$$

Wave-function for Laughlin state!

Low energy theory

Symmetry transf. that pr

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Low energy theory

Symmetry transf. that preserve H_0, C:

$$f_1 \to e^{i\alpha} f_1 \qquad f_1 \to e^{i\lambda} f_1$$

$$f_2 \to e^{-i\alpha} f_2 \qquad f_2 \to f_2$$

$$f_3 \to f_3 \qquad f_3 \to e^{-i\lambda} f_3$$

$$G = U(1) \times U(1)$$

$f_2 \to e^{-i\alpha} f_2$ $\qquad$ $f_2 \to f_2$

$f_3 \to f_3$ $\qquad$ $f_3 \to e^{-i\lambda} f_3$

$G = U(1) \times U(1)$

[strictly speaking, this is only correct if we break the symmetry between f_1, f_2, f_3 in H_0. Otherwise $G = SU(3)$.]

Denote 2 gauge fields by a^1, a^2:

$$H_{eff} = \int f_p^{+}(r) \frac{\left(i\vec{\nabla} + \frac{\vec{A}}{3} + Q_{pq}\vec{a}^{q}\right)^2}{2m} f_p(r) d^2 r$$

$$+ \sum_{q=1}^{2} \left[h\,|\vec{e}^{q}|^2 + K\left(\vec{\nabla} \times a^{q}\right)^2 \right]$$

$$Q_{pq} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Gauge invariance constraint:

$$\mathrm{div}(\vec{e}^{q}) = Q_{pq} \cdot f_p^{+} f_p$$

$$\vdots$$

$$\mathcal{L} = \frac{3}{4\pi} \epsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda$$

Is the parton construction correct?

Focus on slave-fermion construction.

Two issues:

1. Wave function $|\Psi_{spin}\rangle$

2. Low energy theory

There exist "counter examples" where $|\Psi_{spin}\rangle$ is unphysical or in wrong phase.

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1. Wave function $|\Psi_{spin}\rangle$

2. Low energy theory

There exist "counter examples" where $|\Psi_{spin}\rangle$ is unphysical or in wrong phase.

So let's focus on low energy theory.

Conjecture 1: Every low energy theory obtained from slave fermion construction is "physical", i.e. it correctly describes the low energy excitations of <u>some</u> spin Hamiltonian.

To make more precise, let's focus on an example: let H_{eff} be the Hamiltonian defined in the $\mathbb{Z}_2$ spin liquid example.

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<u>Conjecture 2</u>: There exists a symmetry-preserving gapped interpolation between H_{eff} and a Hamiltonian of form

$$H = H_{spin} - \sum_{\langle ij \rangle} \sigma_{ij}^x$$

where H_{spin} is a (gapped) spin Hamiltonian.

Consider $h \gg t, K$