Title: Partons 1

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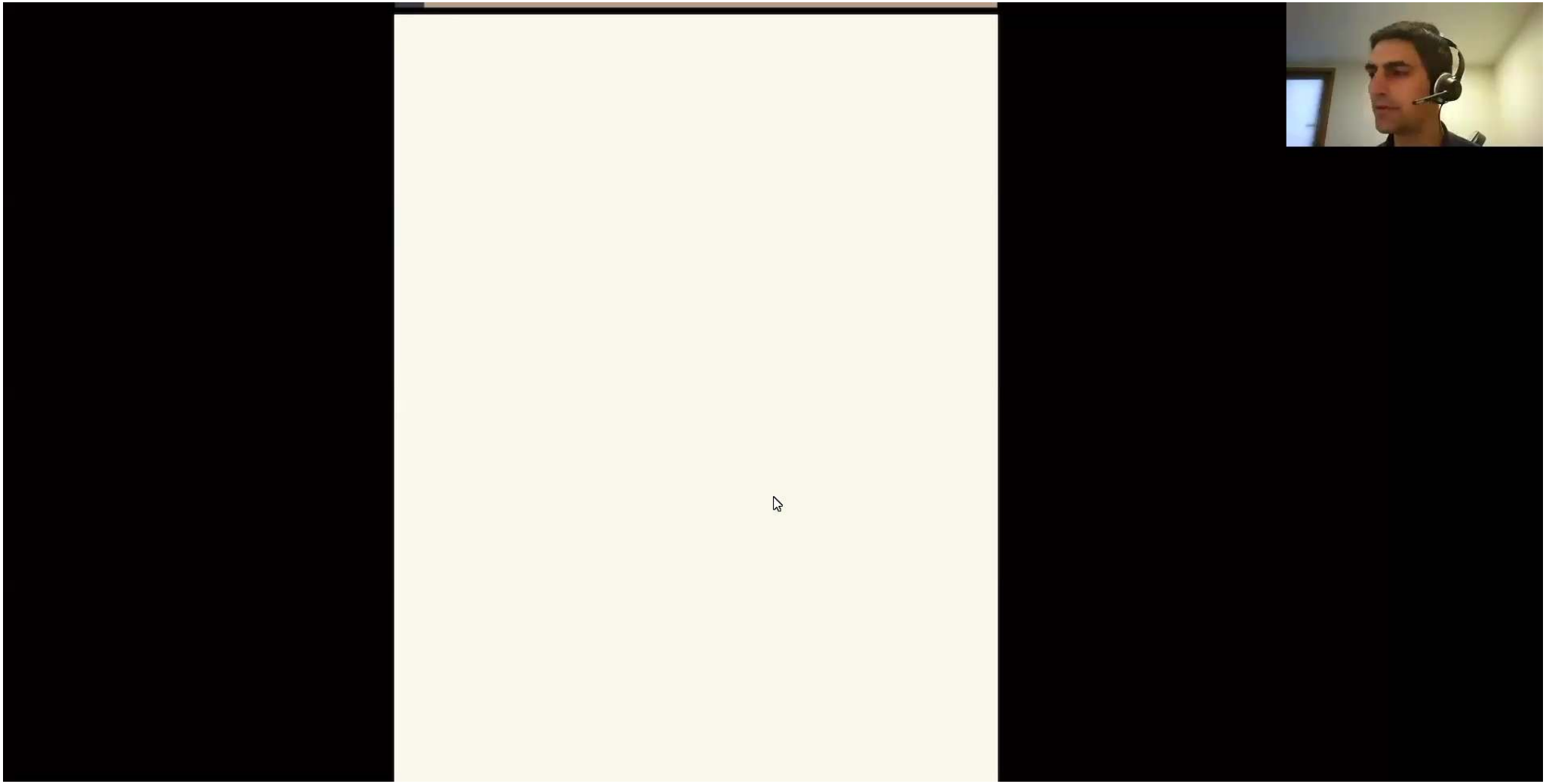

Parton construction

Method for constructing interesting many-body states and low energy theories. (~1980's)

Applications

1. Spin liquids
2. Quantum Hall states
3. ⋮

1. Spin liquids
2. Quantum Hall states
⋮

References:

CL
1. Wen, QFT of many-body systems
2. Balents + Savary, 1601.03742

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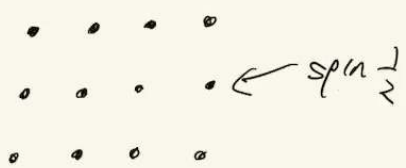
3. Blok + Wen, PRB 42, 8133 (1990)
4. Wen, PRL 66, 802 (1991)

Parton construction of spin liquids

Setting: spin $\frac{1}{2}$ model on square lattice

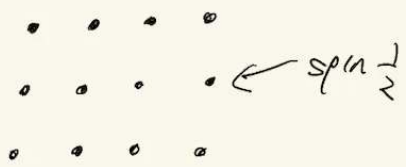


How can we construct interesting spin states?

Parton construction of

Spin liquids

Setting: spin $\frac{1}{2}$ model on square lattice



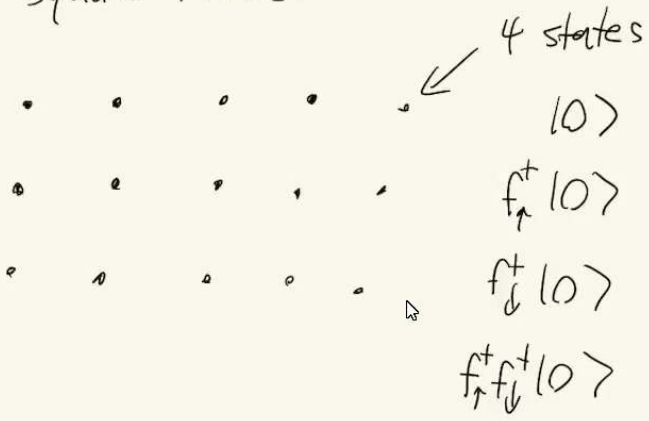
How can we construct interesting spin states?

2 parton-based approaches:

1. "slave fermions"
2. "slave bosons" or "Schwinger bosons"

Slave fermion construction

Consider an enlarged Hilbert space describing spin-$\frac{1}{2}$ fermions on square lattice.



Consider an enlarged Hilbert space describing spin-$\frac{1}{2}$ fermions on square lattice.

$|0\rangle$

$f_\uparrow^\dagger |0\rangle$

$f_\downarrow^\dagger |0\rangle$

$f_\uparrow^\dagger f_\downarrow^\dagger |0\rangle$

Note that $\mathcal{H}_{spin} \subseteq \mathcal{H}_{fermion}$:

$|\uparrow\rangle \longleftrightarrow f_\uparrow^\dagger |0\rangle$

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In terms of operators:

$$\vec{S} = \frac{1}{2} f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta \qquad \alpha, \beta = \uparrow, \downarrow$$

Now consider a quadratic fermion Hamiltonian:

$$H_0 = \sum_{ij}$$

In terms of operators:

$$\vec{S} = \frac{1}{2} f_\alpha^+ \vec{\sigma}_{\alpha\beta} f_\beta \qquad \alpha, \beta = \uparrow, \downarrow$$

Now consider a quadratic fermion Hamiltonian:

$$H_0 = \sum_{ij} \left(t_{ij}^{\alpha\beta} f_{i\alpha}^+ f_{j\beta} + \Delta_{ij}^{\alpha\beta} f_{i\alpha}^+ f_{j\beta}^+ + h.c. \right) - \sum_i \mu_i f_{i\alpha}^+ f_{i\alpha}$$

Let $|\Psi_0\rangle$ be gd. state of H_0

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Let $|\Psi_0\rangle$ be gd. state of H_0

$|\Psi_0\rangle \notin \mathcal{H}_{spin}$ since it doesn't obey the constraint $f_{i\alpha}^\dagger f_{i\alpha} = 1$

$$- \sum_i \mu_i f_{i\alpha}^\dagger f_{i\alpha}$$

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$|\Psi_0\rangle \notin \mathcal{H}_{spin}$ since it doesn't obey the constraint $f_{i\alpha}^\dagger f_{i\alpha} = 1$.

We will be interested in case where $|\Psi_0\rangle$ obeys the constraint on average:

$$\langle f_{i\alpha}^\dagger f_{i\alpha} \rangle_{\Psi_0} = 1$$

Any H_0 with this property is called a "mean field ansatz."

Slave-fermion construction takes ansatz H_0 as input and produces:

1. A spin state $|\Psi_{spin}\rangle$
2. A low energy theory H_{eff}.

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2. A low energy theory H_{eff}.

The main claim/hope:

1. $|\Psi_{spin}\rangle$ is the gd. state of <u>some</u> local spin Hamiltonian.

2. H_{eff} correctly describes the low energy excitations of such a spin Hamiltonian.

Construction of $|\Psi_{spin}\rangle$

We construct $|\Psi_{spin}\rangle$ by projecting $|\Psi_0\rangle$ into $\mathcal{H}_{spin}$:

$$|\Psi_{spin}\rangle = \rho \cdot |\Psi_0\rangle$$

ρ: projection into $\mathcal{H}_{spin}$

Equivalently:

$\Psi_{spin}(\alpha,$

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Equivalently:

$$\Psi_{spin}(\alpha_1, \cdots, \alpha_M) = \langle 0 | f_{1\alpha_1} f_{2\alpha_2} \cdots f_{M\alpha_M} | \Psi_0 \rangle$$

$\alpha_i = \uparrow, \downarrow$

$M = \#$ spins

$|0\rangle$: vacuum

Construction of low energy theory H_{eff}

First, find all onsite unitary symmetry transformations U such that:

(1) $U^\dagger H_0 U = H_0$

(2) U acts trivially with H_{spin}, i.e. $U^\dagger \vec{S}_i U = \vec{S}_i$ for all i.

$\{U\}$ = "invariant gauge group" G

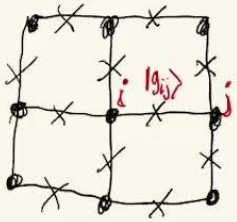
Notation: $\{U^g, g \in G\}$

To construct low energy theory, we couple H_0 to a weakly-fluctuating G-gauge field.

Recipe:

Step 1: Introduce gauge fields on links

$|g_{ij}\rangle$, $g \in G$



G - gauge field.

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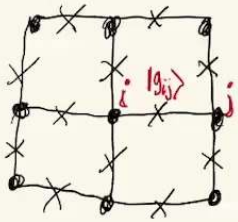


Step 2: Write down gauge theory Hamiltonian

$H_{eff} = \tilde{H}_0 + H_{gauge}$

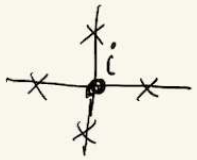
$\tilde{H}_0$ minimally coupled to G-gauge field

Step 3: Restrict to gauge invariant subspace

Specifically, restrict to states $|\Psi\rangle$ s.t.

$$\prod_{\langle ij \rangle} L_{ij}^{g} \cdot \left(e^{-i\theta_i^g} U_i^g \right) |\Psi\rangle = |\Psi\rangle$$

for each lattice site i and each $g \in G$.



Here:

$$L_{ij}^{g} |h_{ij}\rangle = |(gh)_{ij}\rangle$$

U_i^g, θ_i^g defined by

$$U^g = \prod_i U_i^g$$

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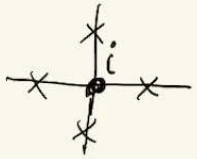

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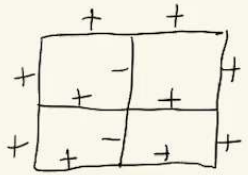
$$U^g = \prod_i U^g_i \qquad U^g_i \Big|_{\mathcal{H}_{phys}} = e^{i\theta^g_i} \cdot \mathbb{1}$$

Example 1: "π-flux state"

$$H_0 = \sum_{\langle ij \rangle} \left(t_{ij} f^{+}_{i\alpha} f_{j\alpha} + h.c. \right)$$



$t_{i, i+\hat{x}} = t$

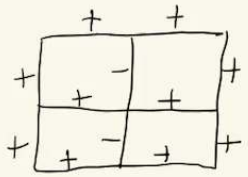
$t_{i, i+\hat{y}} = (-1)^{ix} \cdot t$

Let's construct low energy theory.

Symmetry transformations that leave H_0, $\vec{S}$ invariant:

$$f_{i\alpha} \rightarrow e^{i\theta} f_{i\alpha}$$

$H_0 = \sum_{\langle ij \rangle} (\quad)$



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Symmetry transformations that leave H_0, $\vec{S}$ invariant:

$$f_{i\alpha} \to e^{i\chi} f_{i\alpha}$$

$\Rightarrow G = U(1)$

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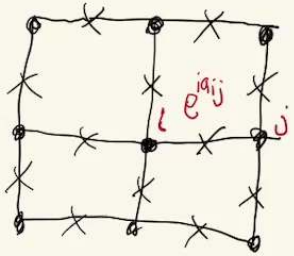
Introduce $U(1)$ gauge fields on links: $e^{ia_{ij}}$



Low energy theory:

$$H_{eff} = \sum_{\langle ij \rangle} \left(t_{ij} e^{ia_{ij}} f^{\dagger}_{i\alpha} f_{j\alpha} + h.c. \right) + H_{gauge}$$

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$$H_{eff} = \sum_{\langle ij \rangle} \left(t_{ij} e^{i a_{ij}} f^{+}_{i\alpha} f_{j\alpha} + h.c. \right) + H_{gauge}$$

$$H_{gauge} = h \sum_{\langle ij \rangle} e_{ij}^2 - K \sum_{\square} \cos\left(a_{pq} + a_{qr} + a_{rs} + a_{sp}\right)$$

Here: $[a_{ij}, e_{ij}] = i$

vector potential → a_{ij}, e_{ij} ← electric field

$h \ll$

$$H_{gauge} = h \sum_{\langle ij \rangle} e_{ij}^2 - K \sum_{\square} \cos(a_{pq} + a_{qr} + a_{rs} + a_{sp})$$

Here: $[a_{ij}, e_{ij}] = i$

a_{ij}: vector potential

e_{ij}: electric field

$h \ll t \ll K$ (a_{ij} weakly fluctuating)

Let's find gauge invariance constraint.

$U^{\chi} =$

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a_{ij}: vector potential

e_{ij}: electric field

$h \ll t \ll K$ (a_{ij} weakly fluctuating)

Let's find gauge invariance constraint.

$$U^{\chi} = e^{i\chi \sum_i f_{i\alpha}^{+} f_{i\alpha}}$$

$$\Rightarrow U_i^{\chi} = e^{i\chi f_{i\alpha}^{+} f_{i\alpha}}$$

$$\Rightarrow U_i^{\chi}\big|_{H_{spin}} = e^{i\chi} \cdot \mathbb{1}$$

$$\Rightarrow e^{i\theta_i^{\chi}} = e^{i\chi}$$

$$U^{\chi} = e^{\cdots}$$

$$\Rightarrow U_i^{\chi} = e^{i\chi f_{i\alpha}^{+} f_{i\alpha}}$$

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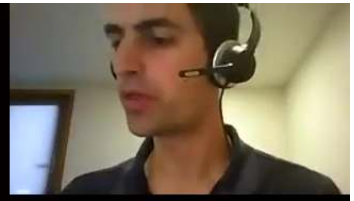

Substitute into (*):

$$e^{-i\chi(e_{ij}+e_{ik}+e_{il}+e_{im})} \cdot e^{i\chi(f_{i\alpha}^{+}f_{i\alpha}-1)} = 1$$

$$\Rightarrow e_{ij}+e_{ik}+e_{il}+e_{im} = f_{i\alpha}^{+}f_{i\alpha}-1$$

$$\Rightarrow (\text{dive})_i = f_{i\alpha}^{+}f_{i\alpha}-1$$

Properties of π-flux state

1. $|\Psi_{spin}\rangle$ invariant under spin rotation and lattice symmetries.

   "spin liquid"

2. Low energy theory consists of 4 2-component Dirac fermions coupled to U(1) gauge field.

"compact $(QED)_3$"

G - gauge field.

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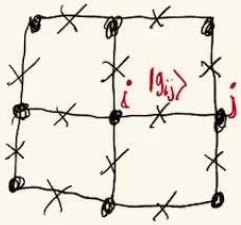
$|g_{ij}\rangle = |(g^{-1})_{ji}\rangle$



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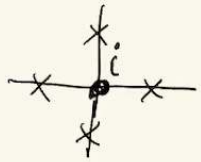

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