Title: Partons 1

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Parton construction

Method for constructing interesting many-body states and low energy theories. (~1980s)

Applications

1. Spin liquids
2. Quantum Hall states

$\vdots$

1. Spin liquids
2. Quantum Hall states

⋮

References:

SL
1. Wen, QFT of many-body systems
2. Balents + Savary, 1601.03742

QH
3. Blok + Wen, PRB 42, 8133 (1990)
4. Wen, PRL 66, 802 (1991)

Parton construction of spin liquids

Setting: spin $\frac{1}{2}$ model on square lattice

How can we construct interesting spin states?

parton construction of

spin liquids

Setting: spin $\frac{1}{2}$ model on square lattice

How can we construct interesting spin states?

2 parton-based approaches:

1. "slave fermions"
2. "slave bosons" or "schwinger bosons"

Slave fermion construction

Consider an enlarged Hilbert space describing spin-$\frac{1}{2}$ fermions on square lattice.

4 states

$|0\rangle$

$f_\uparrow^\dagger |0\rangle$

$f_\downarrow^\dagger |0\rangle$

$f_\uparrow^\dagger f_\downarrow^\dagger |0\rangle$

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Note that $\mathcal{H}_{spin} \subseteq \mathcal{H}_{fermion}$:

$$|\uparrow\rangle \longleftrightarrow f^\dagger_\uparrow |0\rangle$$
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In terms of operators:

$$\vec{S} = \frac{1}{2} f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta \qquad \alpha, \beta = \uparrow, \downarrow$$

Now consider a quadratic fermion Hamiltonian:

$$H_0 = \sum_{ij}$$

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Now consider a quadratic fermion Hamiltonian:

$$H_0 = \sum_{ij} \left(t_{ij}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta} + \Delta_{ij}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger + h.c. \right) - \sum_i \mu_i f_{i\alpha}^\dagger f_{i\alpha}$$

Let $|\Psi_0\rangle$ be gd. state of H_0

Now consider a quadratic fermion Hamiltonian:

$$H_0 = \sum_{ij} \left(t_{ij}^{\alpha\beta} f_{i\alpha}^{+} f_{j\beta} + \Delta_{ij}^{\alpha\beta} f_{i\alpha}^{+} f_{j\beta}^{+} + h.c. \right) - \sum_{i} \mu_i f_{i\alpha}^{+} f_{i\alpha}$$

Let $|\Psi_0\rangle$ be gd. state of H_0

$|\Psi_0\rangle \notin \mathcal{H}_{spin}$ since it doesn't obey the constraint $f_{i\alpha}^{+} f_{i\alpha} = 1$

$$- \sum_i \mu_i f_{i\alpha}^{+} f_{i\alpha}$$

Let $|\Psi_0\rangle$ be gd. state of H_0

$|\Psi_0\rangle \notin \mathcal{H}_{spin}$ since it doesn't obey the constraint $f_{i\alpha}^{+} f_{i\alpha} = 1$.

We will be interested in case where $|\Psi_0\rangle$ obeys the constraint <u>on average</u>:

$$\langle f_{i\alpha}^{+} f_{i\alpha} \rangle_{\Psi_0} = 1$$

Any H_0 with this property is called a "mean field ansatz."

Slave-fermion construction takes ansatz H_0 as input and produces:

1. A spin state $|\Psi_{spin}\rangle$
2. A low energy theory H_{eff}.

1. A spin state $|\Psi_{spin}\rangle$

2. A low energy theory H_{eff}.

The main claim/hope:

1. $|\Psi_{spin}\rangle$ is the gd. state of some local spin Hamiltonian.

2. H_{eff} correctly describes the low energy excitations of such a spin Hamiltonian,

Construction of $|\Psi_{spin}\rangle$

We construct $|\Psi_{spin}\rangle$ by projecting $|\Psi_0\rangle$ into $\mathcal{H}_{spin}$:

$$|\Psi_{spin}\rangle = P \cdot |\Psi_0\rangle$$

projection into $\mathcal{H}_{spin}$

Equivalently:

$\Psi_{spin}(\alpha_1$

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Equivalently:

$$\Psi_{spin}(\alpha_1, \dots, \alpha_M) = \langle 0| f_{1\alpha_1} f_{2\alpha_2} \cdots f_{M\alpha_M} |\Psi_0\rangle$$

vacuum

$\alpha_i = \uparrow, \downarrow$

$M = \#$ spins

Construction of low energy theory H_{eff}

First, find all onsite unitary symmetry transformations U such that:

(1) $U^{\dagger} H_0 U = H_0$

(2) U acts trivially with $\mathcal{H}_{spin}$,
i.e. $U^{\dagger} \vec{S}_i U = \vec{S}_i$ for all i.

$\{U\}$ = "invariant gauge group" G

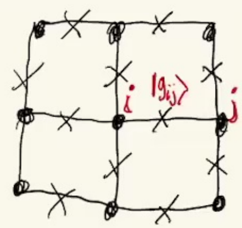
Notation: $\{U^g,\ g \in G\}$

To construct low energy theory, we couple H_0 to a weakly-fluctuating G-gauge field.

Recipe:

Step 1: Introduce gauge fields on links

$$|g_{ij}\rangle, \quad g \in G$$



G-gauge field.

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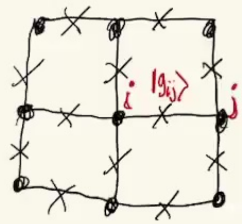


Step 2: Write down gauge theory Hamiltonian

$$H_{eff} = \tilde{H}_0 + H_{gauge}$$

$\tilde{H}_0$ ← H_0 minimally coupled to G-gauge field

Step 3: Restrict to gauge invariant subspace

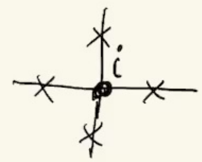
Specifically, restrict to states $|\Psi\rangle$

s.t.

$$\prod_{\langle ij\rangle} L_{ij}^{g} \cdot \left(e^{-i\theta_i^g} U_i^g \right) |\Psi\rangle = |\Psi\rangle$$

for each lattice site i

and each $g \in G$.



Here: $L_{ij}^{g} \, |h_{ij}\rangle = |(gh)_{ij}\rangle$

U_i^g, θ_i^g defined by

$$U^g = \prod_i U_i^g$$

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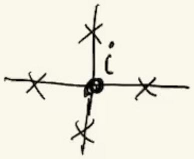


Here: $L^{g}_{ij} |h_{ij}\rangle = |(gh)_{ij}\rangle$

U^{g}_{i}, θ^{g}_{i} defined by

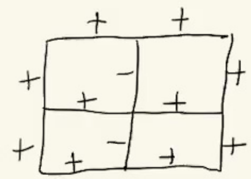
$$U^{g} = \prod_{i} U^{g}_{i} \qquad U^{g}_{i}\Big|_{\mathcal{H}_{spin}} = e^{i\theta^{g}_{i}} \cdot \mathbb{1}$$

Example 1: "π-flux state"

$$H_0 = \sum_{\langle ij \rangle} \left(t_{ij} f^{+}_{i\alpha} f_{j\alpha} + h.c. \right)$$

$$t_{i,i+\hat{x}} = t$$

$$t_{i,i+\hat{y}} = (-1)^{i_x} \cdot t$$

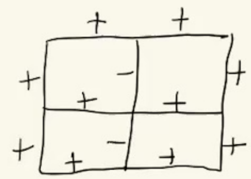


Let's construct low energy theory.

Symmetry transformations that leave H_0, $\vec{S}$ invariant:

$$f_{i\alpha} \rightarrow e^{i\alpha} f_{i\alpha}$$

$$H_0 = \sum_{\langle ij\rangle} \left(t_{ij} f^\dagger_{i\alpha} f_{j\alpha} + h.c.\right)$$



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Symmetry transformations that leave $H_0, \vec{S}$ invariant:

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$$\Rightarrow G = U(1)$$

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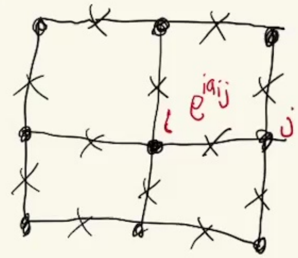
Introduce $U(1)$ gauge fields on links: $e^{ia_{ij}}$



Low energy theory:

$$H_{eff} = \sum_{\langle ij \rangle} \left(t_{ij} e^{ia_{ij}} f^{+}_{i\alpha} f_{j\alpha} + h.c. \right)$$

$$+ H_{gauge}$$

Low energy theory:

$$H_{eff} = \sum_{\langle ij \rangle} \left(t_{ij} e^{i a_{ij}} f^{\dagger}_{i\alpha} f_{j\alpha} + h.c. \right) + H_{gauge}$$

$$H_{gauge} = h \sum_{\langle ij \rangle} e_{ij}^2 - K \sum_{\square} \cos(a_{pq} + a_{qr} + a_{rs} + a_{sp})$$

Here: $[a_{ij}, e_{ij}] = i$

vector potential electric field

$h <<$

$$H_{gauge} = h \sum_{\langle ij \rangle} e_{ij}^2 - K \sum_{\square} \cos(a_{pq} + a_{qr} + a_{rs} + a_{sp})$$

Here: $[a_{ij}, e_{ij}] = i$

vector potential → a_{ij}; electric field → e_{ij}

$$h \ll t \ll K \quad (a_{ij} \text{ weakly fluctuating})$$

Let's find gauge invariance constraint.

$$U^{\chi} =$$

Here: $[a_{ij}, e_{ij}] = i$

vector potential

electric field

$$h \ll t \ll K$$

(a_{ij} weakly fluctuating)

Let's find gauge invariance constraint.

$$U^{\chi} = e^{i\chi \sum_i f^{\dagger}_{i\alpha} f_{i\alpha}}$$

$$\Rightarrow U_i^{\chi} = e^{i\chi f^{\dagger}_{i\alpha} f_{i\alpha}}$$

$$\Rightarrow U_i^{\chi}\Big|_{\mathcal{H}_{spin}} = e^{i\chi} \cdot \mathbb{1}$$

$$\Rightarrow e^{i\theta_i^{\chi}} = e^{i\chi}$$

$$U^{\chi} = e^{i\chi \sum_i f^{\dagger}_{i\alpha} f_{i\alpha}}$$

$$\Rightarrow \; U_i^{\chi} = e^{i\chi f^{\dagger}_{i\alpha} f_{i\alpha}}$$

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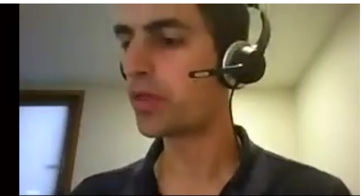

Substitute into (*):

$$e^{-i\chi(e_{ij}+e_{ik}+e_{il}+e_{im})}\cdot e^{i\chi(f_{i\alpha}^{+}f_{i\alpha}-1)}=1$$

$$\Rightarrow e_{ij}+e_{ik}+e_{il}+e_{im}=f_{i\alpha}^{+}f_{i\alpha}-1$$

$$\Rightarrow (\mathrm{div}\, e)_i = f_{i\alpha}^{+}f_{i\alpha}-1$$

Properties of π-flux state

1. $|\Psi_{spin}\rangle$ invariant under spin rotation and lattice symmetries.

   "spin liquid"

2. Low energy theory consists of 4 2-component Dirac fermions coupled to U(1) gauge field.

"compact $(QED)_3$"

G-gauge field.

Recipe:

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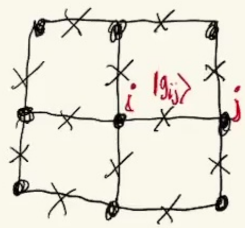

$$|g_{ij}\rangle, \quad g \in G$$

$$|g_{ij}\rangle = |(g^{-1})_{ji}\rangle$$

Step 2: Write down gauge theory Hamiltonian

$$H_{eff} = \tilde{H}_0 + H_{gauge}$$ ← gauge field terms

$\tilde{H}_0$: H_0 minimally coupled to G-gauge field

Step 3: Restrict to gauge invariant subspace

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$\hookrightarrow$ H_0 minimally coupled to G-gauge field

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s.t.

$$(*) \quad \prod_{\langle ij \rangle} L_{ij}^{g} \cdot \left(e^{-i\theta_i^g} U_i^g \right) |\Psi\rangle = |\Psi\rangle$$

for each lattice site i

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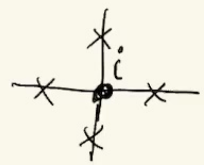

$a_{ij} \to a_{ij} + \chi$

$f_i \to e^{i\chi} f_i$

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