Title: Dualities 2

Speakers: Andreas Karch

Collection: Online School on Ultra Quantum Matter

Date: August 12, 2020 - 11:00 AM

URL: http://pirsa.org/20080007

DUALITIES — LECTURE 2

GOALS: 90's WISDOM:

TOOLS FROM PT

HOW TO DERIVE/

TEST / FIND DUALITIES

(A) T-DUALITY (B) SEIBERG

WARNING: SUSY MAGIC WILL BE USED W/O PROOF

× (A) T- DUALITY IN $d=2$

$d=2 = 1+1 \to$ 1SPACE ONE TIME

" PARTICLE- VORTEX "

FIND RIGHT VARIABL.

DUALITY:

$$Z = \int_{-\infty}^{\infty} dx\, e^{-x^2/a} = \int dy\,(1+2y)\, e^{-(y+y^2)^2/a}$$

" FREE "

$x = y + y^2$ INTER.

PI: INTEGR VAR

$\Leftarrow$ DEGREES OF FREEDOM

Ex $\quad Z = \int \mathcal{D}\phi \, e^{-S}$

<u>1+1 → 2</u> $\quad S = \frac{1}{4\pi} \int d^2x \, \frac{R^2}{2} (\partial\phi)^2$

$(\alpha' = 2)$ $\quad \phi \cong \phi + 2\pi$

WINDING: $\quad \phi(x+2\pi) = \phi(x) + \omega 2\pi$

$x, t \,, \quad x \cong x + 2\pi \qquad \omega \in \mathbb{Z}$

SPECTRUM:

$$X = \underbrace{pt + wX}_{\text{ZERO MODES}} + \text{OSCILL.}$$

QM $\quad P = \frac{n}{R} \Rightarrow E = \frac{n}{R}$

WINDING : $\quad E = \frac{wR}{2}$

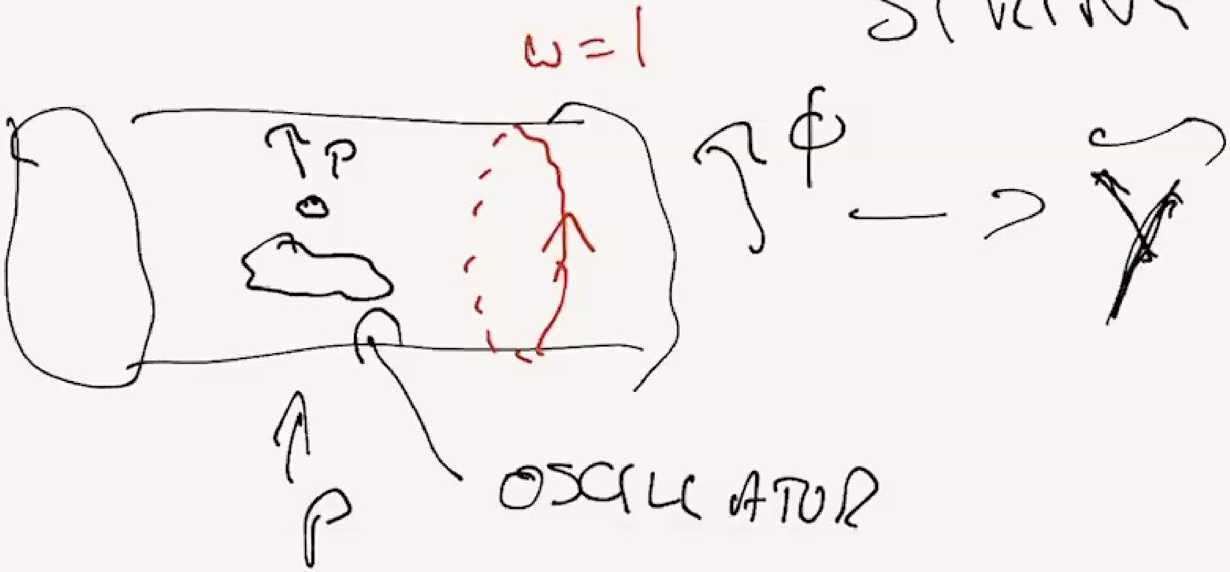
SEE: $\quad R \longleftrightarrow \frac{2}{R} \qquad n \longleftrightarrow w$

SAME SPECTRUM

ϕ : COMPACT

→ COORDINATE IN TARGET SPACE Y

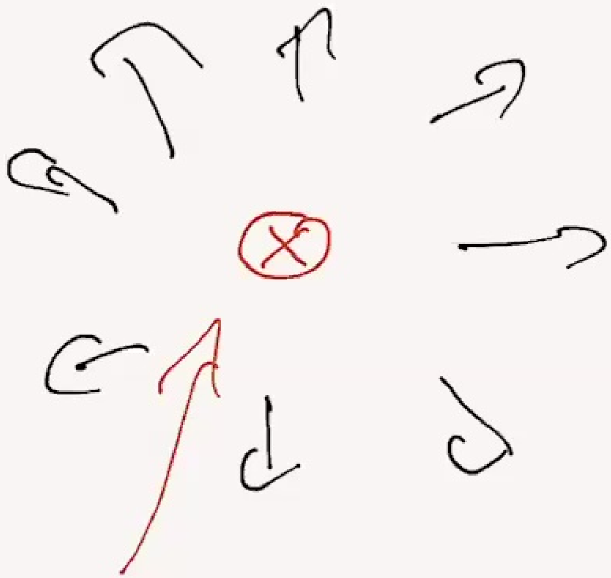
$\phi(x,t)$ → EMBEDDING OF STRING INTO TARGET



T-DUALITY EXT. TO OSCILLATOR SPECTRUM

T-DUALITY & PT-VORTEX

$\mathbb{R}^2$: ϕ WIND AROUND ∞ $\qquad \uparrow \simeq \phi$



VORTEX $\Leftrightarrow$ $\omega = 1$

SING. IN CONT. LIMIT

REMOVE ORIGIN $\rightarrow$ CYLINDER

KEEP VORTEX

T-DUALITY $\Leftrightarrow$ PT-VORT. DUALITY

IDEA OF DERIVATION

$$Z = \int \mathcal{D}\phi \, e^{-S(\phi)}$$

$$= \int \mathcal{D}\phi \, \mathcal{D}\tilde{\phi} \, e^{-S(\phi, \tilde{\phi})}$$

$$= \int \mathcal{D}\tilde{\phi} \, e^{-\tilde{S}(\tilde{\phi})}$$

TOOL: EXPLOIT SHIFT SYMM.

* $\phi \to \phi + c$

* GAUGE: $\partial_\mu \phi \to \mathcal{D}_\mu \phi = \partial_\mu \phi + A_\mu$

$\phi \to \phi + c(x,t)$

$A_\mu \to A_\mu - \partial_\mu c$

x CHANGED THEORY: A_μ

x MAKE SURE: $A_\mu = 0 \to$ BACK

$$\int \mathcal{D}\phi \to \int \mathcal{D}\phi \underbrace{\mathcal{D}A_\mu}_{\text{GAUGE}} \delta(F_{\mu\nu})$$

$$\int \mathcal{D}\phi\, \mathcal{D}A_\mu\, \delta(F_{\mu\nu}) = \int \mathcal{D}\phi\, \mathcal{D}A\, \mathcal{D}\lambda\; e^{\frac{i}{2\pi}\int \lambda\, \epsilon^{\mu\nu}\partial_\mu A_\nu}$$

EUCL: i IN ACTION

LOR: i FOR FREE

S REAL

$F_{\mu\nu} = 0 \implies A_\mu = 0$ ON $\mathbb{R}^2$

CYLINDER? TORUS?

$$\oint_{\gamma_a} dx_\mu A^\mu = \theta_a \rightarrow$$ GAUGE INV.

CLOSED LOOP

CLAIM: λ PERIODIC $\Rightarrow$ $\lambda \equiv \tilde{\phi}$

$\Rightarrow$ $\theta_a = 0$

$$S = \cdots + \frac{i}{2\pi} \int \lambda \, \varepsilon^{\mu\nu} \partial_\mu A_\nu \equiv S_{WIND}$$

$$S_{WIND} = n_a \theta_b + n_b \theta_a$$

$\Uparrow$

WINDING OF λ

$\oint \partial \lambda$ $\quad n_a, n_b \in \mathbb{Z}$

$\int \mathcal{D}\lambda$ INCLUDES $\sum_{n_a, n_b}$ $\Rightarrow$ S_{WIND} DISCRETE

$$\int \mathcal{D}\phi \; e^{-S(\phi)} = \int \mathcal{D}\phi \, \mathcal{D}A_\mu \, \mathcal{D}\tilde{\phi} \; e^{-\bar{S}}$$

$$\bar{S} = \underline{S(\partial_\mu \phi)} + \frac{i}{2\pi} \int \tilde{\phi} \, \varepsilon^{\mu\nu} \underline{\partial_\mu A_\nu}$$

NOW: $\int \mathcal{D}\phi \, \mathcal{D}A_\mu \longrightarrow \tilde{S}(\tilde{\phi})$

ACTION QUADRATIC !

<u>GAUSSIAN</u> $\cong$ SOLVING EOM + PLUGGING IN

EOM: $\frac{R^2}{4\pi} A_1 = \frac{i}{2\pi} \partial_2 \tilde{\phi}$

$$\frac{R^2}{4\pi} A_2 = -\frac{i}{2\pi} \partial_1 \tilde{\phi}$$

$$\longrightarrow \quad \tilde{S} = \frac{1}{4\pi} \int \frac{\tilde{R}^2}{2} \partial \tilde{\phi}^2$$

$$\boxed{\tilde{R} = \frac{2}{R}}$$

NOTES: CAN DO FOR MANY OTHER DUALITIES

$$S = \int \overset{F}{dH_p} \wedge {}^*dH_p$$

p=0: SCALAR [illegible]

p=1: MAXWELL

p=2: [illegible] $H_{\mu\nu} \to F_{\mu\nu\rho}$

$$\int \mathcal{D}H \to \int \mathcal{D}F \mathcal{D}\lambda \, e^{i\int \lambda dF}$$

↑ BIANCHI! $dF = 0$

$\to$ DO $\int \mathcal{D}F \to S(\lambda)$

2d: SCALAR → SCALAR

3d: VECTOR → COMPACT SCALAR

4d: EM DUALITY

* ALSO WORKS FOR SOME INTERACT. THEORIES

$$S = \frac{1}{4\pi}\int d^2x \Big\{ G^{\mu\nu}(\phi)\,\partial_\mu\phi\,\partial_\nu\phi + B^{\mu\nu}(\phi)\,\partial_\mu\phi\,\partial_\nu\phi + \Phi(\phi)\,R \Big\}\sqrt{g}$$

$$G^{\mu\nu} = G^{\nu\mu}, \qquad B^{\mu\nu} = -B^{\nu\mu}$$

$+ \dots + \phi^{\text{POWER}} (\partial \phi)^2 + \dots$

$\uparrow$

INTERACTIONS

$\partial \phi \to D\phi \to$ STILL QUADRATIC IN A's

GAUGE-FIX: $\phi = 0$

PI $\int \mathcal{D}A$ DO ABLE

$\longrightarrow$ DUAL ACTION

SEE $\tilde{g}_{\mu\nu}$, $\tilde{B}_{\mu\nu}$, $\tilde{\phi}$

EX 2: SEIBERGS DUALITY

T-DUALITY TOO GOOD TO BE ~~TRUE~~ INTEREST.

CLAIM:

$$SU(N_c) + N_f$$

$$\Uparrow$$

$$SU(N_f - N_c) + N_f$$

$\uparrow$ MATTER IN FUND.

HOW DO WE KNOW THIS IS TRUE?

IR – DUALITY

(A) $\neq$ (B)

RG FLOW

SAME

SOMETIMES

(B) = FREE

(A) NOT

$$\beta^{1\text{-LOOP}} = -\frac{g^3}{16\pi^2}\, b_0$$

$$b_0 = \frac{11}{3}\mu_{adj} - \frac{\mu_S}{6} - \frac{2}{3}\mu_F$$

Say $\mu = \mathrm{tr}\,(T_a T_b)$ $\qquad \mu = \frac{1}{2}$ FUND

$\mu = N_c$ adj.

SUSY: A_μ, λ $\qquad \psi, \phi$

$$b_0 = N_c\left(\frac{11}{3} - \frac{2}{3}\right) - N_f\left(\frac{1}{3} + \frac{2}{3}\right)$$

$$= 3N_c - N_f$$

WEAKL. CPLD

3/2 N_c

MESS

FREE

CONFINING

CONFORM

$3N_c$

N_f

CL

3/2 N_c

FREE

wk

$3N_c$

CONFM.

MESS

$$SU(\underbrace{N_f - N_c}_{\tilde{N}_c})$$

$$\boxed{N_f = \frac{3}{2} N_c}$$

$$\frac{3}{2}\tilde{N}_c = N_f = 3\tilde{N}_c$$

$$N_f = 3N_f - 3N_c$$

HOW TO CHECK?

* OPERATOR MATCHING
  (SUSY MAGIC)

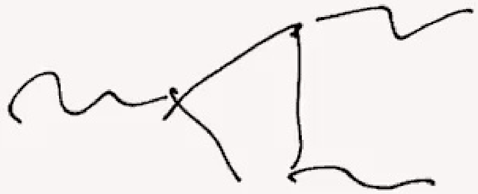
* ANOMALIES

* DEFORMATIONS

ELECTRIC

	$SU(N_C)$	$SU(N_F)$	$\times\, SU(N_F)$	$\times\, U(1)_A$	$\times\, U(1)_B$	$\times\, U(1)_R$
Q	$\square$	$\square$		$+1$	$+1$	r_Q
ψ	$\square$	$\square$		$+1$	$+1$	$r_Q - 1$
$\tilde{Q}$	$\overline{\square}$		$\square$	$+1$	-1	r_Q
$\overline{\psi}$	$\overline{\square}$		$\square$	$+1$	-1	$r_Q - 1$
λ	adj					$+1$
A_M	adj					

ANOMALIES

 $\neq 0$

$$\partial_\mu j^\mu = \underset{\mathrm{tr}(T^a T^b T^c)}{A} \; F_{\mu\nu} F_{\rho\sigma} \varepsilon^{\mu\nu\rho\sigma}$$

L : LOCAL

G = GLOBAL

	J	F	
	L	L	→ TRASH
$U(1)_A$ ∈	G	L	→ DUST OUT SYMM
't HOOFT	G	G	

el; α^3

$$\tilde{q}, q: \; N_f N_c \left(-\frac{N_c}{N_f}\right)^3 \times 2 + (N_c^2-1)\, 1^3$$

$$= N_f (N_f - N_c) \left(\frac{N_c}{N_f} - 1\right)^3 \times 2 + N_f^2 \left(1 - \frac{2N_c}{N_f}\right)^3 + \left[(N_f - N_c)^2 - 1\right] \cdot 1^3$$

SINGLET

DEFORMATIONS:

m : $\quad N_f \to N_f - 1 \quad \checkmark$

DUAL: POTENTIAL:

MASS $\to$ FORCE HIGGSING

$\tilde{N}_c \to \tilde{N}_c - 1 \qquad N_f$

$\qquad\qquad\qquad\qquad\qquad \downarrow \quad \checkmark$

$N_f - N_c \to N_f - N_c - 1 \qquad N_f - 1$