Title: Exactly Solvable Topological and Fracton Models as Gauge Theories 1

Speakers: Xie Chen

Collection: Online School on Ultra Quantum Matter

Date: August 11, 2020 - 1:45 PM

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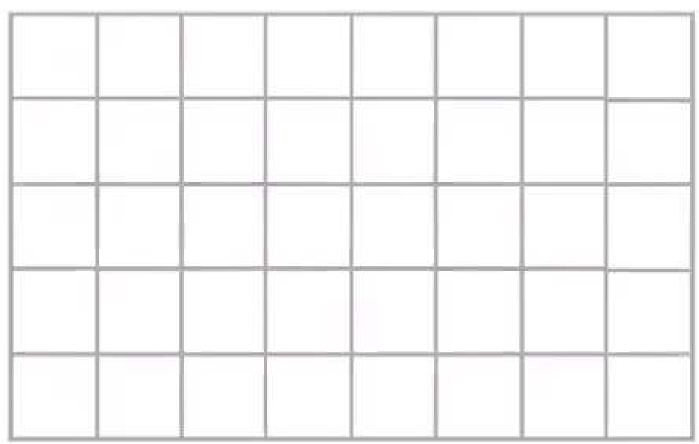
Topo-Fracton

Monday, August 10, 2020 10:19 AM

Exactly Solvable Topological and Fracton Models as Gauge Theories

1. Toric Code
2. Toric Code as Gauge Theory
3. Toric Code from Gauging
4. Double Semion from Gauging
5. X-cube from Gauging

1. Toric Code.



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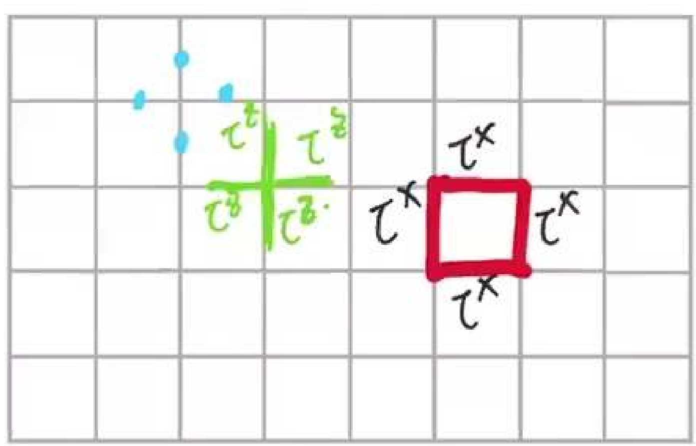
- qubit $|0\rangle, |1\rangle$
  τ.



$|\psi\rangle =$

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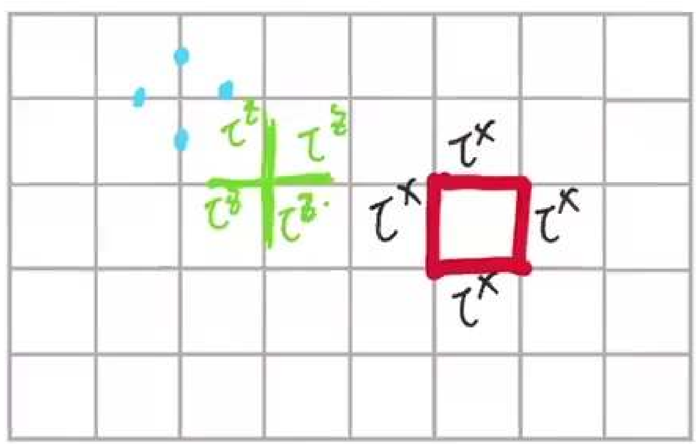
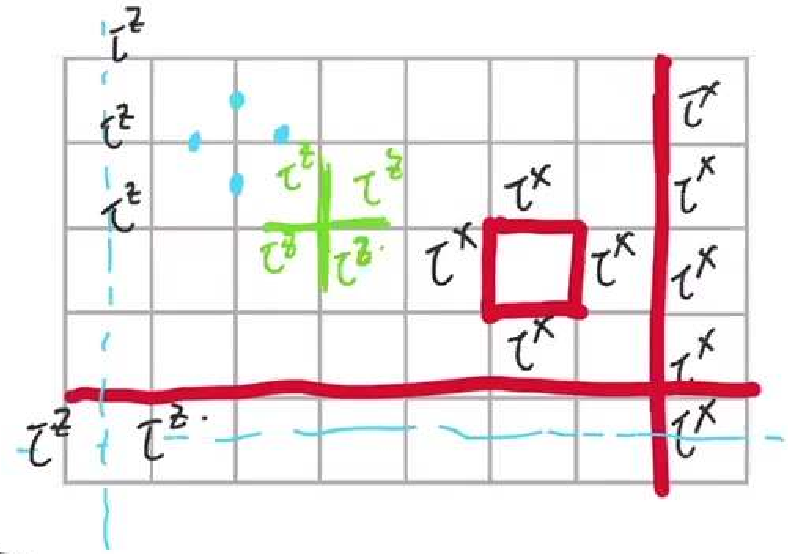
$$|\psi\rangle = \sum_c |c\rangle$$

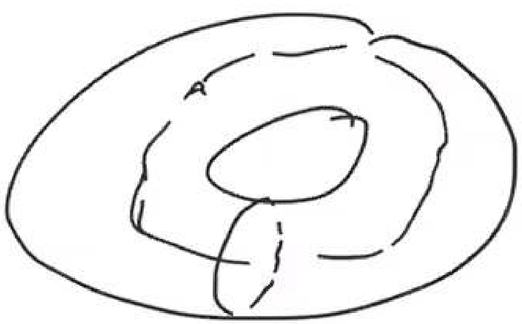


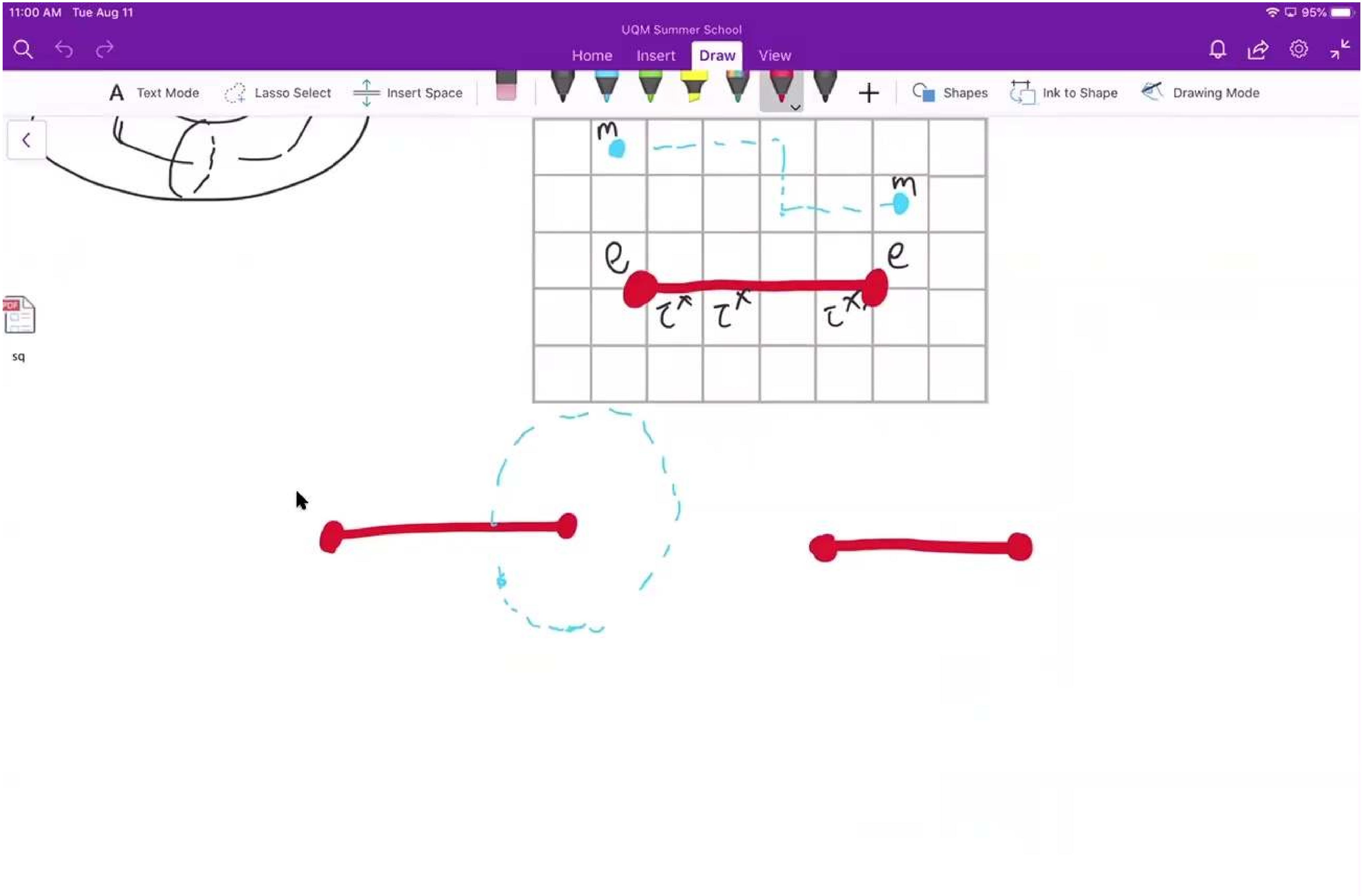
1. Toric Code.

- qubit τ. $|0\rangle, |1\rangle$



$$|\psi\rangle = \sum_c |c\rangle$$

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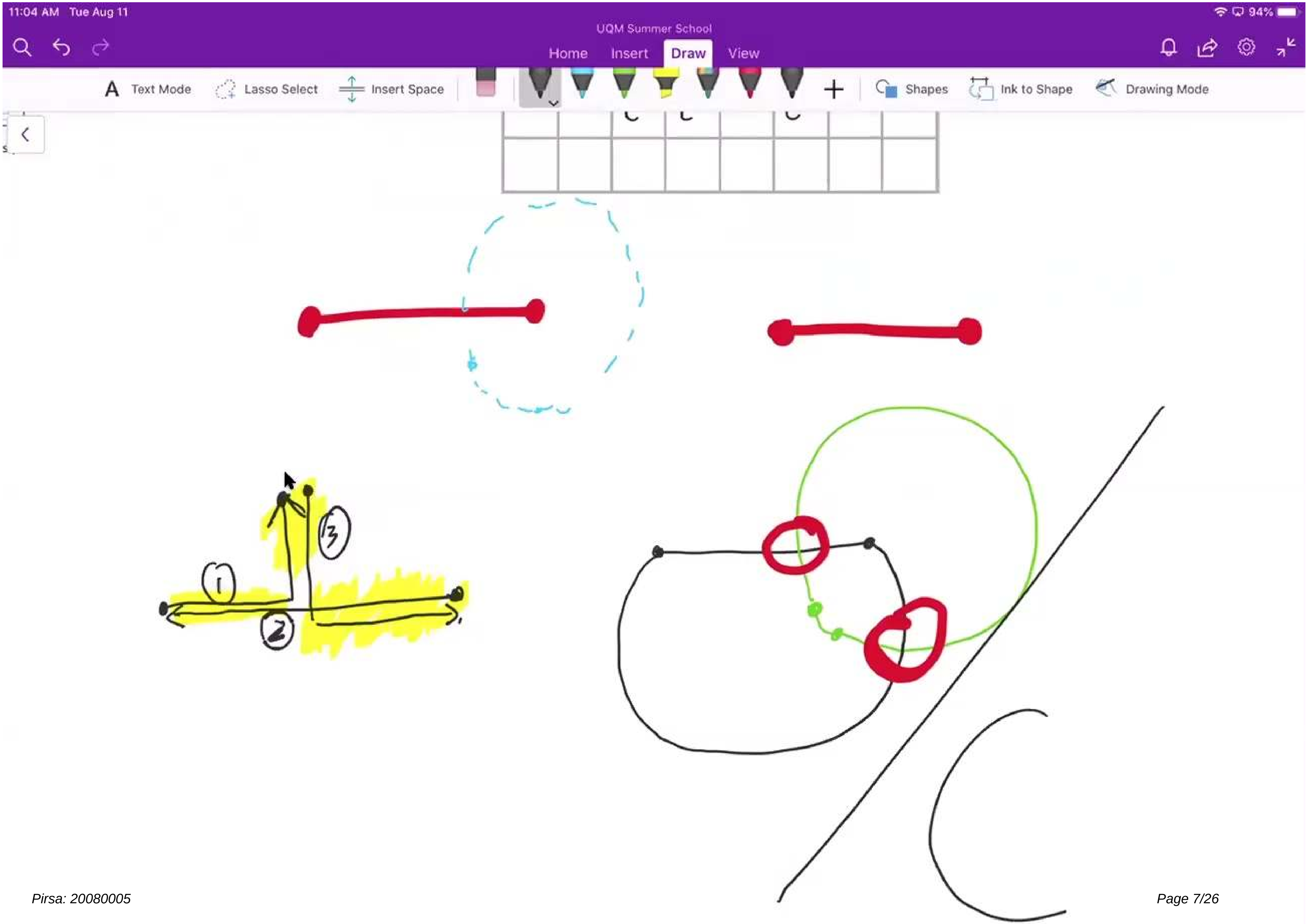
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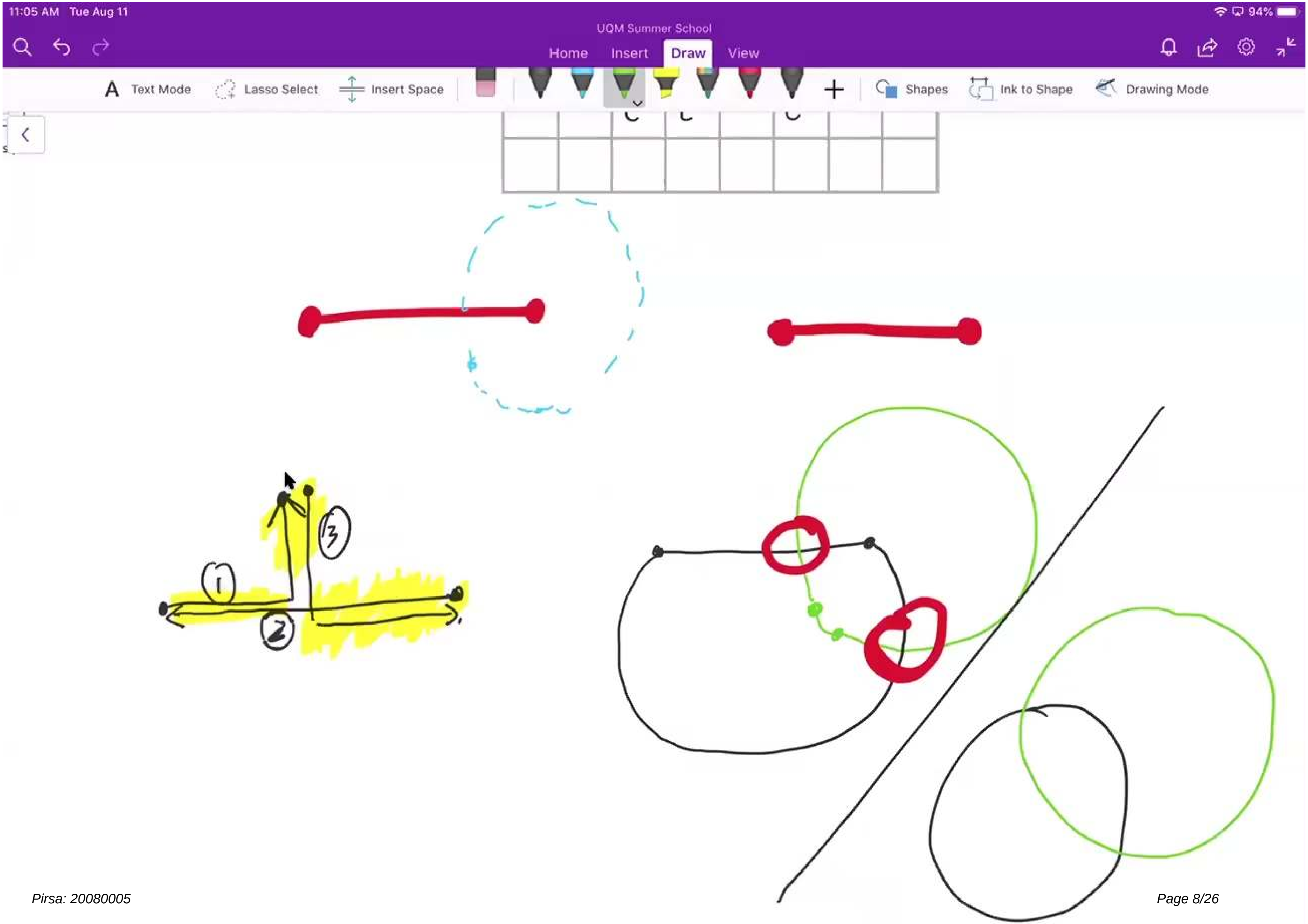
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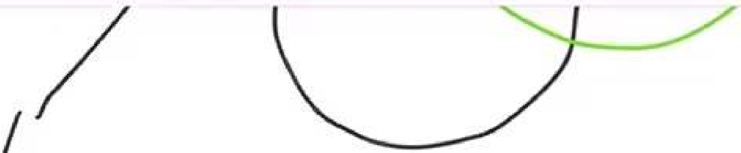

2. TC as Gauge Theory.

Classical E&M

$$\vec{B} = \nabla \times \vec{A} \qquad \vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t}.$$

2. TC as gauge th'y.

Classical E & M

$$\vec{B} = \nabla \times \vec{A} \qquad \vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t}.$$

$$\vec{A} \to \vec{A} + \nabla f \,, \quad \varphi \to \varphi - \frac{\partial f}{\partial t}$$

$f(\vec{r}, t)$.

"gauge" → "Local"

$\vec{A} \to \vec{A} + \nabla f, \quad \varphi \to \varphi - \frac{\partial f}{\partial t}$

$f(\vec{r}, t).$

"gauge" → "Local"

Lattice Quantum E&M

rotor DOF

Lattice Quantum E&M

rotor DOF $\phi \sim [0, 2\pi)$ $\quad A$

$L \sim \mathbb{Z}$ $\quad E$

$$[E, A] = -i$$

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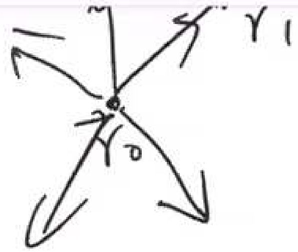
$$\vec{A} \to \vec{A} + \nabla f(\vec{r}),$$

$$f(\vec{r}) = f_0\, \delta_{\vec{r},\vec{r}_0}.$$

$$A\langle \vec{r}_0 \vec{r}_1 \rangle \longrightarrow A\langle \vec{r}_0 \vec{r}_1 \rangle - f_0$$



$$U_{\vec r_0}(f_0) = \prod_{\vec r_1} e^{i f_0 E_{\langle \vec r_0 \vec r_1 \rangle}}$$

$A \in [0, 2\pi)$ $f_0 \in [0, 2\pi)$.

$U_{\vec r_0}(f_0)$ forms a $U(1)$ group.

$U(1)$ gauge symmetry.

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$\mathbb{Z} \bmod 2.$

$e^{iA}: +1, -1 \qquad e^{i\pi E}: +1, -1$

$$e^{iA} e^{i\pi E} = - e^{i\pi E} e^{iA}.$$

rotor $\rightarrow$ qubit.

$e^{iA} \longrightarrow \tau^x \qquad e^{iE} \longrightarrow \tau^z.$

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$U_{\vec{\gamma}_0}(f_0)$ forms a $U(1)$ group.

$U(1)$ gauge symmetry.

$$U \sim e^{i f_0 \int_s d\vec{s} \cdot \vec{E}}.$$

$$U_{\vec{\gamma}_0} = 1 \Longleftrightarrow \nabla \cdot \vec{E} = 0.$$

Break down $U(1) \to Z_2$.

$[0, 2\pi) \to 0, \pi$ $\quad \mathbb{Z} \to 0, 1$

$\mathbb{Z} \bmod 2$.

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Break down $U(1) \to Z_2$.

$[0, 2\pi) \to 0, \pi$ $\qquad$ $\mathbb{Z} \to 0, 1$

$\mathbb{Z} \bmod 2$.

e^{iA}: $+1, -1$ $\qquad$ $e^{i\pi E}$: $+1, -1$

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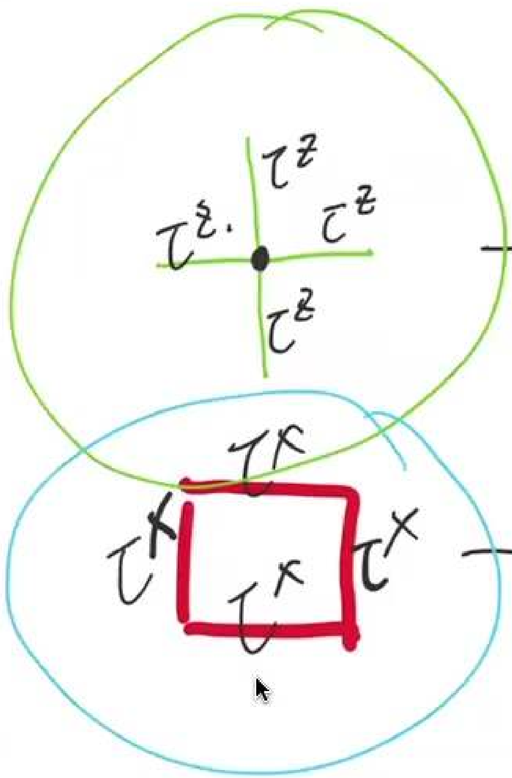

$$\to e^{i\pi\Sigma E} = U_v. \sim U_{\vec{r}_0}$$

$$\to e^{i\Sigma A}.$$

$$e^{i\Sigma A} + e^{-i\Sigma A}$$

$$= \cos(\Sigma A).$$

$$= \cos(\phi).$$

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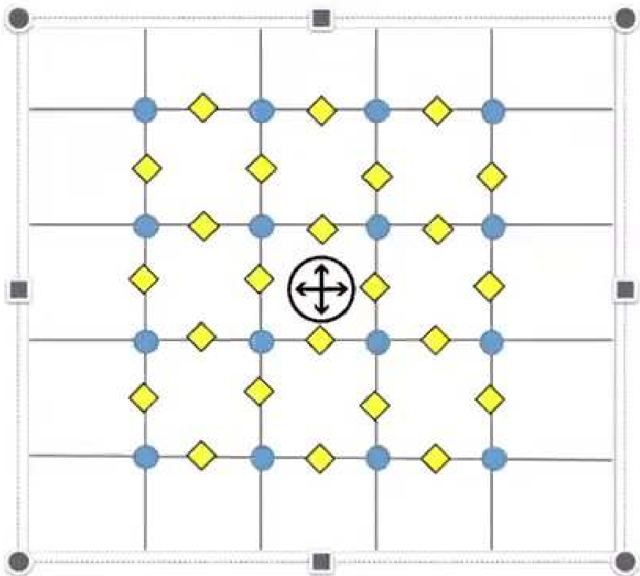
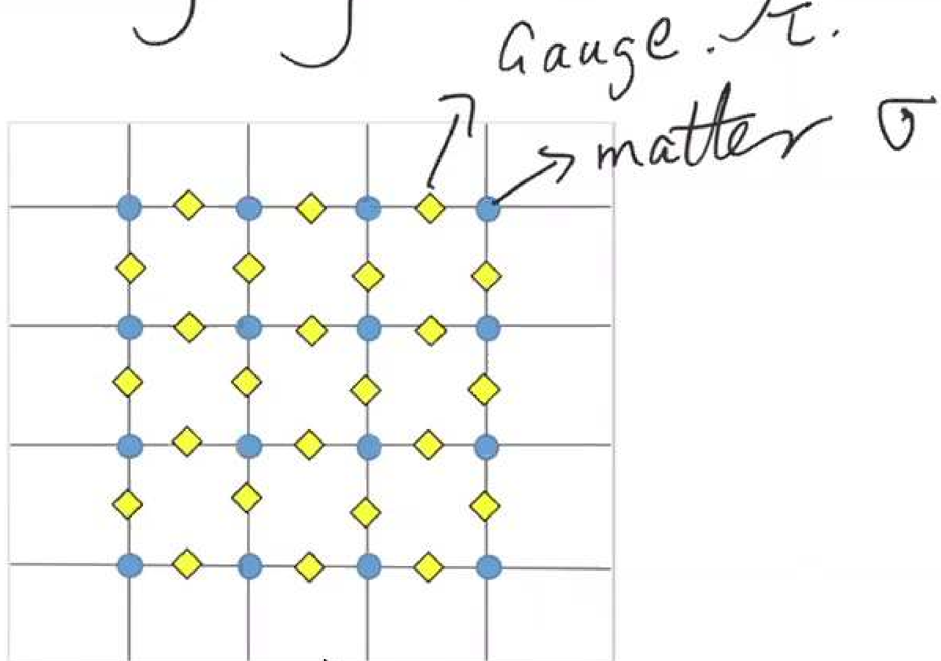
$= \cos(\phi).$

4. TC from Gauging the Ising Paramagnet.





$$H = -J \sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j + B \sum_i \sigma^x_i$$

$$U = \prod_i \sigma^x_i$$

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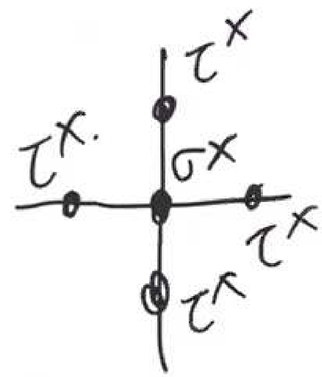
$B >> J \qquad J = 0,$

Gauge $\longrightarrow$ Local Z_2 sym.

τ : qubit

Gauge sym :



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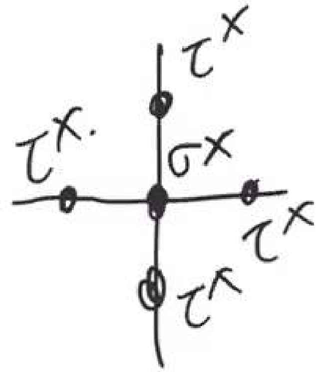
τ : qubit

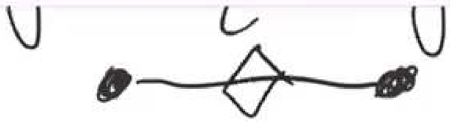
Gauge sym:



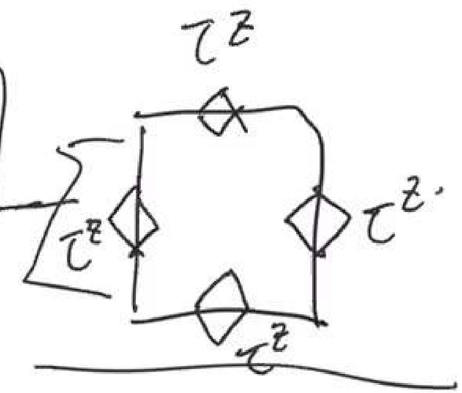
$\rightarrow e^{i\pi(\Sigma E - n_v)}$

$\sigma^x \sim e^{i\pi n_v}$.

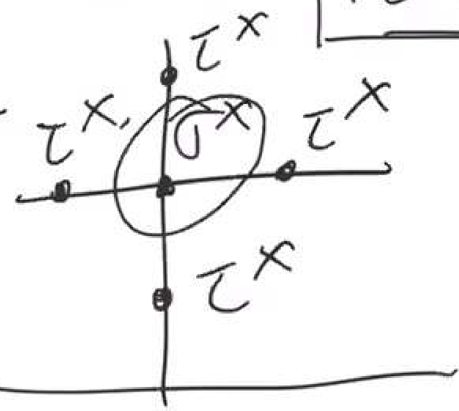
$$\Sigma E = n_v \pmod 2,$$



$$H_g = -J\sum \sigma^z \tau^z \sigma^z + B\sum \sigma^x - \sum$$



$$-\sum$$



$$J = 0.$$

$$H_g(\sigma^x = 1) \longrightarrow H_{TC}.$$

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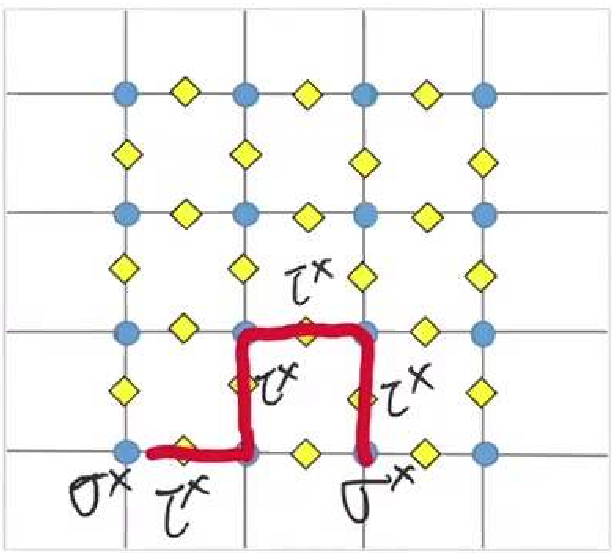
$$\mathcal{H}_g(\sigma^x = 1) \longrightarrow \underline{\mathcal{H}_{TC}}.$$



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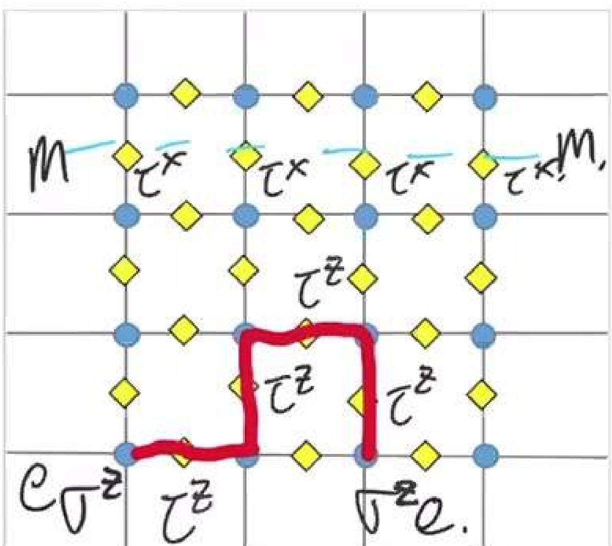
$$H_g(\sigma^x = 1) \longrightarrow \underline{H_{TC}}.$$



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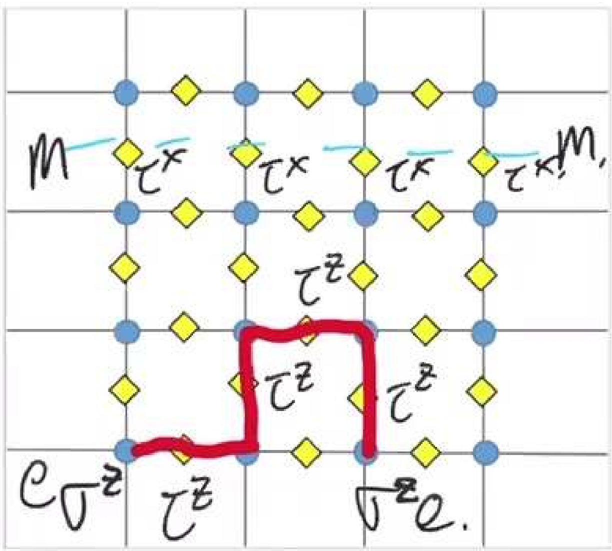

m bosonic

e bosonic.

e~m braiding $-1 = e^{i\pi}$.

$e^{i\phi}$.