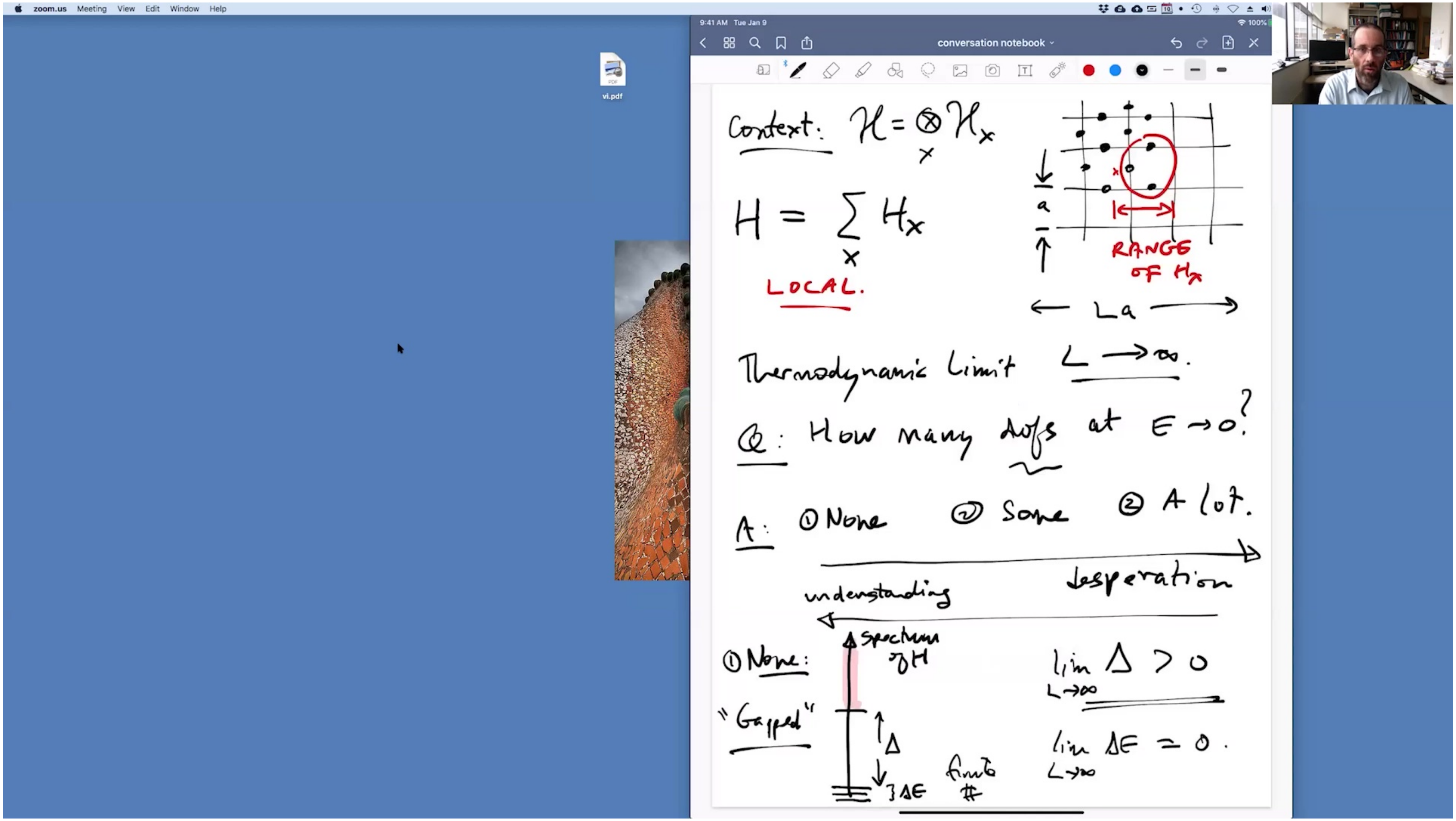
Title: Quantum Phases of Matter and Entanglement Basics

Speakers: John McGreevy

Collection: Online School on Ultra Quantum Matter

Date: August 10, 2020 - 11:10 AM

URL: http://pirsa.org/20080001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vi.pdf
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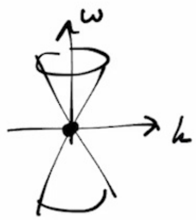
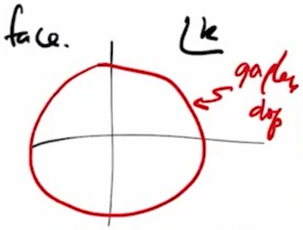
conversation notebook
Context: $\mathcal{H} = \bigotimes_x \mathcal{H}_x$
$H = \sum_x H_x$
LOCAL.
a
RANGE OF H_x
$\leftarrow La \rightarrow$
Thermodynamic Limit $L \to \infty$.
Q: How many dofs at $E \to 0$?
A: ① None ② Some ③ A lot.
understanding
desperation
① None:
"Gapped"
spectrum of H
Δ
$\exists \Delta E$
finite #
$\lim_{L\to\infty} \Delta > 0$
$\lim_{L\to\infty} \Delta E = 0$.

② Some : gapless dof at isolated pts in momentum space.

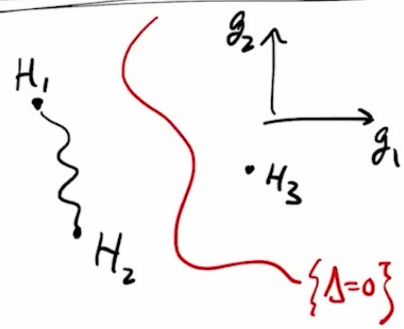


③ A lot : A Fermi surface. (FL or NFL)



Gapped Phase

$[H_1] = [H_2]$
$\neq [H_3].$



Topological Order :

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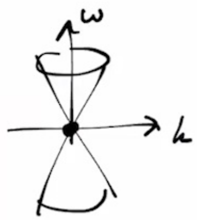

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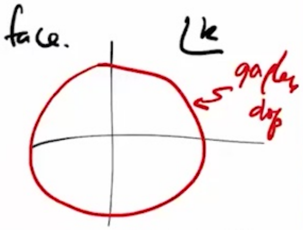

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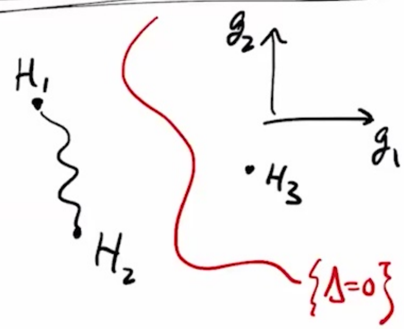
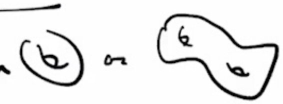
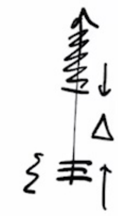


Topological Order:

Place system on 

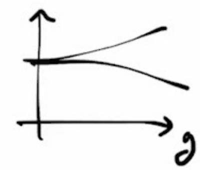
# of gs depends on topology of space.

stability



Necessary Condition: $\langle gs2 | O_{local} | gs1 \rangle \xrightarrow{(L\to\infty)} 0$

IF NOT: $\Delta H = \sum O_{local} g$



gs subspace is a Q. error correcting code w/ code distance $\propto L^{>0}$.

Another symptom: excitations which cannot be created by a local op.

"anyons".

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Topological Order :

Place system on (torus) or (blob)

# of gs depends on topology of space. stably

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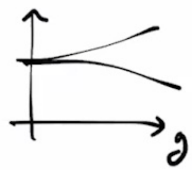

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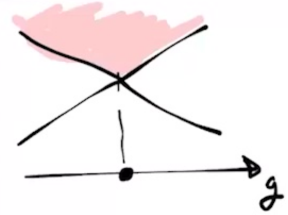
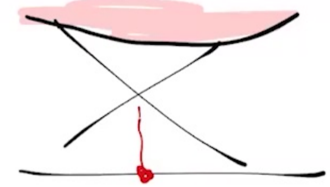


or



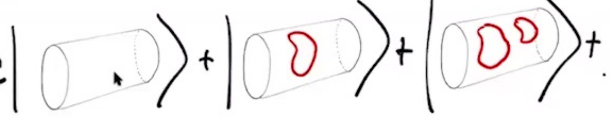
continuous transition

first order transition.

eg of TO: toric code or $\mathbb{Z}_2$ gauge theory.

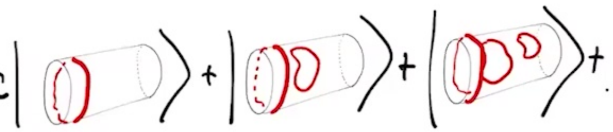
A groundstate:

$|gs\,0\rangle \propto$



$= \sum_{\text{loops}, C} | C \rangle$

$|gs\,1\rangle \propto$



$|gs1\rangle \perp |gs0\rangle$

not a local op.

$|gs1\rangle = O(\;)\, |gs0\rangle$

continuous transition | first order transition.

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A groundstate:

$$|gs\,0\rangle \propto |\;\rangle + |\;\rangle + |\;\rangle + \dots$$

$$= \sum_{\text{loops}, C} |C\rangle$$

$$|gs\,1\rangle \propto |\;\rangle + |\;\rangle + |\;\rangle + \dots$$

$|gs\,1\rangle \perp |gs\,0\rangle$ — not a local op.

$$|gs\,1\rangle = \mathcal{O}(\;)\,|gs\,0\rangle$$

$$\mathcal{O}(x \sim y)\,|gs\,0\rangle$$

$= |$ state w/ anyons at x & $y\rangle$

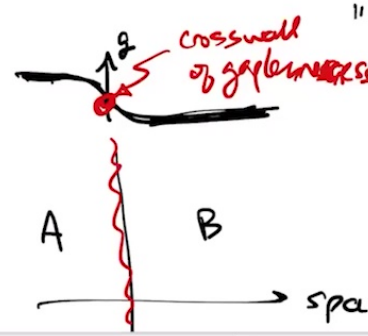
Sometimes: $\exists$ a theory of the top. groundstates in terms of TFT.

$\equiv$ LIQUID.

vs: fractons.

Edge modes: eg invertible states $\supset$ SPT, top. insulators.

$\equiv$ characterized by its edge modes. "anomaly".



But eg: TC can have gapped boundaries.

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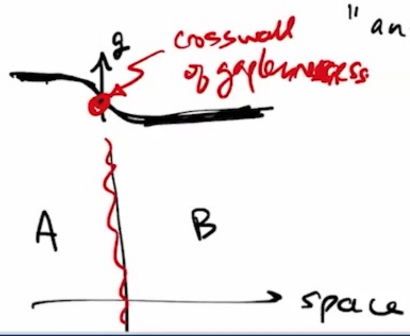

But eg:
TC can have gapped boundaries.

Gaplessness needs explanation:

- tuning to a critical point.
- broken continuous symmetry
- Fermi surface
- edge of an invertible phase
- symmetry forbids mass terms (eg chiral symmetry)
- CFT w/ no relevant operators.

Gapless phases: region of $\{H\}$ within which perturbation theory works.

* Refinement by symmetry

Restrict to H preserved by G.

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Restrict to H preserved by G.

Entanglement.

$|\psi\rangle = \bigotimes_x |s_x\rangle$ is not entangled.

Mean field theory. Rep(G).

$\bar{A} \quad A \quad \bar{A}$ → space

$\mathcal{H} = A \otimes \bar{A} \ni |\psi\rangle$

$\rho_A = \mathrm{tr}_{\bar{A}} |\psi\rangle\langle\psi|$

$S_A = -\mathrm{tr}\, \rho_A \log \rho_A$. EE.

Interlude on adiabatic continuation:

claim 1: A quantum phase is a property of g.s.

claim 2: $[H_0] = [H_1] \Leftrightarrow |\psi(1)\rangle = U |\psi(0)\rangle$

finite depth local unitary.

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$\Rightarrow$ $H_0 \overset{H(s)}{\sim} H_1$ $H(s)$ gapped $\forall s$. Δ ind. of L.

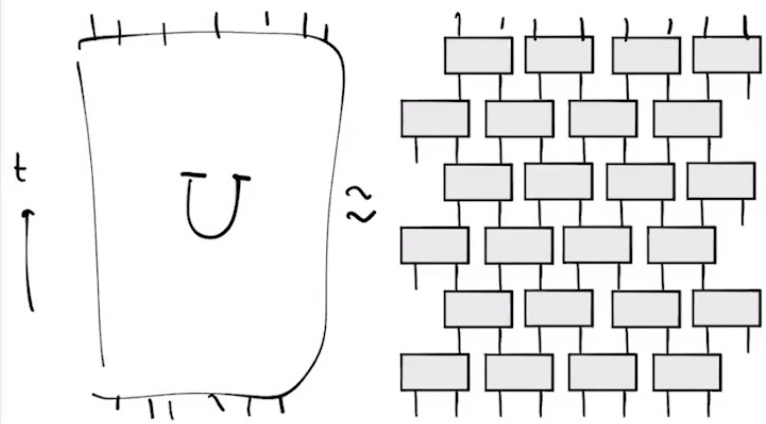
$$T e^{i\int_0^1 dt\, H(t)} |\psi(0)\rangle \propto |\psi(1)\rangle + \text{errors}$$

good: finite time evolution.

bad: errors are extensive.

quasiadiabatic filtering: $\underset{\text{local}}{H(s)} \rightarrow \underset{\text{local}}{\tilde{H}(s)}$.

$$\leadsto |\psi(1)\rangle = T e^{i\int_0^1 dt\, \tilde{H}(t)} |\psi(0)\rangle.$$



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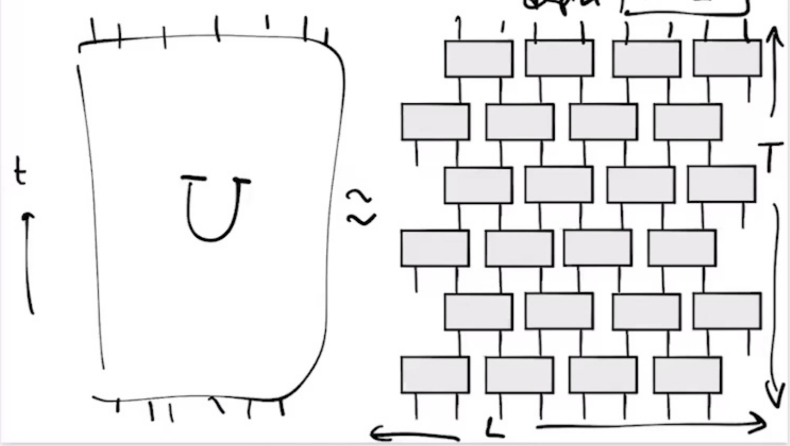
depth $\boxed{T \sim L^0}$



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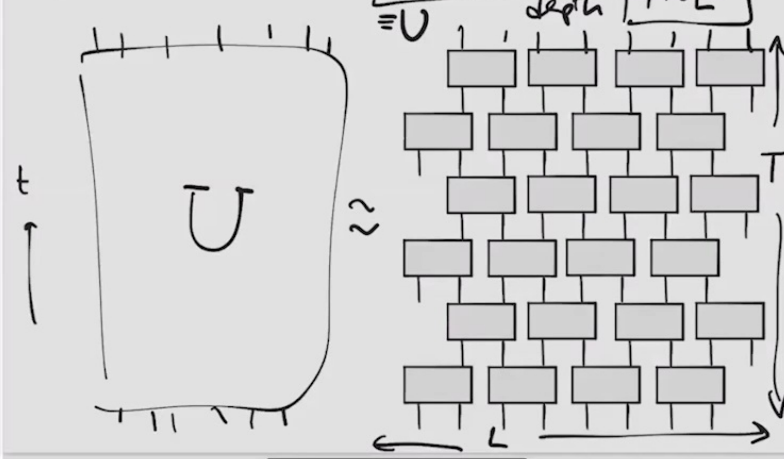

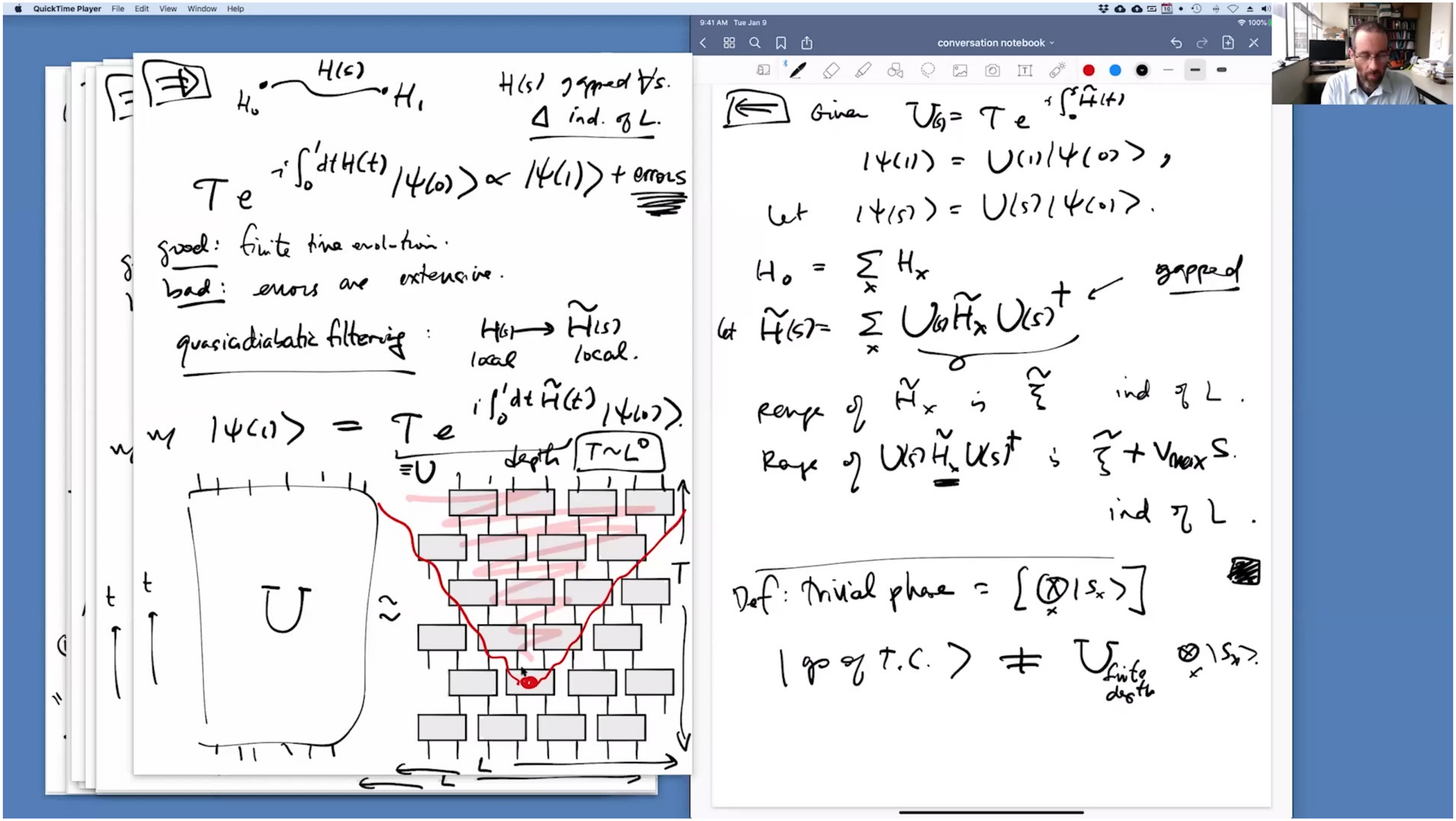
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⇒
H(s)
H0 H1
H(s) gapped ∀s.
Δ ind. of L.
T e^{-i∫_0^1 dt H(t)} |ψ(0)⟩ ∝ |ψ(1)⟩ + errors
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local local.
|ψ(1)⟩ = T e^{i∫_0^1 dt H̃(t)} |ψ(0)⟩.
≡ U depth T~L^0
U
t
T
L
⇐ Given U(s) = T e^{i∫_0^s H̃(t)}
|ψ(1)⟩ = U(1)|ψ(0)⟩,
let |ψ(s)⟩ = U(s)|ψ(0)⟩.
H0 = Σ_x Hx gapped
let H̃(s) = Σ_x U(s) H̃x U(s)†
range of H̃x is ξ̃ ind of L.
Range of U(s) H̃x U(s)† is ξ̃ + Vmax S.
ind of L.
Def: Trivial phase = [⊗_x |sx⟩]
|gs of T.C.⟩ ≠ U_finite depth ⊗_x |sx⟩.

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Given $U(s) = \mathcal{T} e^{i\int_0^s \tilde{H}(t)}$

$|\psi(1)\rangle = U(1)|\psi(0)\rangle$,

Let $|\psi(s)\rangle = U(s)|\psi(0)\rangle$.

$H_0 = \sum_x H_x$ ← gapped

Let $\tilde{H}(s) = \underbrace{\sum_x U(s)\tilde{H}_x U(s)^\dagger}_{\sigma}$

Range of $\tilde{H}_x$ is $\tilde{\xi}$ ind of L.

Range of $U(s)\underline{\tilde{H}_x}U(s)^\dagger$ is $\tilde{\xi} + V_{max} s$ ind of L.

Def: Trivial phase $= [\otimes_x |s_x\rangle]$

$$\underbrace{|gs\ of\ T.C.\rangle}_{\text{LRE}} \neq \underbrace{U_{\text{finite depth}} \otimes_x |s_x\rangle}_{\text{SRE}}$$

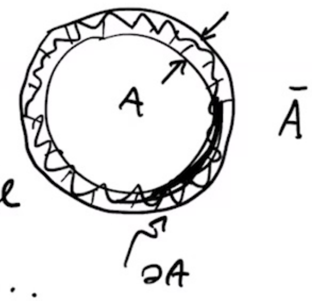
What is S_A for $U \otimes_x |s_x\rangle$

SRE ⇒

$$S_A = \int_{\partial A} s\, d\ell$$

$$= \oint [\Lambda + b\underline{K} + cK^2 + \dots] d\ell$$

$$= \Lambda \ell(\partial A) + \cancel{\tilde{b}}^{\,0} + \frac{\tilde{c}}{\ell(\partial A)} + \dots$$



$\boxed{d = 2}$

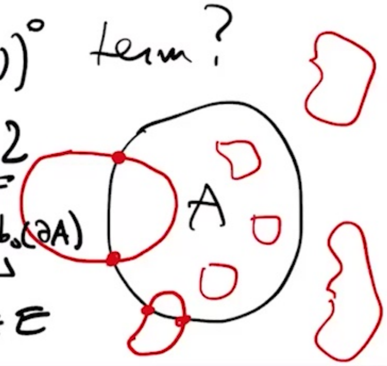
- for pure states $S(A) = S(\bar{A})$
- $K(\partial A) = -K(\partial \bar{A})$

K = extrinsic curvature of ∂A.

Whence such a $(\ell(\partial A))^0$ term?

$$\Rightarrow S_A^{TC} = |\partial A|\Lambda - \underline{\log 2}$$

$$\equiv \underbrace{|\partial A|\Lambda}_{} - \underbrace{\gamma b_0(\partial A)}_{\text{TEE}}$$



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SRE $\Rightarrow$

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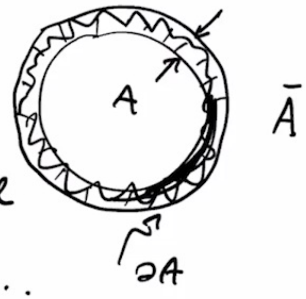

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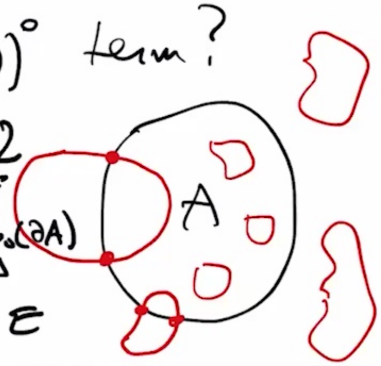

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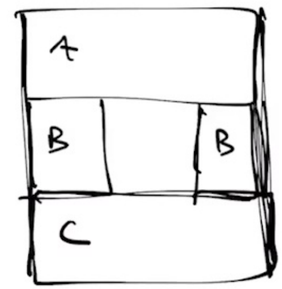
$$\partial(AB) + \partial(BC) = \partial(B) + \partial(ABC)$$



$$I(A:C|B) \equiv S_{AB} + S_{BC} - S_B - S_{ABC}$$

area law bits cancel.

$$\text{SSA} \Leftrightarrow I(A:C|B) \geq 0.$$

$$I(A:C|B) = 2\gamma.$$

$$S_A = |\partial A|\Lambda - \gamma$$

If $\gamma = 0$ then ρ_{ABC} is determined by $\rho_A = \mathrm{tr}_{BC}\, \rho_{ABC}$, ρ_B, ρ_C.

For liquid phases, TEE $= \frac{1}{2}\log\left(\text{total quantum dimension}\right)$

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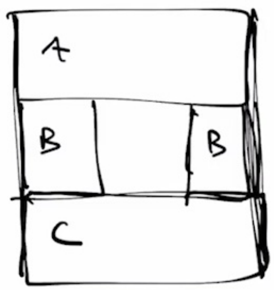
$$\partial(AB)+\partial(BC)=\partial(B)+\partial(ABC)$$



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$$|\psi\rangle = \sum C_{s_1\ldots s_{2L^d}} |s_1 \ldots s_{2L^d}\rangle$$

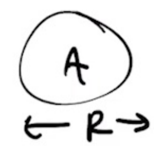
2^{L^d} #s.

$$|\psi_{MFT}\rangle = \otimes \sum C_{s_x}|s_x\rangle$$

L^d #s.

<u>expectation</u>: for g.s. of local H

$$\underline{S_A \sim aR^{d-1} + \text{smaller.}}$$



<u>Exceptions to area law</u>: critical pts in $d=1$

$$S_A \sim \frac{c}{3}\log R/\epsilon$$

<u>metals</u>: $S_A \sim (k_F R)^{d-1}\log k_F R$.

$$|\psi\rangle = \sum C_{s_1 \ldots s_{2^{L^d}}} \, | s_1 \ldots s_{2^{L^d}} \rangle$$

2^{L^d} #s.

$$|\psi_{MFT}\rangle = \otimes \sum C_{s_x} |s_x\rangle$$

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A

$\leftarrow R \rightarrow$

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Nongroundstates: S_A (thermal states) $\sim R^d s_{th}(T)$

volume law.

local entanglement in both.

Universal terms in EE of subregions (like γ, c) are RG monotones.

(SSA, Lorentz invariance)

omissions:
- $T \neq 0$.
- Non-eqbm matter.