Title: Controlled access to the low-energy physics of critical Fermi surfaces

Speakers: Ipsita Mandal

Series: Quantum Matter

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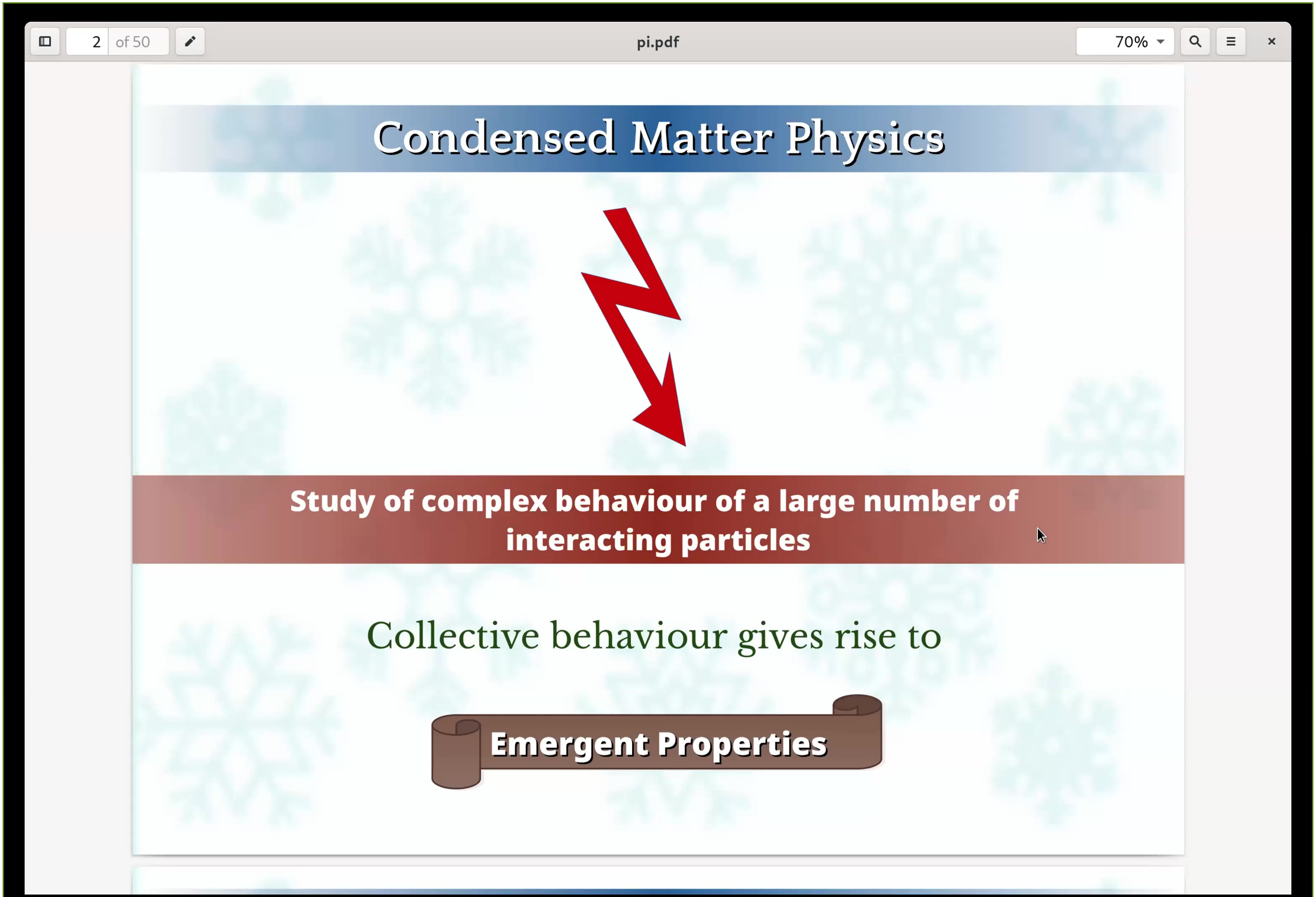
Abstract: Condensed matter physics is the study of the complex behaviour of a large number of interacting particles such that their collective behaviour gives rise to emergent properties. We will discuss some interesting quantum condensed matter systems where their intriguing emergent phenomena arise due to strong coupling. We will revisit the Landau paradigm of Fermi liquid theory and hence understand the properties of the non-Fermi liquid systems which cannot be described within the Landau framework, due to the destruction of the Landau quasiparticles. In particular, we will focus on critical Fermi surface states, where there is a well-defined Fermi surface, but no quasiparticles, as a result of the strong interactions between the Fermi surface and some massless boson(s). We will outline a framework to extract the low-energy physics of such systems in a controlled approximation, using the tool of dimensional regularization.

Controlled Access to Low-Energy Physics of Critical Fermi Surfaces

IPSITA MANDAL

Universitetet
i Stavanger

NORDITA
The Nordic Institute for
Theoretical Physics

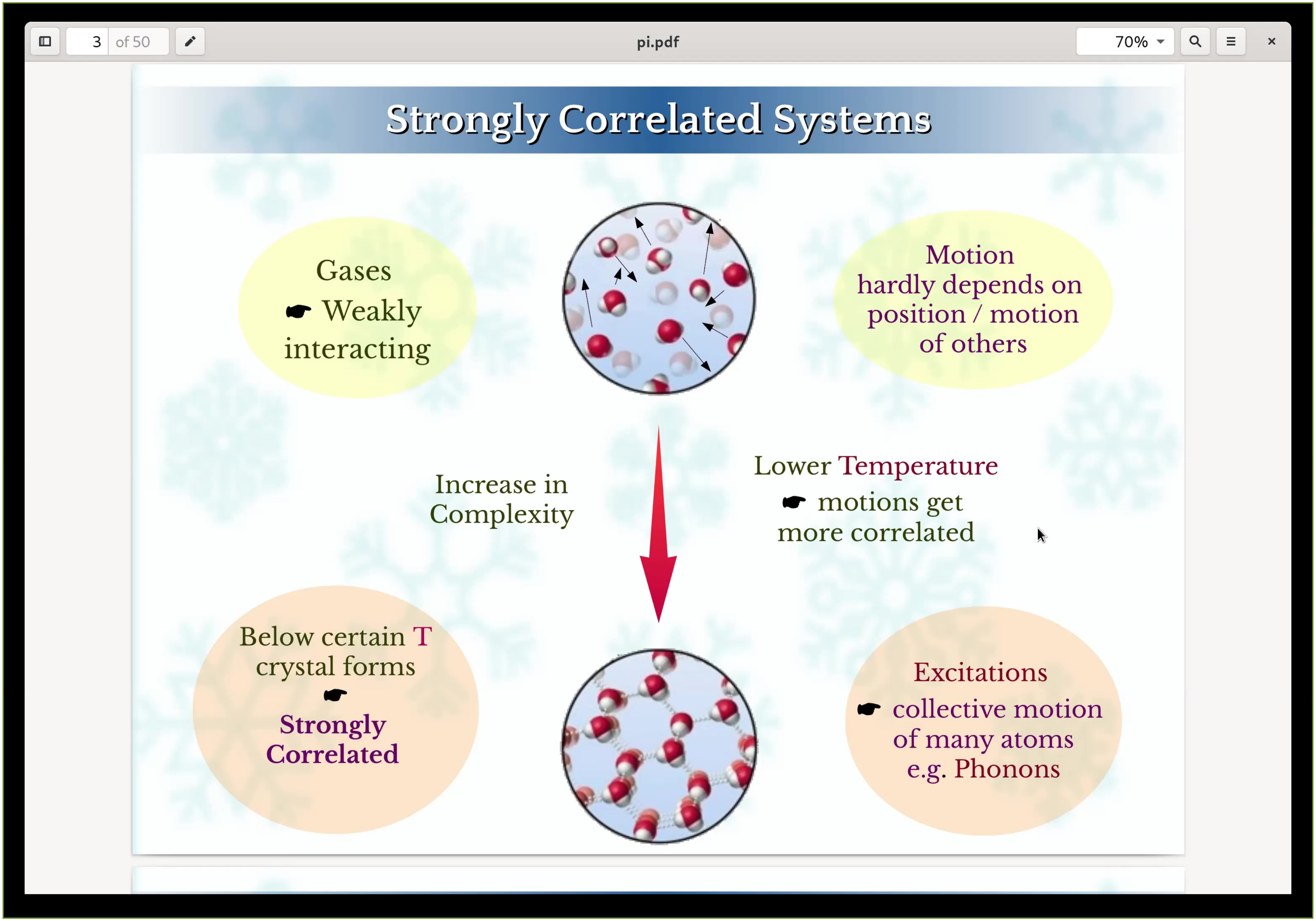


Condensed Matter Physics

Study of complex behaviour of a large number of interacting particles

Collective behaviour gives rise to

Emergent Properties



Strongly Correlated Systems

Gases
☛ Weakly interacting

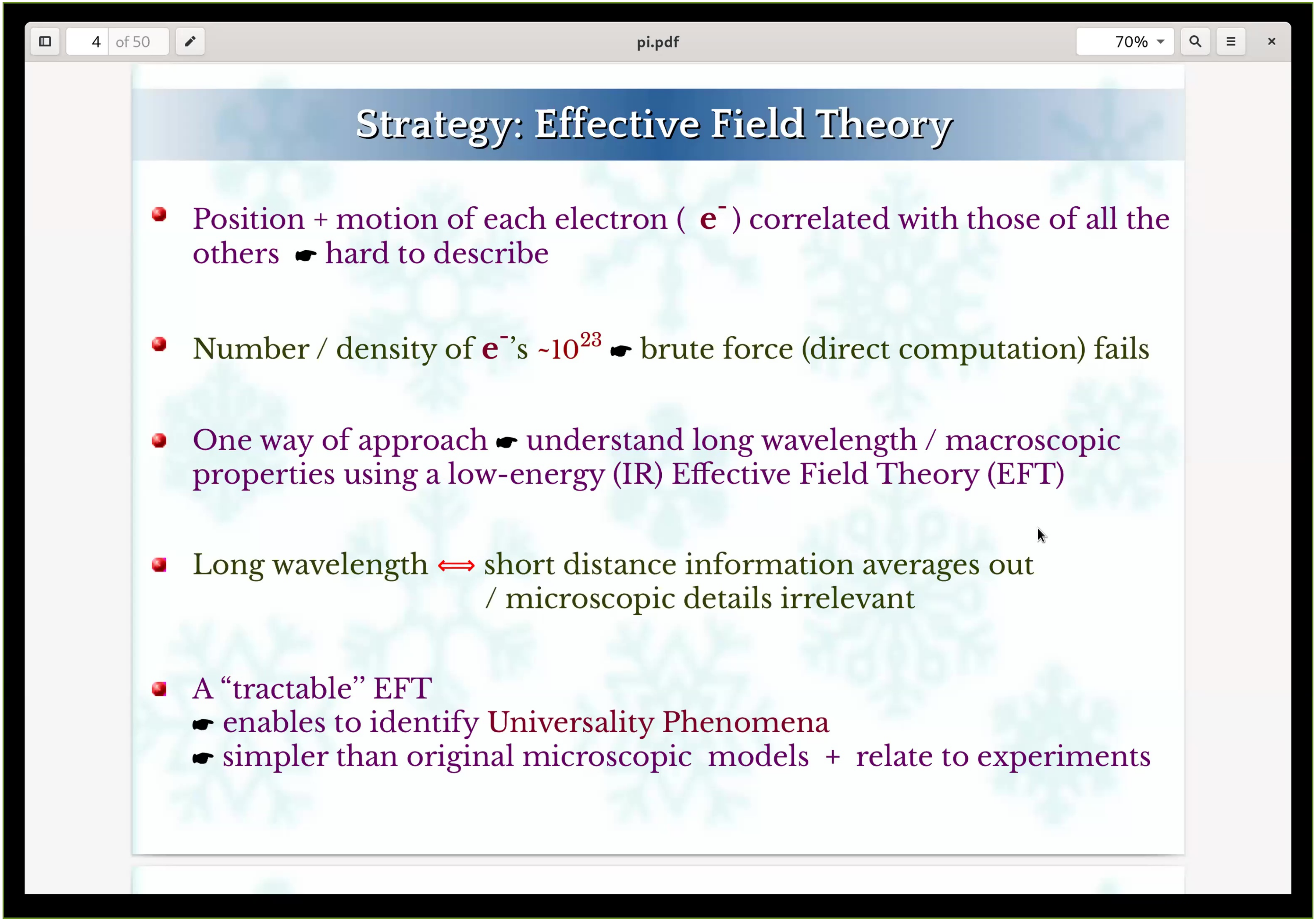
Motion hardly depends on position / motion of others

Increase in Complexity

Lower Temperature
☛ motions get more correlated

Below certain T crystal forms
☛
Strongly Correlated

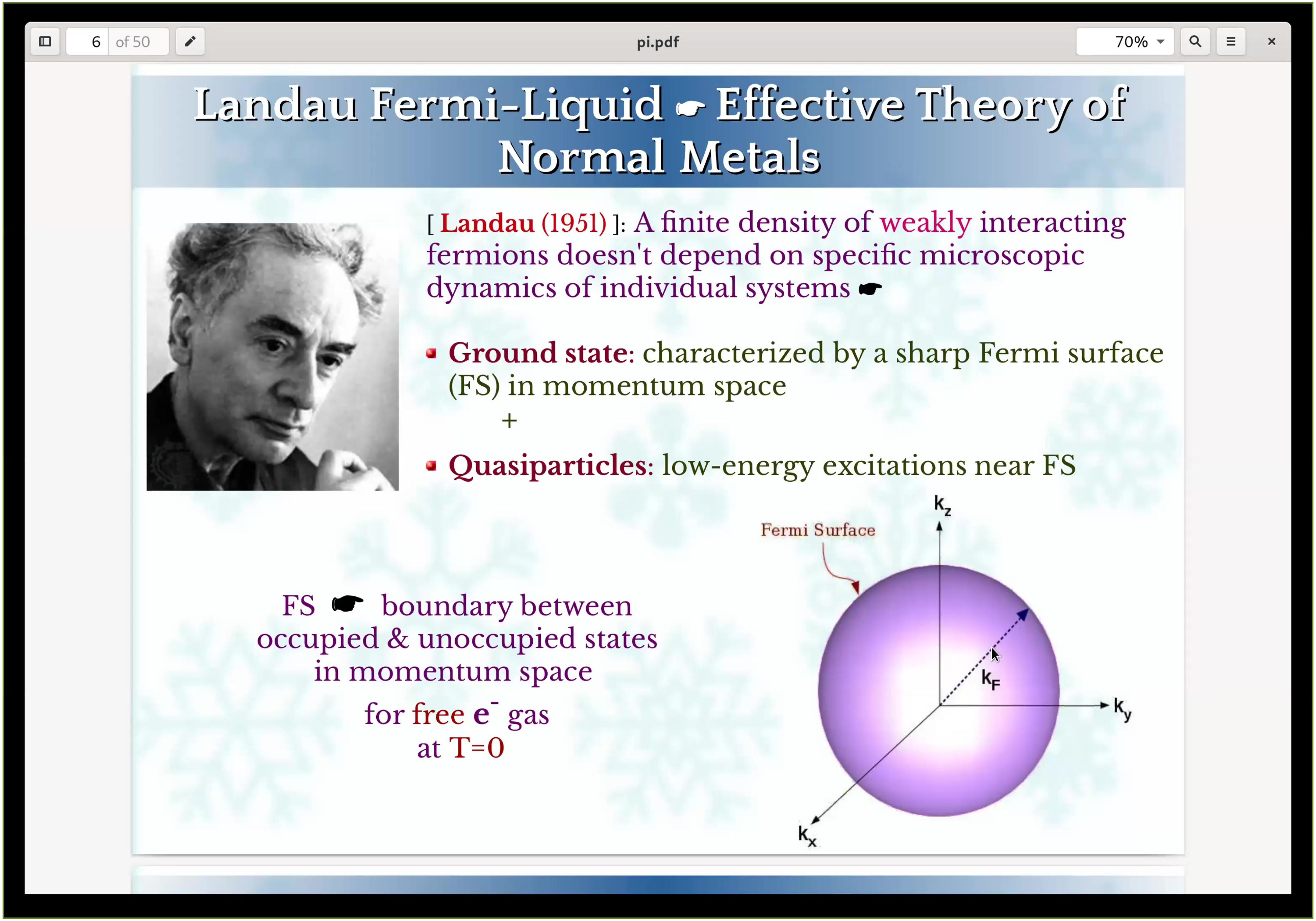
Excitations
☛ collective motion of many atoms e.g. Phonons



Strategy: Effective Field Theory

- Position + motion of each electron ($\mathbf{e}^-$) correlated with those of all the others ☛ hard to describe
- Number / density of $\mathbf{e}^-$'s $\sim 10^{23}$ ☛ brute force (direct computation) fails
- One way of approach ☛ understand long wavelength / macroscopic properties using a low-energy (IR) Effective Field Theory (EFT)
- Long wavelength ⟺ short distance information averages out
  / microscopic details irrelevant
- A "tractable" EFT
  ☛ enables to identify Universality Phenomena
  ☛ simpler than original microscopic models + relate to experiments

Non-Fermi Liquids (NFL)



Landau Fermi-Liquid ☛ Effective Theory of Normal Metals

[**Landau** (1951)]: A finite density of weakly interacting fermions doesn't depend on specific microscopic dynamics of individual systems ☛

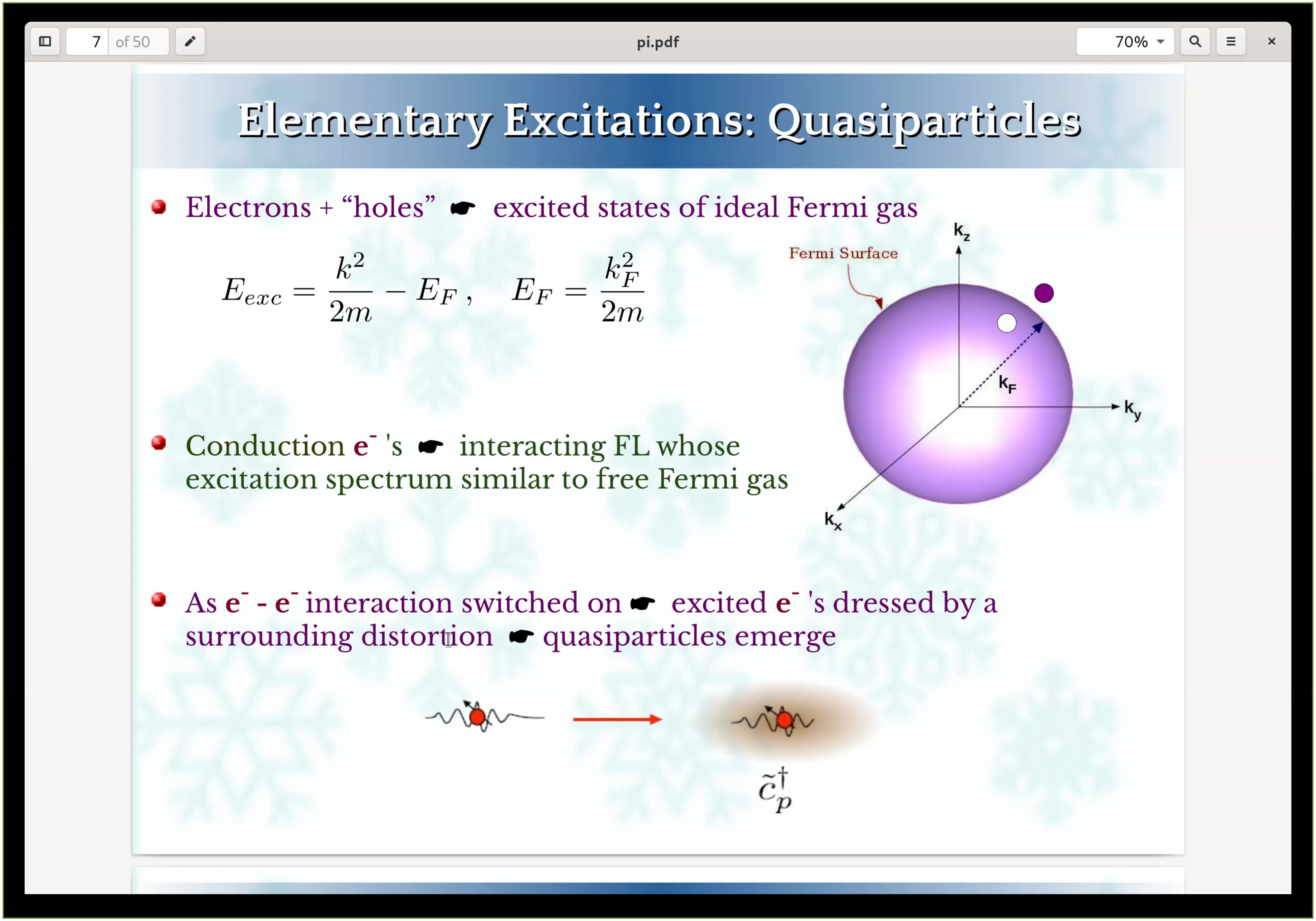
- **Ground state**: characterized by a sharp Fermi surface (FS) in momentum space

  +

- **Quasiparticles**: low-energy excitations near FS

FS ☛ boundary between
occupied & unoccupied states
in momentum space

for free e^- gas
at T=0



Elementary Excitations: Quasiparticles

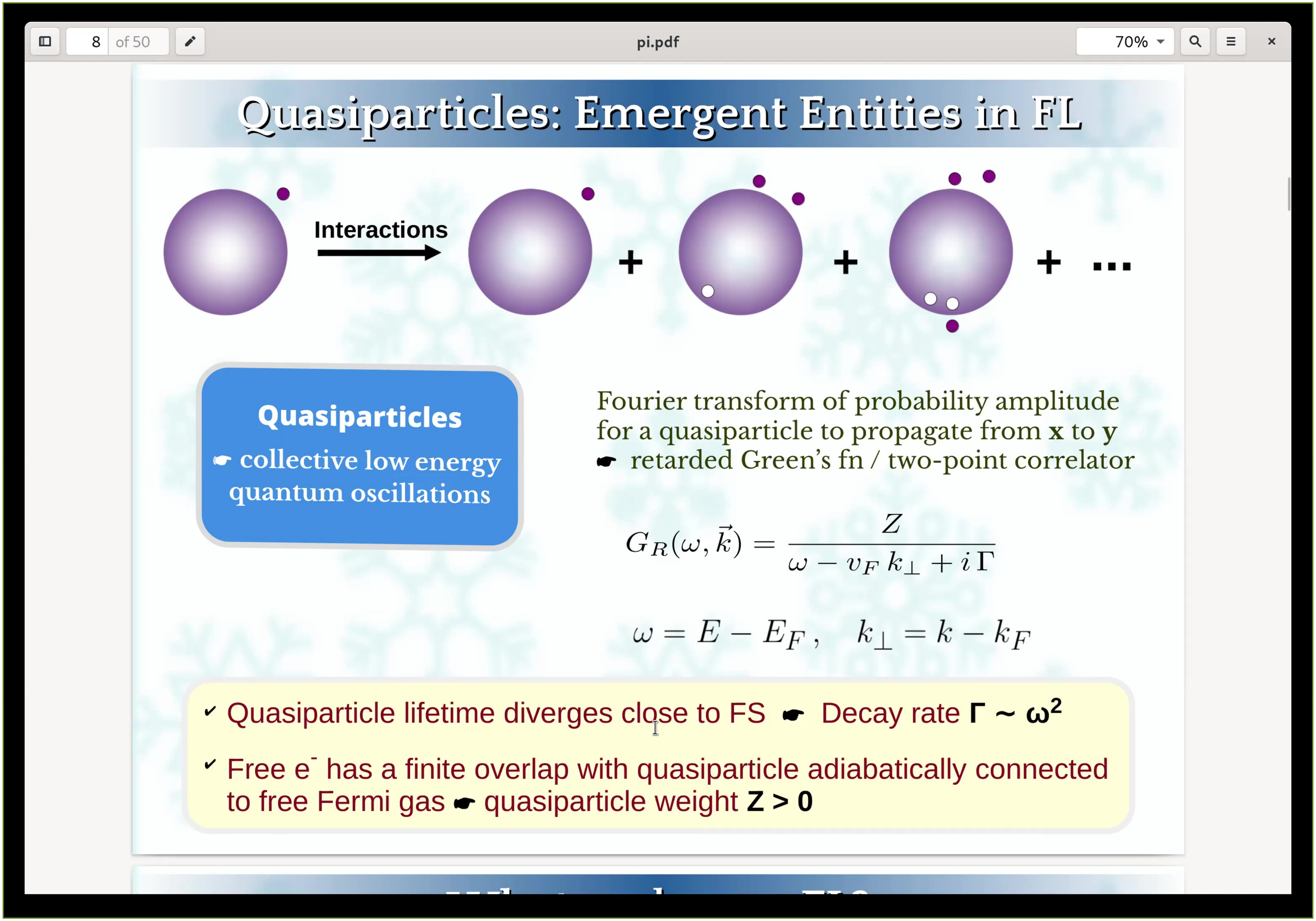
- Electrons + “holes” ☛ excited states of ideal Fermi gas

$$E_{exc} = \frac{k^2}{2m} - E_F \,, \quad E_F = \frac{k_F^2}{2m}$$

- Conduction e⁻'s ☛ interacting FL whose excitation spectrum similar to free Fermi gas

- As e⁻ - e⁻ interaction switched on ☛ excited e⁻'s dressed by a surrounding distortion ☛ quasiparticles emerge

$$\tilde{c}_p^\dagger$$



Quasiparticles: Emergent Entities in FL

Quasiparticles

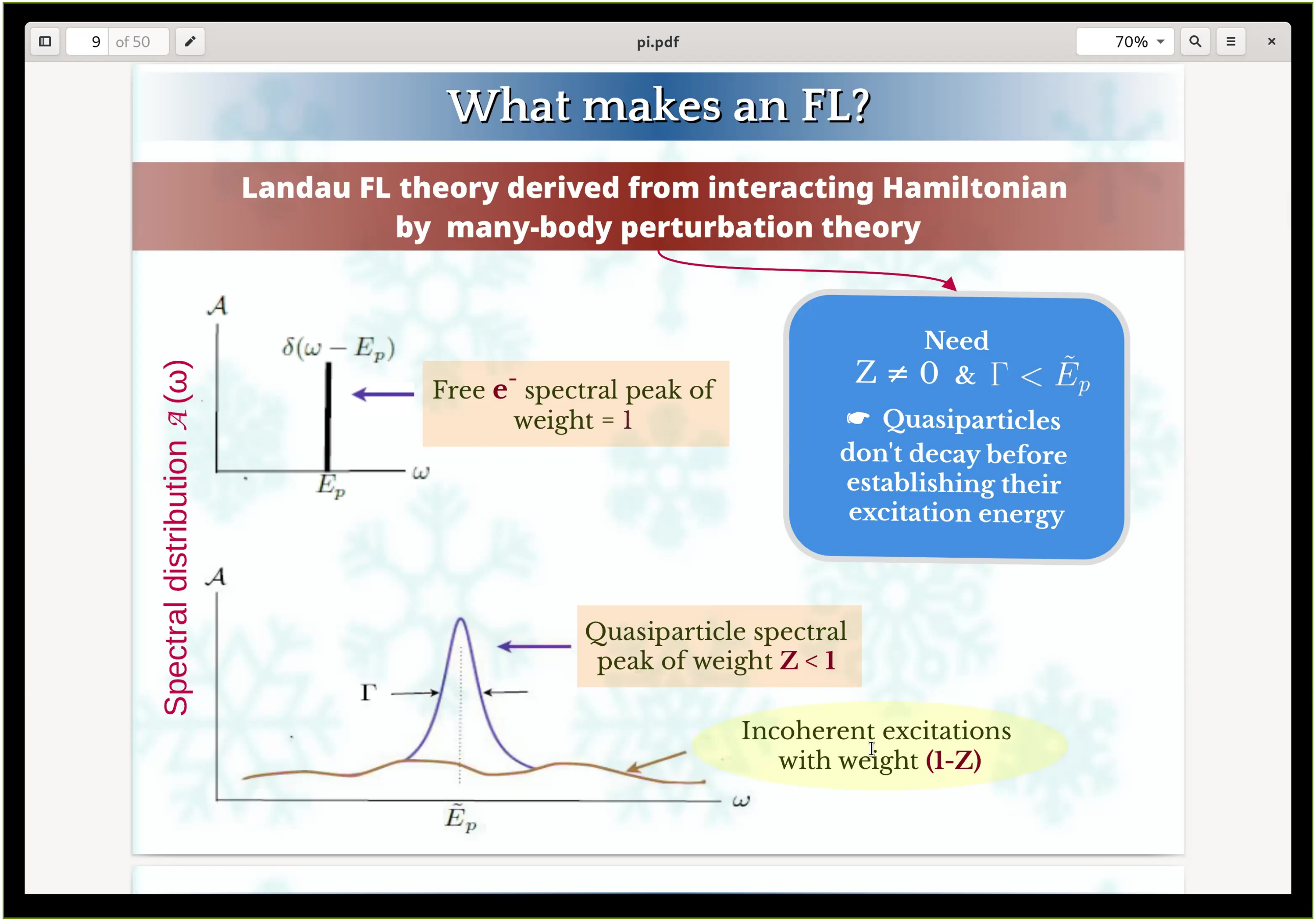
☛ collective low energy quantum oscillations

Fourier transform of probability amplitude for a quasiparticle to propagate from **x** to **y**
☛ retarded Green's fn / two-point correlator

$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F\, k_\perp + i\,\Gamma}$$

$$\omega = E - E_F\,, \quad k_\perp = k - k_F$$

- ✓ Quasiparticle lifetime diverges close to FS ☛ Decay rate **Γ ~ ω²**
- ✓ Free e⁻ has a finite overlap with quasiparticle adiabatically connected to free Fermi gas ☛ quasiparticle weight **Z > 0**



What makes an FL?

Landau FL theory derived from interacting Hamiltonian by many-body perturbation theory

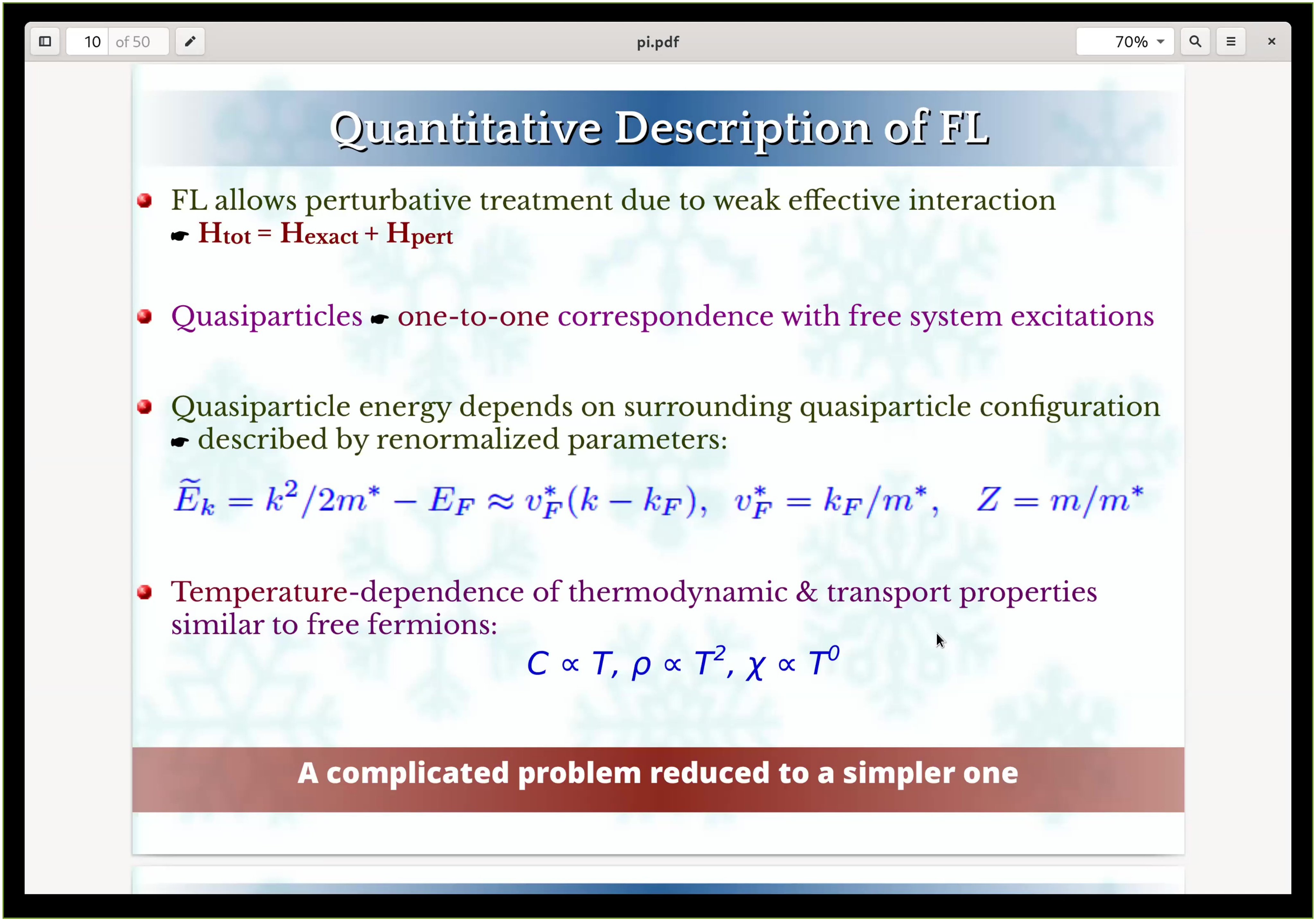
Need
$Z \neq 0$ & $\Gamma < \tilde{E}_p$

☛ Quasiparticles don't decay before establishing their excitation energy

Free **e$^-$** spectral peak of weight = 1

Quasiparticle spectral peak of weight **Z < 1**

Incoherent excitations with weight **(1-Z)**



Quantitative Description of FL

- FL allows perturbative treatment due to weak effective interaction
  ☛ $H_{tot} = H_{exact} + H_{pert}$

- Quasiparticles ☛ one-to-one correspondence with free system excitations

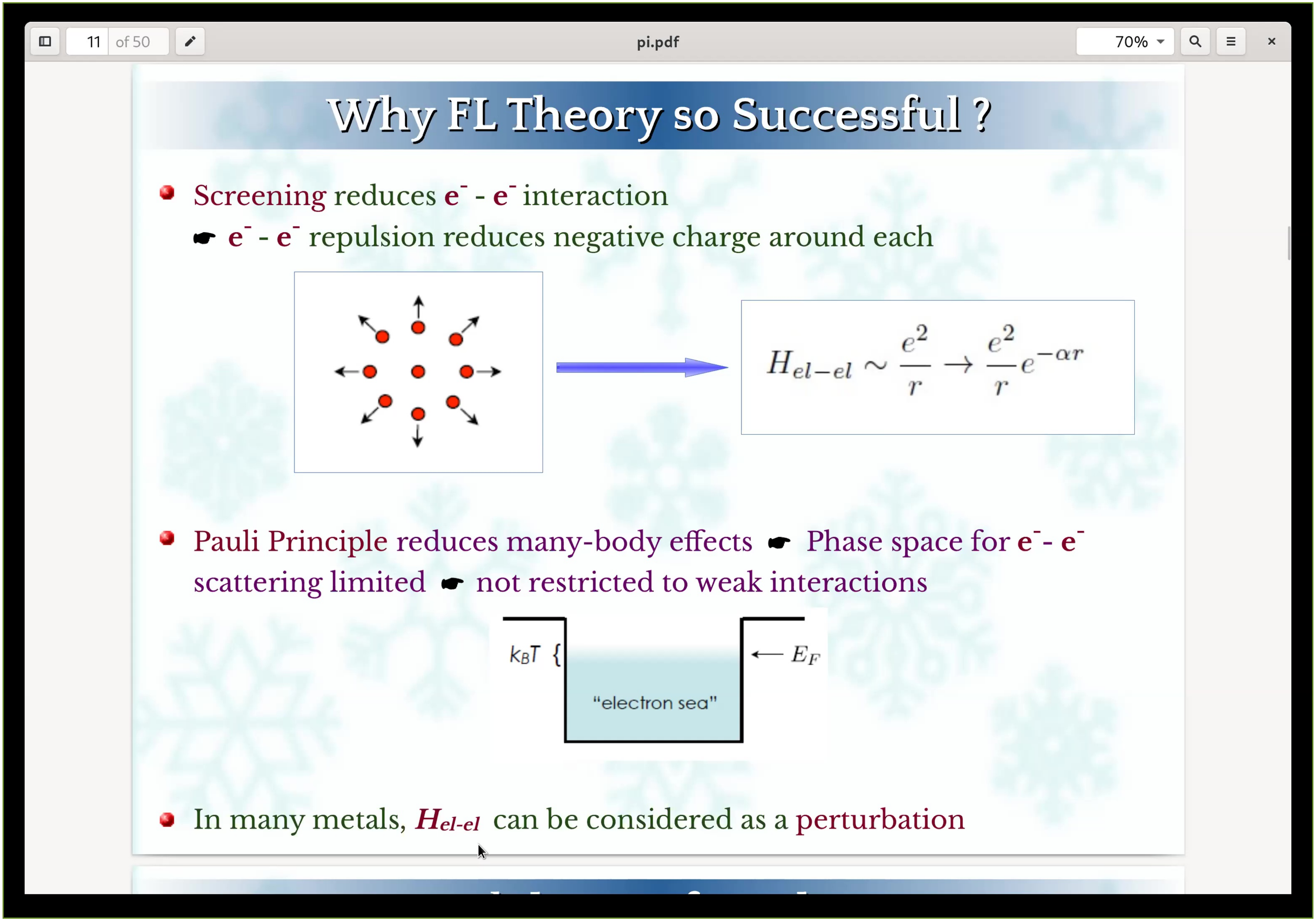
- Quasiparticle energy depends on surrounding quasiparticle configuration
  ☛ described by renormalized parameters:

$$\widetilde{E}_k = k^2/2m^* - E_F \approx v_F^*(k - k_F), \quad v_F^* = k_F/m^*, \quad Z = m/m^*$$

- Temperature-dependence of thermodynamic & transport properties similar to free fermions:

$$C \propto T,\ \rho \propto T^2,\ \chi \propto T^0$$

A complicated problem reduced to a simpler one



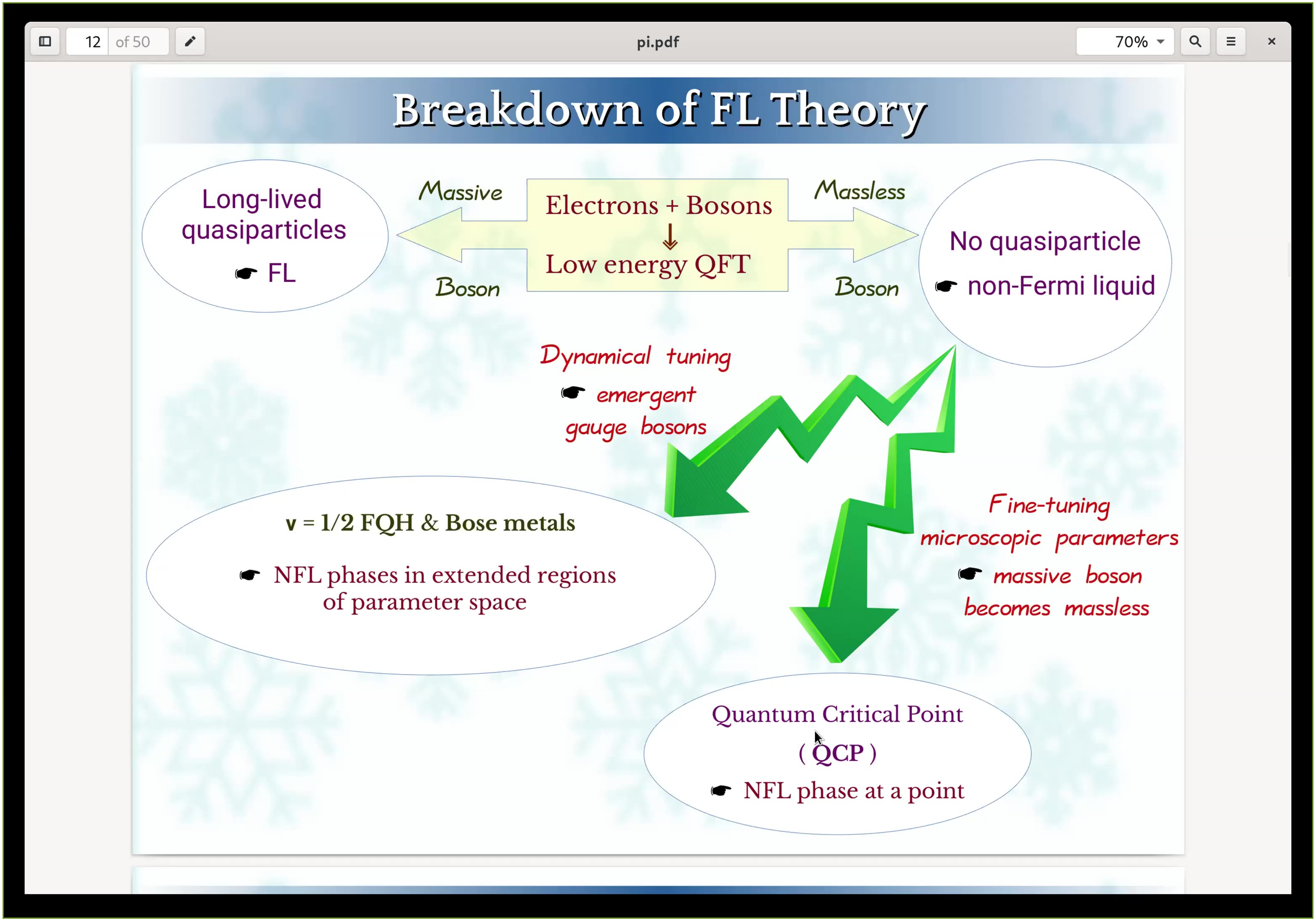
Why FL Theory so Successful ?

- Screening reduces **e⁻ - e⁻** interaction
  ☛ **e⁻ - e⁻** repulsion reduces negative charge around each

$$H_{el-el} \sim \frac{e^2}{r} \to \frac{e^2}{r} e^{-\alpha r}$$

- Pauli Principle reduces many-body effects ☛ Phase space for **e⁻- e⁻** scattering limited ☛ not restricted to weak interactions

- In many metals, ***$H_{el\text{-}el}$*** can be considered as a perturbation



Breakdown of FL Theory

Electrons + Bosons
↓
Low energy QFT

Massive
Boson

Long-lived quasiparticles
☛ FL

Massless
Boson

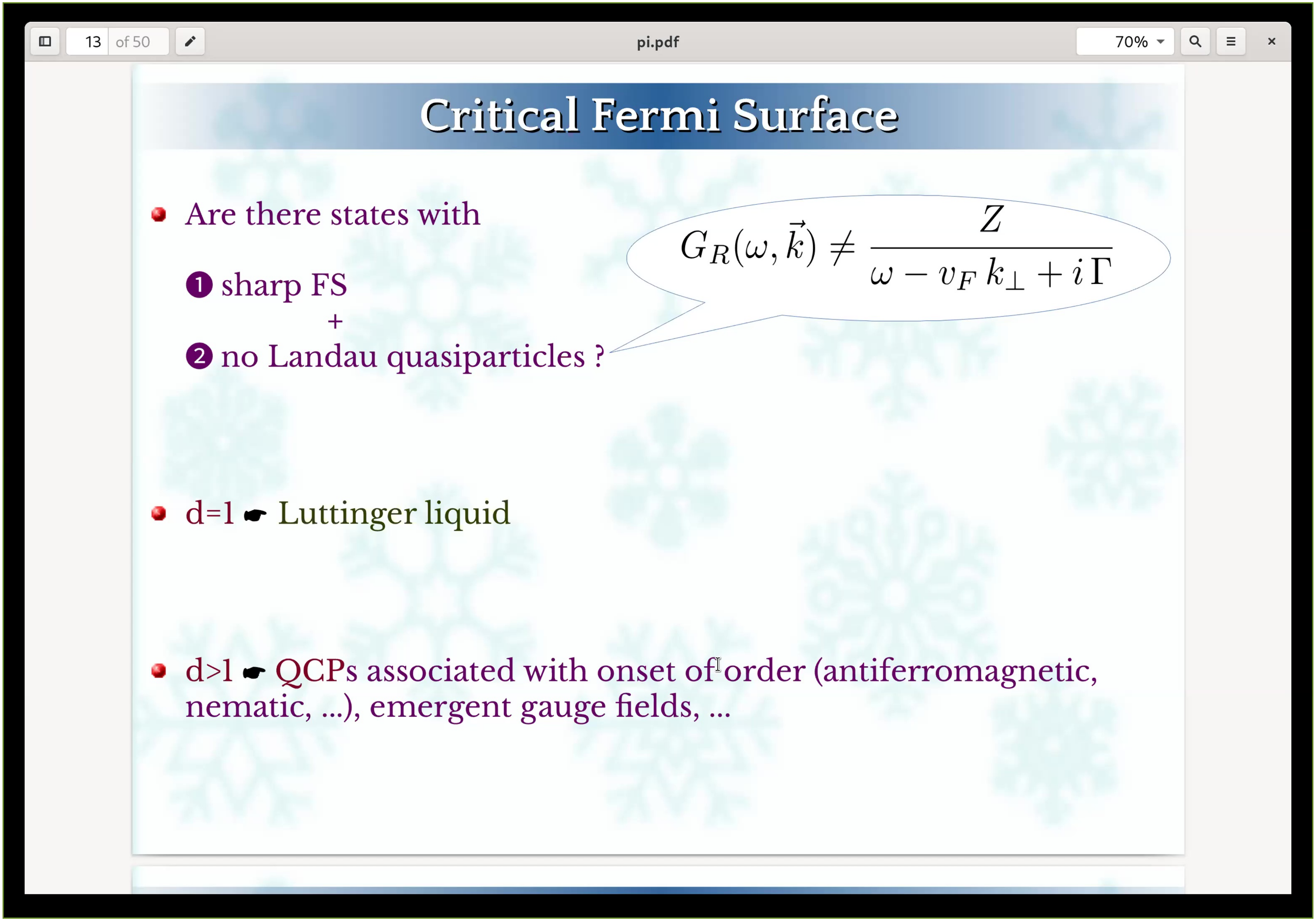
No quasiparticle
☛ non-Fermi liquid

Dynamical tuning
☛ *emergent gauge bosons*

ν = 1/2 FQH & Bose metals
☛ NFL phases in extended regions of parameter space

Fine-tuning microscopic parameters
☛ *massive boson becomes massless*

Quantum Critical Point
(QCP)
☛ NFL phase at a point



Critical Fermi Surface

- Are there states with

  ❶ sharp FS

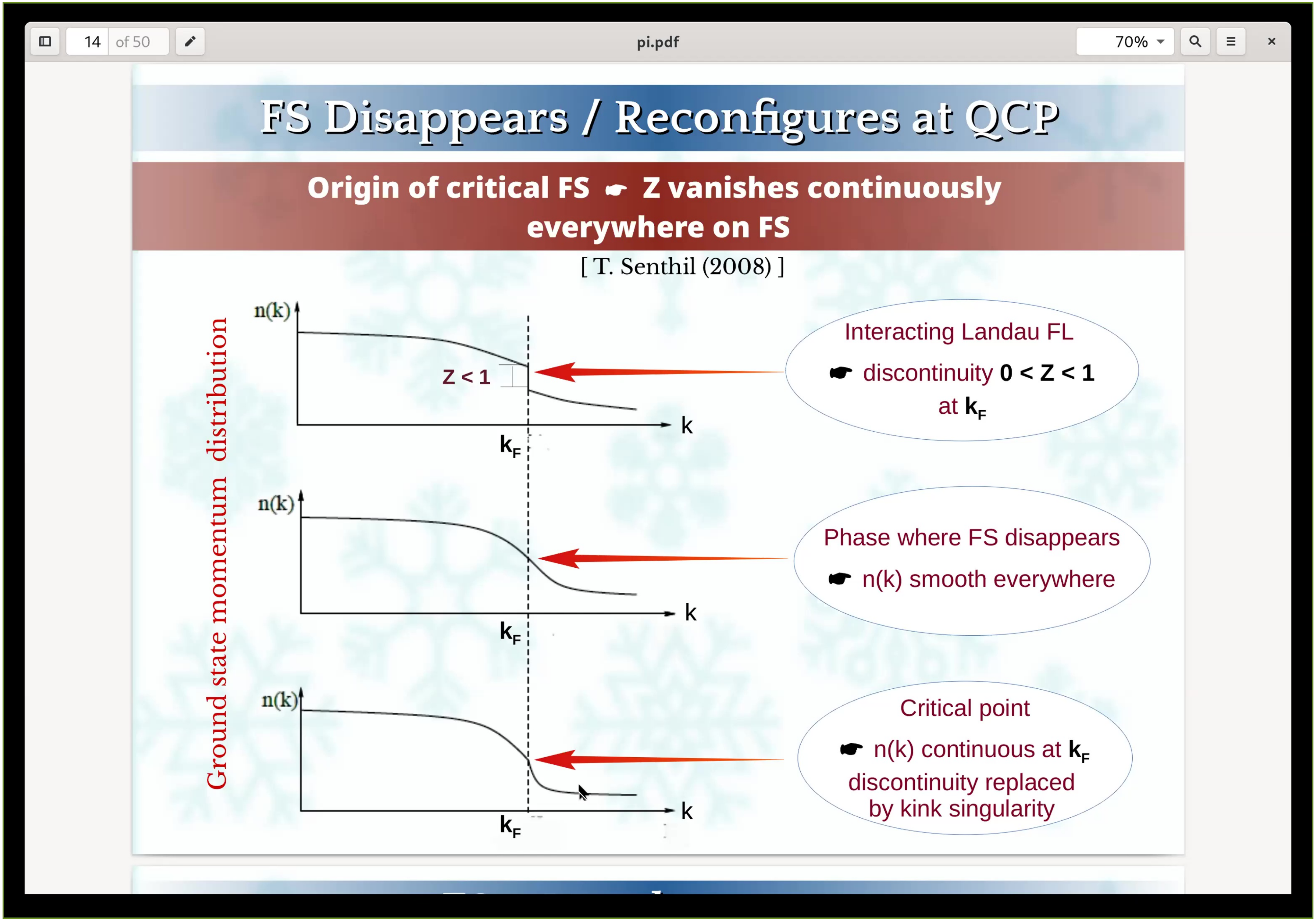
  +

  ❷ no Landau quasiparticles ?

$$G_R(\omega, \vec{k}) \neq \frac{Z}{\omega - v_F\, k_\perp + i\,\Gamma}$$

- d=1 ☛ Luttinger liquid

- d>1 ☛ QCPs associated with onset of order (antiferromagnetic, nematic, ...), emergent gauge fields, ...



FS Disappears / Reconfigures at QCP

Origin of critical FS ☛ Z vanishes continuously everywhere on FS

[T. Senthil (2008)]

Interacting Landau FL

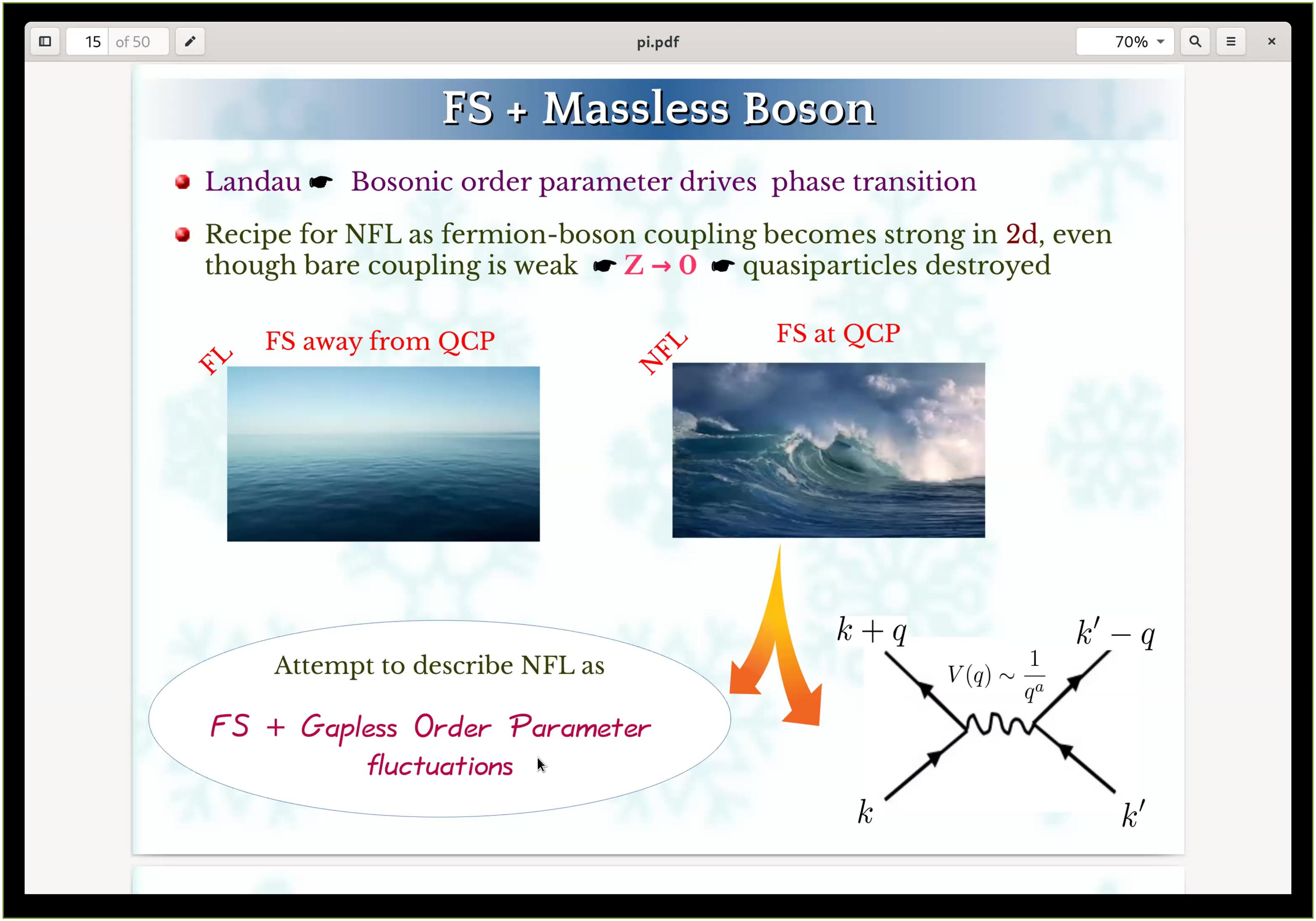
☛ discontinuity **$0 < Z < 1$** at **k_F**

Phase where FS disappears

☛ n(k) smooth everywhere

Critical point

☛ n(k) continuous at **k_F**
discontinuity replaced
by kink singularity



FS + Massless Boson

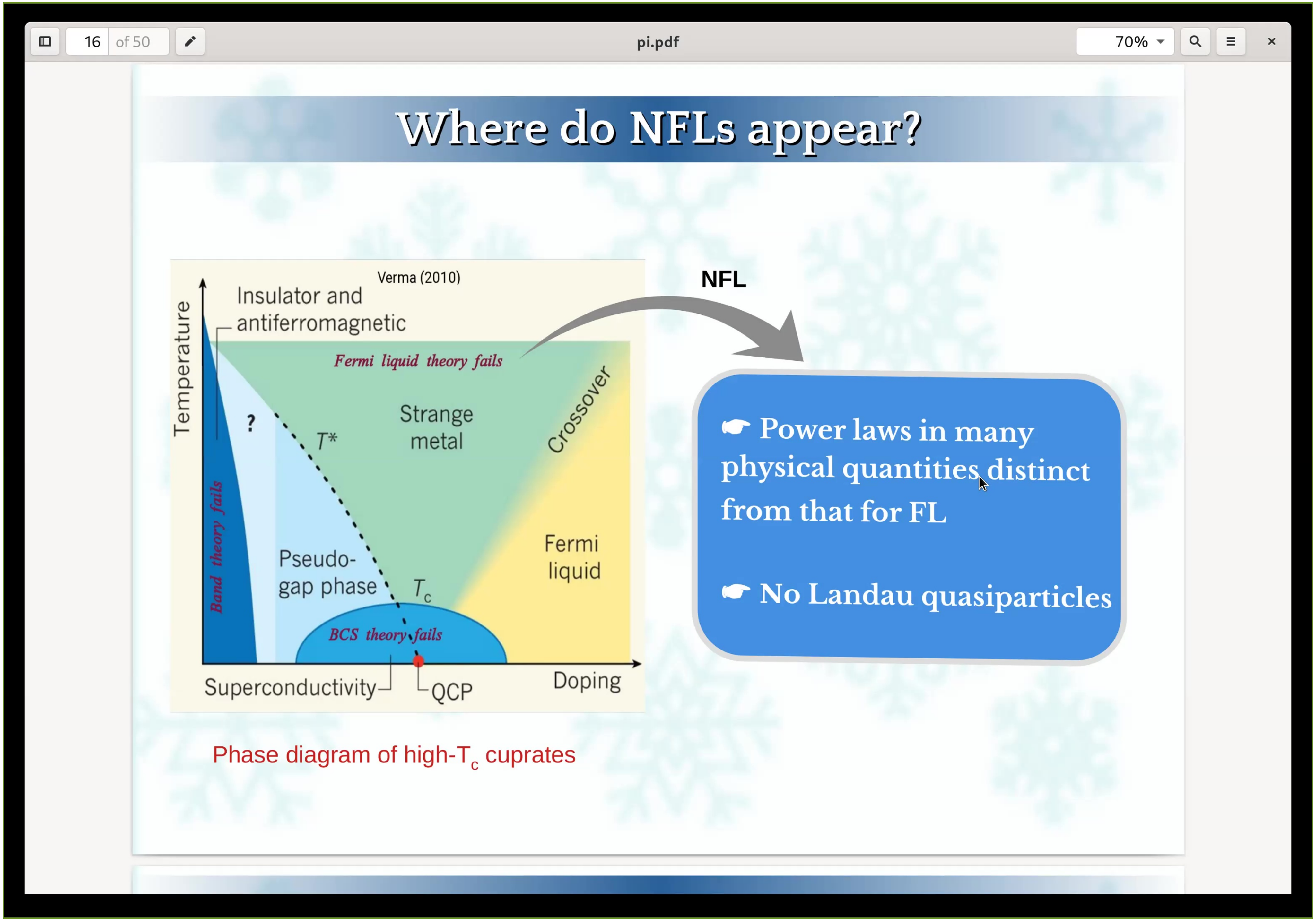
- Landau ☛ Bosonic order parameter drives phase transition
- Recipe for NFL as fermion-boson coupling becomes strong in 2d, even though bare coupling is weak ☛ Z → 0 ☛ quasiparticles destroyed

FS away from QCP

FS at QCP

Attempt to describe NFL as

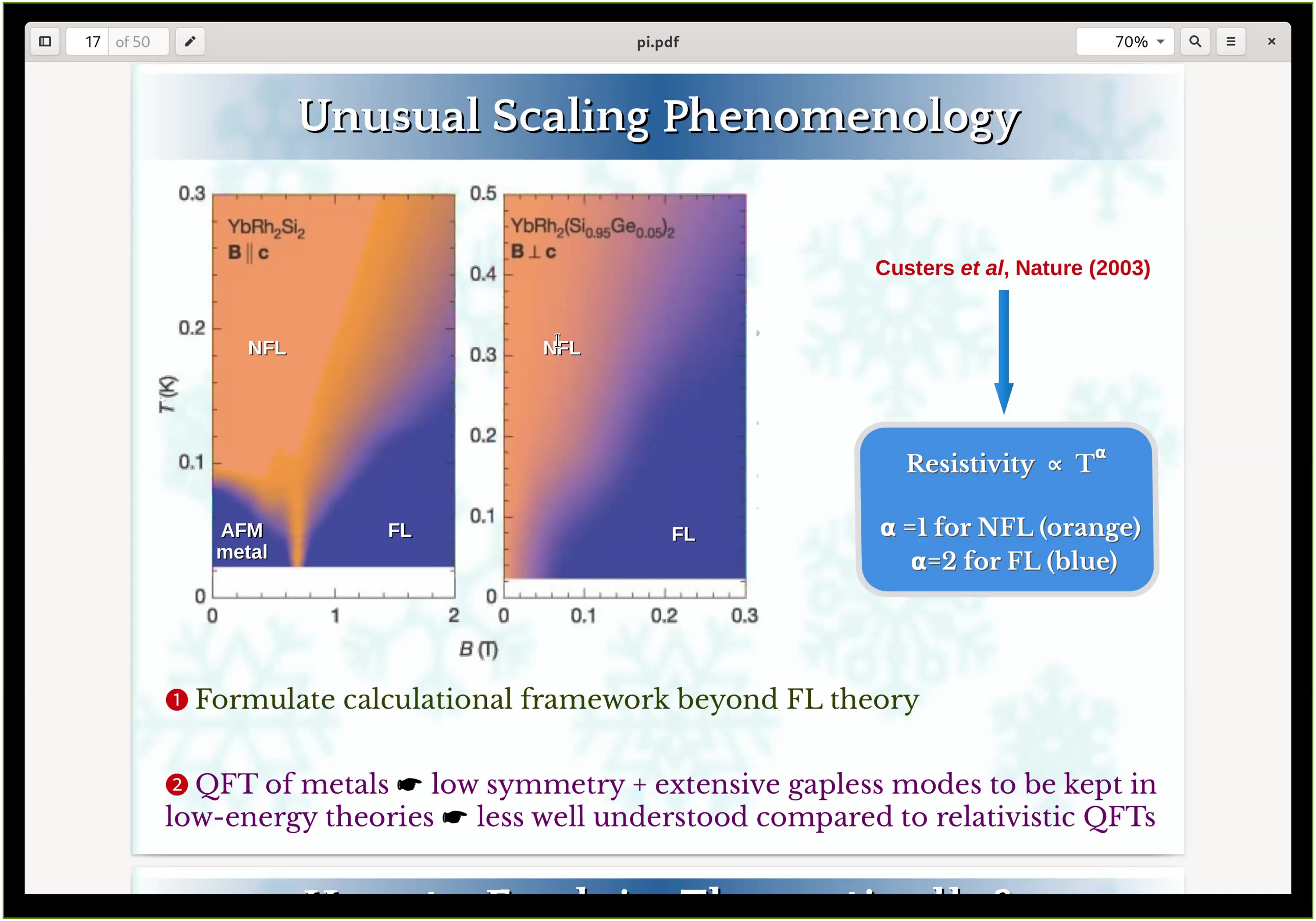
FS + Gapless Order Parameter fluctuations



Where do NFLs appear?

Phase diagram of high-T_c cuprates

- ☛ Power laws in many physical quantities distinct from that for FL
- ☛ No Landau quasiparticles



Unusual Scaling Phenomenology

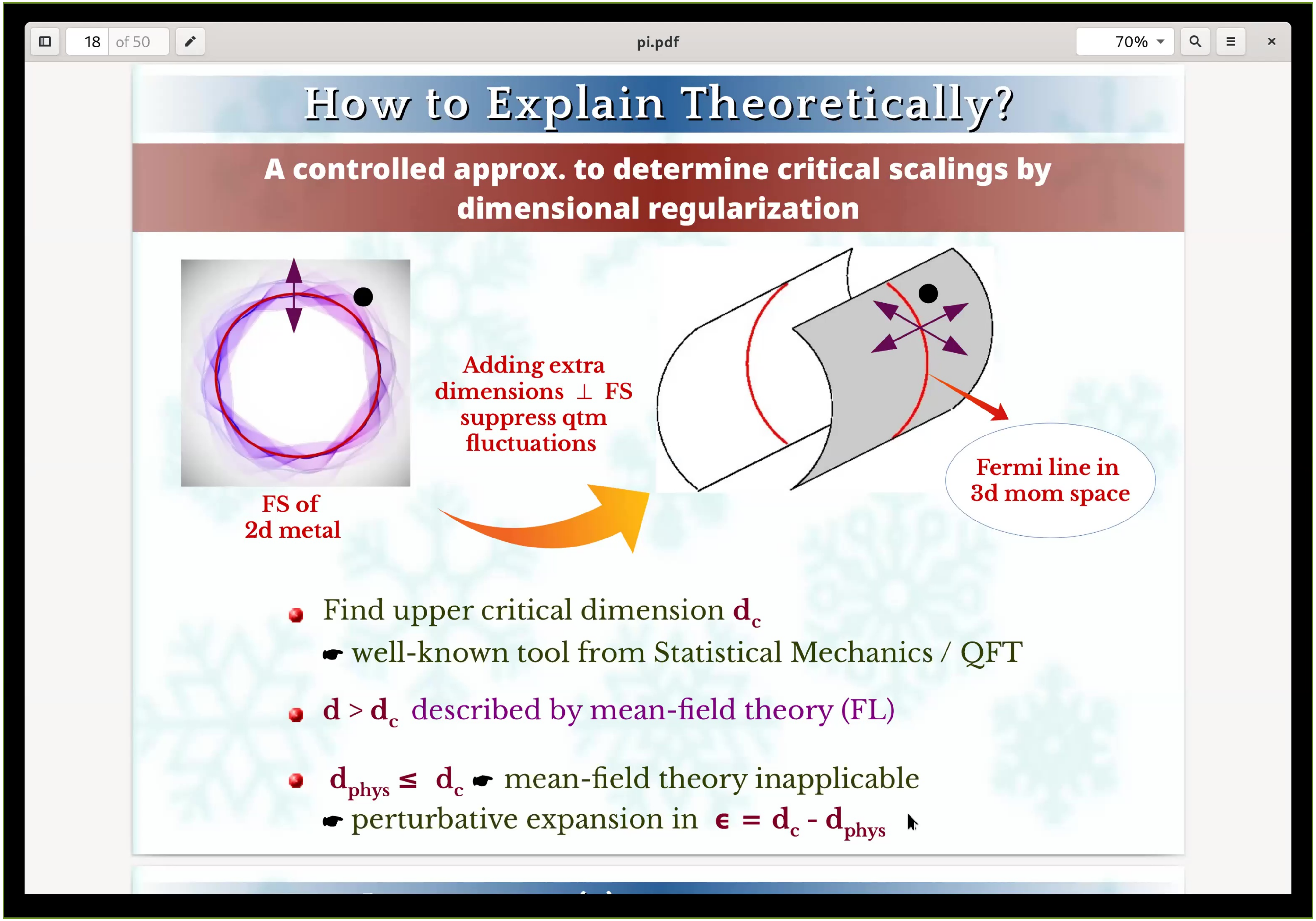
Custers *et al*, Nature (2003)

Resistivity $\propto T^{\alpha}$

α =1 for NFL (orange)
α=2 for FL (blue)

❶ Formulate calculational framework beyond FL theory

❷ QFT of metals ☛ low symmetry + extensive gapless modes to be kept in low-energy theories ☛ less well understood compared to relativistic QFTs



How to Explain Theoretically?

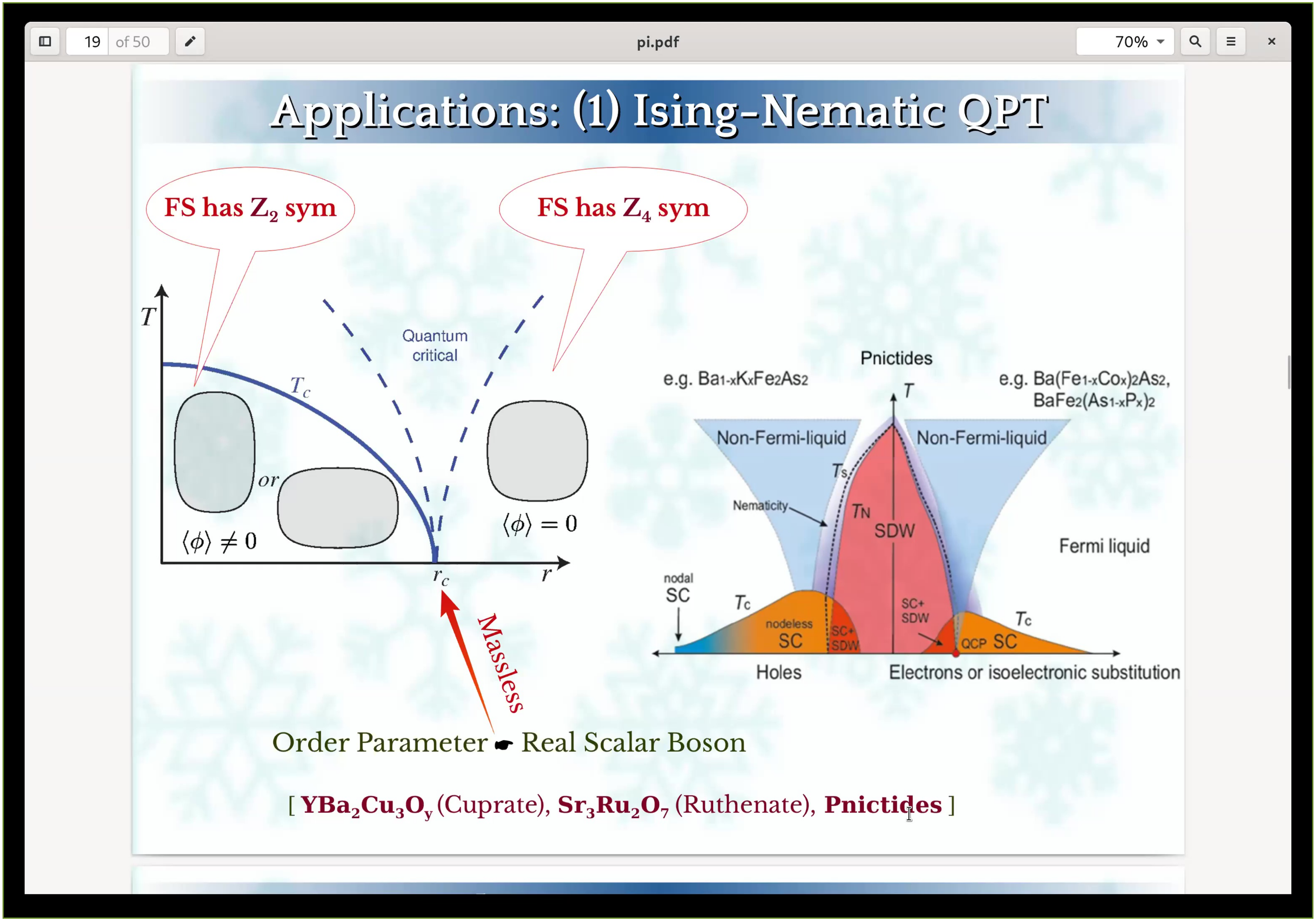
A controlled approx. to determine critical scalings by dimensional regularization

FS of 2d metal

Adding extra dimensions ⊥ FS suppress qtm fluctuations

Fermi line in 3d mom space

- Find upper critical dimension d_c
  - ☛ well-known tool from Statistical Mechanics / QFT
- $d > d_c$ described by mean-field theory (FL)
- $d_{phys} \leq d_c$ ☛ mean-field theory inapplicable
  - ☛ perturbative expansion in $\epsilon = d_c - d_{phys}$



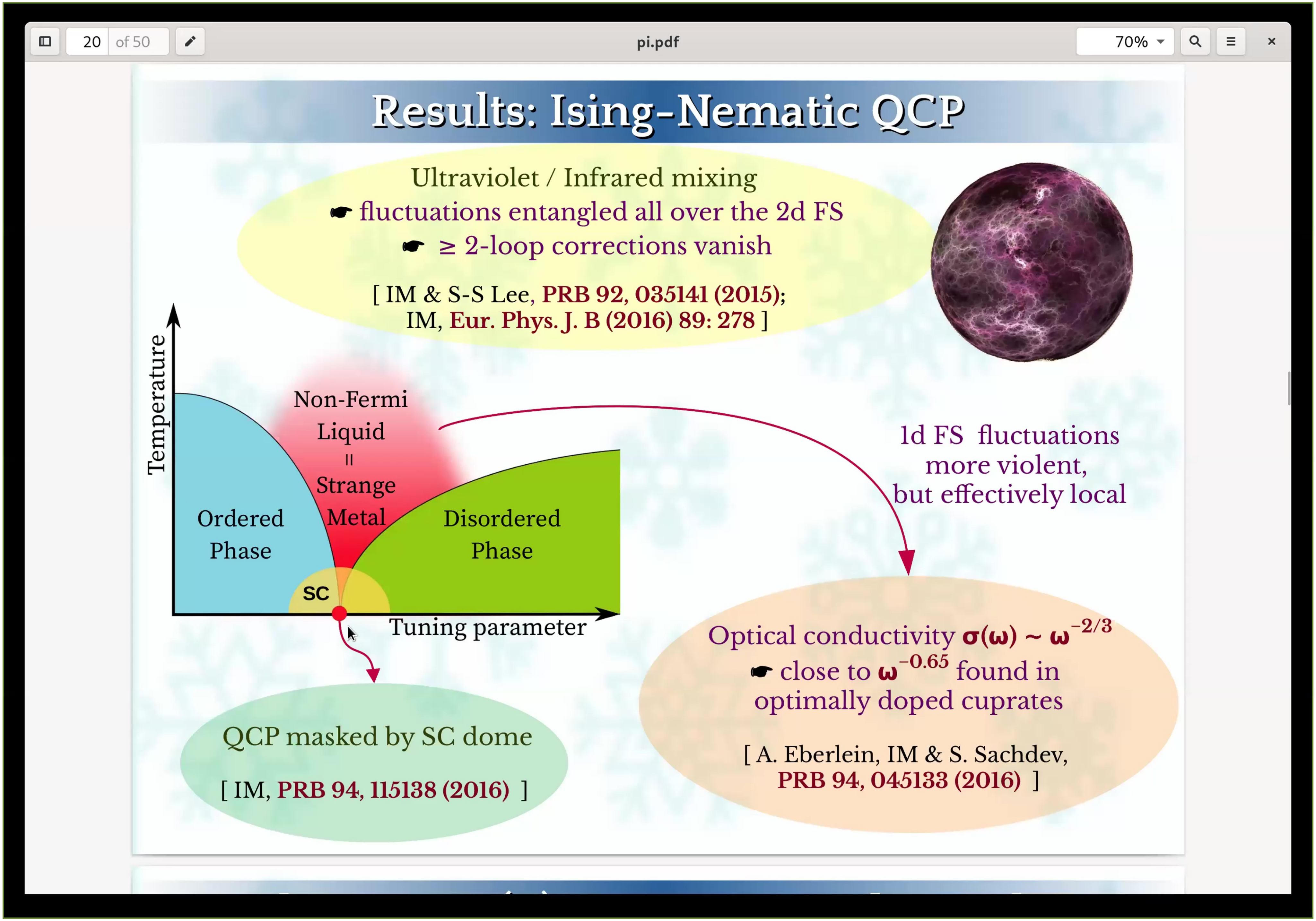
Applications: (1) Ising-Nematic QPT

FS has Z_2 sym

FS has Z_4 sym

Order Parameter ☛ Real Scalar Boson

[**$YBa_2Cu_3O_y$** (Cuprate), **$Sr_3Ru_2O_7$** (Ruthenate), **Pnictides**]



Results: Ising-Nematic QCP

Ultraviolet / Infrared mixing

☛ fluctuations entangled all over the 2d FS

☛ ≥ 2-loop corrections vanish

[IM & S-S Lee, **PRB 92, 035141 (2015)**;
IM, **Eur. Phys. J. B (2016) 89: 278**]

1d FS fluctuations
more violent,
but effectively local

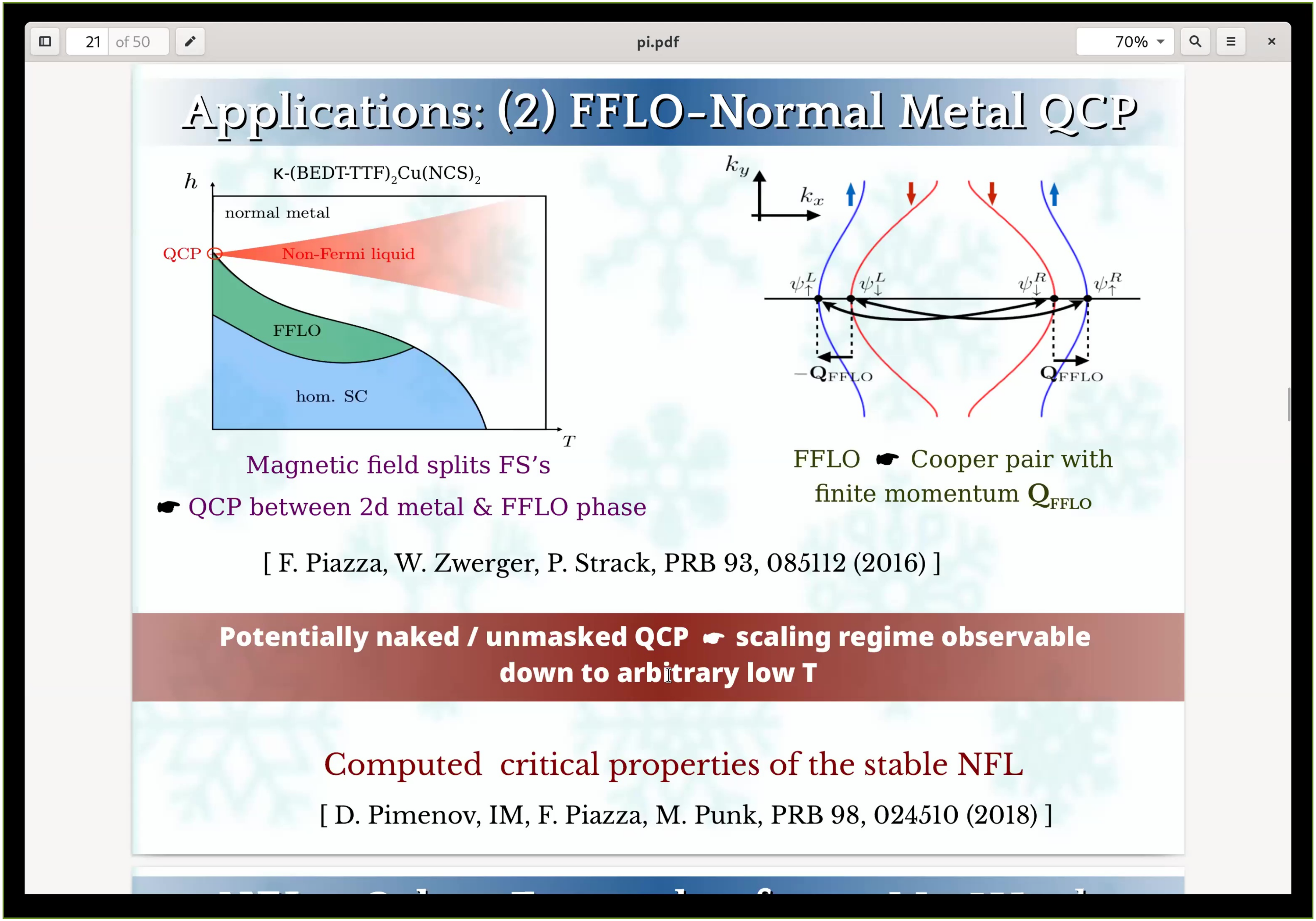
QCP masked by SC dome

[IM, **PRB 94, 115138 (2016)**]

Optical conductivity $\boldsymbol{\sigma(\omega)} \sim \boldsymbol{\omega}^{-2/3}$

☛ close to $\boldsymbol{\omega}^{-0.65}$ found in optimally doped cuprates

[A. Eberlein, IM & S. Sachdev,
PRB 94, 045133 (2016)]



Applications: (2) FFLO-Normal Metal QCP

Magnetic field splits FS's

☛ QCP between 2d metal & FFLO phase

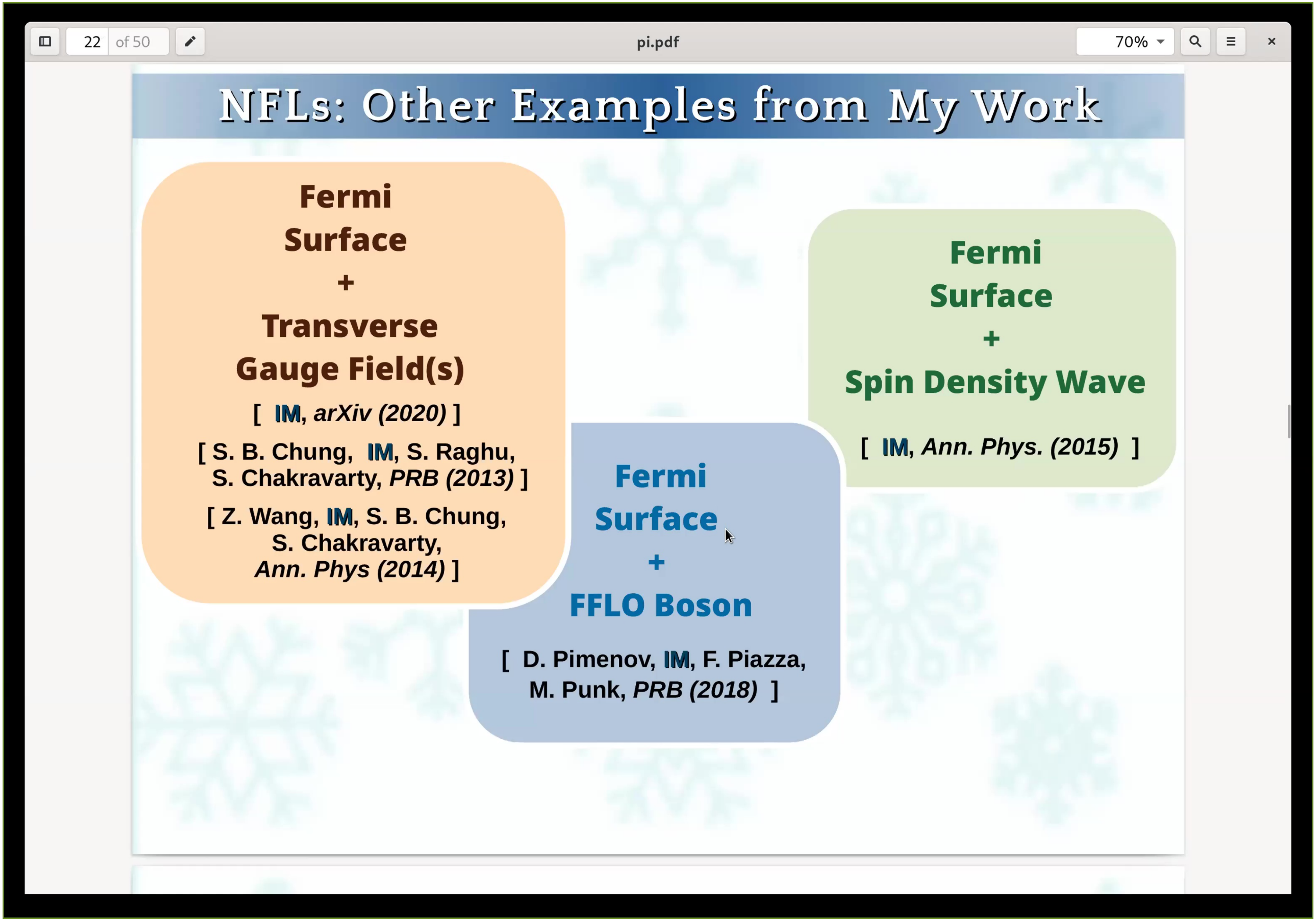
FFLO ☛ Cooper pair with finite momentum $\mathbf{Q}_{\mathrm{FFLO}}$

[F. Piazza, W. Zwerger, P. Strack, PRB 93, 085112 (2016)]

Potentially naked / unmasked QCP ☛ scaling regime observable down to arbitrary low T

Computed critical properties of the stable NFL

[D. Pimenov, IM, F. Piazza, M. Punk, PRB 98, 024510 (2018)]



NFLs: Other Examples from My Work

**Fermi
Surface
+
Transverse
Gauge Field(s)**

[**IM**, *arXiv (2020)*]

[S. B. Chung, **IM**, S. Raghu,
S. Chakravarty, *PRB (2013)*]

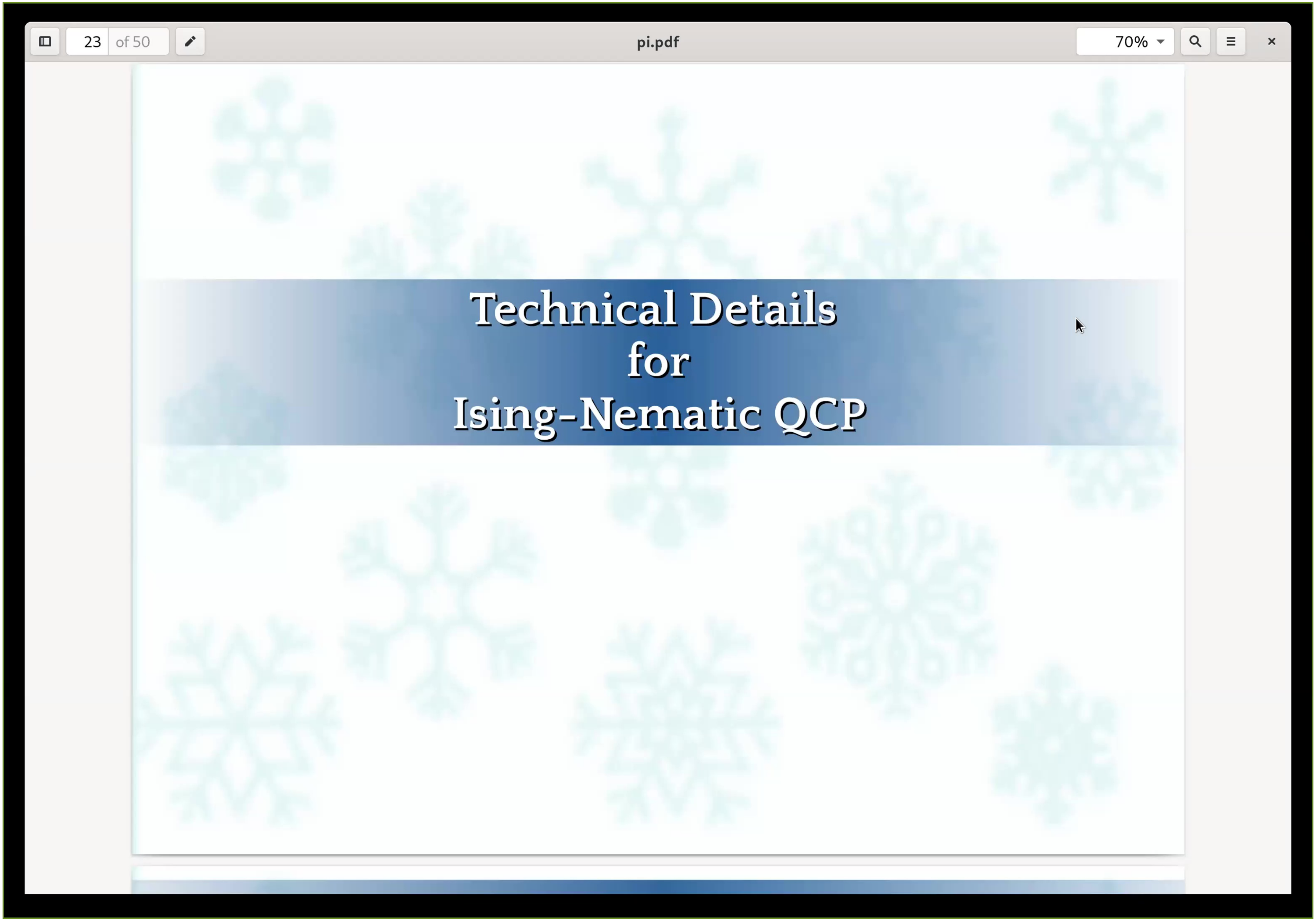
[Z. Wang, **IM**, S. B. Chung,
S. Chakravarty,
Ann. Phys (2014)]

**Fermi
Surface
+
FFLO Boson**

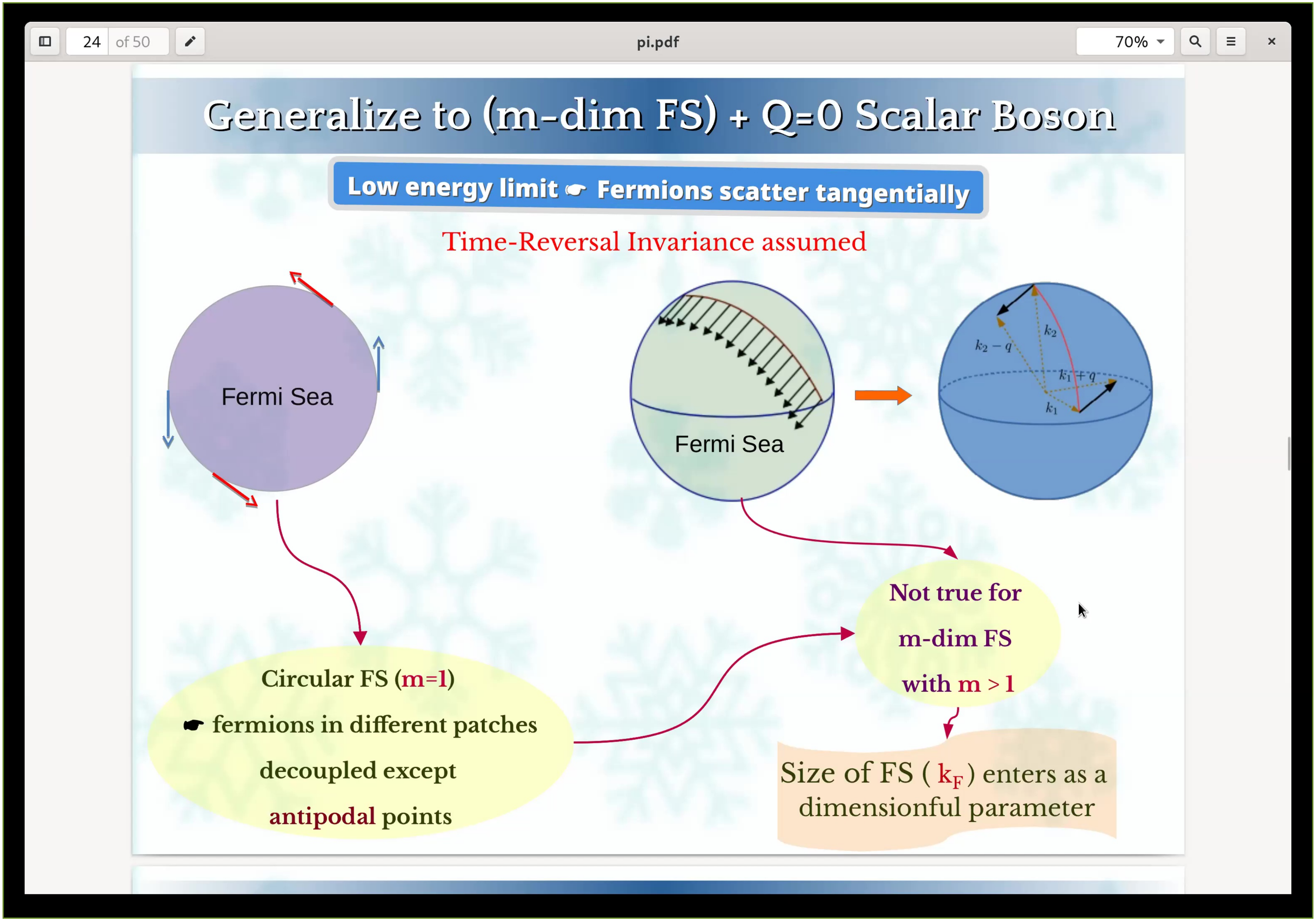
[D. Pimenov, **IM**, F. Piazza,
M. Punk, *PRB (2018)*]

**Fermi
Surface
+
Spin Density Wave**

[**IM**, *Ann. Phys. (2015)*]



Technical Details
for
Ising-Nematic QCP



Generalize to (m-dim FS) + Q=0 Scalar Boson

Low energy limit ☛ Fermions scatter tangentially

Time-Reversal Invariance assumed

Circular FS (m=1)

☛ fermions in different patches

decoupled except

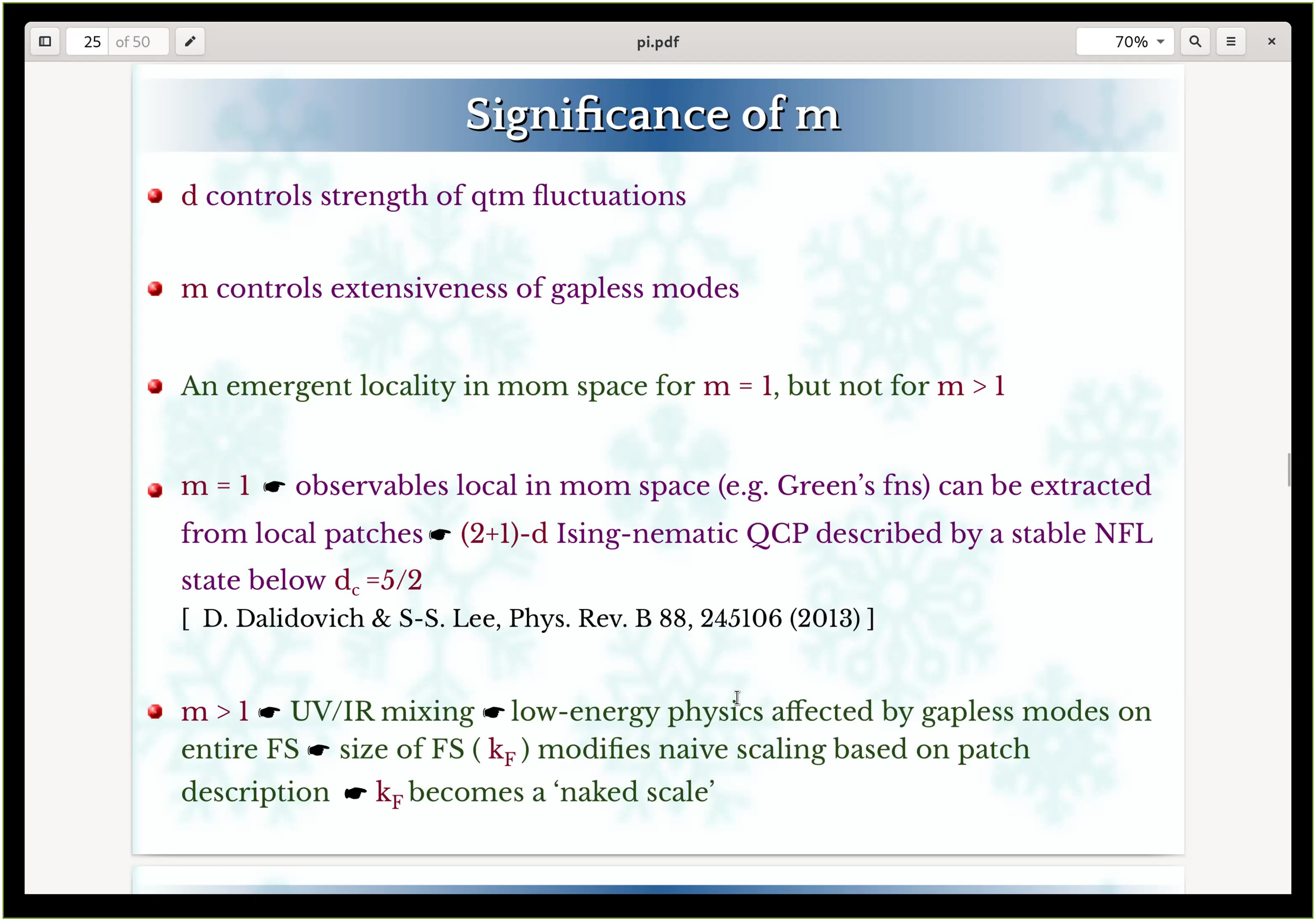
antipodal points

Not true for

m-dim FS

with m > 1

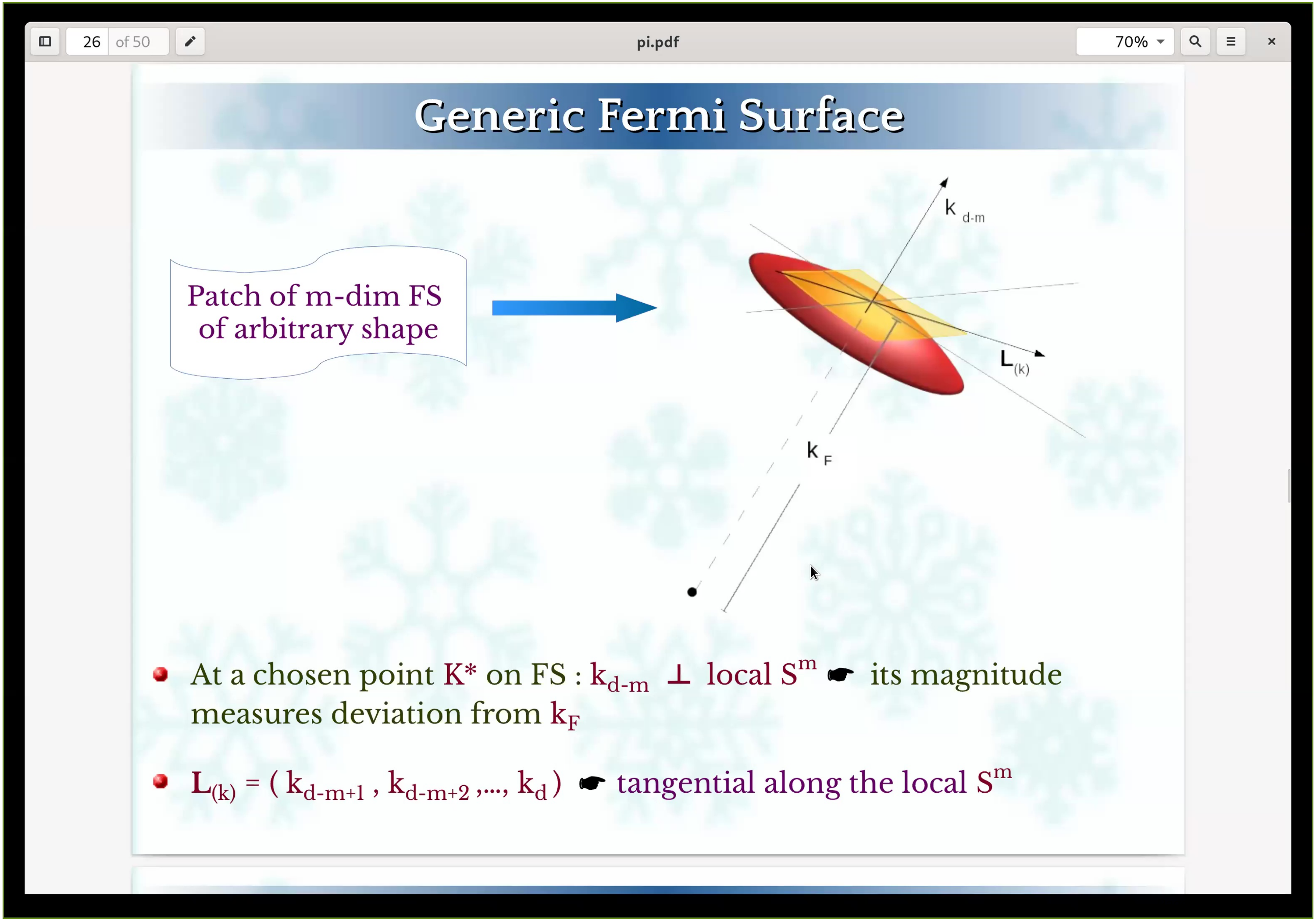
Size of FS (k_F) enters as a dimensionful parameter



Significance of m

- d controls strength of qtm fluctuations
- m controls extensiveness of gapless modes
- An emergent locality in mom space for m = 1, but not for m > 1
- m = 1 ☛ observables local in mom space (e.g. Green’s fns) can be extracted from local patches ☛ (2+1)-d Ising-nematic QCP described by a stable NFL state below d_c =5/2

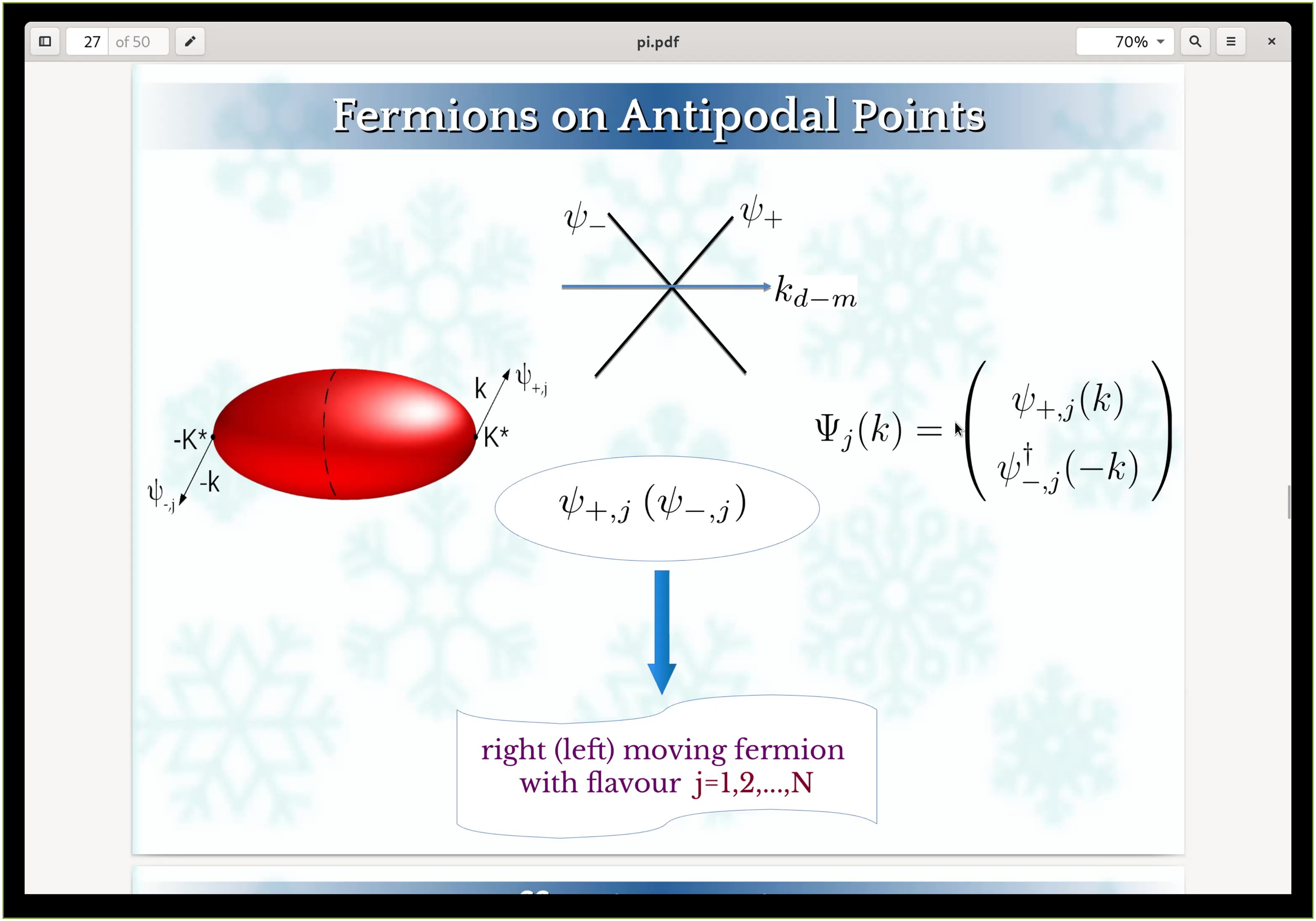
  [D. Dalidovich & S-S. Lee, Phys. Rev. B 88, 245106 (2013)]
- m > 1 ☛ UV/IR mixing ☛ low-energy physics affected by gapless modes on entire FS ☛ size of FS (k_F) modifies naive scaling based on patch description ☛ k_F becomes a ‘naked scale’



Generic Fermi Surface

Patch of m-dim FS of arbitrary shape

- At a chosen point K* on FS : k_{d-m} ⊥ local S^m ☛ its magnitude measures deviation from k_F
- $L_{(k)} = (k_{d-m+1} , k_{d-m+2} ,..., k_d)$ ☛ tangential along the local S^m

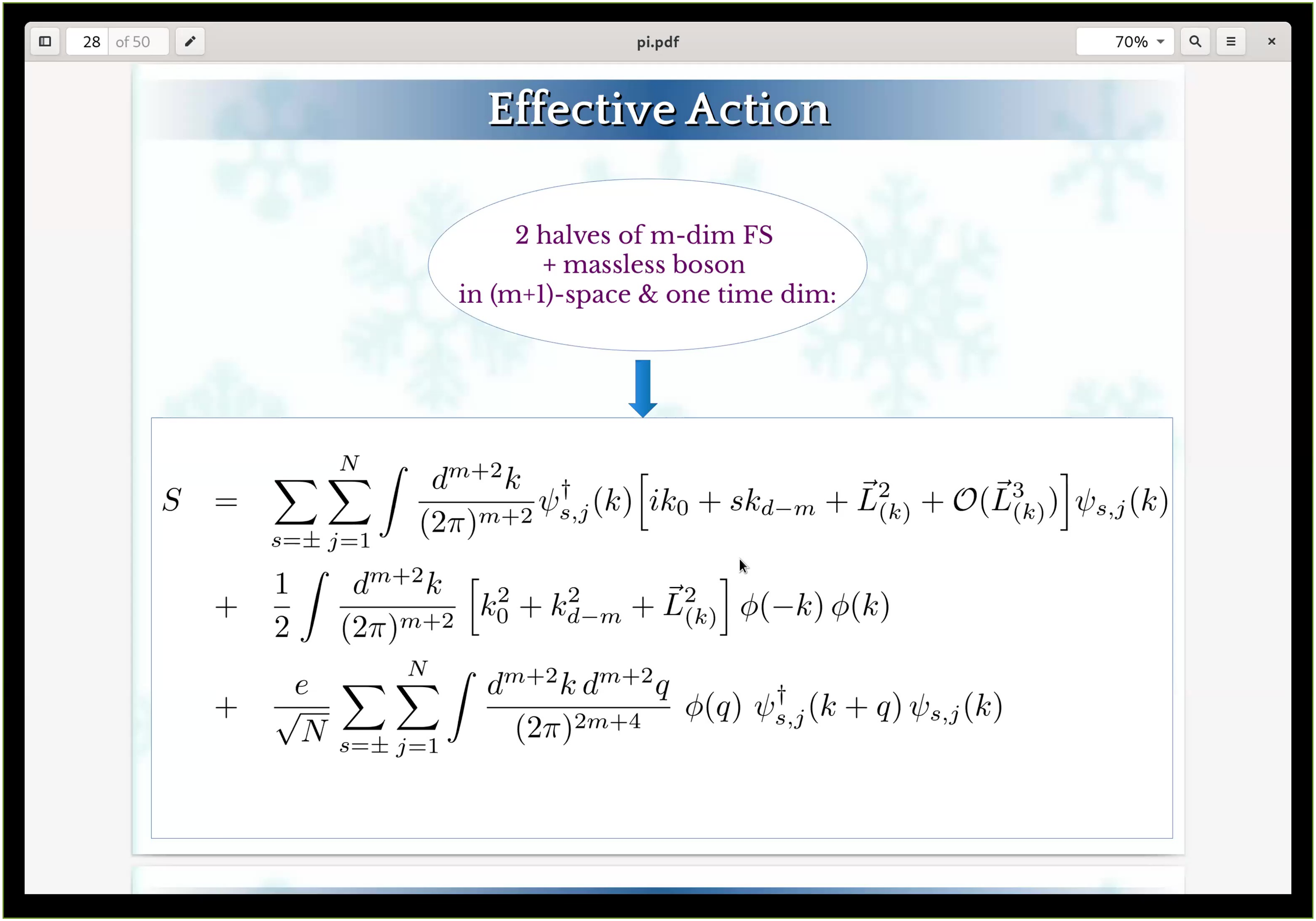


Fermions on Antipodal Points

$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^\dagger_{-,j}(-k) \end{pmatrix}$$

$$\psi_{+,j} \; (\psi_{-,j})$$

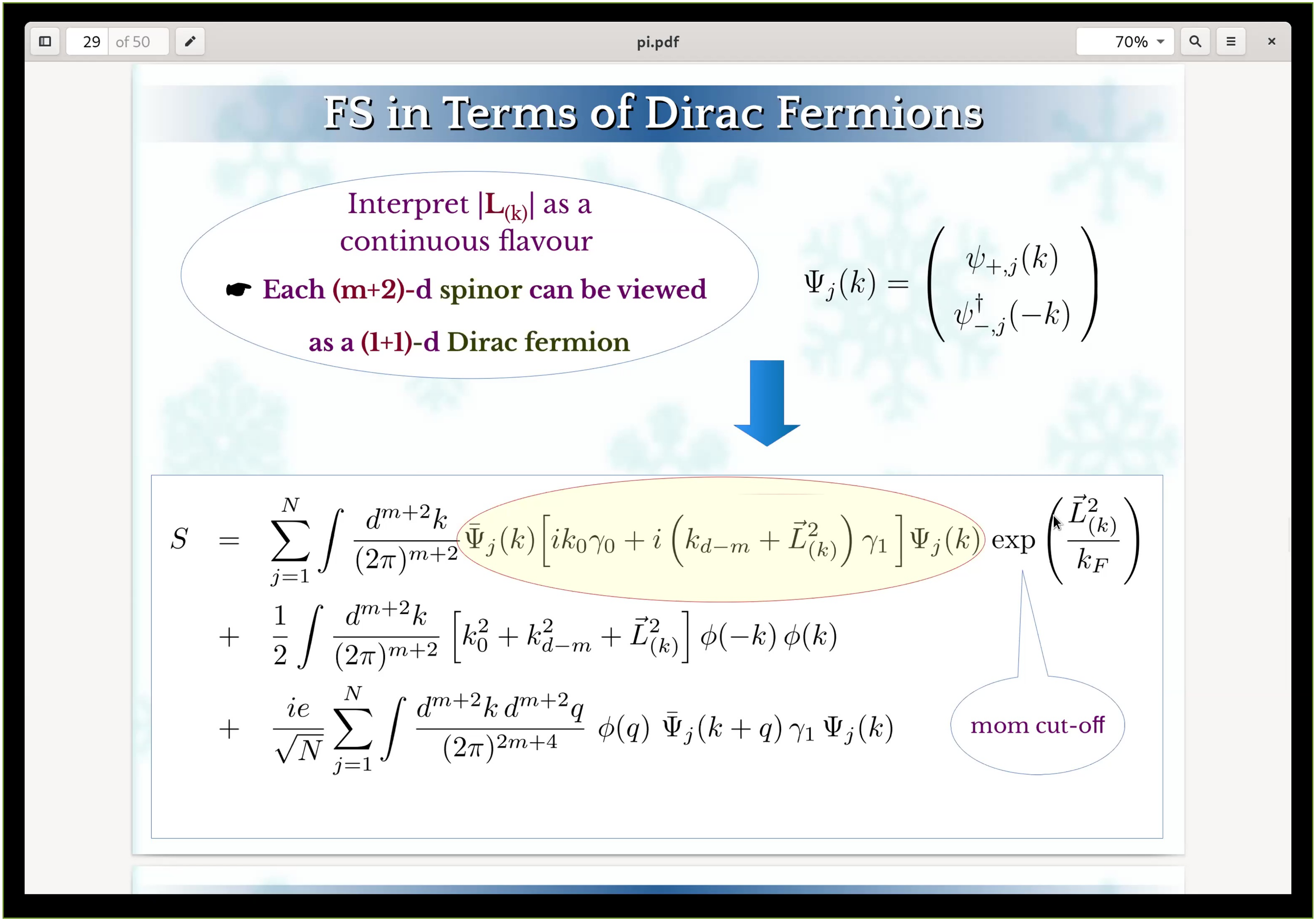
right (left) moving fermion
with flavour j=1,2,...,N



Effective Action

2 halves of m-dim FS
+ massless boson
in (m+1)-space & one time dim:

$$
\begin{aligned}
S \;=\; & \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^{m+2}k}{(2\pi)^{m+2}}\,\psi^{\dagger}_{s,j}(k)\left[ik_0 + s k_{d-m} + \vec{L}^2_{(k)} + \mathcal{O}(\vec{L}^3_{(k)})\right]\psi_{s,j}(k) \\
+ \; & \frac{1}{2}\int \frac{d^{m+2}k}{(2\pi)^{m+2}}\left[k_0^2 + k_{d-m}^2 + \vec{L}^2_{(k)}\right]\phi(-k)\,\phi(k) \\
+ \; & \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^{m+2}k\,d^{m+2}q}{(2\pi)^{2m+4}}\;\phi(q)\;\psi^{\dagger}_{s,j}(k+q)\,\psi_{s,j}(k)
\end{aligned}
$$



FS in Terms of Dirac Fermions

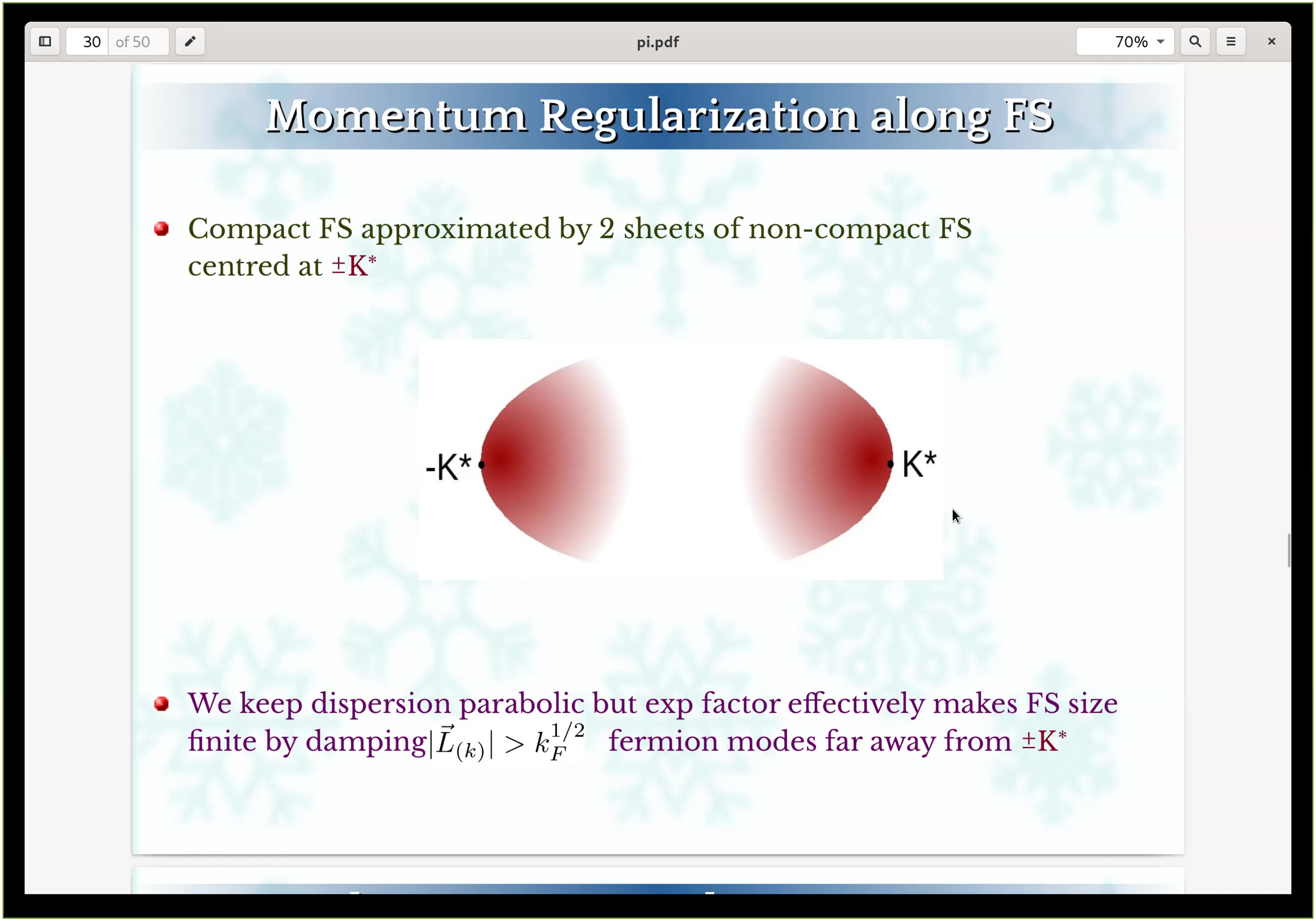
Interpret $|\mathbf{L}_{(k)}|$ as a continuous flavour

☛ **Each (m+2)-d spinor can be viewed as a (1+1)-d Dirac fermion**

$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^{\dagger}_{-,j}(-k) \end{pmatrix}$$

$$\begin{aligned} S \;=\;& \sum_{j=1}^{N} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \bar{\Psi}_j(k) \left[i k_0 \gamma_0 + i \left(k_{d-m} + \vec{L}^2_{(k)} \right) \gamma_1 \right] \Psi_j(k) \, \exp\left(\frac{\vec{L}^2_{(k)}}{k_F} \right) \\ +\;& \frac{1}{2} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \left[k_0^2 + k_{d-m}^2 + \vec{L}^2_{(k)} \right] \phi(-k)\, \phi(k) \\ +\;& \frac{ie}{\sqrt{N}} \sum_{j=1}^{N} \int \frac{d^{m+2}k \, d^{m+2}q}{(2\pi)^{2m+4}} \, \phi(q) \; \bar{\Psi}_j(k+q) \, \gamma_1 \, \Psi_j(k) \end{aligned}$$

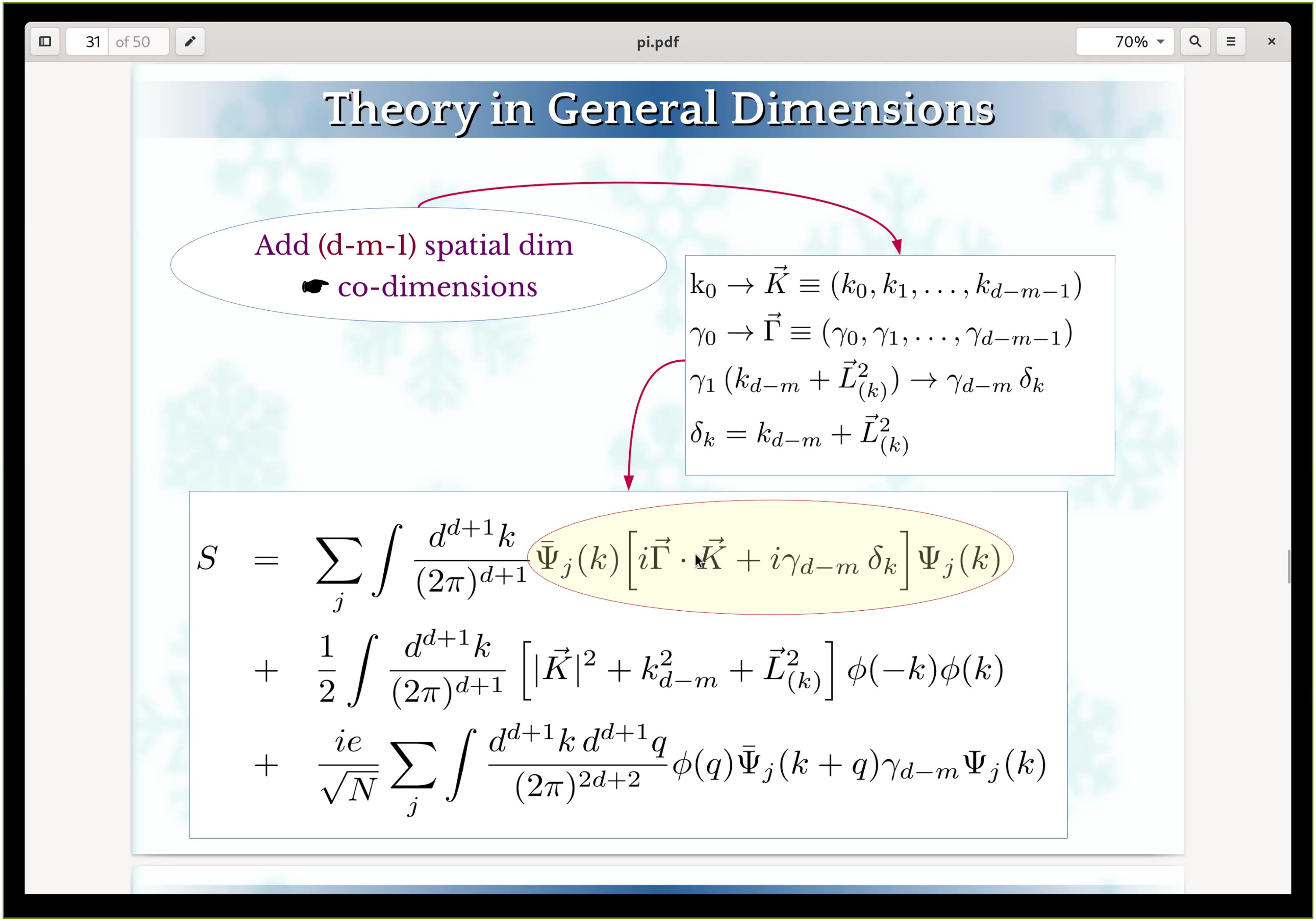
mom cut-off



Momentum Regularization along FS

- Compact FS approximated by 2 sheets of non-compact FS centred at ±K^*

- We keep dispersion parabolic but exp factor effectively makes FS size finite by damping $|\vec{L}_{(k)}| > k_F^{1/2}$ fermion modes far away from ±K^*



Theory in General Dimensions

Add (d-m-1) spatial dim
☛ co-dimensions

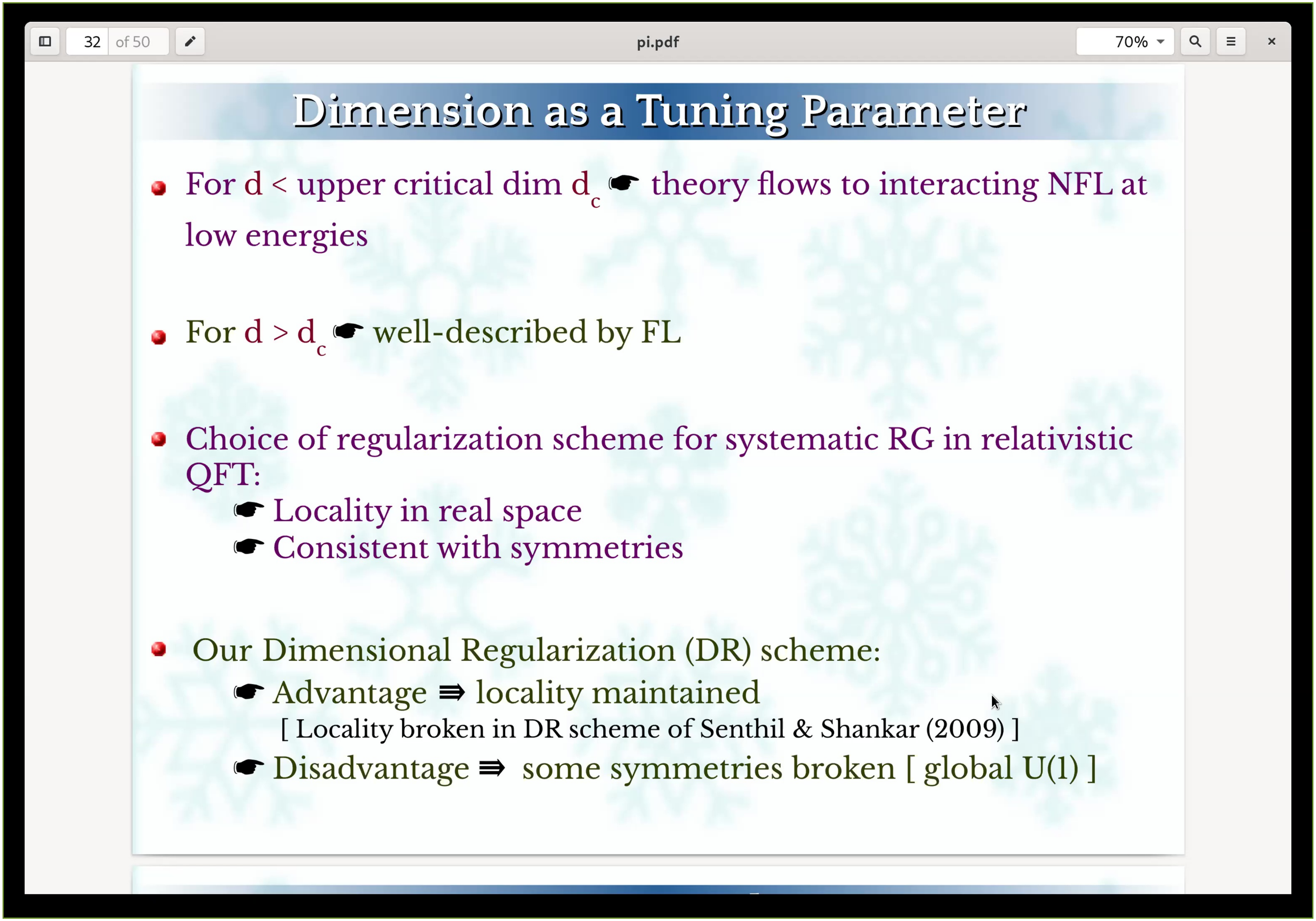
$$k_0 \rightarrow \vec{K} \equiv (k_0, k_1, \ldots, k_{d-m-1})$$

$$\gamma_0 \rightarrow \vec{\Gamma} \equiv (\gamma_0, \gamma_1, \ldots, \gamma_{d-m-1})$$

$$\gamma_1 \, (k_{d-m} + \vec{L}^2_{(k)}) \rightarrow \gamma_{d-m} \, \delta_k$$

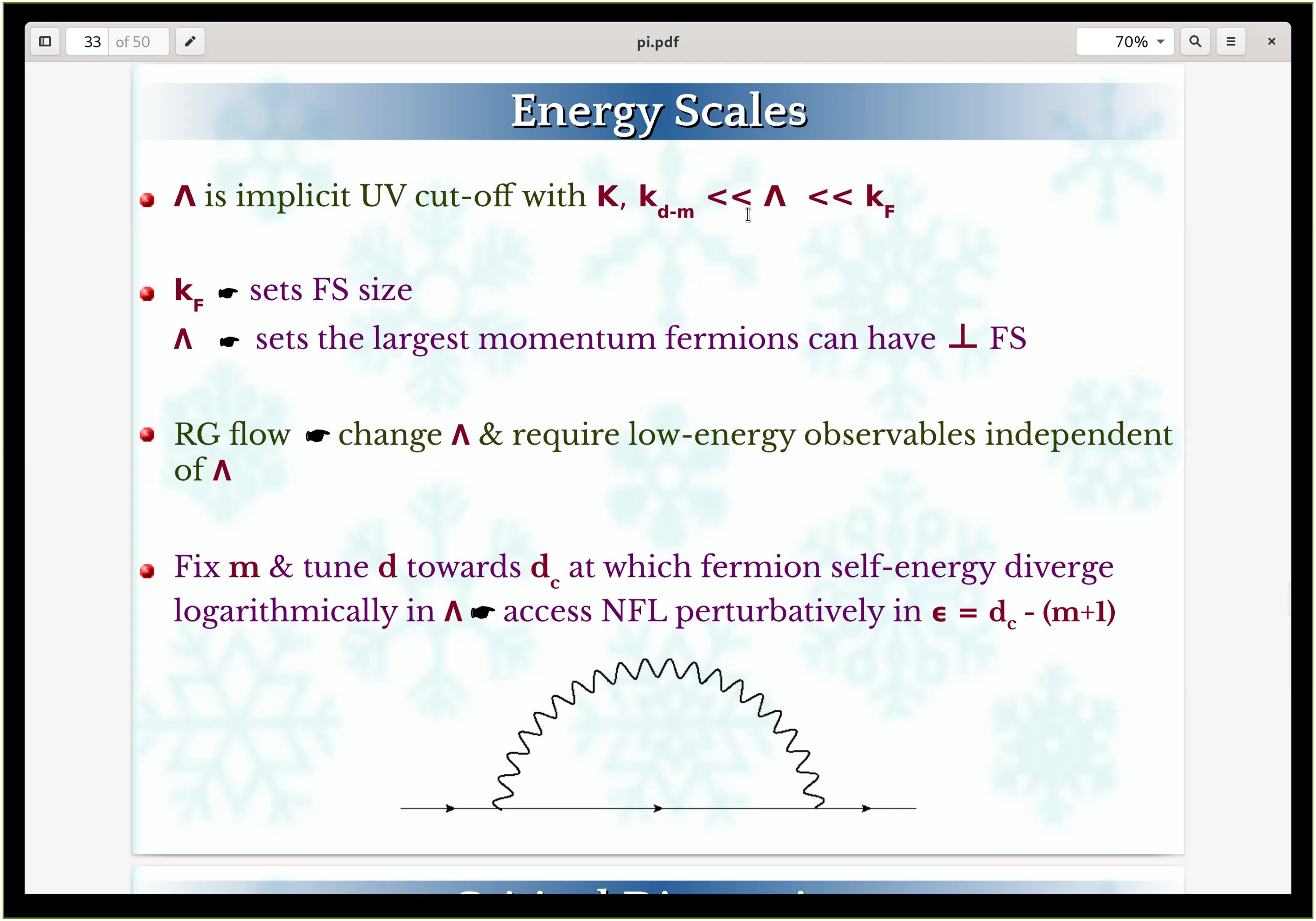
$$\delta_k = k_{d-m} + \vec{L}^2_{(k)}$$

$$\begin{aligned} S &= \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \left[i\vec{\Gamma} \cdot \vec{K} + i\gamma_{d-m} \, \delta_k \right] \Psi_j(k) \\ &+ \frac{1}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k^2_{d-m} + \vec{L}^2_{(k)} \right] \phi(-k)\phi(k) \\ &+ \frac{ie}{\sqrt{N}} \sum_j \int \frac{d^{d+1}k \, d^{d+1}q}{(2\pi)^{2d+2}} \phi(q) \bar{\Psi}_j(k+q) \gamma_{d-m} \Psi_j(k) \end{aligned}$$



Dimension as a Tuning Parameter

- For d < upper critical dim d_c ☛ theory flows to interacting NFL at low energies
- For d > d_c ☛ well-described by FL
- Choice of regularization scheme for systematic RG in relativistic QFT:
  - ☛ Locality in real space
  - ☛ Consistent with symmetries
- Our Dimensional Regularization (DR) scheme:
  - ☛ Advantage ⇛ locality maintained
    [Locality broken in DR scheme of Senthil & Shankar (2009)]
  - ☛ Disadvantage ⇛ some symmetries broken [global U(1)]



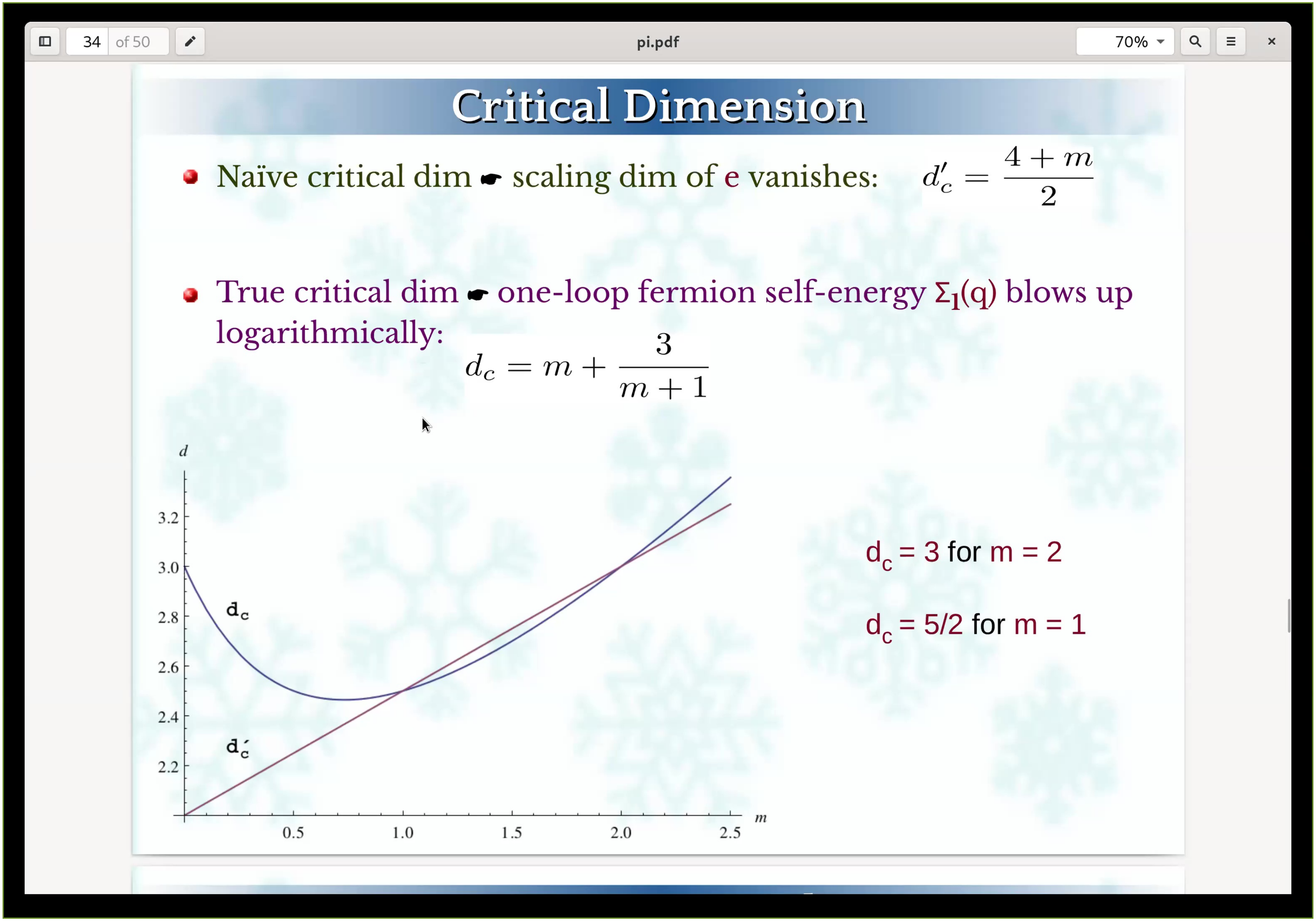
Energy Scales

- Λ is implicit UV cut-off with K, $k_{d-m} << \Lambda << k_F$

- k_F ☛ sets FS size
  Λ ☛ sets the largest momentum fermions can have $\perp$ FS

- RG flow ☛ change Λ & require low-energy observables independent of Λ

- Fix **m** & tune **d** towards d_c at which fermion self-energy diverge logarithmically in Λ ☛ access NFL perturbatively in $\epsilon = d_c - (m+1)$



Critical Dimension

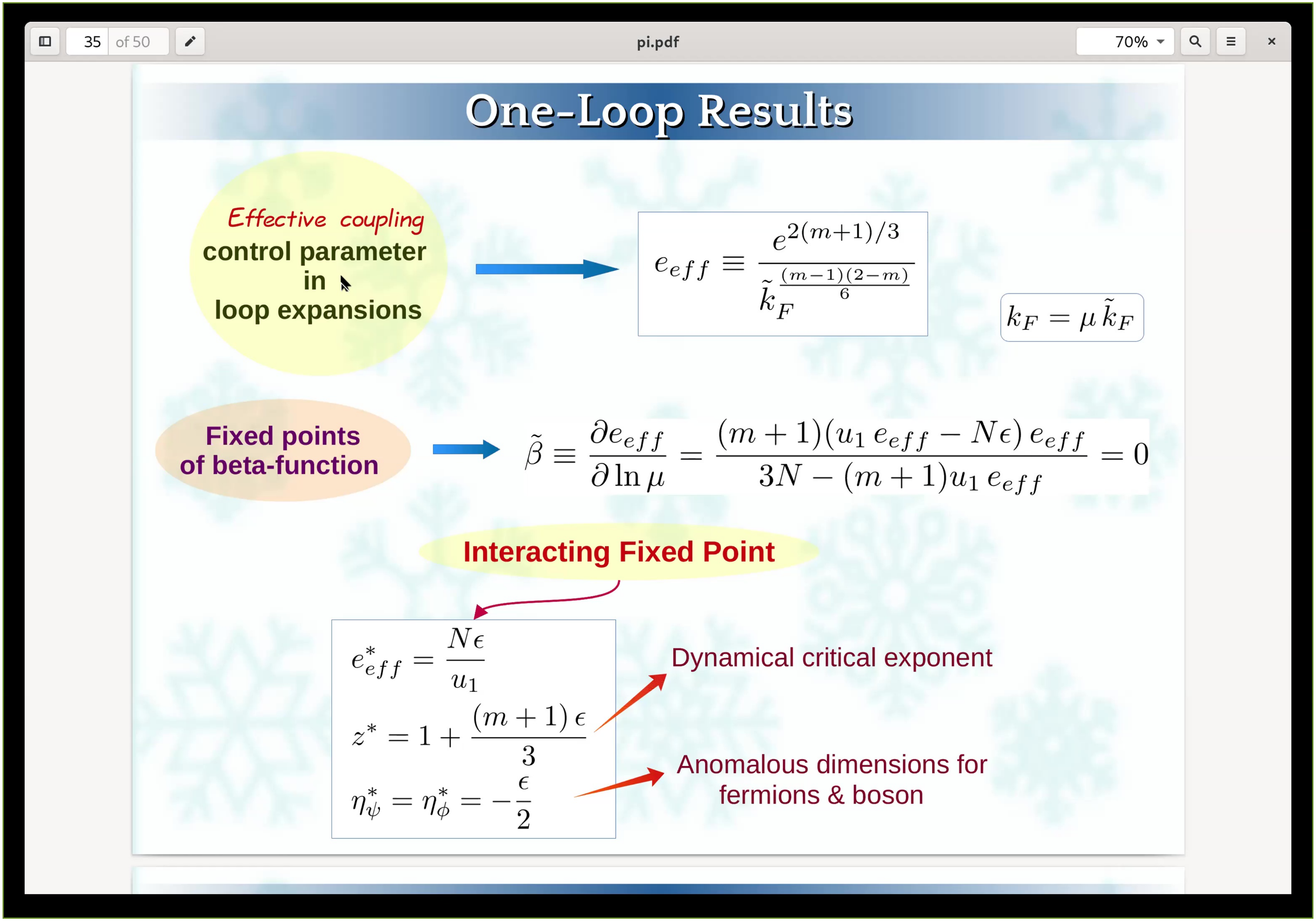
- Naïve critical dim ☛ scaling dim of e vanishes: $d_c' = \dfrac{4+m}{2}$

- True critical dim ☛ one-loop fermion self-energy $\Sigma_1(q)$ blows up logarithmically:

$$d_c = m + \frac{3}{m+1}$$

d_c = 3 for m = 2

d_c = 5/2 for m = 1



One-Loop Results

Effective coupling
control parameter in loop expansions

$$e_{eff} \equiv \frac{e^{2(m+1)/3}}{\tilde{k}_F^{\frac{(m-1)(2-m)}{6}}}$$

$$k_F = \mu\, \tilde{k}_F$$

Fixed points of beta-function

$$\tilde{\beta} \equiv \frac{\partial e_{eff}}{\partial \ln \mu} = \frac{(m+1)(u_1\, e_{eff} - N\epsilon)\, e_{eff}}{3N - (m+1) u_1\, e_{eff}} = 0$$

Interacting Fixed Point

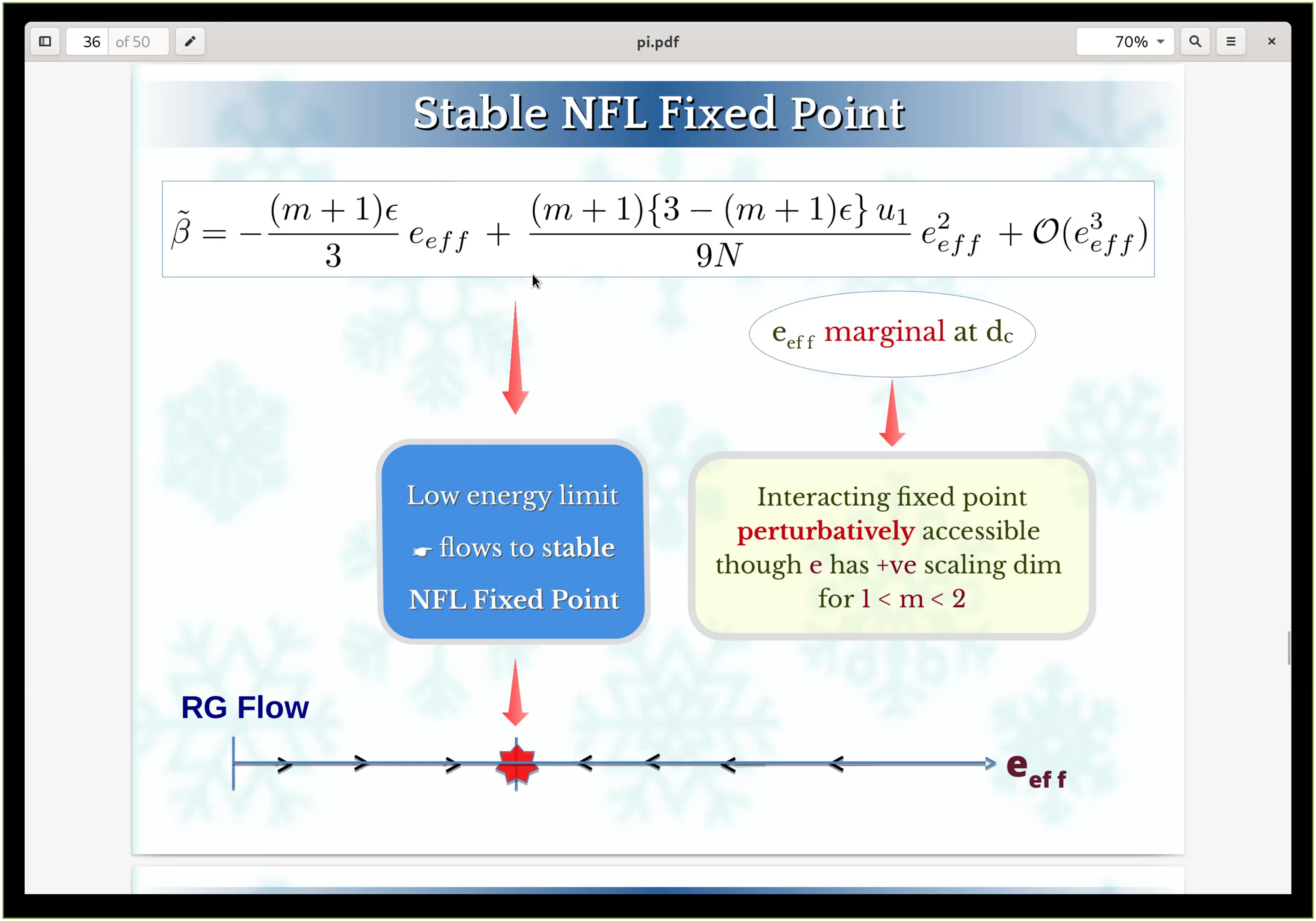
$$e^*_{eff} = \frac{N\epsilon}{u_1}$$

$$z^* = 1 + \frac{(m+1)\,\epsilon}{3}$$

$$\eta^*_\psi = \eta^*_\phi = -\frac{\epsilon}{2}$$

Dynamical critical exponent

Anomalous dimensions for fermions & boson



Stable NFL Fixed Point

$$\tilde{\beta} = -\frac{(m+1)\epsilon}{3}\, e_{eff} + \frac{(m+1)\{3-(m+1)\epsilon\}\, u_1}{9N}\, e_{eff}^2 + \mathcal{O}(e_{eff}^3)$$

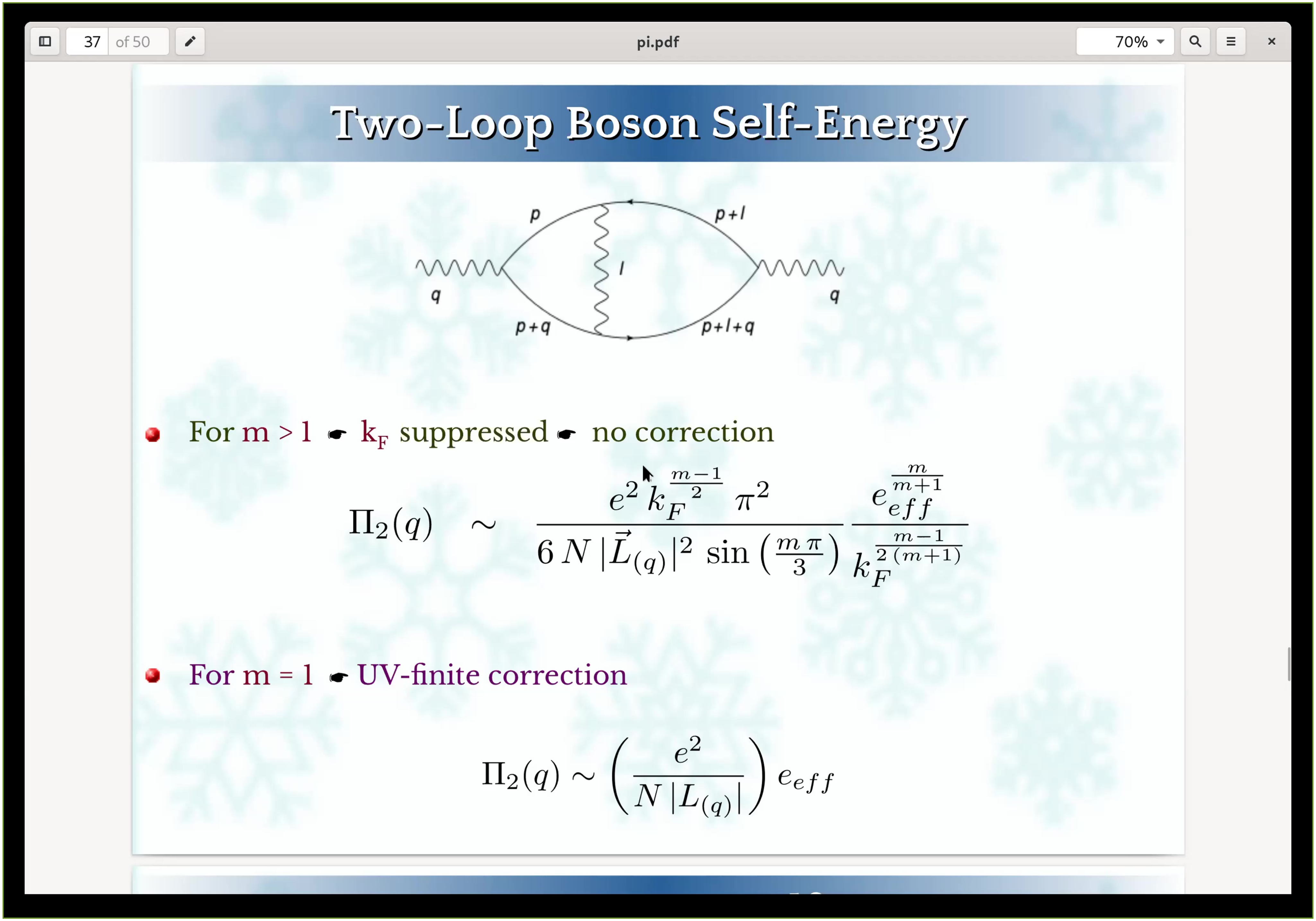
e_{eff} marginal at d_c

Low energy limit

☛ flows to stable

NFL Fixed Point

Interacting fixed point **perturbatively** accessible though e has +ve scaling dim for 1 < m < 2



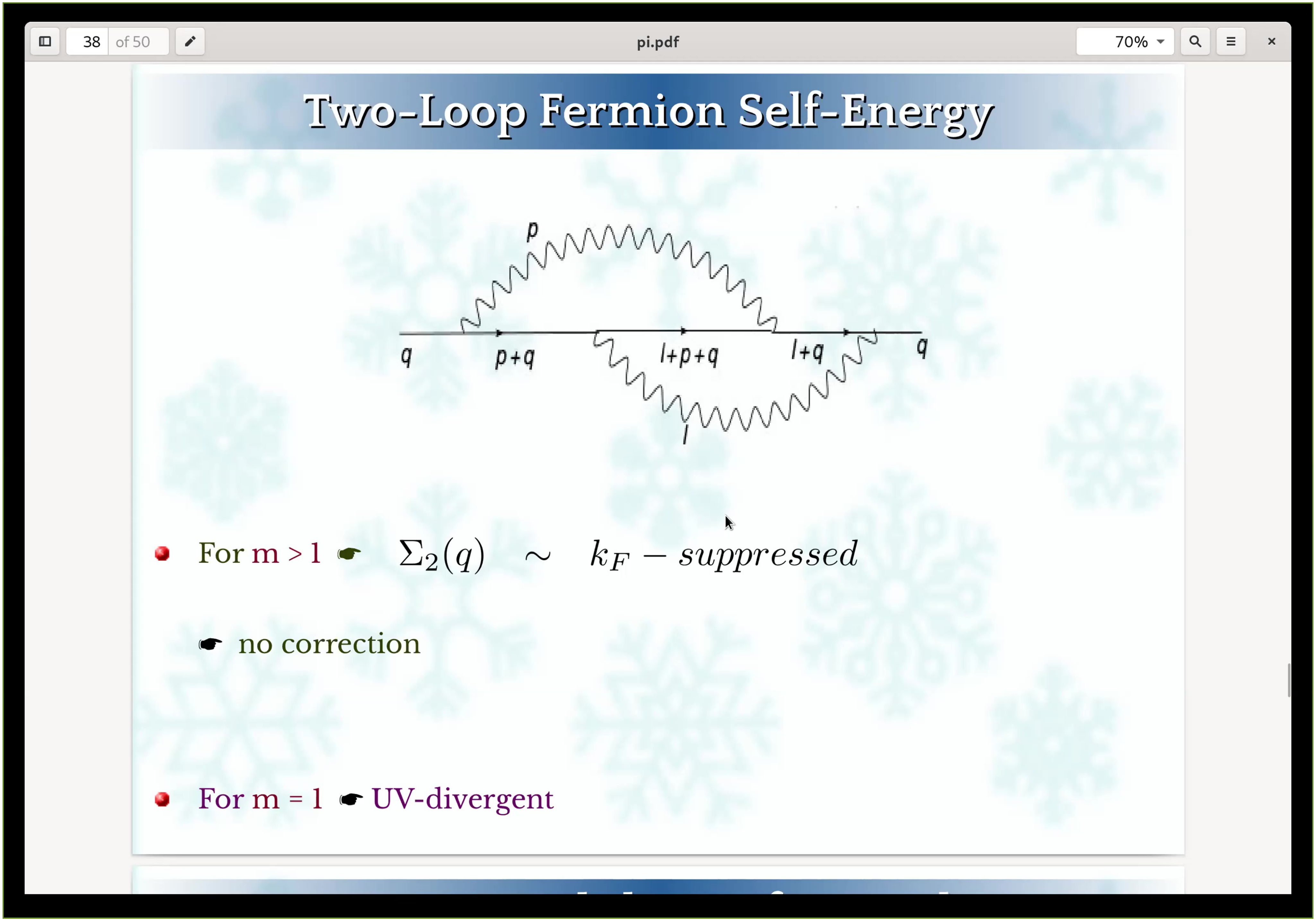
Two-Loop Boson Self-Energy

- For m > 1 ☛ k_F suppressed ☛ no correction

$$\Pi_2(q) \quad \sim \quad \frac{e^2\, k_F^{\frac{m-1}{2}}\ \pi^2}{6\, N\, |\vec{L}_{(q)}|^2\ \sin\left(\frac{m\,\pi}{3}\right)}\, \frac{e_{eff}^{\frac{m}{m+1}}}{k_F^{\frac{m-1}{2\,(m+1)}}}$$

- For m = 1 ☛ UV-finite correction

$$\Pi_2(q) \sim \left(\frac{e^2}{N\,|L_{(q)}|}\right) e_{eff}$$

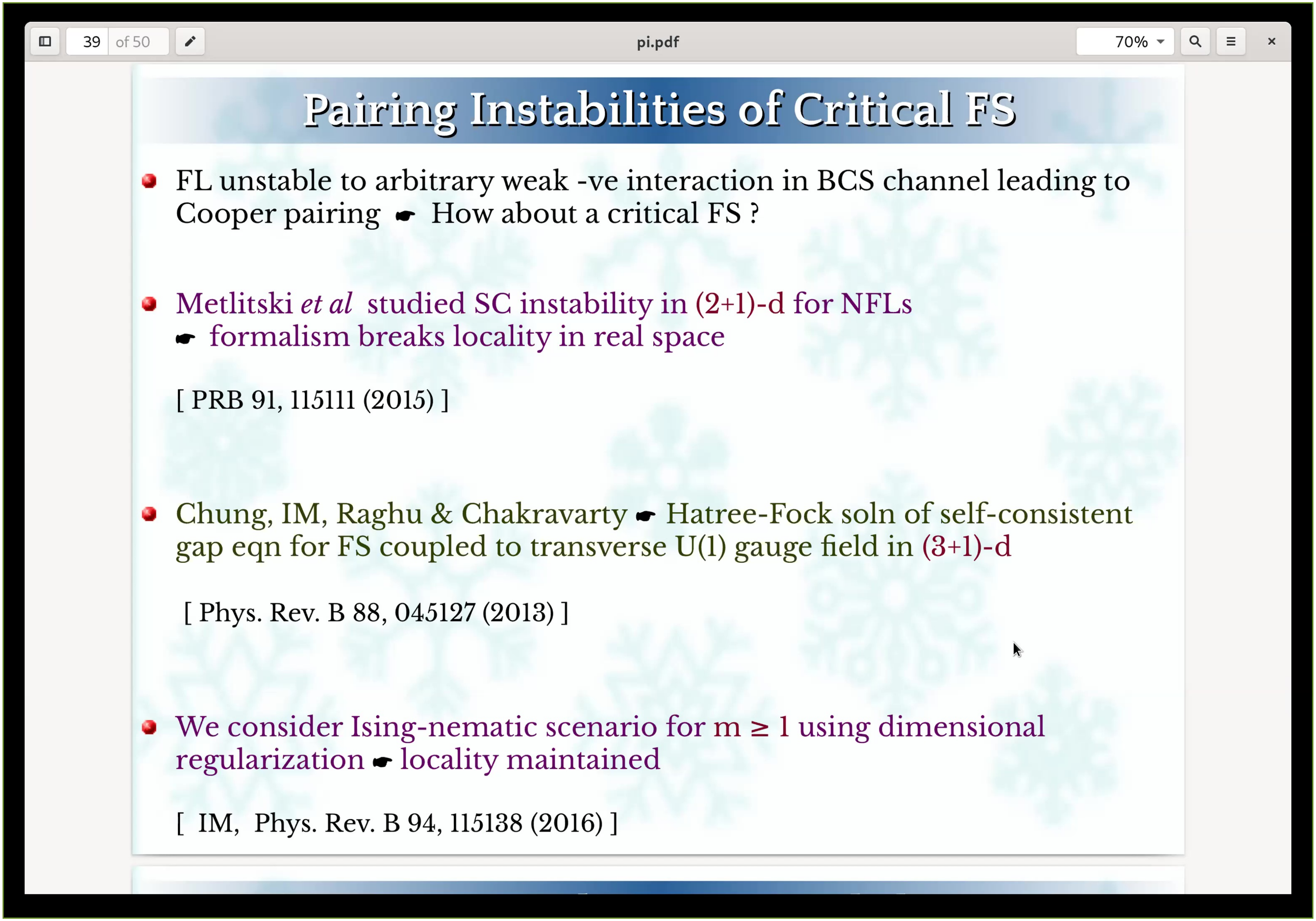


Two-Loop Fermion Self-Energy

- For m > 1 ☛ $\Sigma_2(q) \quad \sim \quad k_F - suppressed$

  ☛ no correction

- For m = 1 ☛ UV-divergent



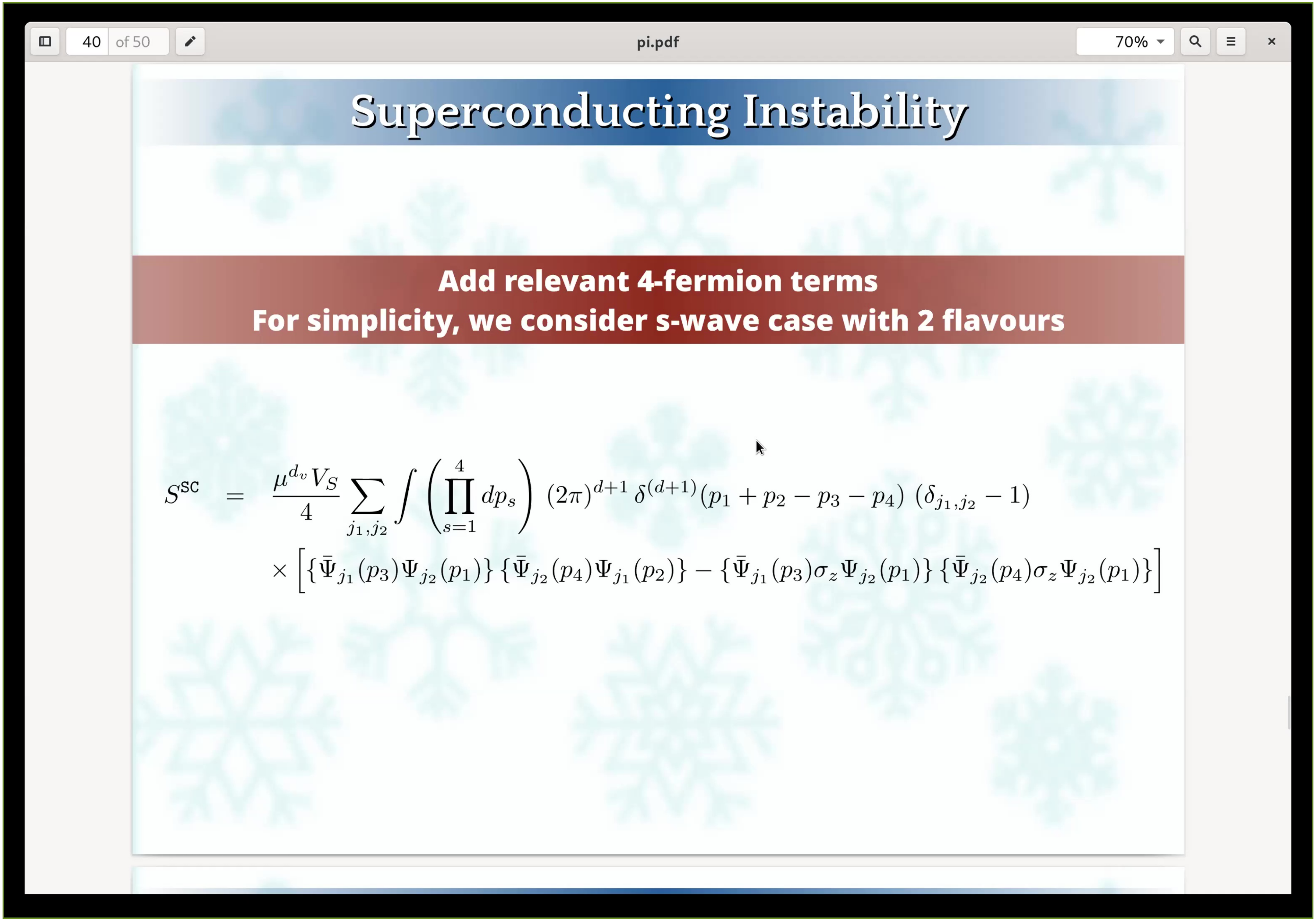
Pairing Instabilities of Critical FS

- FL unstable to arbitrary weak -ve interaction in BCS channel leading to Cooper pairing ☛ How about a critical FS ?
- Metlitski *et al* studied SC instability in (2+1)-d for NFLs
  ☛ formalism breaks locality in real space

  [PRB 91, 115111 (2015)]
- Chung, IM, Raghu & Chakravarty ☛ Hatree-Fock soln of self-consistent gap eqn for FS coupled to transverse U(1) gauge field in (3+1)-d

  [Phys. Rev. B 88, 045127 (2013)]
- We consider Ising-nematic scenario for m ≥ 1 using dimensional regularization ☛ locality maintained

  [IM, Phys. Rev. B 94, 115138 (2016)]

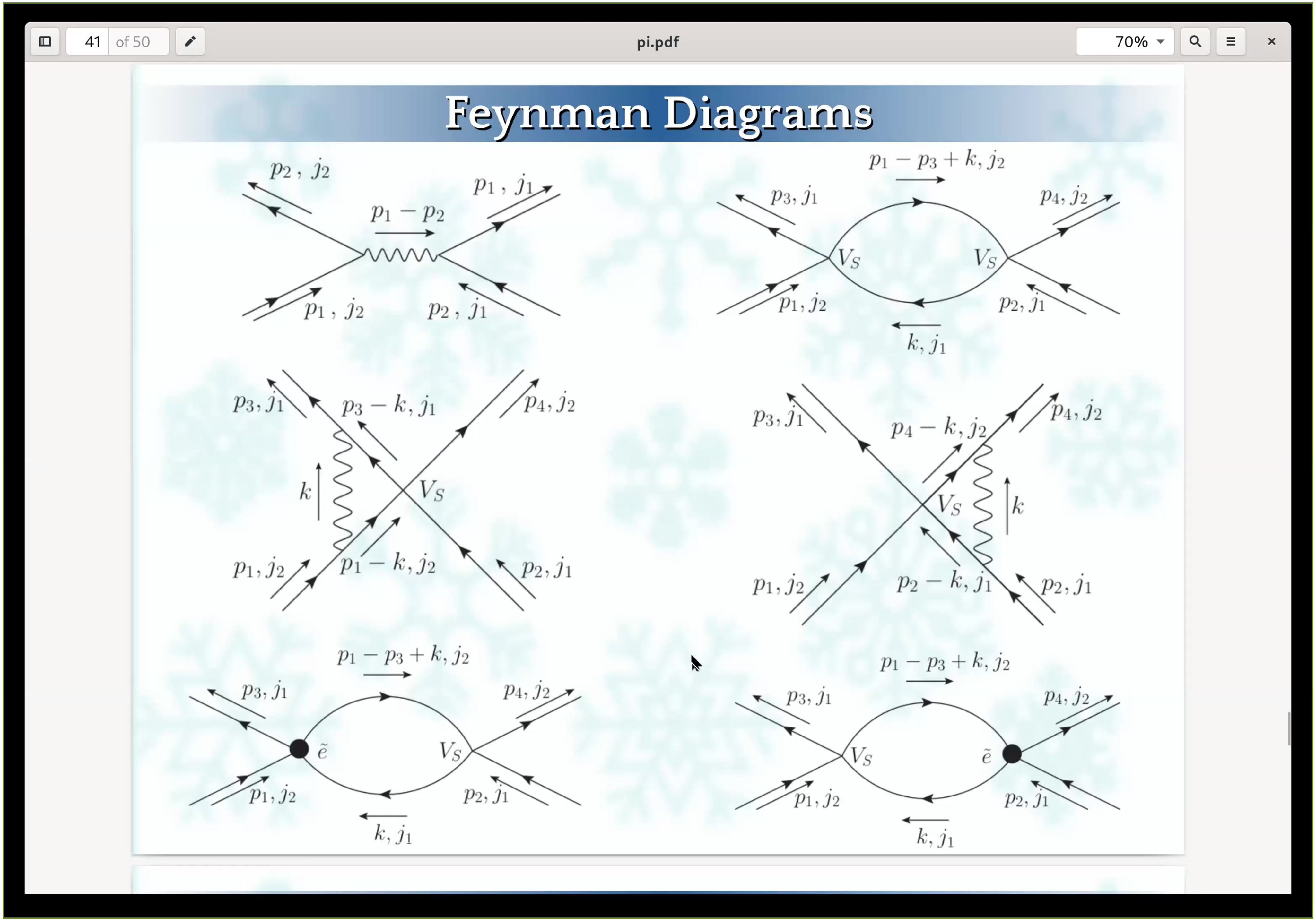


Superconducting Instability

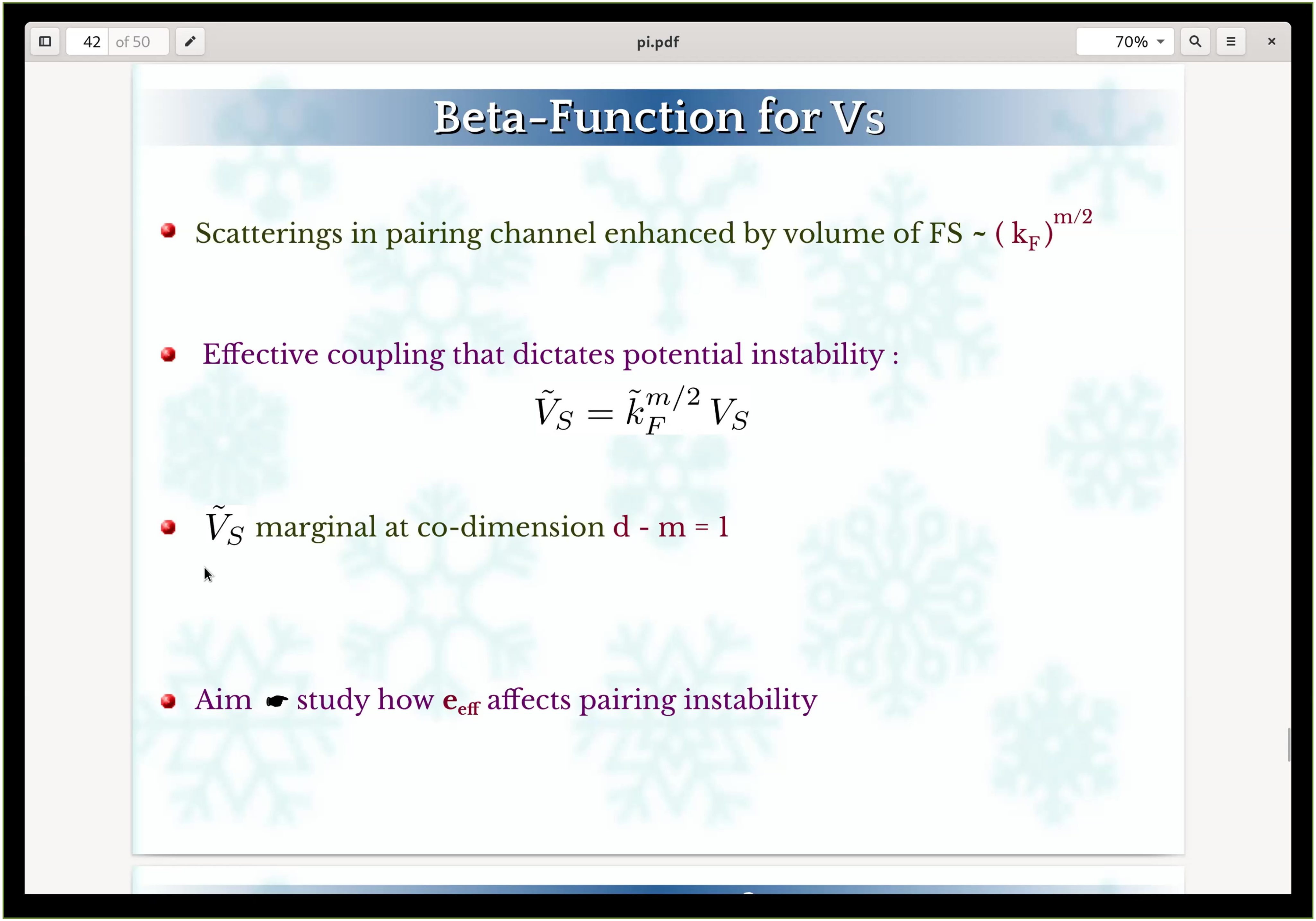
Add relevant 4-fermion terms
For simplicity, we consider s-wave case with 2 flavours

$$S^{\mathrm{SC}} = \frac{\mu^{d_v} V_S}{4} \sum_{j_1,j_2} \int \left(\prod_{s=1}^{4} dp_s \right) (2\pi)^{d+1} \, \delta^{(d+1)}(p_1 + p_2 - p_3 - p_4) \, (\delta_{j_1,j_2} - 1)$$

$$\times \left[\{\bar{\Psi}_{j_1}(p_3) \Psi_{j_2}(p_1)\} \{\bar{\Psi}_{j_2}(p_4) \Psi_{j_1}(p_2)\} - \{\bar{\Psi}_{j_1}(p_3) \sigma_z \Psi_{j_2}(p_1)\} \{\bar{\Psi}_{j_2}(p_4) \sigma_z \Psi_{j_2}(p_1)\} \right]$$



Feynman Diagrams



Beta-Function for Vs

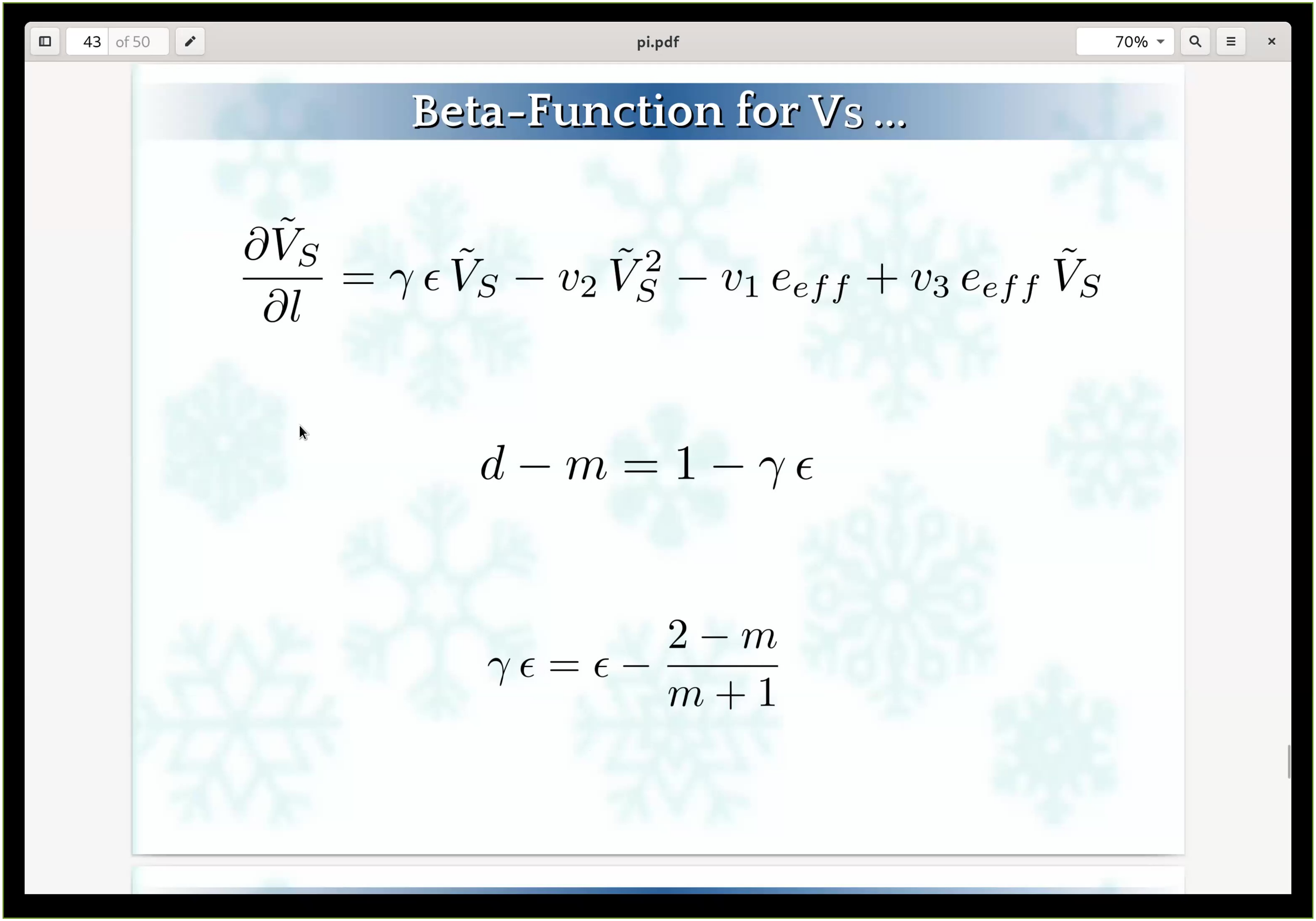
- Scatterings in pairing channel enhanced by volume of FS ~ $(k_F)^{m/2}$

- Effective coupling that dictates potential instability :

$$\tilde{V}_S = \tilde{k}_F^{m/2} V_S$$

- $\tilde{V}_S$ marginal at co-dimension d - m = 1

- Aim ☛ study how $\mathbf{e}_{\mathrm{eff}}$ affects pairing instability

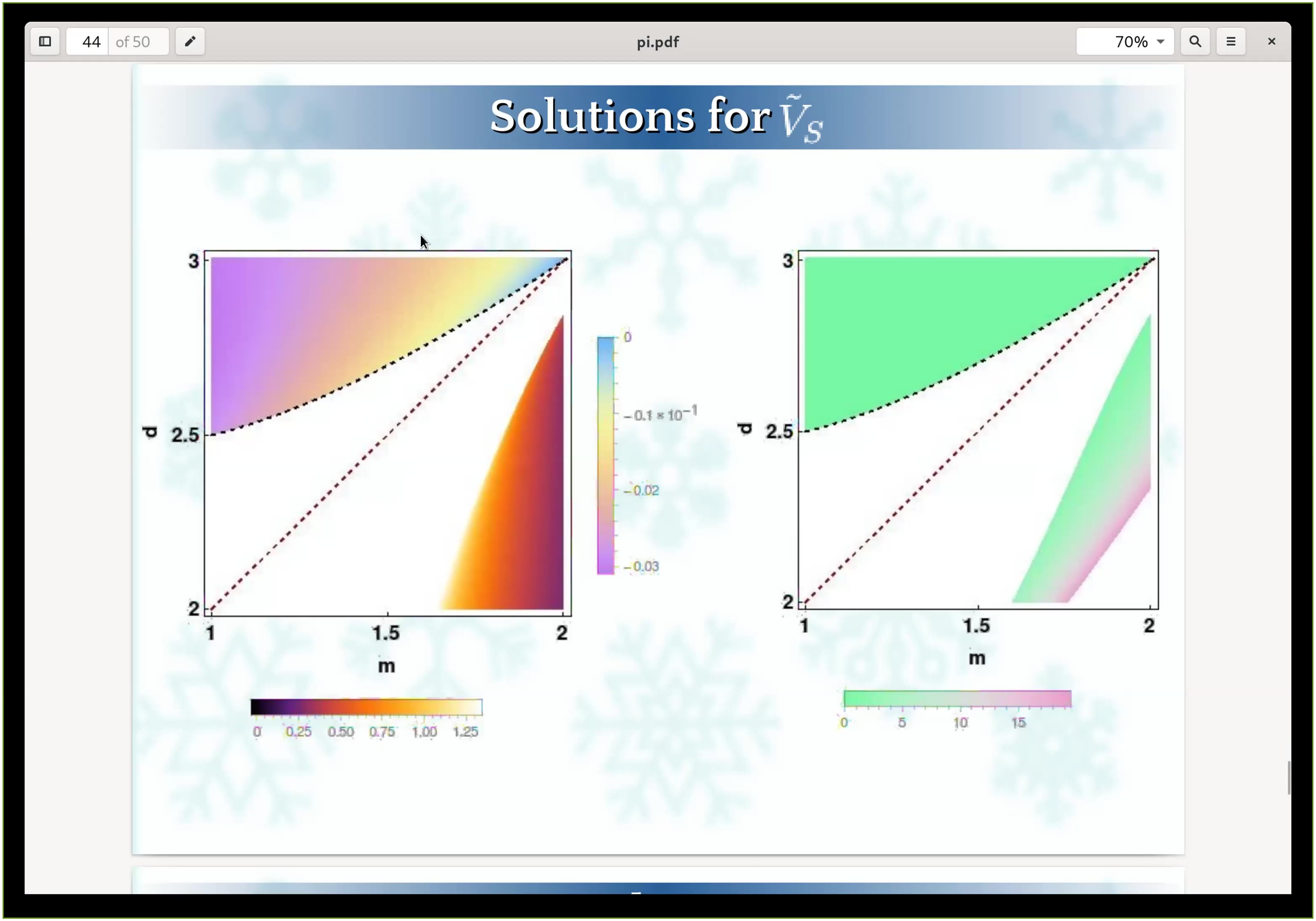


Beta-Function for Vs ...

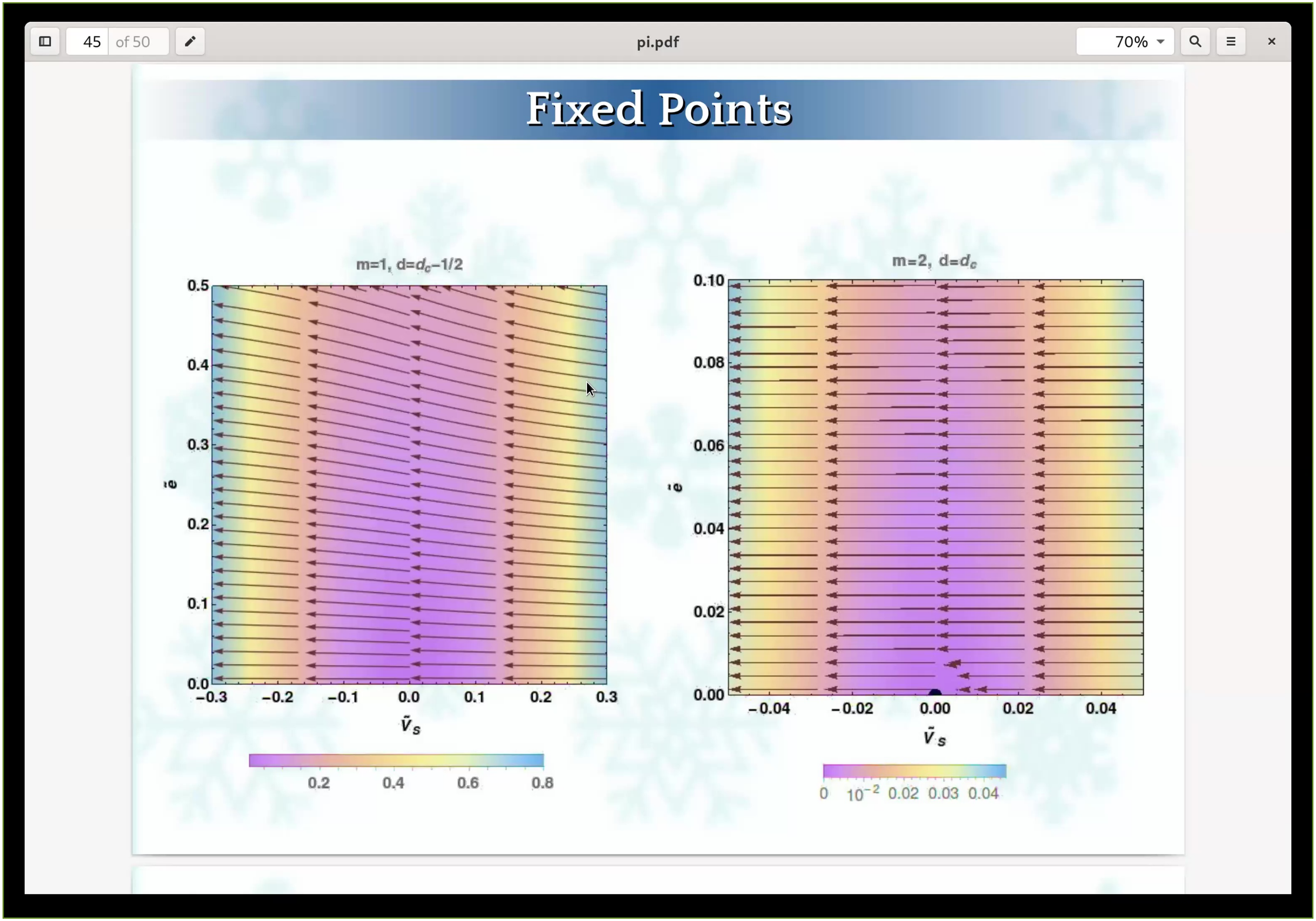
$$\frac{\partial \tilde{V}_S}{\partial l} = \gamma\,\epsilon\,\tilde{V}_S - v_2\,\tilde{V}_S^2 - v_1\,e_{eff} + v_3\,e_{eff}\,\tilde{V}_S$$

$$d - m = 1 - \gamma\,\epsilon$$

$$\gamma\,\epsilon = \epsilon - \frac{2-m}{m+1}$$

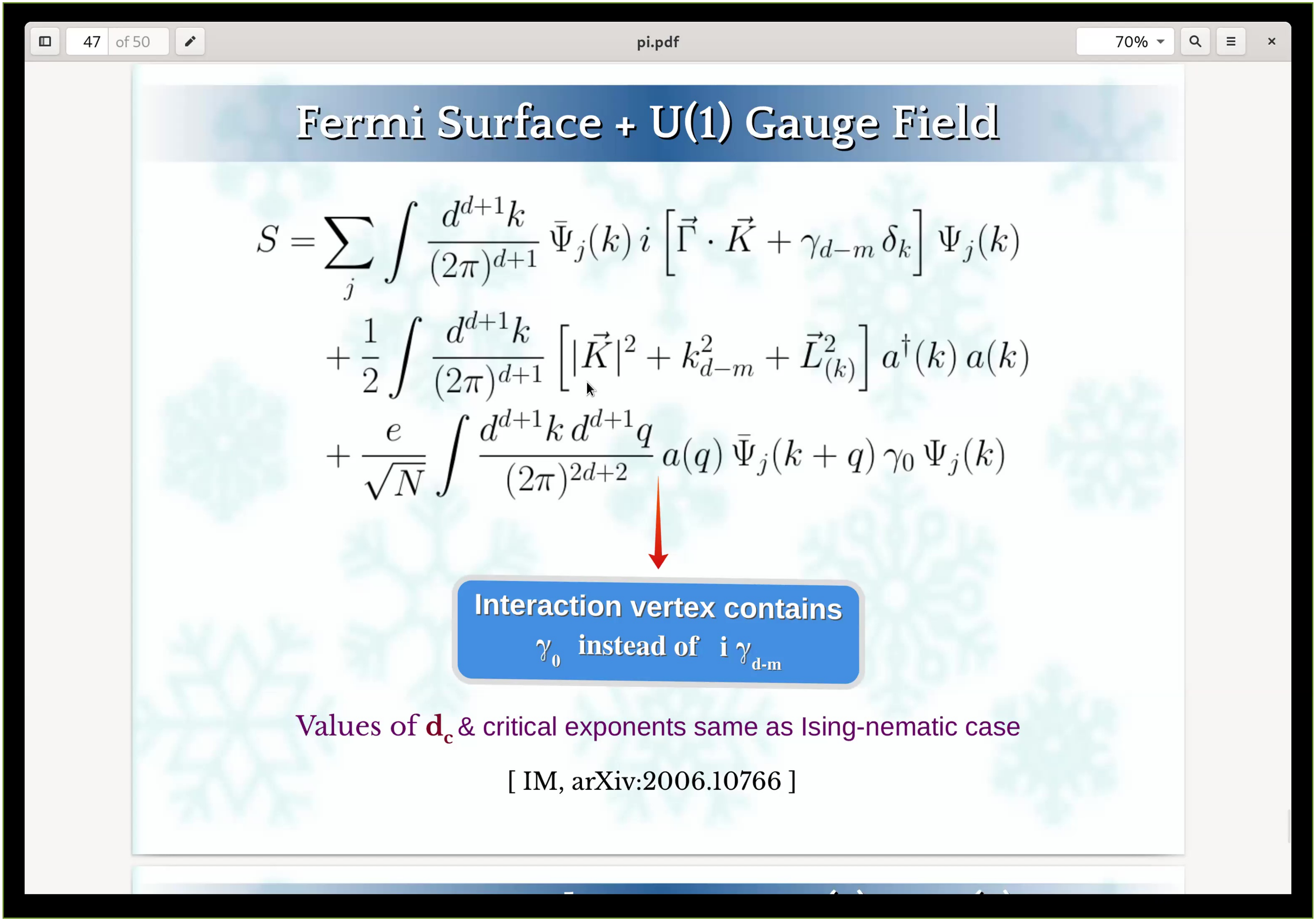


Solutions for $\tilde{V}_S$



Fixed Points

Recent Work involving Transverse Gauge Fields



Fermi Surface + U(1) Gauge Field

$$S = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}}\, \bar{\Psi}_j(k)\, i \left[\vec{\Gamma}\cdot\vec{K} + \gamma_{d-m}\,\delta_k\right] \Psi_j(k)$$

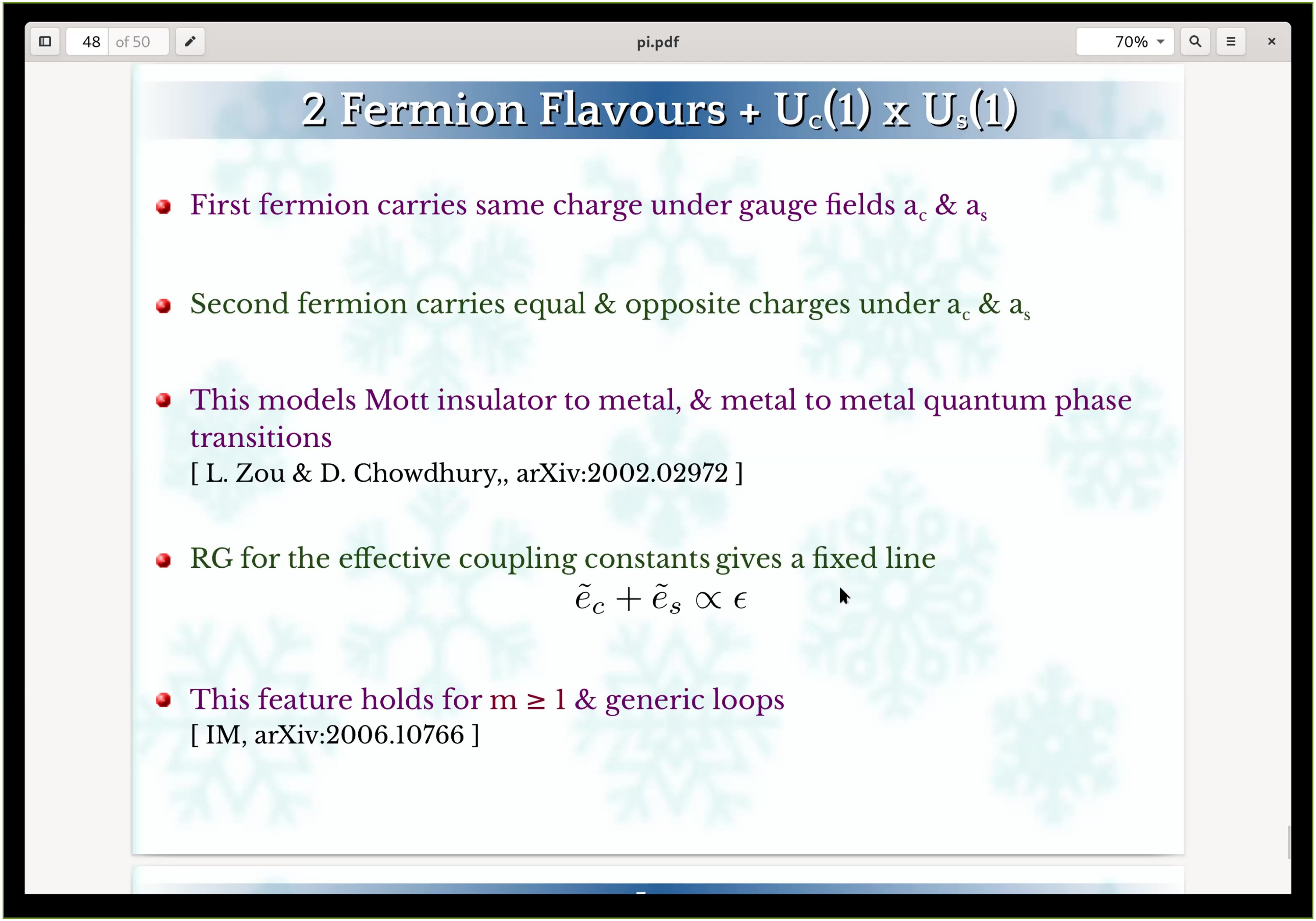
$$+ \frac{1}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k_{d-m}^2 + \vec{L}_{(k)}^2\right] a^\dagger(k)\, a(k)$$

$$+ \frac{e}{\sqrt{N}} \int \frac{d^{d+1}k\, d^{d+1}q}{(2\pi)^{2d+2}}\, a(q)\, \bar{\Psi}_j(k+q)\, \gamma_0\, \Psi_j(k)$$

Interaction vertex contains
γ_0 instead of $i\,\gamma_{d\text{-}m}$

Values of $\mathbf{d_c}$ & critical exponents same as Ising-nematic case

[IM, arXiv:2006.10766]

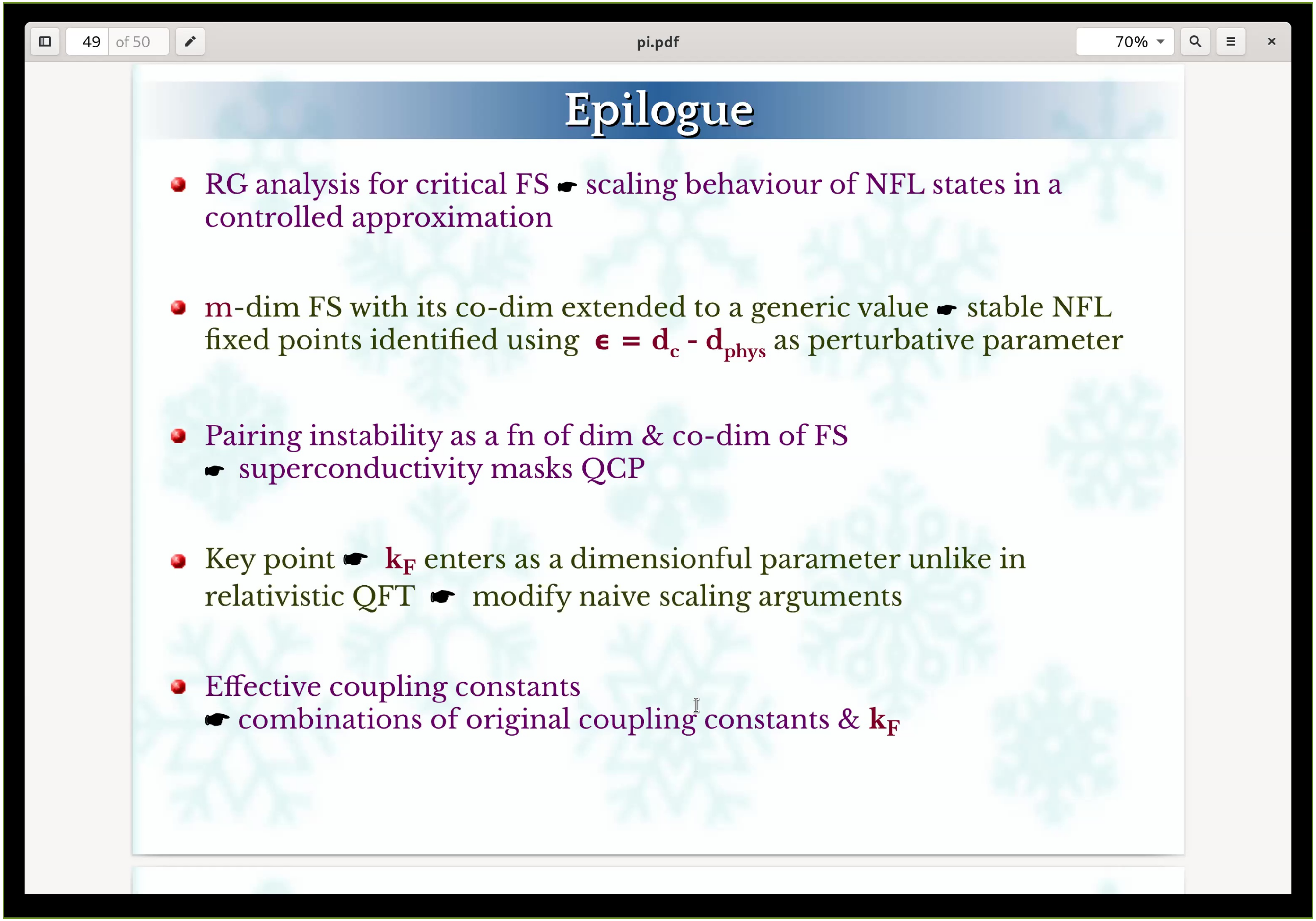


2 Fermion Flavours + $U_c(1)$ x $U_s(1)$

- First fermion carries same charge under gauge fields a_c & a_s
- Second fermion carries equal & opposite charges under a_c & a_s
- This models Mott insulator to metal, & metal to metal quantum phase transitions
  [L. Zou & D. Chowdhury,, arXiv:2002.02972]
- RG for the effective coupling constants gives a fixed line

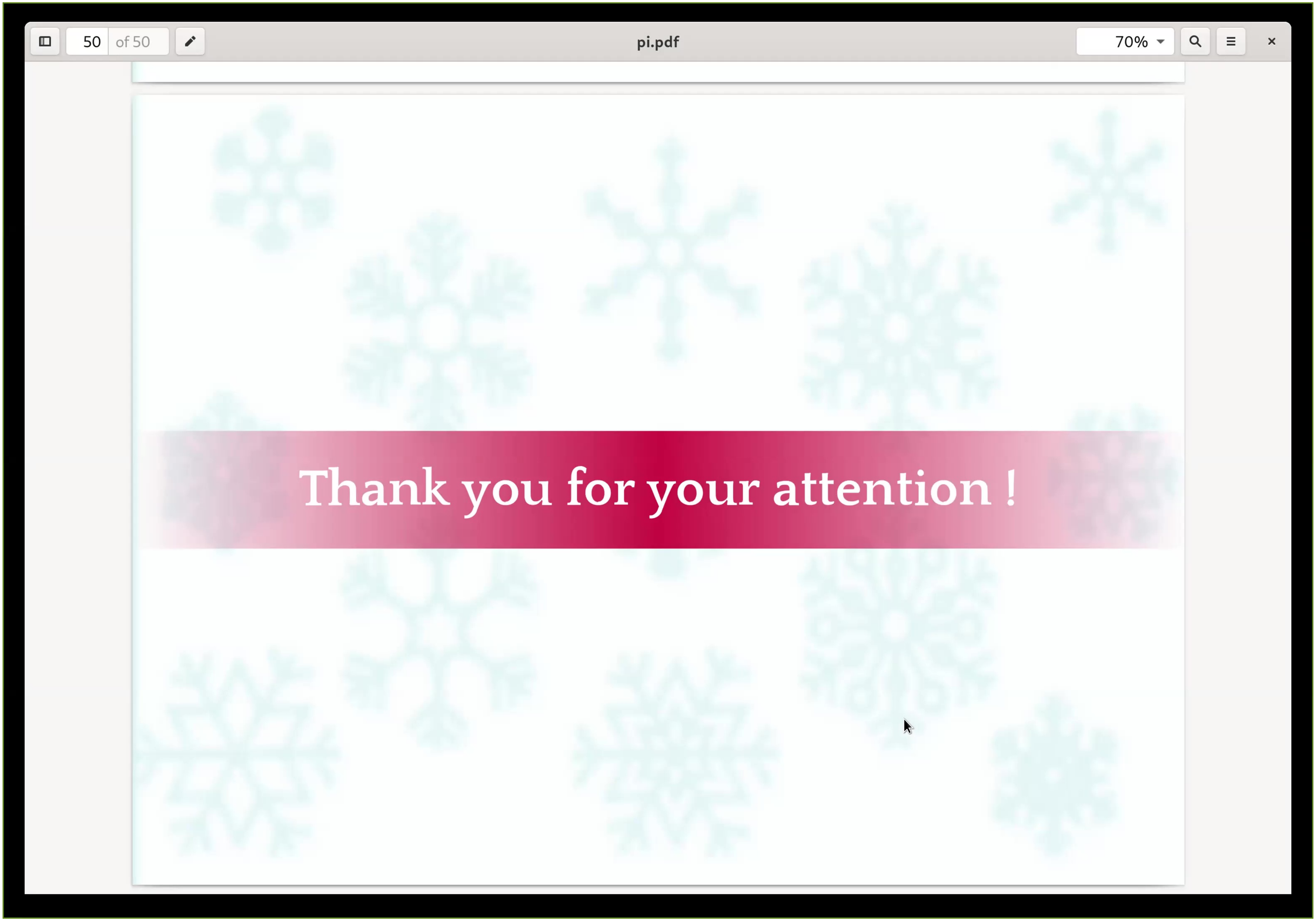
$$\tilde{e}_c + \tilde{e}_s \propto \epsilon$$

- This feature holds for m ≥ 1 & generic loops
  [IM, arXiv:2006.10766]



Epilogue

- RG analysis for critical FS ☛ scaling behaviour of NFL states in a controlled approximation
- m-dim FS with its co-dim extended to a generic value ☛ stable NFL fixed points identified using $\epsilon = d_c - d_{phys}$ as perturbative parameter
- Pairing instability as a fn of dim & co-dim of FS ☛ superconductivity masks QCP
- Key point ☛ k_F enters as a dimensionful parameter unlike in relativistic QFT ☛ modify naive scaling arguments
- Effective coupling constants ☛ combinations of original coupling constants & k_F



Thank you for your attention !