Title: Sprinklings in Causal Set Theory and Local Structures to Discretize Field Propagators

Speakers: Christoph Minz

Collection: Quantum Gravity 2020

Date: July 17, 2020 - 9:20 AM

URL: http://pirsa.org/20070017

Sprinklings in Causal Set Theory and Local Structures to Discretize Field Propagators

Christoph Minz
(in joined work with Christopher J. Fewster, Eli Hawkins, and Kasia Rejzner)[1]

[1]Department of Mathematics
University of York

Quantum Gravity, July 17, 2020

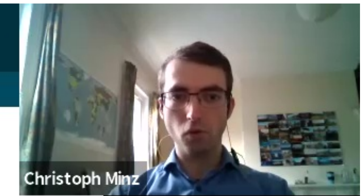
- It is a framework for quantum gravity, see introductions by Henson [Hen09] or Sorkin [Sor11].
- Continuum spacetime manifold is replaced by a locally finite partial ordered set $(\mathscr{S}, \preceq)$, i.e. it fulfills the axioms

$$\begin{aligned} \text{transitivity:} \quad & x \preceq z \preceq y \Rightarrow x \preceq y, \\ \text{acyclicity (anti-symmetry):} \quad & (x \preceq y \wedge y \preceq x) \Leftrightarrow x = y, \\ \text{local finiteness:} \quad & \left| \{ z \in \mathscr{S} | x \preceq z \preceq y \} \right| < \infty. \end{aligned}$$

- The *causal interval* or *Alexandrov subset* of two causally related events is denoted by

$$I(x, y) := \{ z \in \mathscr{S} | x \preceq z \preceq y \} .$$

- We write $x \prec y$ if and only if $x \preceq y$ and $x \neq y$.
- An event y is *linked* to x, $x \prec\!* \, y$ (or $x \prec\!* \, y$) if and only if $I(x, y) = \{x, y\}$ and $x \neq y$.



Sprinkling is the Poisson process of obtaining a sprinkled causal set (*causet*) from a spacetime:

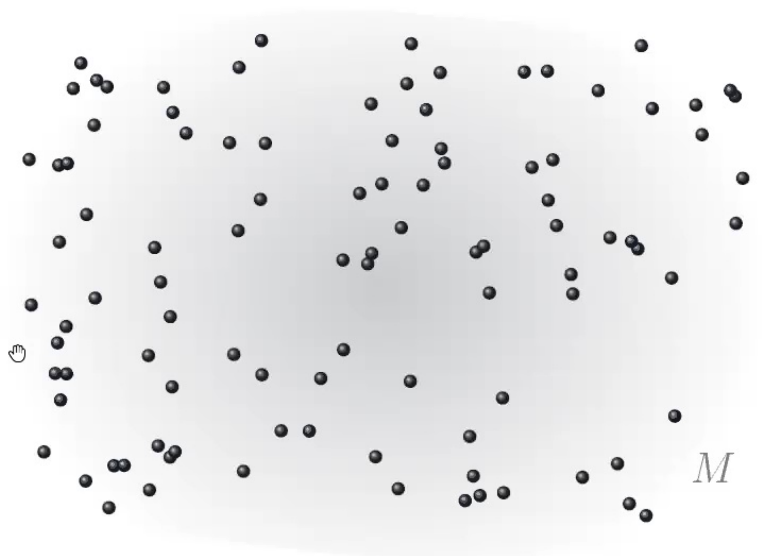
1. Let (M, g) be a smooth spacetime manifold with metric g.
2. Select a locally finite set called *sprinkle* of spacetime points $S \underset{\text{loc. finite}}{\subset} M$ by a Poisson process.
3. Restrict the causal relation from the spacetime to the sprinkle

$$x \preceq y \Leftrightarrow x \in J^{-}(y)$$

We will derive the corresponding probability measure in the following.

- Let $U \in L$ be a subset of M in the set L of open, pre-compact subsets.
- The configuration space of sprinkles on the manifold M is

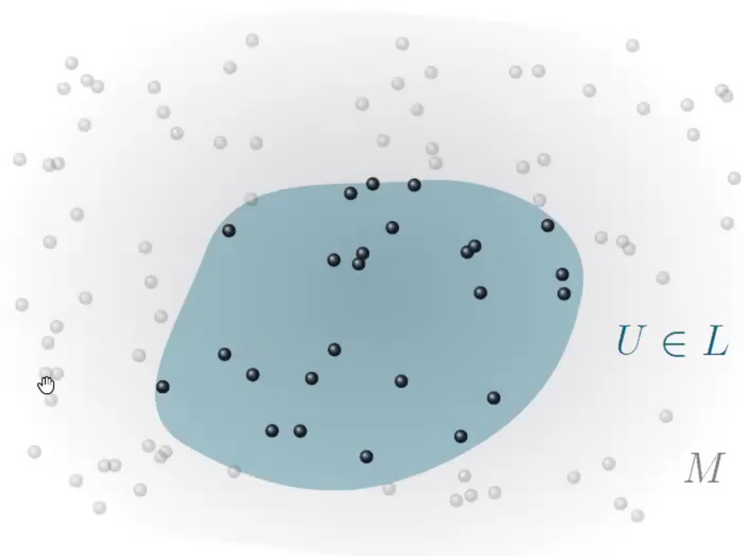
$$Q := \{S \subset M \mid \forall U \in L : |S \cap U| < \infty\} .$$



Christoph Minz

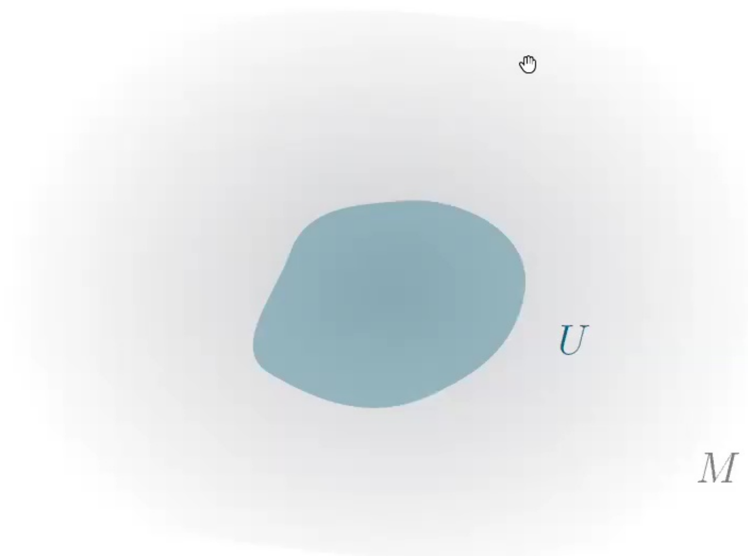
- Let $U \in L$ be a subset of M in the set L of open, pre-compact subsets.
- The configuration space of sprinkles on the manifold M is

$$Q := \{S \subset M \mid \forall U \in L : |S \cap U| < \infty\}.$$



- For the construction [AKR98], consider n-tuples of non-identical events in U:

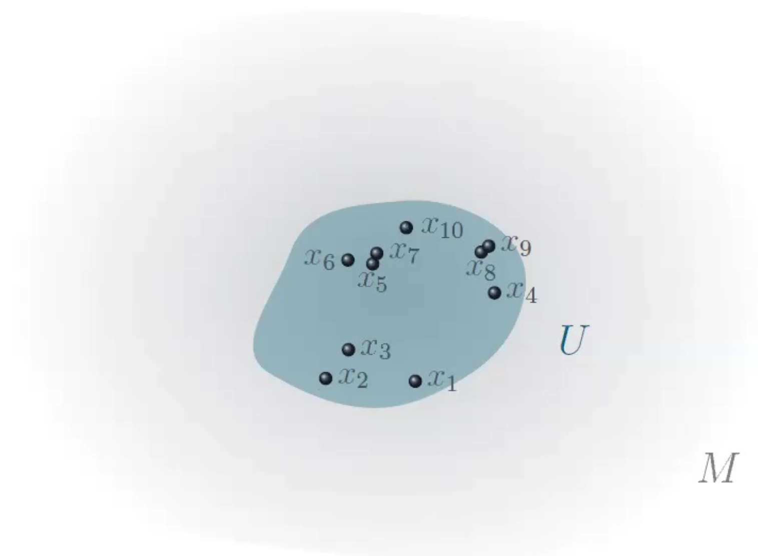
$$\widetilde{Q_{U,n}} := \{(x_1, x_2, \ldots, x_n) \in U^n \mid \forall i, j \in [1, n] : x_i = x_j \Leftrightarrow i = j\}$$



Christoph Minz

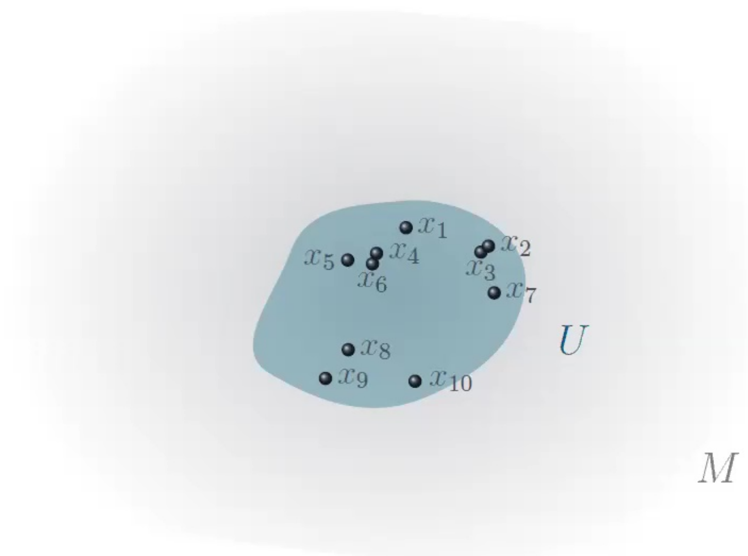
- For the construction [AKR98], consider n-tuples of non-identical events in U:

$$\widetilde{Q_{U,n}} := \{(x_1, x_2, \ldots, x_n) \in U^n \mid \forall i, j \in [1, n] : x_i = x_j \Leftrightarrow i = j\}$$



- For the construction [AKR98], consider n-tuples of non-identical events in U:

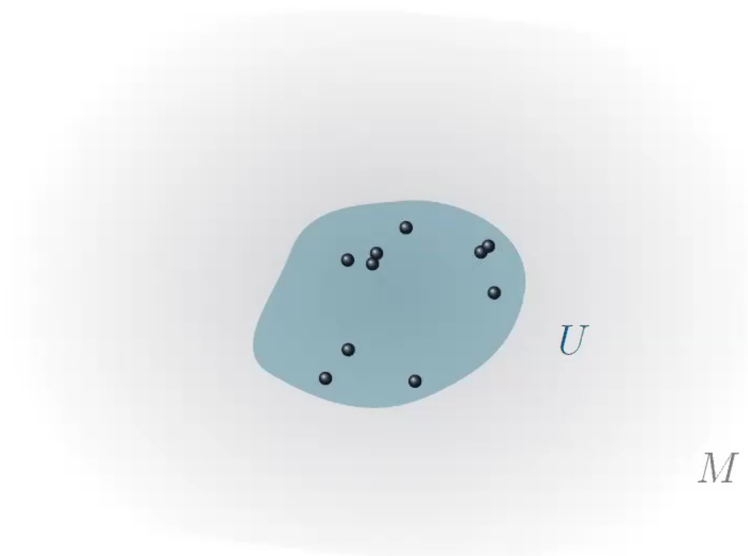
$$\widetilde{Q_{U,n}} := \{(x_1, x_2, \dots, x_n) \in U^n \mid \forall i, j \in [1, n] : x_i = x_j \Leftrightarrow i = j\}$$



- Every n-tuple with the same events x_i corresponds to the same sprinkling:

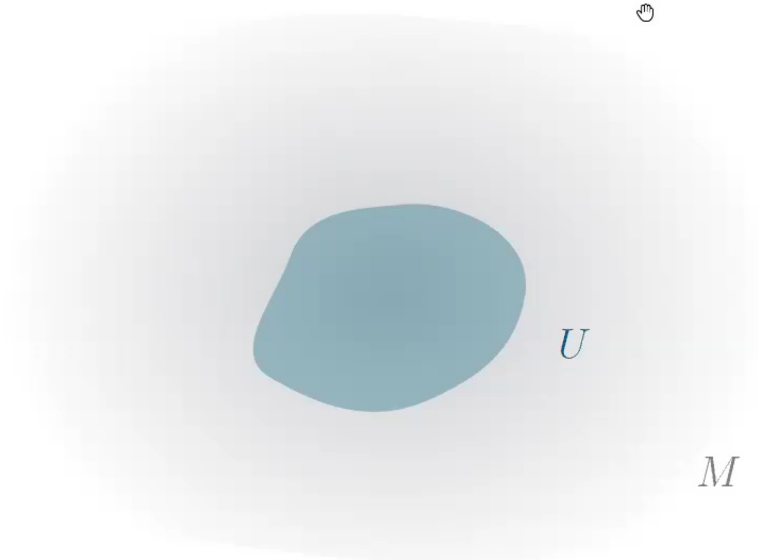
$$\Sigma_{U,n} : \widetilde{Q_{U,n}} \to Q_{U,n}, \qquad (x_1, x_2, \dots, x_n) \mapsto \{x_1, x_2, \dots, x_n\}.$$

$$Q_{U,n} := \{S \subset U \mid |S| = n\}.$$



- Take the union over all cardinalities to get the config. space Q_U.
- The intensity measure for every open subset of M, derived from its volume and the sprinkling density parameter ρ

$$\lambda(U) = \rho \int_U \sqrt{|g|}\, \mathrm{d}^d x$$



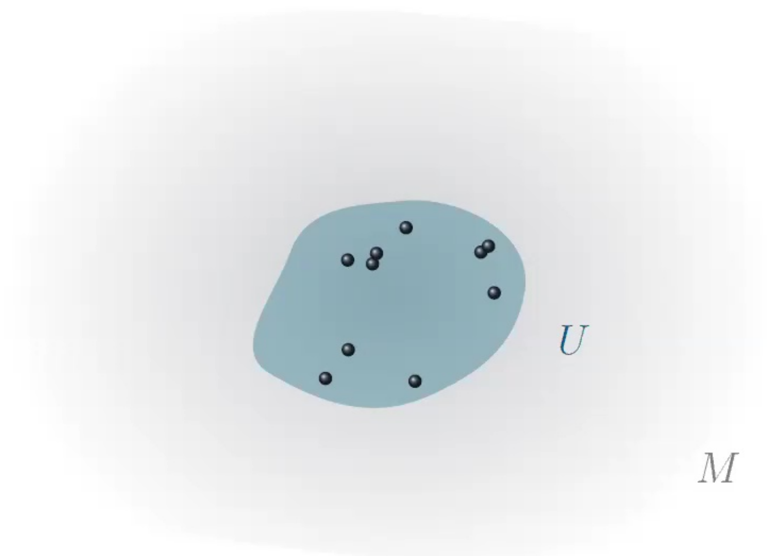
Christoph Minz

- Poisson (probability) measure on U [AKR98] assigns a probability

$$\mu_U(B) := \mathrm{e}^{-\lambda(U)} \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^{\otimes n} \circ \Sigma_{U,n}^{-1}(B_n)$$

for any measureable subset $B = (B_n)_{n \in \mathbb{N}_0}$ such that $B_n \in \mathcal{B}(Q_{U,n})$.

- There exists a unique Poisson measure on the entire manifold M.

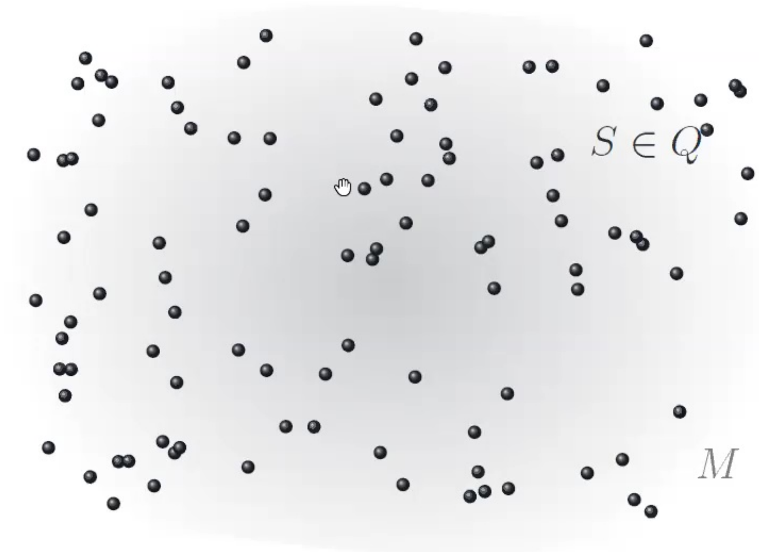


- Poisson (probability) measure on U [AKR98] assigns a probability

$$\mu_U(B) := \mathrm{e}^{-\lambda(U)} \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^{\otimes n} \circ \Sigma_{U,n}^{-1}(B_n)$$

for any measureable subset $B = (B_n)_{n \in \mathbb{N}_0}$ such that $B_n \in \mathcal{B}(Q_{U,n})$.

- There exists a unique Poisson measure on the entire manifold M.

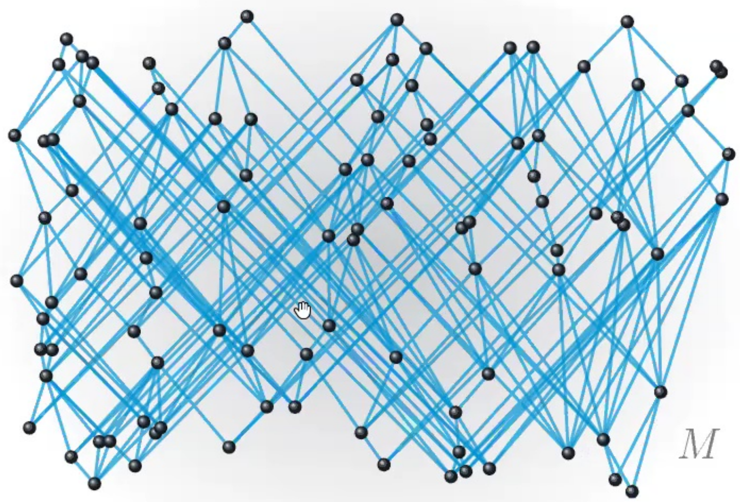


- Poisson (probability) measure on U [AKR98] assigns a probability

$$\mu_U(B) := \mathrm{e}^{-\lambda(U)} \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^{\otimes n} \circ \Sigma_{U,n}^{-1}(B_n)$$

for any measureable subset $B = (B_n)_{n \in \mathbb{N}_0}$ such that $B_n \in \mathcal{B}(Q_{U,n})$.

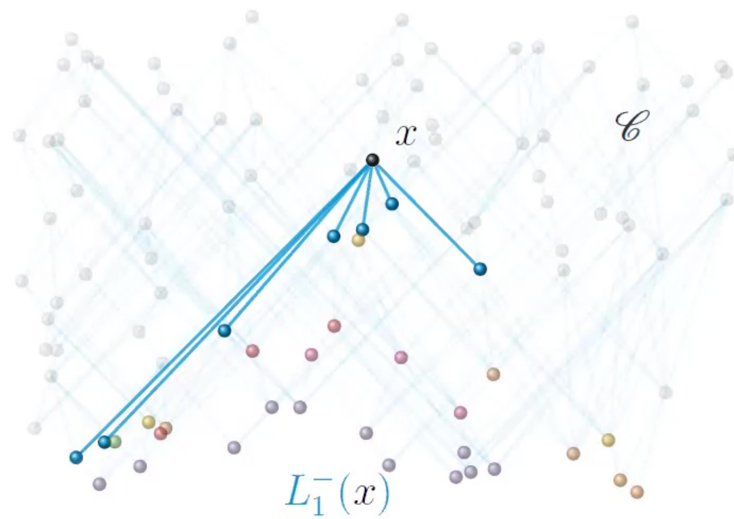
- There exists a unique Poisson measure on the entire manifold M.



Christoph Minz

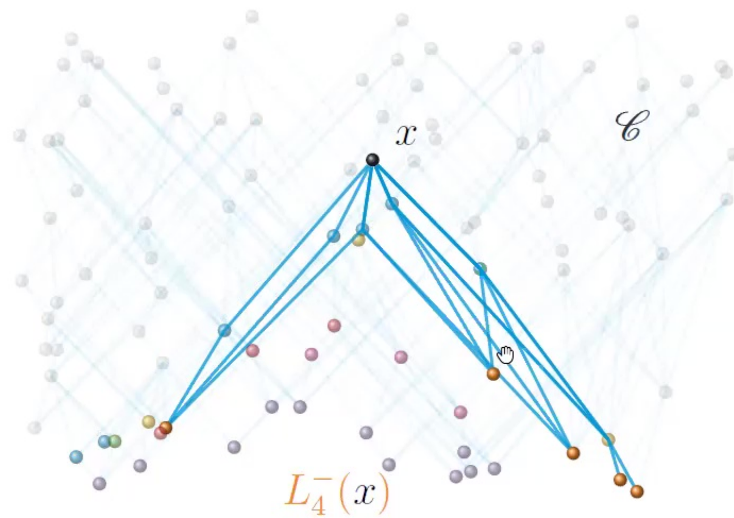
- Classical and quantum fields on causal sets are described by discretized counterparts to the eom. on a continuum manifold.
- Most approaches to discretization are based on past k-*layers* [Sor09, ASS14],

$$L_k^-(x) := \{y \in \mathscr{C} \mid k = |I(y,x)| - 1\}.$$



- Classical and quantum fields on causal sets are described by discretized counterparts to the eom. on a continuum manifold.
- Most approaches to discretization are based on past k-*layers* [Sor09, ASS14],

$$L_k^-(x) := \{y \in \mathscr{C} \mid k = |I(y,x)| - 1\}.$$



- These discretizations need the spacetime dimension as an input, but in general the spacetime dimension of a given causal set is an emergent (local) property and not pre-defined.
- A more recently proposed discretization method [DHFRW20] has the potential to be independent of the dimension, but needs a supplementary structure to a causal set called a *preferred past* as defined in the following.
- We define a *path* from a causet event $x \in \mathscr{C}$ to an event in its future $y \succ x$ as the set of events $\mathscr{P}$ that forms the linked chain

$$x \prec\!* \, x_1 \prec\!* \, x_2 \prec\!* \cdots \prec\!* \, x_{n-2} \prec\!* \, y.$$

- We denote the set of all *paths from* x *to* y by $\mathrm{paths}(x, y)$.
- The length (number of links) of the shortest path is called the *rank*

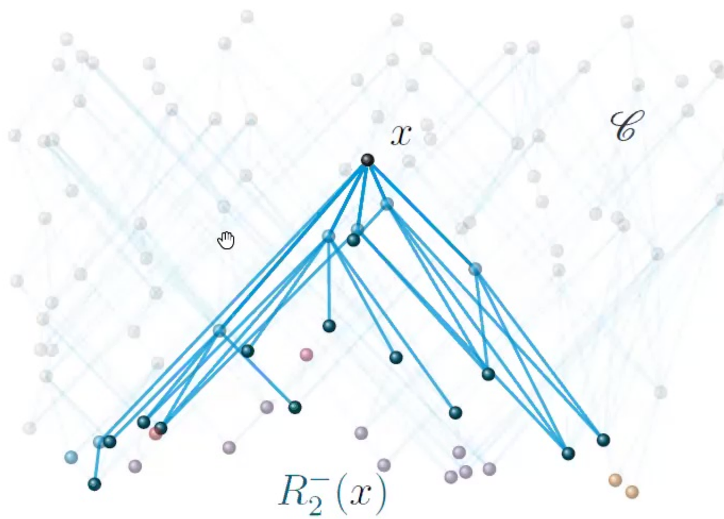
$$\mathrm{rk}(y, x) := \begin{cases} \min\limits_{\mathscr{P} \in \mathrm{paths}(x,y)} |\mathscr{P}| - 1 & x \preceq y, \\ \infty & \text{otherwise.} \end{cases}$$

Christoph Minz

- The *rank k past* of an event $x \in \mathscr{C}$ is the set

$$R_k^-(x) := \{y \in \mathscr{C} \mid \mathrm{rk}(x, y) = k\}.$$

- A *preferred past structure* is a map $\Lambda^- : \mathscr{C} \setminus C_2^- \to \mathscr{C}$ such that $\Lambda^-(x) \in R_2^-(x)$ for all $x \in \mathscr{C} \setminus C_2^-$.

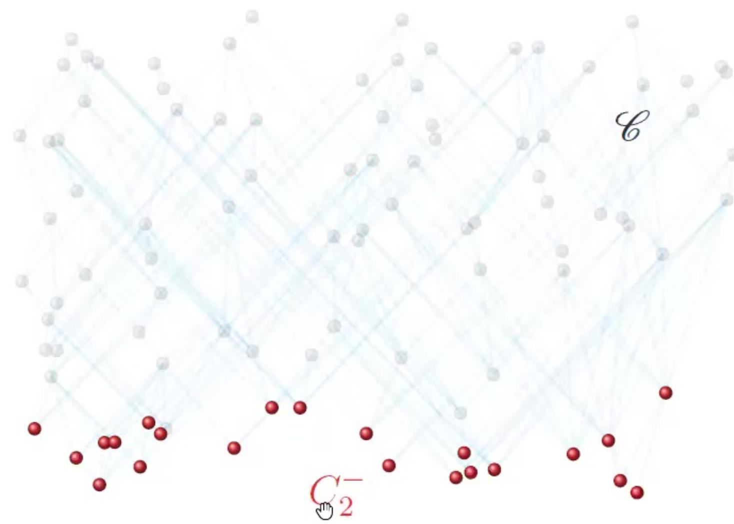


Christoph Minz

- The *rank* k *past* of an event $x \in \mathscr{C}$ is the set

$$R_k^-(x) := \{y \in \mathscr{C} \mid \mathrm{rk}(x, y) = k\} .$$

- A *preferred past structure* is a map $\Lambda^- : \mathscr{C} \setminus C_2^- \to \mathscr{C}$ such that $\Lambda^-(x) \in R_2^-(x)$ for all $x \in \mathscr{C} \setminus C_2^-$.



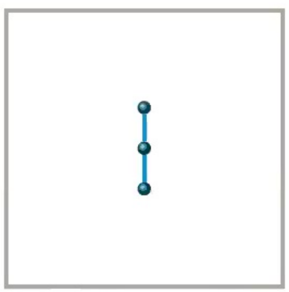
Diamonds

- For an event x and one of its rank 2 past events y, we call the Alexandrov set $I(y, x)$ a k-*diamond* if its *size* is

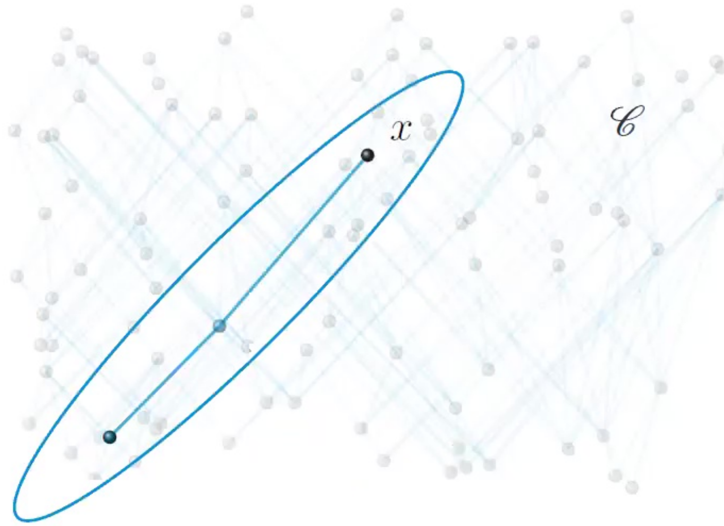
$$k = |I(y, x) \setminus \{x, y\}|.$$

- The *number of internal events* for a given k-diamond $I(y, x)$ is

$$\mathrm{itn}(x, y) := k - |\{z \in I(y, x) \mid y \prec\!* \, z \prec\!* \, x\}| .$$



1-diamond
0 internal events

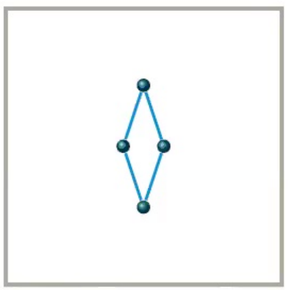


- For an event x and one of its rank 2 past events y, we call the Alexandrov set $I(y,x)$ a k-*diamond* if its *size* is

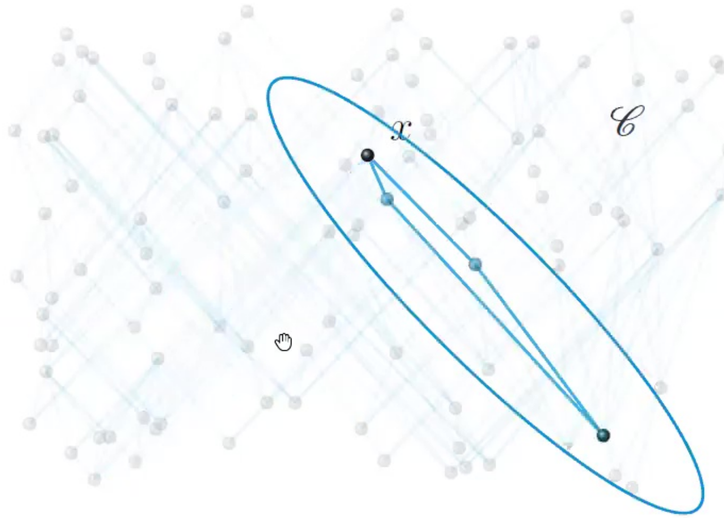
$$k = |I(y,x) \setminus \{x,y\}|.$$

- The *number of internal events* for a given k-diamond $I(y,x)$ is

$$\mathrm{itn}(x,y) := k - |\{z \in I(y,x) \mid y \prec\!* \, z \prec\!* \, x\}| \,.$$



2-diamond
0 internal events

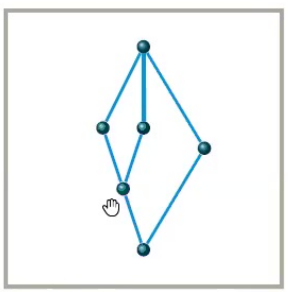


- For an event x and one of its rank 2 past events y, we call the Alexandrov set $I(y, x)$ a k-*diamond* if its *size* is

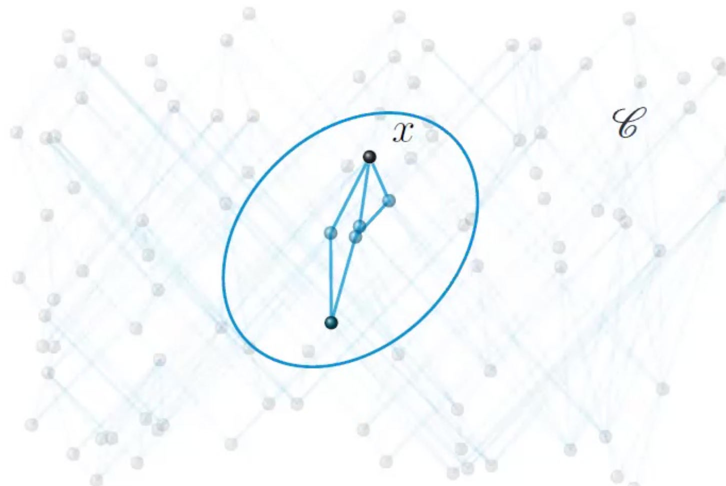
$$k = |I(y, x) \setminus \{x, y\}|.$$

- The *number of internal events* for a given k-diamond $I(y, x)$ is

$$\mathrm{itn}(x, y) := k - |\{z \in I(y, x) \mid y \prec\!* \, z \prec\!* \, x\}|\,.$$



4-diamond
3 internal events

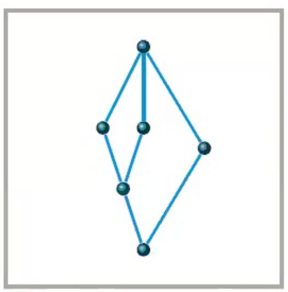


- For an event x and one of its rank 2 past events y, we call the Alexandrov set $I(y, x)$ a k-*diamond* if its *size* is

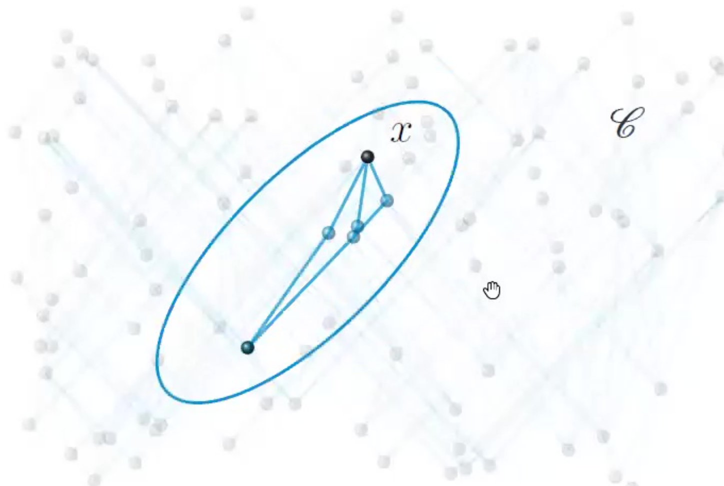
$$k = |I(y, x) \setminus \{x, y\}|.$$

- The *number of internal events* for a given k-diamond $I(y, x)$ is

$$\mathrm{itn}(x, y) := k - |\{z \in I(y, x) \mid y \prec\!* \, z \prec\!* \, x\}| \, .$$



4-diamond
3 internal events



- In order to find the rank 2 past events to prefer, we numerically investigated 6 criteria and compared their distributions of the number of rank 2 past events and their proper time separation.

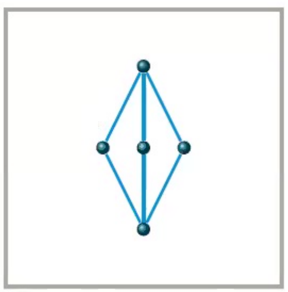
- 1: largest pure diamonds,
- 2: diamonds with the least internal events of the diamonds with the most rank 2 paths,
- 3: smallest diamonds,
- 4: diamonds with the most internal events of the diamonds with the most rank 2 paths,
- 5: consider all diamonds with the smallest number i of internal events of those diamonds with the greatest number $p_{\max}$ of rank 2 paths (as in criterion 2); furthermore, include diamonds that have $(p_{\max} - j)$ rank 2 paths and up to $(i - j)$ internal events (or are pure), $\forall j \in [1, p_{\max} - 1]$. Split this set of diamonds and sort increasingly by the number of rank 2 paths, then count the diamonds in each subset. Select the first subset that has only one diamond so that it is a unique element - or choose the largest diamonds of the selection.
- 6: largest diamonds.

- For an event x and one of its rank 2 past events y, we call the Alexandrov set $I(y, x)$ a k-*diamond* if its *size* is

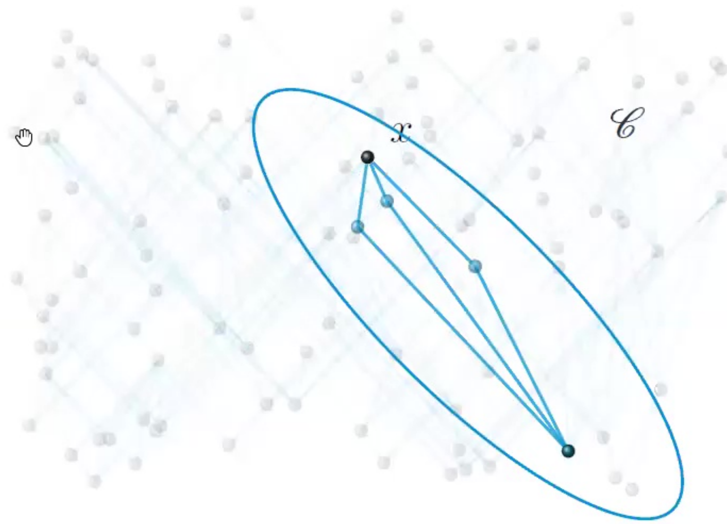
$$k = |I(y, x) \setminus \{x, y\}|.$$

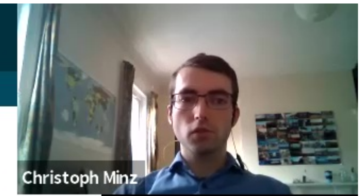
- The *number of internal events* for a given k-diamond $I(y, x)$ is

$$\mathrm{itn}(x, y) := k - |\{z \in I(y, x) \mid y \prec\!* \, z \prec\!* \, x\}| .$$



3-diamond
0 internal events

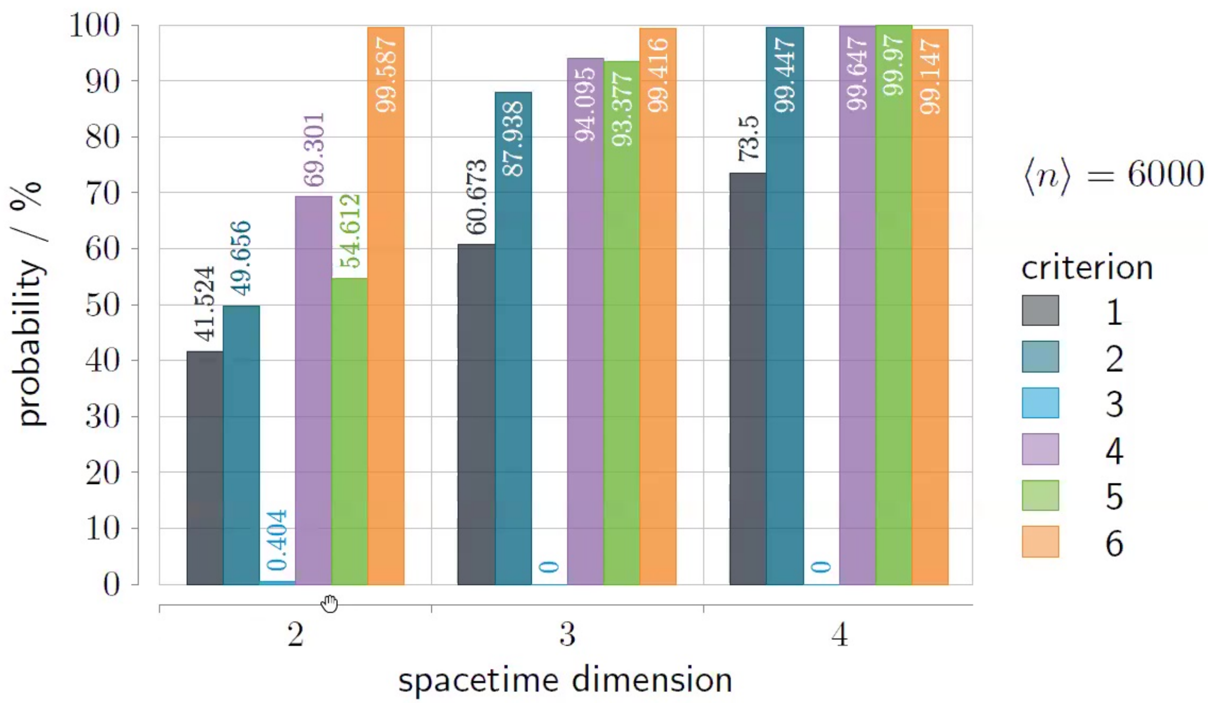




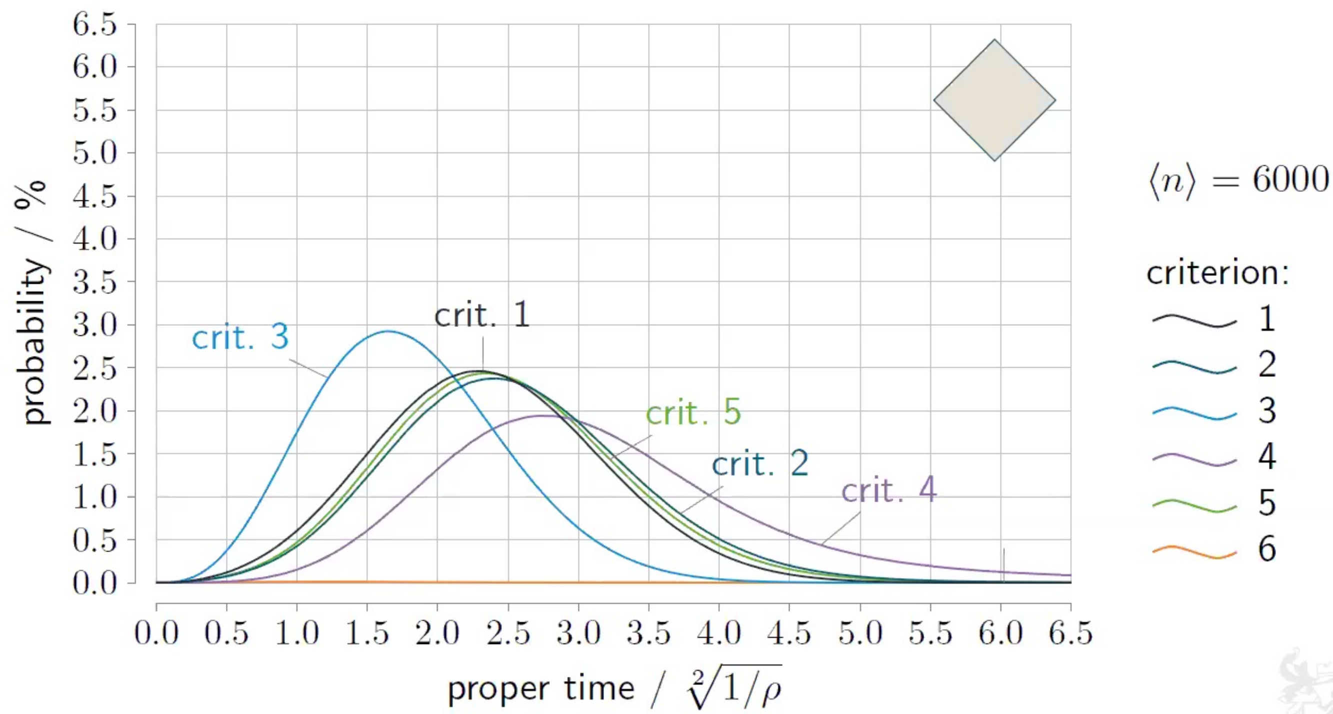
- In order to find the rank 2 past events to prefer, we numerically investigated 6 criteria and compared their distributions of the number of rank 2 past events and their proper time separation.

- 1: largest pure diamonds,
- 2: diamonds with the least internal events of the diamonds with the most rank 2 paths,
- 3: smallest diamonds,
- 4: diamonds with the most internal events of the diamonds with the most rank 2 paths,
- 5: consider all diamonds with the smallest number i of internal events of those diamonds with the greatest number $p_{\max}$ of rank 2 paths (as in criterion 2); furthermore, include diamonds that have $(p_{\max} - j)$ rank 2 paths and up to $(i - j)$ internal events (or are pure), $\forall j \in [1, p_{\max} - 1]$. Split this set of diamonds and sort increasingly by the number of rank 2 paths, then count the diamonds in each subset. Select the first subset that has only one diamond so that it is a unique element - or choose the largest diamonds of the selection.
- 6: largest diamonds.

- Probability that the criterion selects a unique event from the rank 2 past (for 10,000 causet ensembles in Alexandrov subsets of 1 + 1, 1 + 2 and 1 + 3 dimensional Minkowski spacetime)

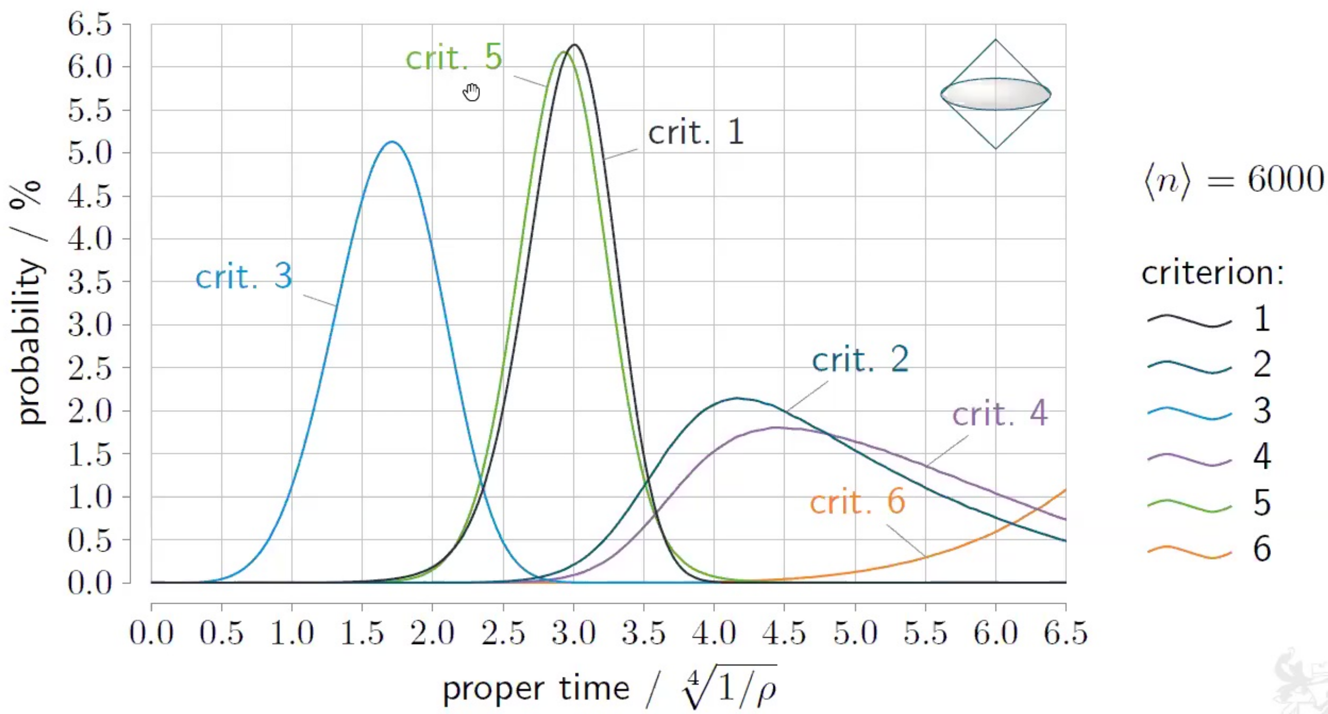


- Proper time separation between the past and future event of the diamond, for flat spacetime dimension $1+1$, $1+2$, $1+3$

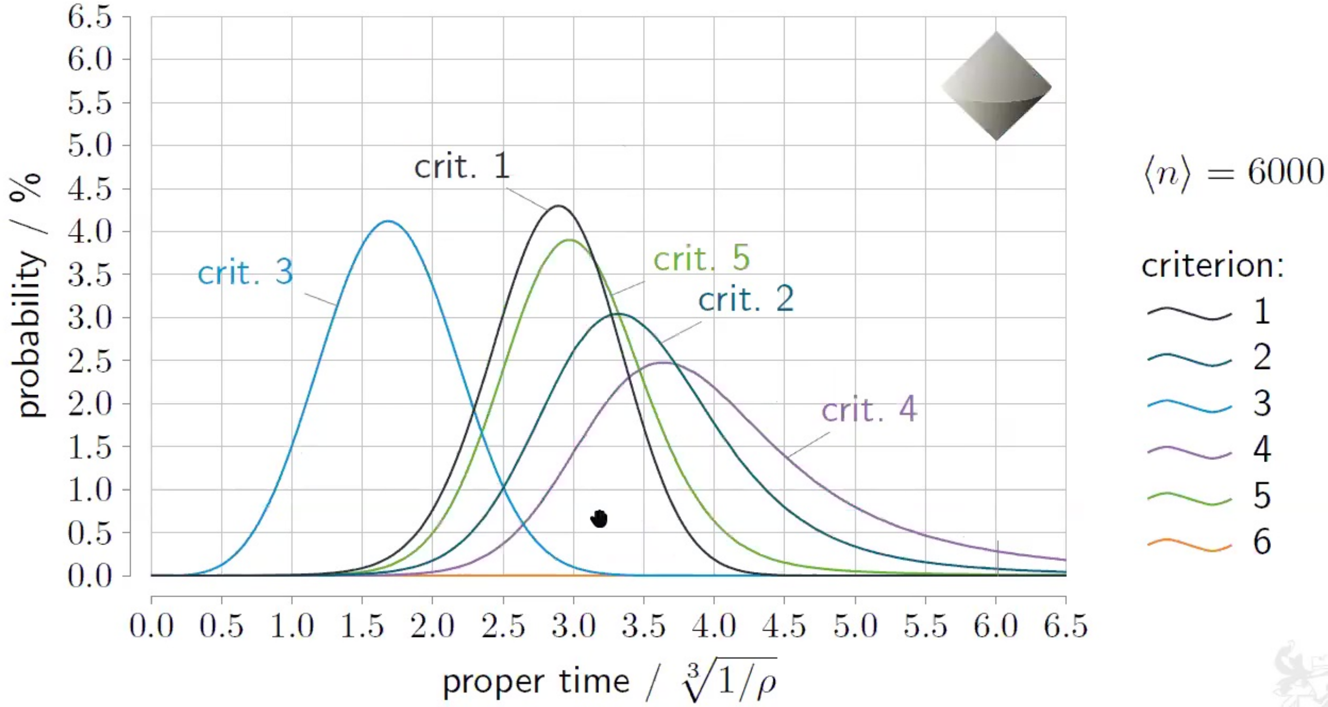


Christoph Minz

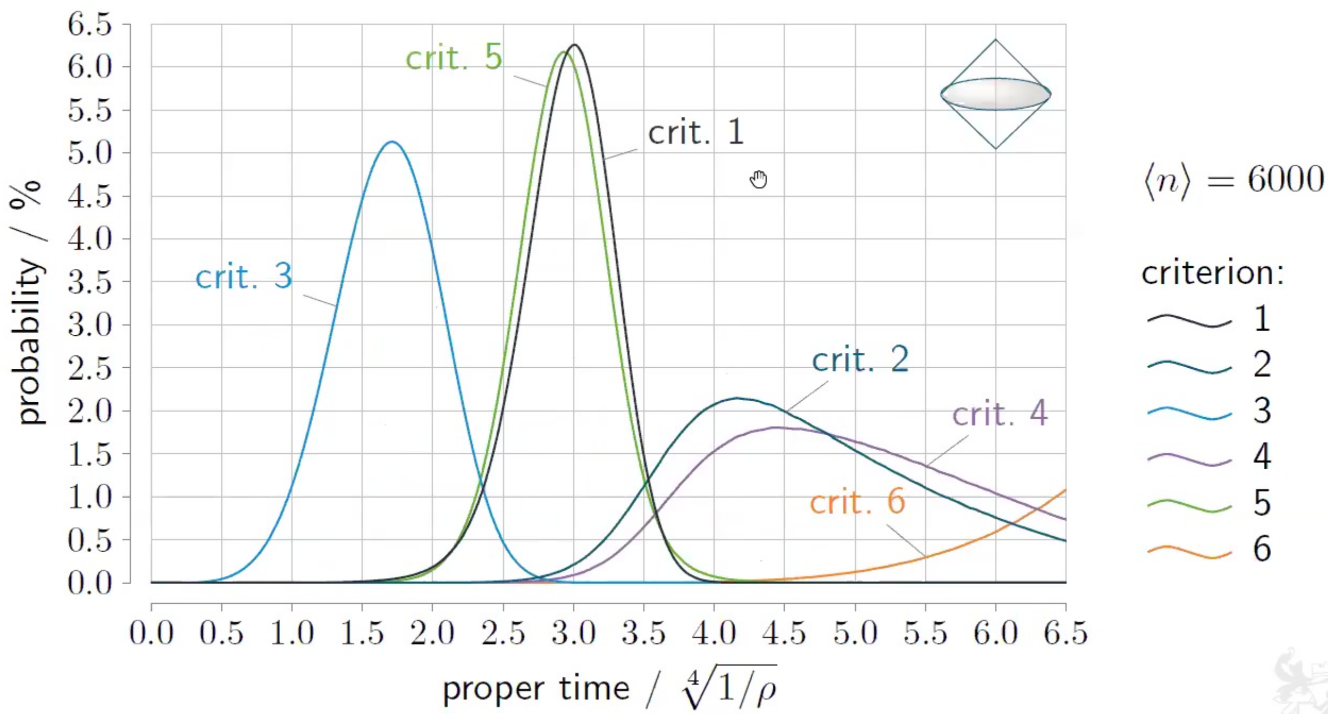
- Proper time separation between the past and future event of the diamond, for flat spacetime dimension $1+1$, $1+2$, $1+3$



- Proper time separation between the past and future event of the diamond, for flat spacetime dimension 1 + 1, 1 + 2, 1 + 3



- Proper time separation between the past and future event of the diamond, for flat spacetime dimension $1+1$, $1+2$, $1+3$



Sergio Albeverio, Yu G Kondratiev, and Michael Röckner.
Analysis and Geometry on Configuration Spaces.
Journal of Functional Analysis, 154:444–500, 1998.

Siavash Aslanbeigi, Mehdi Saravani, and Rafael D. Sorkin.
Generalized Causal Set d'Alembertians.
Journal of High Energy Physics, 2014(6):24, Jun 2014.

Edmund Dable-Heath, Christopher J Fewster, Kasia Rejzner, and Nick Woods.
Algebraic Classical and Quantum Field Theory on Causal Sets.
Physical Review D, 101(6):065013, 2020.

Joe Henson.
The Causal Set Approach to Quantum Gravity.
Approaches to Quantum Gravity: Towards a New Understanding of Space, Time and Matter, 393, 2009.

Rafael D. Sorkin.
Does Locality Fail at Intermediate Length-Scales.
Approaches to Quantum Gravity, pages 26–43, 2009.

Rafael D Sorkin.
Scalar Field Theory on a Causal Set in Histories Form.
In *Journal of Physics: Conference Series*, volume 306, 2011.