Title: Virasoro hair and entropy for axisymmetric Killing horizons

Speakers: Lin-Qing Chen

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Virasoro Hair and Entropy for Axisymmetric Killing Horizons

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Lin-Qing Chen (OIST)

with Wan Zhen Chua, Shuwei Liu, Antony J. Speranza, Bruno de S. L. Torres (PI)

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What is the microscopic origin of horizon entropy?

$$S_{BH} = \frac{A}{4G\hbar}$$

- One perspective: It was suggested by Carlip in 94' that the present of horizon as a boundary makes the would-be-gauge degrees of freedom physical. The black hole entropy is governed by the horizon symmetries.

- The existence of a spacetime boundary promotes gauge symmetry to be physical symmetry on the phase space. (It has been studied under different names: asymptotic symmetries, large diffeomorphisms, edge modes, soft hairs, boundary symmetries etc)

[Ashtekar, Barnich, Brandt, Brown, Compere, Donnay, Donnelly, Freidel, Geiller, Grumiller, Henneaux, Pranzetti, Perry, Strominger, Speranza, etc.]

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Deriving horizon entropy from symmetries

- **The Cardy formula:**
  For unitary and modular invariant 2-d CFT, $T_L \gg 1, T_R \gg 1$ the entropy in the canonical ensemble

$$S_{Cardy} = \frac{\pi^2}{3}(c_R T_R + c_L T_L)$$

  It also has higher dimensions, chiral CFT, warped algebra, BMS_3 versions

- It was initially realized in Strominger's derivation of BTZ entropy by using Brown-Henneaux result of asymptotic AdS_3 symmetries. Much subsequent work on generalization.

- Derivation of the microstate counting for extremal Kerr leads to the paradigm of Kerr/CFT.

- In particular, Carlip demonstrated the idea for generic Killing horizon through analyzing the horizon symmetry which preserves the structure of near horizon R-T plane. [Carlip 99', etc.]

- Our work is a generalization of [Black Hole Entropy and Soft Hair, by Haco, Hawking, Perry, Strominger, 18'] to any bifurcate axisymmetric Killing horizon.

This entropy counting approach is completely **theory or model independent**, without reference to other details of the microscopic dynamics, but it gives us guidance and insights for quantum gravity.

The near horizon geometry

- We consider stationary, bifurcate, axisymmetric Killing horizons in $d \geq 3$ with horizon generator $\chi^a = t^a + \sum_A \Omega_A \psi_A^a$ (by the black hole rigidity theorem)

- Mutual commuting vectors (ρ^a, χ^a, ψ^a) defines near horizon coordinate system, with ρ^a being the radial vector $\rho^a = -\frac{1}{2\kappa}\nabla^a \chi^2$, [Carlip 99']

- Select one rotational Killing vector, the near horizon expansion in the coordinate (x, t, ϕ, θ^A)

  x proper geodesic distance to the bifurcation surface; ϕ comoving angle;
  θ^A all the other transverse dimensions;

$$\begin{aligned} ds^2 = & -\kappa^2 x^2 dt^2 + dx^2 + \psi^2 d\phi^2 + q_{AB} d\theta^A d\theta^B \\ & - 2\kappa x^2 dt \left(N_\phi d\phi + N_A d\theta^A\right) + \mathcal{O}(x^2) \end{aligned}$$

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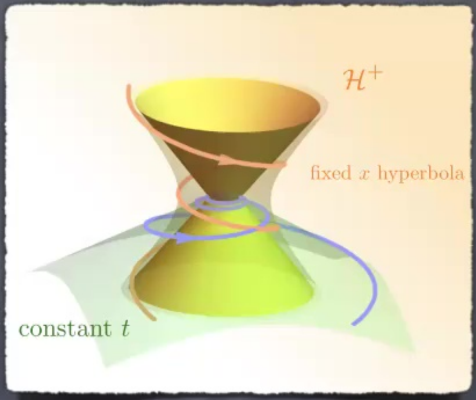
$$ds^2 = -\kappa^2 x^2 dt^2 + dx^2 + \psi^2 d\phi^2 + q_{AB} d\theta^A d\theta^B$$
$$\quad - 2\kappa x^2 dt\left(N_\phi d\phi + N_A d\theta^A\right) + \mathcal{O}(x^2)$$

Zoom in

Conformal Coordinate

$$w^+ = x e^{\alpha\phi + \kappa t}$$
$$w^- = x e^{\beta\phi - \kappa t}$$
$$y = e^{\frac{\alpha+\beta}{2}\phi}$$

$$(w^+, w^-, y) \sim \left(e^{2\pi\alpha} w^+, e^{2\pi\beta} w^-, e^{\pi(\alpha+\beta)} y\right).$$



$$ds^2 = \frac{dw^+ dw^-}{y^2} + \frac{4\psi^2}{(\alpha+\beta)^2}\frac{dy^2}{y^2} + q_{AB} d\theta^A d\theta^B$$
$$- \frac{2dy}{(\alpha+\beta)y^3}\left((\beta + N_\phi) w^- dw^+ + (\alpha - N_\phi) w^+ dw^-\right)$$
$$- \left(\frac{w^- dw^+}{y^2} - \frac{w^+ dw^-}{y^2}\right)\kappa N_A d\theta^A + \dots$$

locally (warped) AdS3 in
the Poincare coordinate

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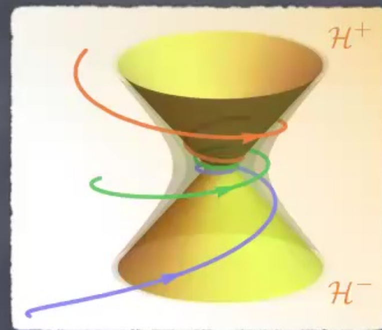
More geometrical intuition

Conformal Coordinate

$$w^+ = xe^{\alpha\phi+\kappa t}$$
$$w^- = xe^{\beta\phi-\kappa t}$$
$$y = e^{\frac{\alpha+\beta}{2}\phi}$$



Three spirals depicting intersections of constant w^+& w^-
From top to bottom $w^+/w^- = (5, 1, 1/5)$

$$ds^2 = \frac{dw^+dw^-}{y^2} + \frac{4\psi^2}{(\alpha+\beta)^2}\frac{dy^2}{y^2} + q_{AB}d\theta^A d\theta^B - \frac{2dy}{(\alpha+\beta)y^3}\Big((\beta+N_\phi)w^-dw^+ + (\alpha-N_\phi)w^+dw^-\Big) + \dots$$

$$x^2 = \frac{w^+w^-}{y^2}$$

How to approach the bifurcation surface ?

- **Two boundaries in the story:**

  The boundary of Cauchy surface: bifurcation surface $w^+ = w^- = 0$

  Asymptotic boundary of AdS3 folia $y \to 0 \quad (\phi \to -\infty)$

- What is the structure we will preserve: asymptotic warped AdS3

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Conformal vector fields

- In Poincare coordinate (conformal coordinate) [Haco, Hawking, Perry, Strominger, 18']

$$\zeta_{\varepsilon} = \varepsilon(w^+)\partial_+ + \frac{1}{2}\varepsilon'(w^+)y\partial_y \qquad \xi_{\bar{\varepsilon}} = \bar{\varepsilon}(w^-)\partial_- + \frac{1}{2}\bar{\varepsilon}'(w^-)y\partial_y$$

Mode expansion from the periodicity condition: $\varepsilon_n(w^+) = \alpha\,(w^+)^{1+\frac{in}{\alpha}}$ $\bar{\varepsilon}_n(w^-) = -\beta\,(w^-)^{1-\frac{in}{\beta}}$

They have singular limit when the coordinate w goes to zero.

Lie brackets form Witt algebra

$[\zeta_m, \zeta_n] = i(n-m)\zeta_{m+n}$ Regular on the future horizon but not the past horizon.

$[\xi_m, \xi_n] = i(n-m)\xi_{m+n}$ Regular on the past horizon but not the future horizon.

- The vector field written in terms of (ρ^a, χ^a, ψ^a)

$$\zeta_n^a = (w^+)^{\frac{in}{\alpha}}\left[\zeta_0^a - \frac{in\rho^a}{2\kappa} + \frac{in}{\alpha+\beta}\left(\frac{\beta-\alpha}{2\kappa}\chi^a + \psi^a\right)\right]$$

Killing

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Symmetries on the covariant phase space

- Presymplectic form associate with a Cauchy surface $\Omega_\Sigma = \int_\Sigma \omega(g, \delta_1 g, \delta_2 g)$

  g_{ab} solutions of Einstein equation, $\partial\Sigma = \mathcal{B}$ the bifurcation surface

- Diffeomorphisms that are pure gauge: $\Omega(\delta g, \pounds_\xi g_{ab}) = 0$

- The Hamiltonian which generates the symmetry (large diffeomorphisms) on the phase space associate with vector field ζ_n

$$\not{\delta} H_n = \Omega(\delta g_{ab}, \pounds_{\zeta_n} g_{ab}) = \int_{\partial\Sigma} \underbrace{\delta Q_{\zeta_n}}_{\text{Noether}} - i_{\zeta_n} \underbrace{\theta(g, \delta g)}_{\text{The symplectic potential}}$$

Witt algebra

$[\zeta_m, \zeta_n] = i(n-m)\zeta_{m+n}$

If the Hamiltonian is integrable: $\delta(\not{\delta} H_\zeta) = \int_{\partial\Sigma} \iota_\zeta \omega = 0$

$$\{H_m, H_n\} = -i\Big[(n-m)H_{m+n} + K(m,n)\Big]$$

Unique nontrivial central extension: $K_{R,L}(m,n) = \frac{c_{R,L}}{12}(m^3 - m)\delta_{m+n,0}$ Virasoro

[The **covariant phase space formalism** was developed by Peierl, Witten, Crnkovic, Ashtekar, Wald, Lee, Iyer, Zoupas, Barnich, Compere, Harlow, Wu etc.]

The central charges

The nontrivial central extension essentially comes from the singular diffeomorphisms under the limit towards bifurcation surface.

$$\partial_+ \zeta_m^y \to \frac{m^2 (w^+)^{-1+im/\alpha}}{y} \subset \pounds_{\zeta_m} g_{\mu\nu}$$

The left and right central charge:

$$c_R = \frac{24}{(\alpha+\beta)^2}\left(\frac{\beta A}{8\pi G} + J_H\right) = \frac{24}{\alpha+\beta}\frac{H_0}{\alpha} \qquad c_L = \frac{24}{(\alpha+\beta)^2}\left(\frac{\alpha A}{8\pi G} - J_H\right) = \frac{24}{\alpha+\beta}\frac{\bar{H}_0}{\beta}.$$

which are the linear combination of Noether charges:

$$J_H \equiv \int_{\mathcal{B}} Q_\psi = \frac{1}{4G}\int d\theta^A \sqrt{q}|\psi| N_\phi \qquad Area = 2\pi \int d\theta^A \sqrt{q}|\psi|$$

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The central charges

$$c_R = \frac{24}{(\alpha+\beta)^2}\left(\frac{\beta A}{8\pi G} + J_H\right); \qquad c_L = \frac{24}{(\alpha+\beta)^2}\left(\frac{\alpha A}{8\pi G} - J_H\right).$$

Boundary condition to ensure integrability of Hamiltonians:

$$\int d\theta^A \sqrt{q}\, |\psi|(\delta_\xi \kappa) \stackrel{!}{=} 0$$

↓

$$\alpha - \beta = \frac{16\pi G J_H}{A},$$

Wald-Zoupus counterterm

Add a (d-1) form to the symplectic potential to subtract the nonintegrable piece, e.o.m. is unimpacted

$$\int_{\partial\Sigma} \iota_\xi(\theta + \mathcal{X}(g,\delta g)) \quad \text{is exact}$$

↓ We adopt the counterterm from [HHPS 18]

$$\Delta c_R = -\Delta c_L = \frac{-12}{(\alpha+\beta)^2}\left((\beta-\alpha)\frac{A}{8\pi G} + 2J_H\right)$$

↓

$$c_R = c_L = \frac{3A}{2\pi G(\alpha+\beta)}$$

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The Cardy formula and the horizon entropy

- The left and right temperatures $(T_R, T_L) = (\alpha/2\pi, \beta/2\pi)$ of a putative CFT identified from the periodicity condition and the Frolov-Thorne vacuum density matrix respect to the Killing vector: [Frolove-Thorne 98'] [Applies to extremal Kerr/CFT by Guica et.al. 08]

$$\rho \sim \exp\left(-\frac{2\pi}{\kappa}\omega_\chi\right) = \exp\left(-\frac{\omega_R}{T_R} - \frac{\omega_L}{T_L}\right) \qquad \omega_R = k_a \zeta_0^a \quad \omega_L = k_a \xi_0^a$$

- Apply the Cardy formula: for a modular invariant putative 2-d CFT, the entropy of canonical ensemble is given by:

$$S_{Cardy} = \frac{\pi^2}{3}(c_R T_R + c_L T_L) = \frac{A}{4G} = S_{BH}$$

The range of validity: $T_L \gg 1, T_R \gg 1$

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Summary

- We studied the stationary bifurcate Killing horizon with axial symmetries in $d \geq 3$ and the general construction directly applicable to Rindler space.

- The gravitational phase space for the near-horizon region admits a 2d conformal symmetry algebra with **central charges proportional to the area** of bifurcation surface.

- For Kerr black hole, it has a well-defined zero spin limit to Schwarzchild.

- The construction allows CFT temperature to be chosen within the validity range of Cardy formula: $T_L \gg 1, T_R \gg 1$ which has been an issue in many previous work.

- The Cardy entropy coincides with the Bekenstein-Hawking entropy, suggesting a microscopic interpretation.

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Remarks & Questions

- What are the physical criteria for further specifying the CFT temperature?

  $(T_R - T_L)A = 8GJ_H$ defines different pair of temperatures compare to [HHPS, 18']

- Two copies of Virasoros are only a subset of the full horizon symmetries, yet strong enough to determine the density of states. We didn't use the possible additional rotational symmetries. What is the implication? Signal of dimensional reduction?

- What type of edge modes are responsible for the horizon microstates? The "surface preserving" edge mode is not enough, we also need the "surface translation" piece, which cannot be fully localized as a boundary piece.

- How to have well-defined charges for the stretched horizon (timelike boundary)?

- The relationship between pole structure appears here and the soft pole of the horizon memory effect.

- Connect our construction to the extremal limit $\kappa = 0$ to compare to NHEK/CFT

- Generalization to arbitrary cross-section of the horizon, causal diamond, quantum extremal surface, even generic subregion.

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The questions I'm curious the most — work in progress

- Why does it work?

Identify the modular invariance, then compare the saddle of the gravitational path integral to the original proof of Cardy formula; Try to understand the crucial role that singular diffeo shall play. Would the two "oracles" are secretly the same?

- The different role of conformal anomaly and gravitational anomaly for horizon entropy counting

$$\int d\theta^A \sqrt{q}\ |\psi|(\delta_\xi \kappa) \stackrel{!}{=} 0$$

It demands the vanishing of this particular type of symplectic flux from the transverse dimensions to the AdS bulk, vanishing of holographic gravitational anomaly:

- Generalizing to the near-adiabatic dynamical horizon with time-dependent surface gravity?

- "Near-horizon/CFT" duality? What type of CFT it would be?

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