Title: Black hole information, spacetime wormholes, and baby universes

Speakers: Henry Maxfield

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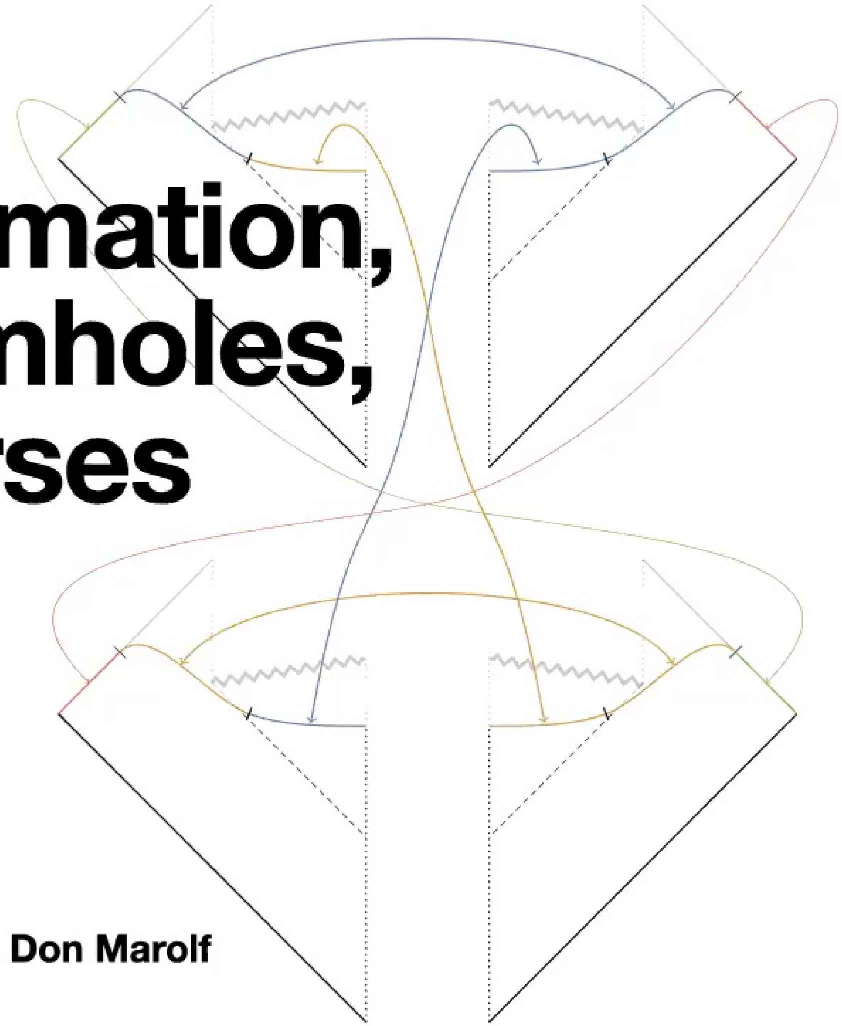
Abstract: Recent discoveries suggest that semiclassical gravity is more consistent with unitarity than previously believed. I will argue that it makes predictions for the measurements of asymptotic observers that are in complete accord with the idea that black holes are ordinary quantum systems, with states counted by the Bekenstein-Hawking formula. The argument uses the semiclassical gravitational path integral, incorporating newly discovered `spacetime wormhole' topologies. These new ideas revive an old paradigm, relating the information problem to the physics of baby universes.

Black hole information, spacetime wormholes, and baby universes

Quantum Gravity 2020



Henry Maxfield, based on work in progress with Don Marolf

Black hole information

Can a distant observer describe black hole formation and evaporation with unitary quantum mechanics?

Should we interpret $S_{\rm BH} = \dfrac{A}{4G_N}$ as counting states in that description?

"Bekenstein-Hawking unitarity":

Measurements of black holes by distant observers are described by an ordinary quantum system with unitary evolution and $\dim \mathscr{H} = e^{S_{\rm BH}}$

Other possibilities:

- Information loss (initial$\longrightarrow$final evolution not unitary)
- Remnants (unitary, but information returned only after black hole is small)

A conservative approach

Using only well-established physics, what should we expect?

- Compute observables at infinity ($\mathscr{I}^+$)
- Only assume gravitational low-energy EFT (semiclassical path integral)
- Use saddle-points in semiclassical regime of validity

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- Are the resulting predictions consistent with BH unitarity?

Make no use of:

- Geometries with large curvatures
- UV completion (strings, loops, …)
- Duality (AdS/CFT) or assumed unitarity

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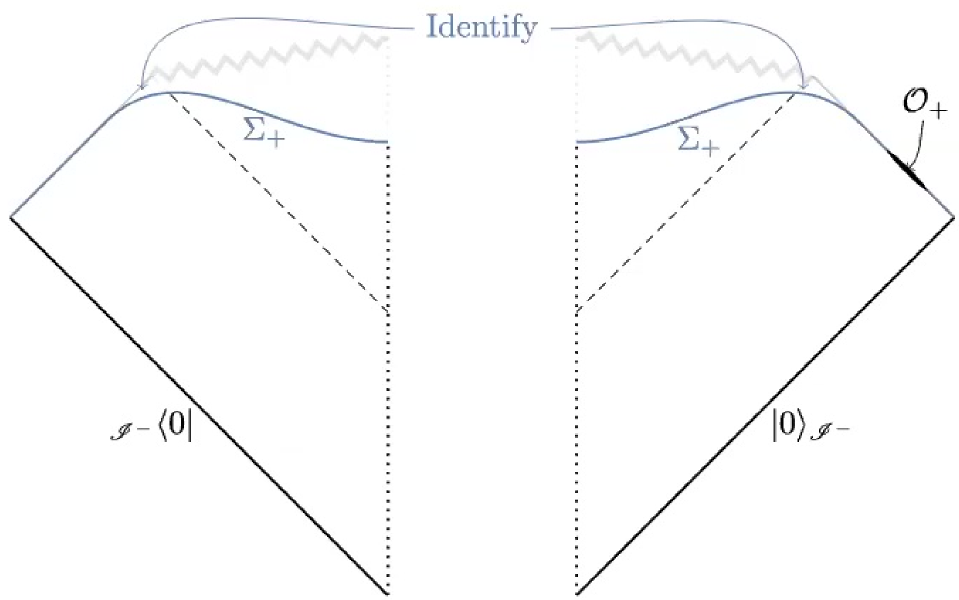
Conclusion:

All observables compatible with BH unitarity!

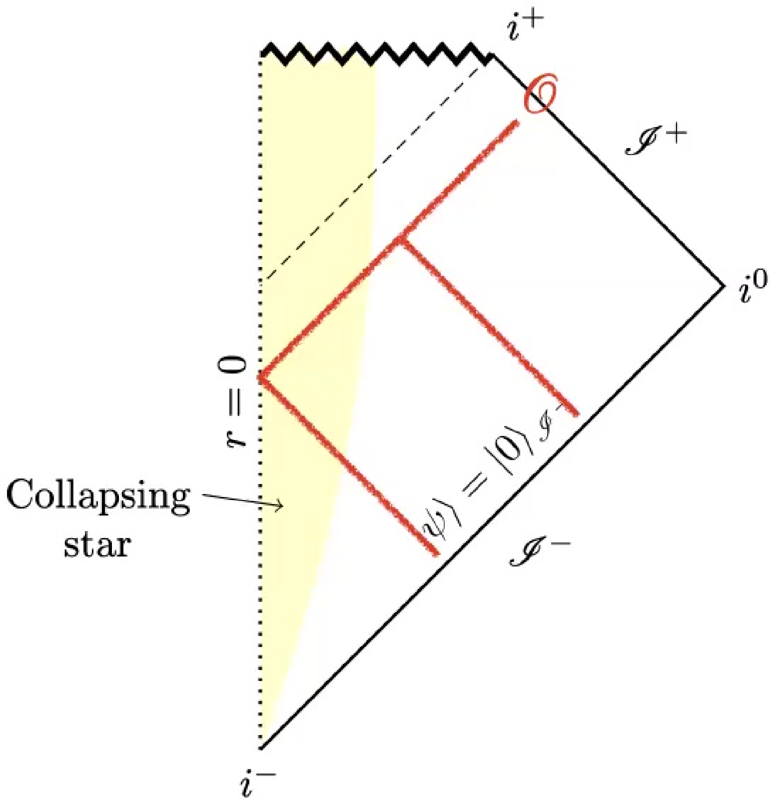
At a cost: predictions are **not unique,** but **statistical**

1. Hawking 1975



Hawking's calculation

QFT on fixed background



Compute EV of operator $\mathscr{O}$ on $\mathscr{I}^+$

Heisenberg evolve back to $\mathscr{I}^-$

e.g. free matter $a_m(\mathscr{I}^+) = \sum_n \left(\alpha_{mn}\, a_n(\mathscr{I}^-) + \beta_{mn}\, a_n^\dagger(\mathscr{I}^-)\right)$

Evaluate in initial state (ingoing vacuum)

e.g. occupation numbers $\left\langle N_m(\mathscr{I}^+)\right\rangle = \sum_n |\beta_{mn}|^2 .$

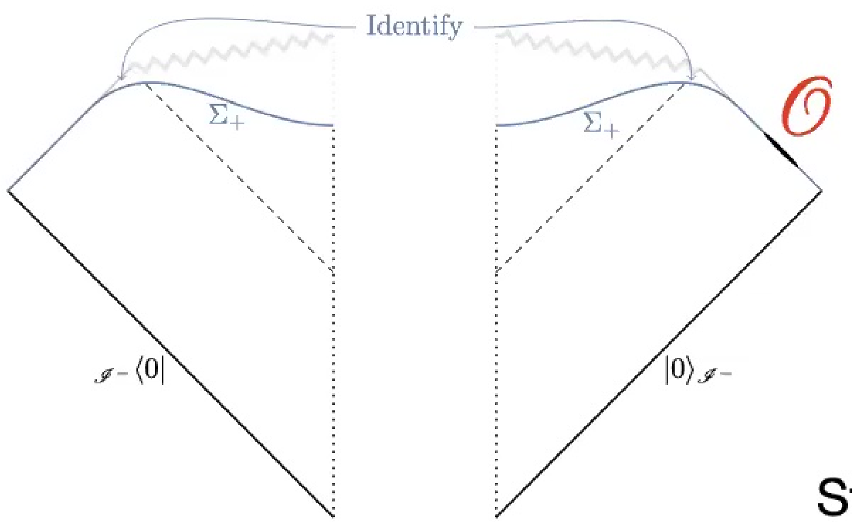
No evolution through strong curvature regions required

Hawking's calculation

QFT on fixed background

Path integral description

Integrate over matter fields
on "doubled" spacetime:



Compute EV of operator $\mathcal{O}$ on $\mathscr{I}^+$

Heisenberg evolve back to $\mathscr{I}^-$

In-in (Schwinger-Keldysh) formalism

Boundary conditions at $\mathscr{I}^-$: initial state

No need to specify final state

Identify "bra" and "ket" spacetimes
on future Cauchy surface Σ_+

Strong curvature regions not part of geometry

Hawking's calculation

QFT on fixed background

Path integral description

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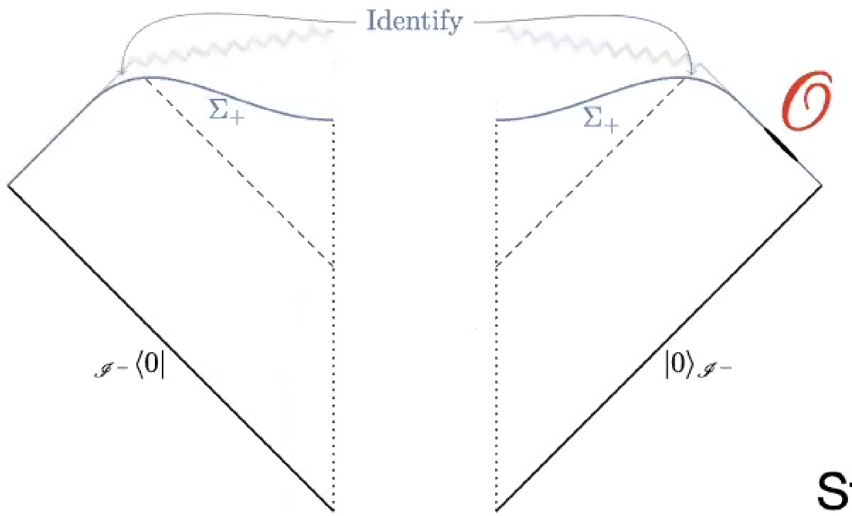

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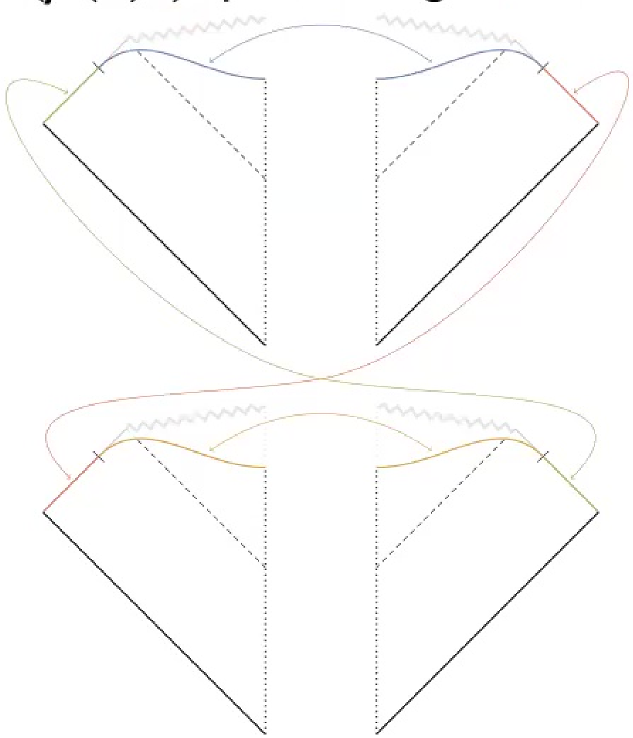
Hawking's calculation: entropy

QFT on fixed background

$\mathrm{Tr}(\rho(u)^2)$: path integral on



Expectation value of "swap operator" on two copies of state:

$$\mathrm{Tr}(\rho(u)^2) = \left\langle \mathcal{S}(\mathcal{I}_u) \right\rangle_{2-\mathrm{replica}} = e^{-S_2(\rho(u))}$$

Generalises to Rényi entropy:

$$S_n(\rho(u)) = -\frac{1}{n-1} \log \mathrm{Tr}(\rho(u)^n)$$

von Neumann entropy as formal limit:

$$S(\rho(u)) = -\mathrm{Tr}(\rho(u) \log \rho(u)) = \lim_{n \to 1} S_n(\rho(u))$$

$S(\rho(u)) \sim$ thermal entropy of Hawking radiation

Quantum gravity: integrate over metrics

Look for saddle-points of gravitational action + matter effective action $S_{\rm EH}[g] + S_{\rm eff}[g]$

$$\int \mathscr{D}g \; e^{iS_{\rm EH}[g]} \int \mathscr{D}\phi \; e^{iS_{\rm matter}[g,\phi]} = \int \mathscr{D}g \; e^{i(S_{\rm EH}[g]+S_{\rm eff}[g])}$$

Boundary conditions: fix asymptotic geometry, otherwise allow general spacetime

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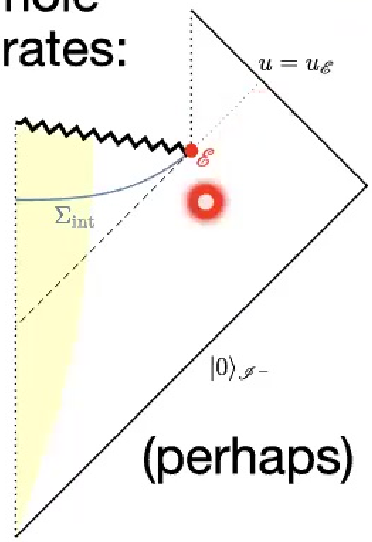
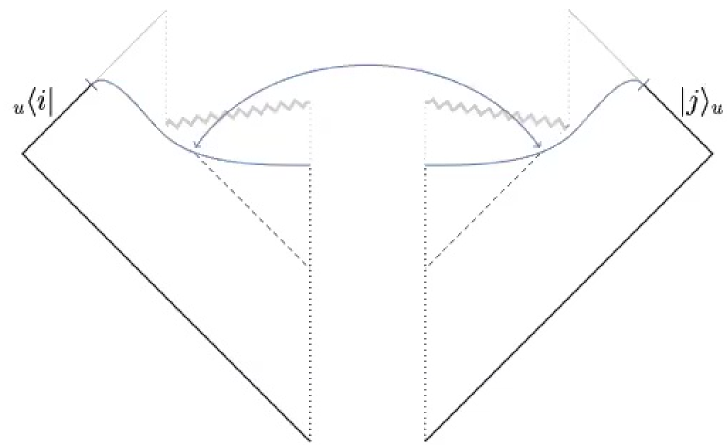
Black hole evaporates:



(perhaps)

Density matrix

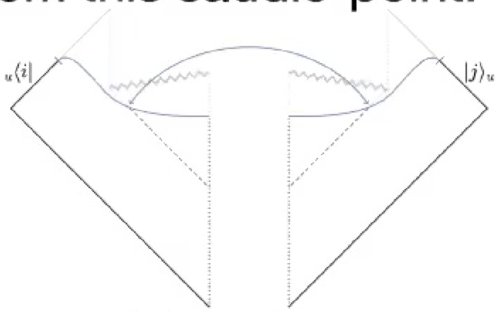
$_u\langle i\,|\,\rho(u)\,|\,j\rangle_u$



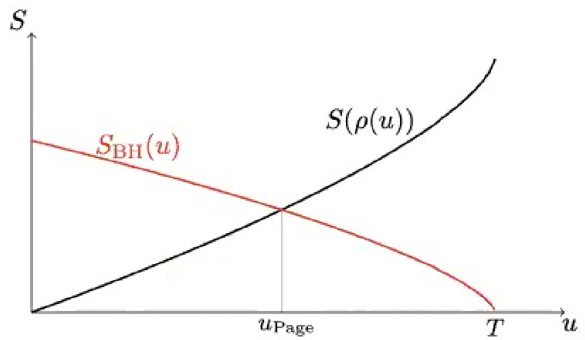
For measurements before evaporation ($u < u_{\mathscr{E}}$), no assumptions about endpoint $\mathscr{E}$ or singularity necessary

A violation of BH unitarity?

Compute density matrix from this saddle-point:



$\rho(u)$ is thermal.
Violates BH unitarity!



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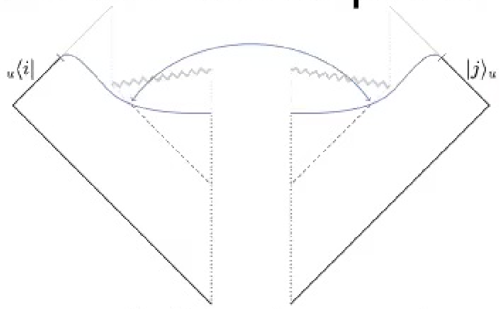

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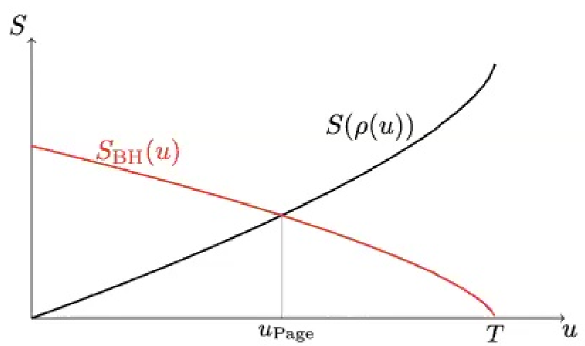


But entropy isn't an observable.
Try to check with a measurement!

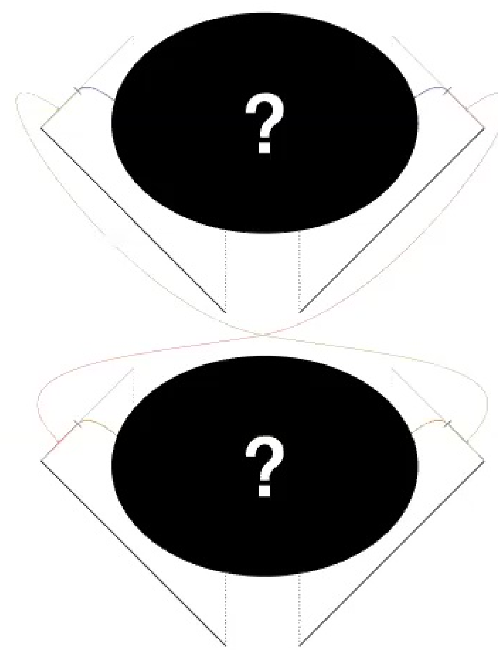
Need to create several copies of
Hawking radiation.

Measure swap (for example):

$$\left\langle \mathcal{S}(\mathcal{I}_u) \right\rangle_{2-\text{replica}} = \mathrm{Tr}\left(\mathcal{S}(\mathcal{I}_u)\rho^{(2)}(u) \right) = e^{-S_2^{\text{swap}}(u)}$$

"Swap entropy" $S_n^{\text{swap}}(u)$**:**

Entropy deduced by asymptotic observer from measurements



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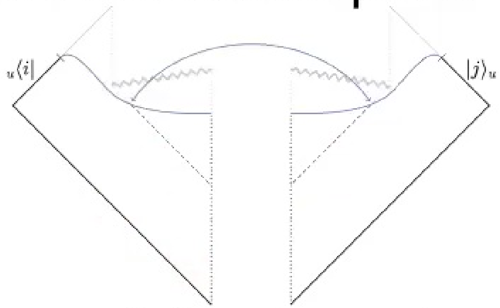

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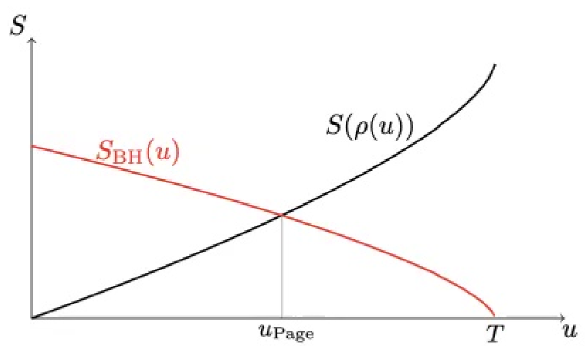

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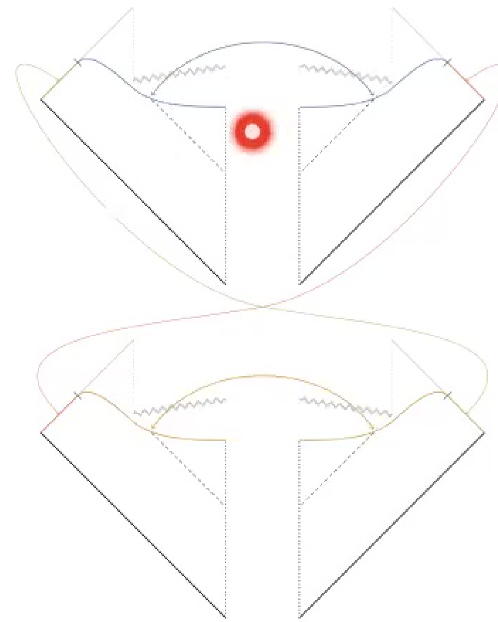


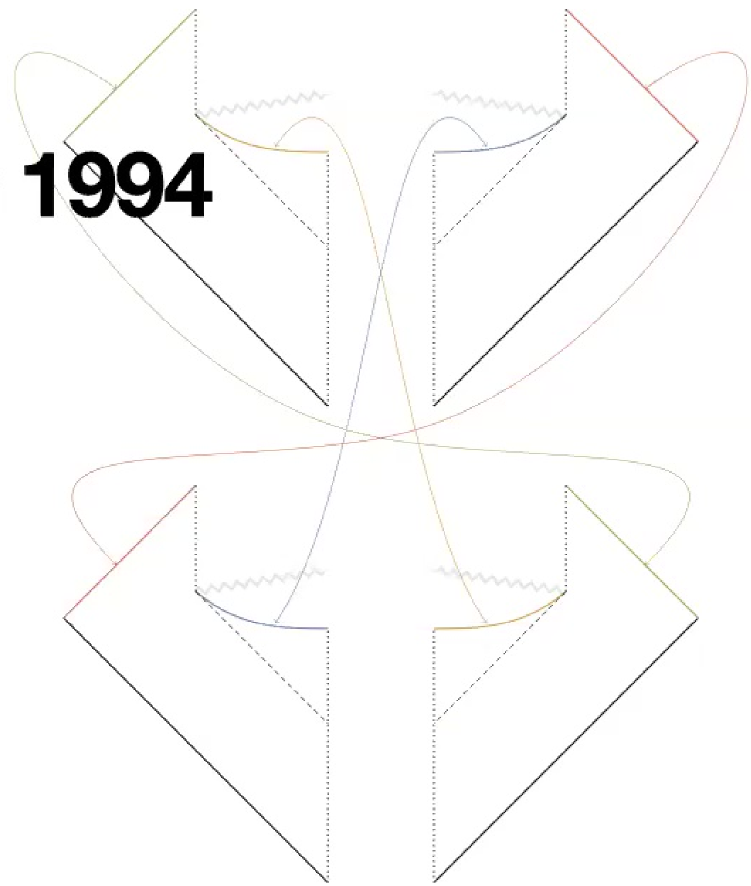
$$\left\langle \mathcal{S}(\mathcal{I}_u)\right\rangle_{2-\text{replica}} = \text{Tr}\left(\mathcal{S}(\mathcal{I}_u)\rho^{(2)}(u)\right)$$
$$= e^{-S_2^{\text{swap}}(u)}$$

Naive answer:

$$\rho^{(2)}(u) = \rho(u) \otimes \rho(u)$$

Same result as before.

2. Polchinski Strominger 1994

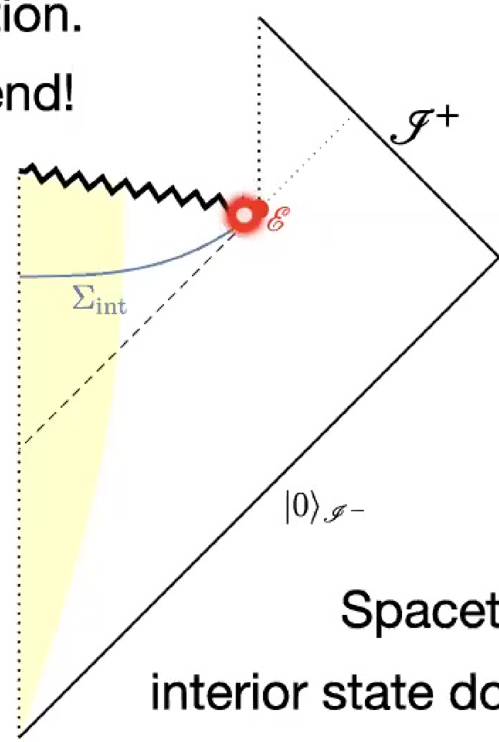


Polchinski-Strominger wormholes

A warm-up

For historical & pedagogical reasons, make some assumptions about evaporation.

This will **not be necessary** in the end!



For now, assume $\Sigma_{\rm int} \cup \mathscr{I}^+$ can be treated like a Cauchy surface

Spacetime splits at $\mathscr{E}$,

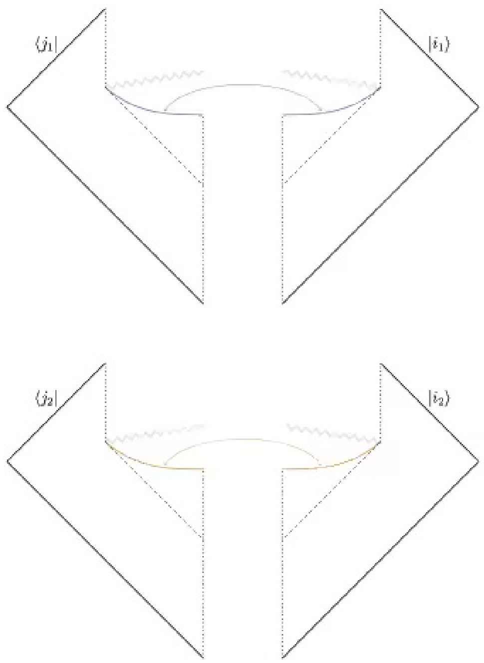
interior state does not propagate to $\mathscr{I}^+$

Polchinski-Strominger wormholes

A warm-up

Two-copy density matrix on all of $\mathscr{I}^+$

$$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle = \langle j_1 | \rho_H | i_1 \rangle \langle j_2 | \rho_H | i_2 \rangle$$



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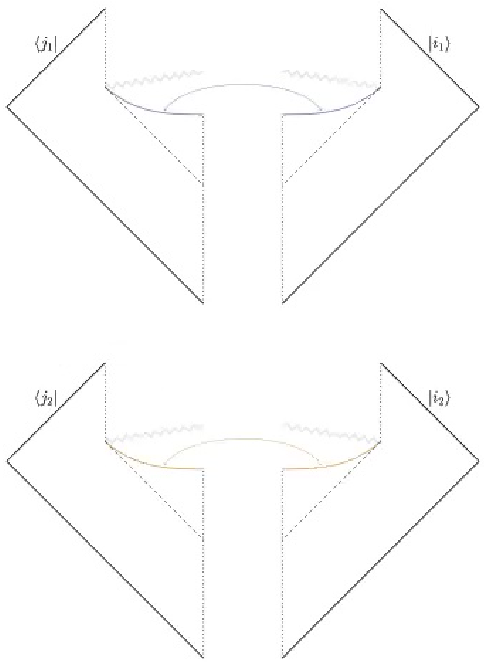

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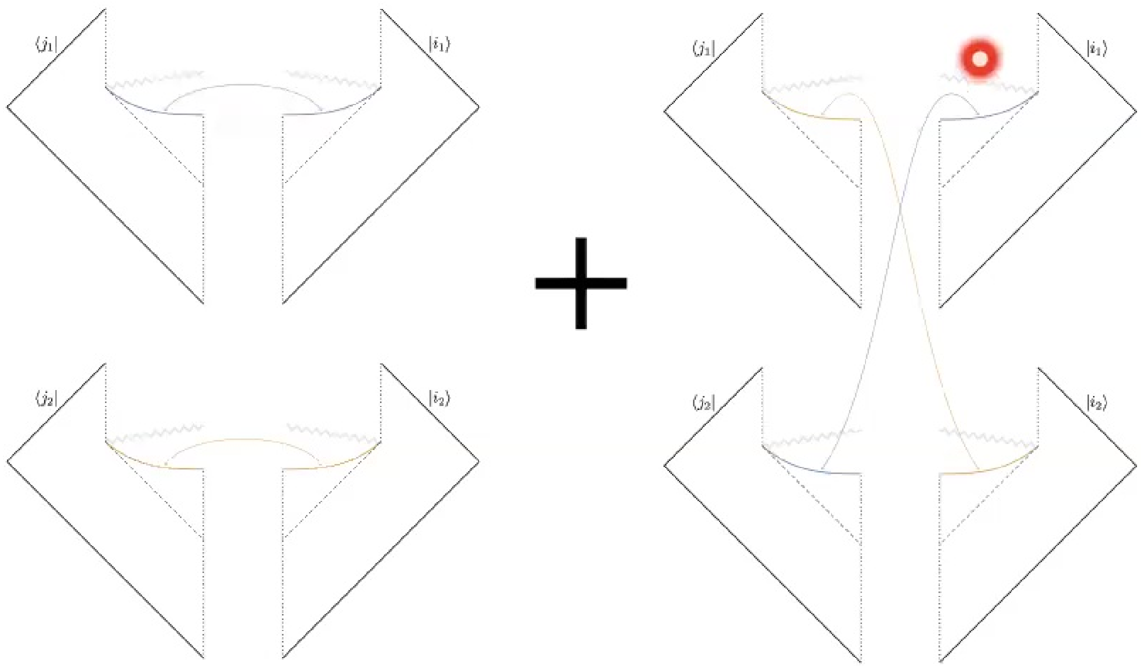
$$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle = \langle j_1 | \rho_H | i_1 \rangle \langle j_2 | \rho_H | i_2 \rangle + \langle j_2 | \rho_H | i_1 \rangle \langle j_1 | \rho_H | i_2 \rangle$$



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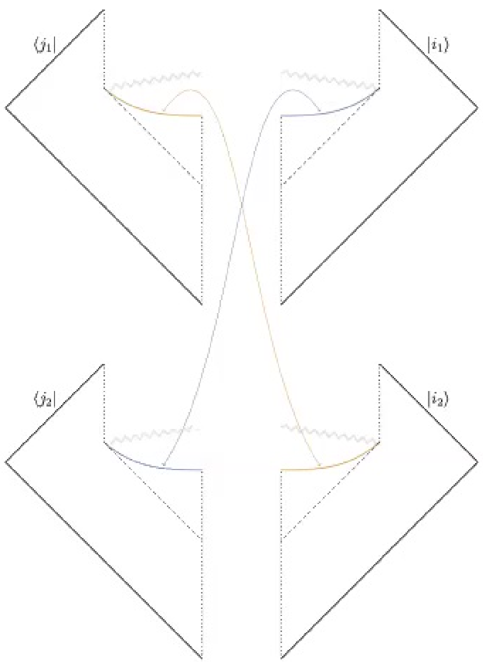


$$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle = \langle j_1 | \rho_H | i_1 \rangle \langle j_2 | \rho_H | i_2 \rangle + \langle j_2 | \rho_H | i_1 \rangle \langle j_1 | \rho_H | i_2 \rangle$$

$$\rho^{(2)} = (1 + \mathscr{S})\, \rho_H \otimes \rho_H$$

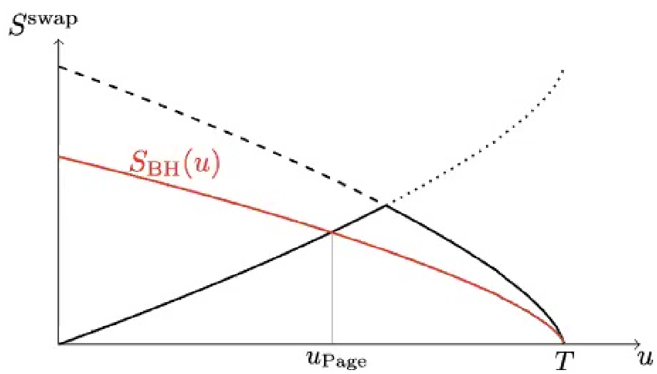
$\rho^{(2)}$ invariant under $\mathscr{S} \implies \left\langle \mathscr{S} \right\rangle = 1 \implies S_2^{\text{swap}} = 0$

Observables match with **pure** state of Hawking radiation!

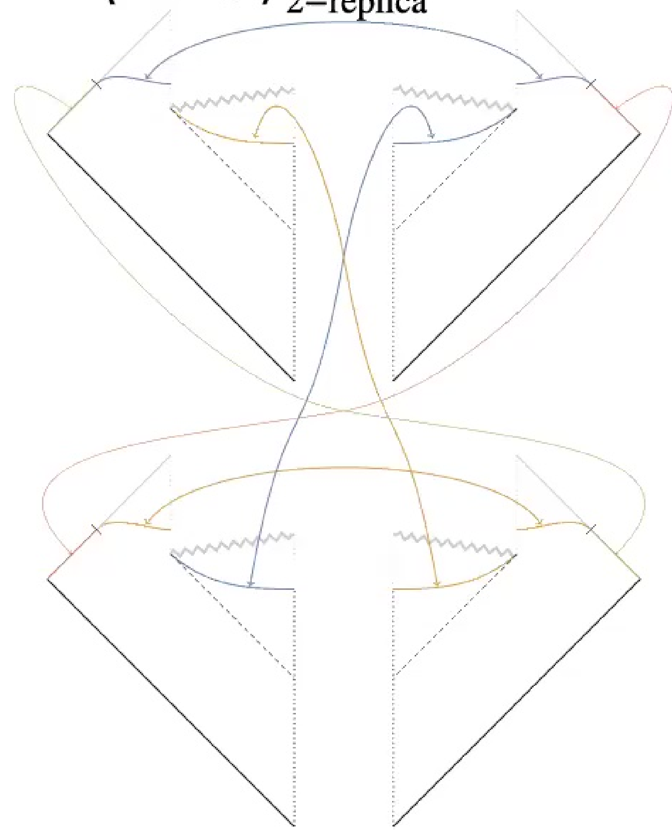
Polchinski-Strominger wormholes

Problems

- Requires assumptions about UV physics
- Does not give BH unitarity
  - Not the Page curve for $S^{\text{swap}}(u)$
  - Worse violations (remnants) if we “feed” the black hole
- Acausality?

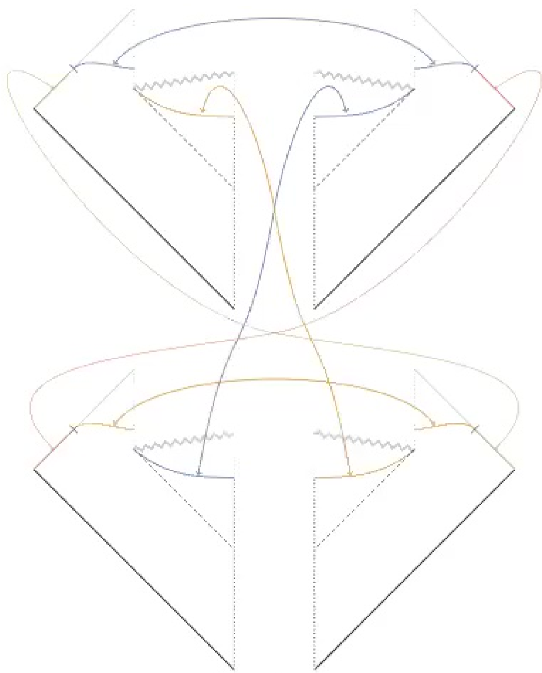


$$\left\langle \mathcal{S}(\mathcal{I}_u) \right\rangle_{2-\text{replica}} = e^{-S_2^{\text{swap}}(u)}$$



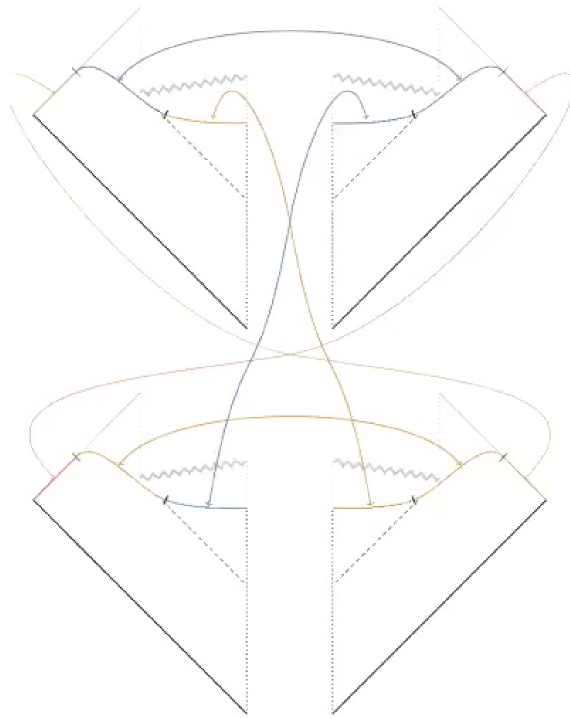
Replica wormholes

PS wormhole contributing to
$\left\langle \rho^{(2)} \mathcal{S}(\mathscr{I}^+) \right\rangle$:



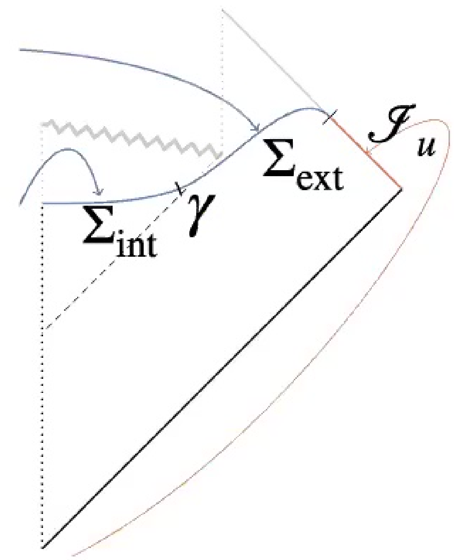
Dynamical swap of
black hole interior

Replica wormhole:



Dynamical swap of
compact partial Cauchy surface

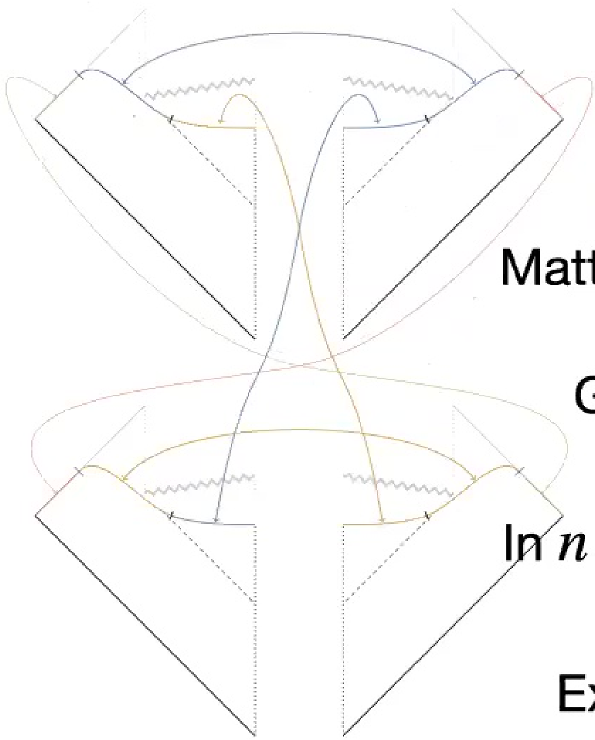
Divide Cauchy surface into
$\mathscr{I}_u \cup \Sigma_{\text{ext}} \cup \Sigma_{\text{int}}$

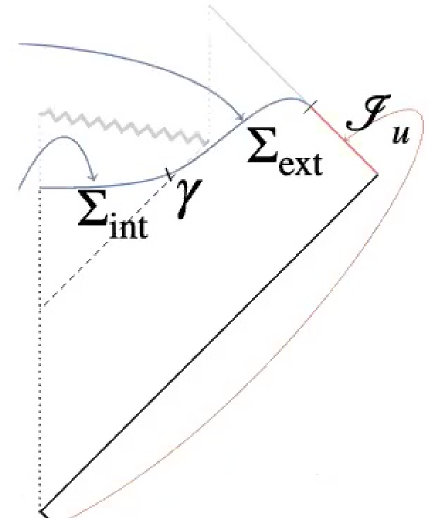


Distinct geometries depend on location of codimension-2 surface $\partial\Sigma_{\text{int}} = \gamma$

Replica wormholes

Saddle-points and quantum extremal surfaces





Generalise to n-replica calculation

Reformulate: make sense of path integral for real $n > 1$

Matter effective action $\longrightarrow S_n^{\text{matter}}(\mathscr{I}_u \cup \Sigma_{\text{int}})$

Gravitational action $\longrightarrow \dfrac{A[\gamma]}{4G_N}$

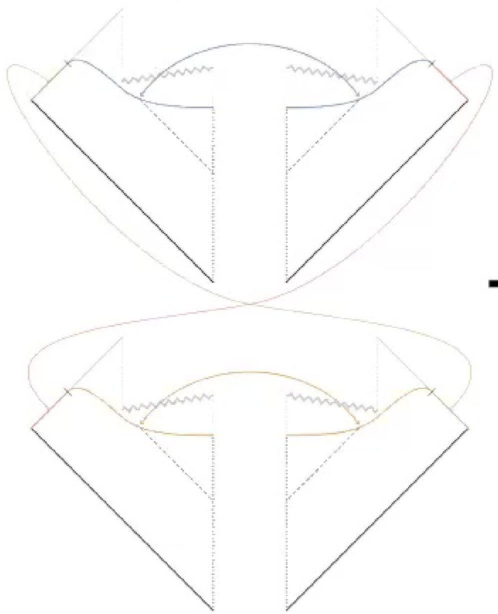
In $n \to 1$ limit, saddle-point if γ is a **QES:**

Extremise $S_{\text{gen}}(\gamma) = \dfrac{A(\gamma)}{4G_N} + S_{\text{matter}}(\mathscr{I}_u \cup \Sigma_{\text{int}})$

Replica wormholes

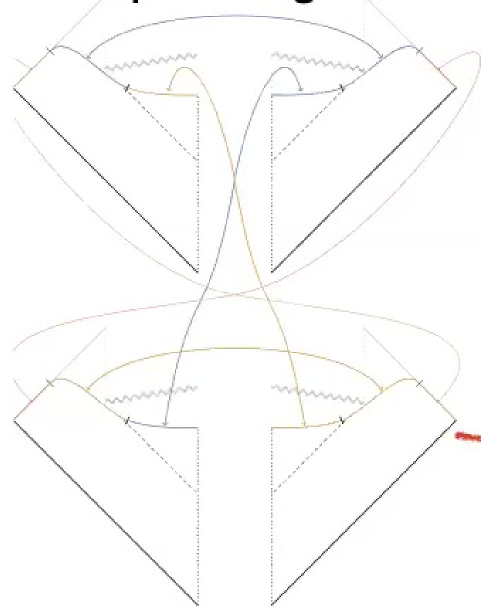
Saddle-points and quantum extremal surfaces

A nontrivial QES exists for evaporating BH:



Dominant at early times

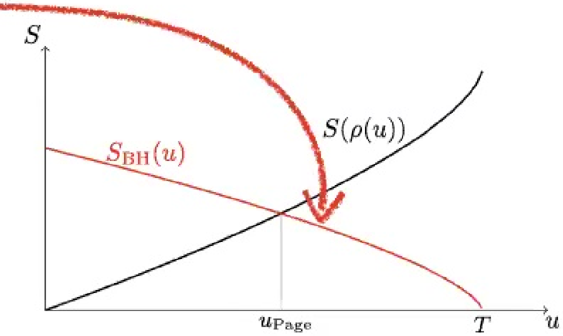
+



Dominant at late times

Together: Page curve for $S^{\mathrm{swap}}(u)$!

Conclusion: all observations will be compatible with BH unitarity



Replica wormholes

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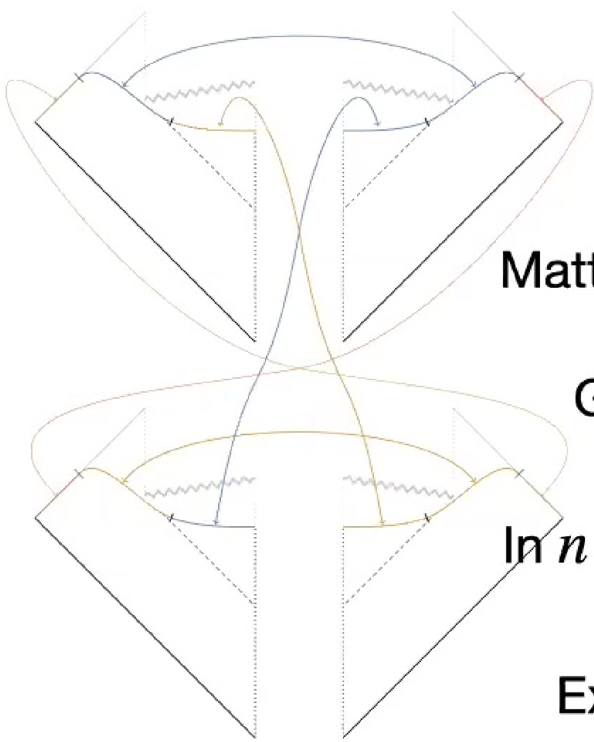

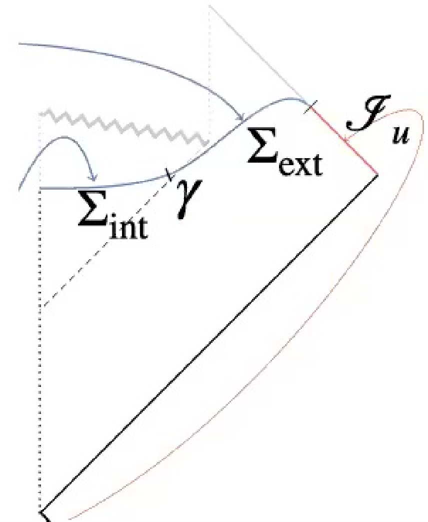

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Replica wormholes

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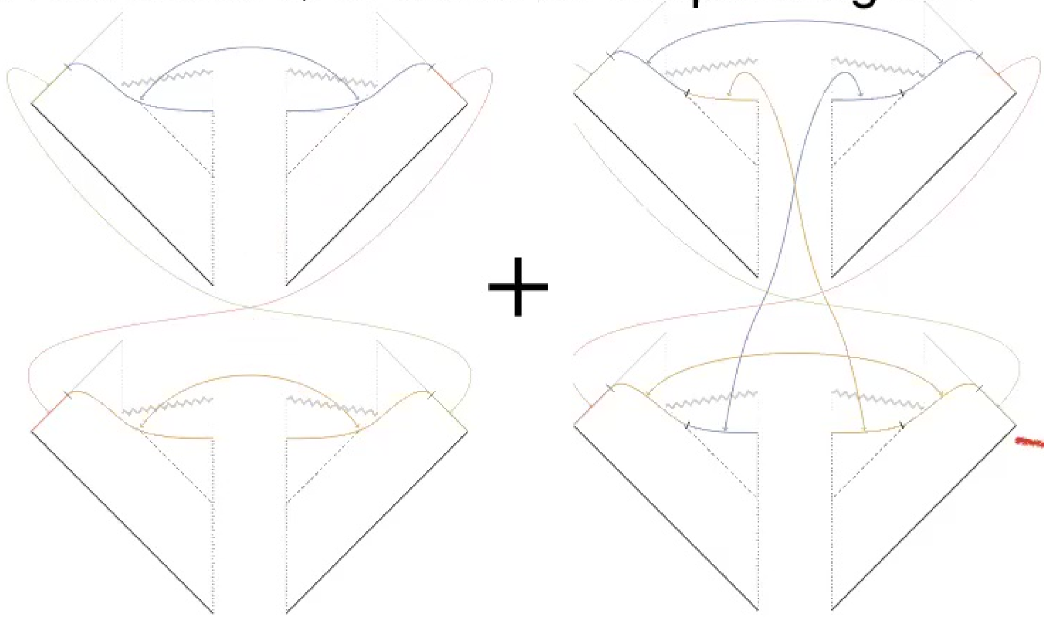

Dominant at early times   Dominant at late times

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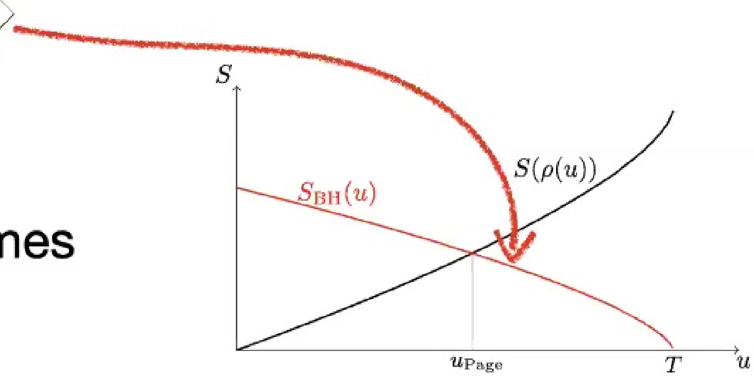
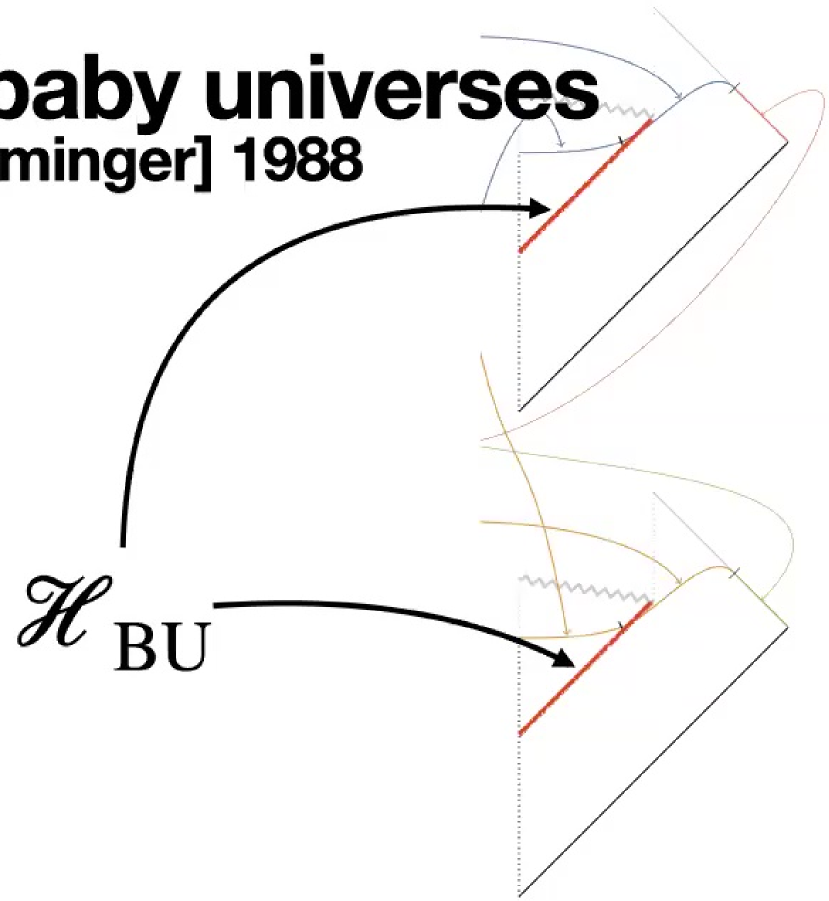


3. Return of the baby universes

[Coleman][GiddingsStrominger] 1988



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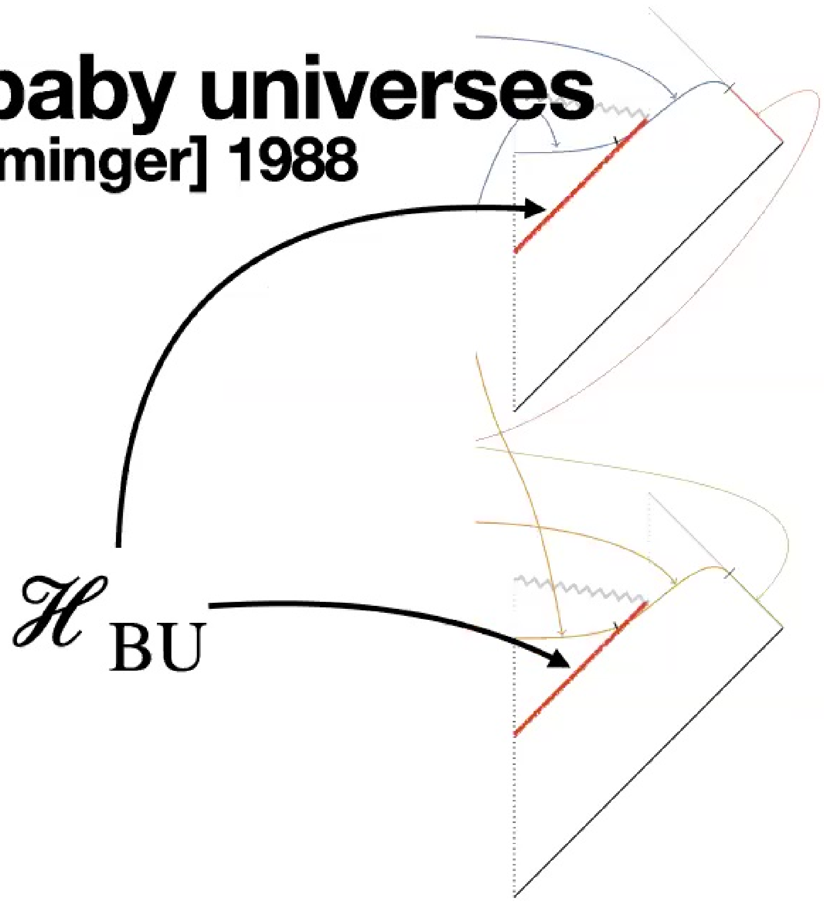

The Hilbert space of baby universes

Observables are compatible with BH unitarity

But density matrix of radiation $\rho(u)$ is Hawking's result

} How are these compatible?

What is the Hilbert space interpretation?

The Hilbert space of baby universes

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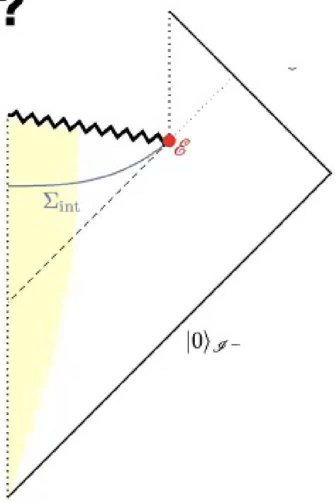
What is the Hilbert space interpretation?

Sum over intermediate states on Σ_{int}

Hilbert space of closed "baby" universes

Superselection sectors for asymptotic observables

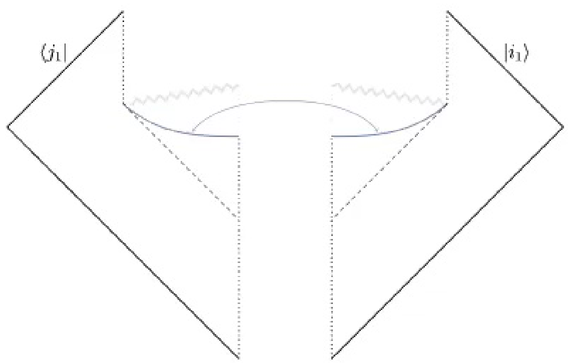


1. Hawking

Hilbert space interpretation: comes from cutting open path integral

One set of Hawking radiation:

$$\langle j_1 | \rho | i_1 \rangle =$$



Pure state on $\mathscr{I}^+$ **and** Σ_{int}:

$$|\psi\rangle = \sum_{i,a} \psi_{ai} \, | i \rangle_{\mathscr{I}^+} \otimes | a \rangle_{\Sigma_{\text{int}}}$$

Sum over intermediate states on Σ_{int}:

"trace out" interior, find

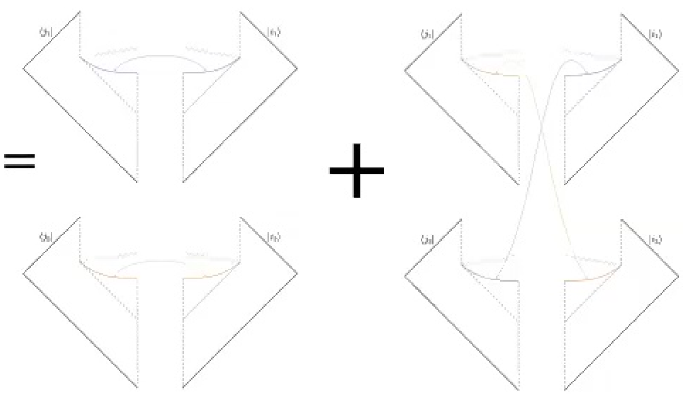
$$\langle j | \rho | i \rangle = \sum_{a,b} \bar{\psi}_{bj} \psi_{ai} \langle b | a \rangle_{\Sigma_{\text{int}}}$$

ρ mixed due to entanglement with "closed universe" Σ_{int}

2. Polchinski-Strominger

Several sets of Hawking radiation:

$$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle =$$



Pure state on n copies of $\mathscr{I}^+$ and Σ_{int}:

$$|\psi^{(2)}\rangle = \sum_{\substack{i_1, i_2 \\ a_1, a_2}} \psi_{a_1 i_1} \psi_{a_2 i_2} | i_1, i_2 \rangle_{\mathscr{I}^+} \otimes | a_1, a_2 \rangle_{\text{BU}}$$

Inner product on "baby universes" induced by PS wormholes:

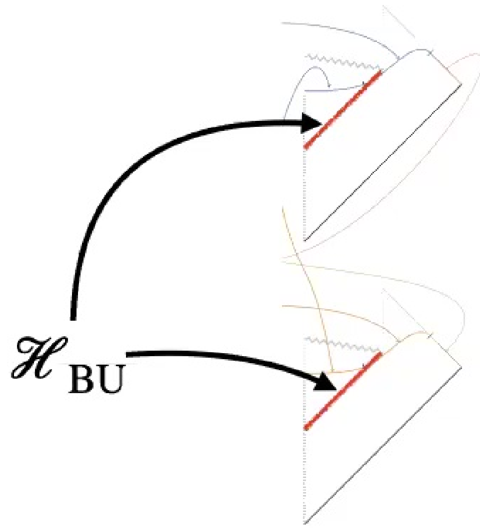
$$\langle b_1, b_2 | a_1, a_2 \rangle_{\text{BU}} = \langle b_1 | a_1 \rangle_{\Sigma_{\text{int}}} \langle b_2 | a_2 \rangle_{\Sigma_{\text{int}}} + \langle b_2 | a_1 \rangle_{\Sigma_{\text{int}}} \langle b_1 | a_2 \rangle_{\Sigma_{\text{int}}}$$

Not just factorised inner product on $\mathscr{H}_{\Sigma_{\text{int}}} \otimes \mathscr{H}_{\Sigma_{\text{int}}}$!

Closed universes are indistinguishable bosons

$$\mathscr{H}_{\text{BU}} = \bigoplus_{n=0}^{\infty} \text{Sym}^n \mathscr{H}_{\Sigma_{\text{int}}}$$

3. Replica wormholes



Replica wormholes modify inner products

$$\langle b_1, b_2 | a_1, a_2 \rangle_{\mathrm{BU}}$$

Modifies Fock space of closed universes: "interactions"

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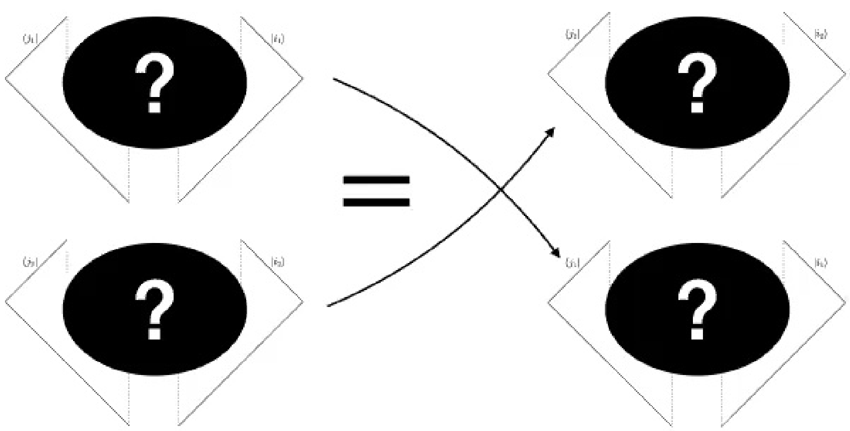
Evolve to Cauchy slice containing γ, take overlap including swap on "island"

$$\langle b_1, b_2 | a_1, a_2 \rangle_{\mathrm{BU}} \supset \langle b_1 | \otimes \langle b_2 | \mathcal{S}(\text{island}) | a_1 \rangle \otimes | a_2 \rangle$$

4. An abstract construction of $\mathscr{H}_{\mathrm{BU}}$

Asymptotic observables form a **commutative** algebra of operators on $\mathscr{H}_{\mathrm{BU}}$



Boundary conditions are not ordered

Construct Hilbert space $\mathscr{H}_{\mathrm{BU}}$ from **expectation values** only

Basis of states $|\alpha\rangle \in \mathscr{H}_{\mathrm{BU}}$ simultaneously diagonalises observables

$\longrightarrow$ **superselection sectors**

Described by **classical probability**

Baby universes provide **statistical** (not quantum) correlations.

A unitary theory with unknown "couplings".

Probability distribution given by state

$$\sum_{\alpha} \sqrt{p_\alpha} e^{i\theta_\alpha} |\alpha\rangle \in \mathscr{H}_{\mathrm{BU}}$$

Conclusion

And open questions

Semiclassical gravity makes predictions in line with Bekenstein-Hawking unitarity
But predictions are statistical, not unique

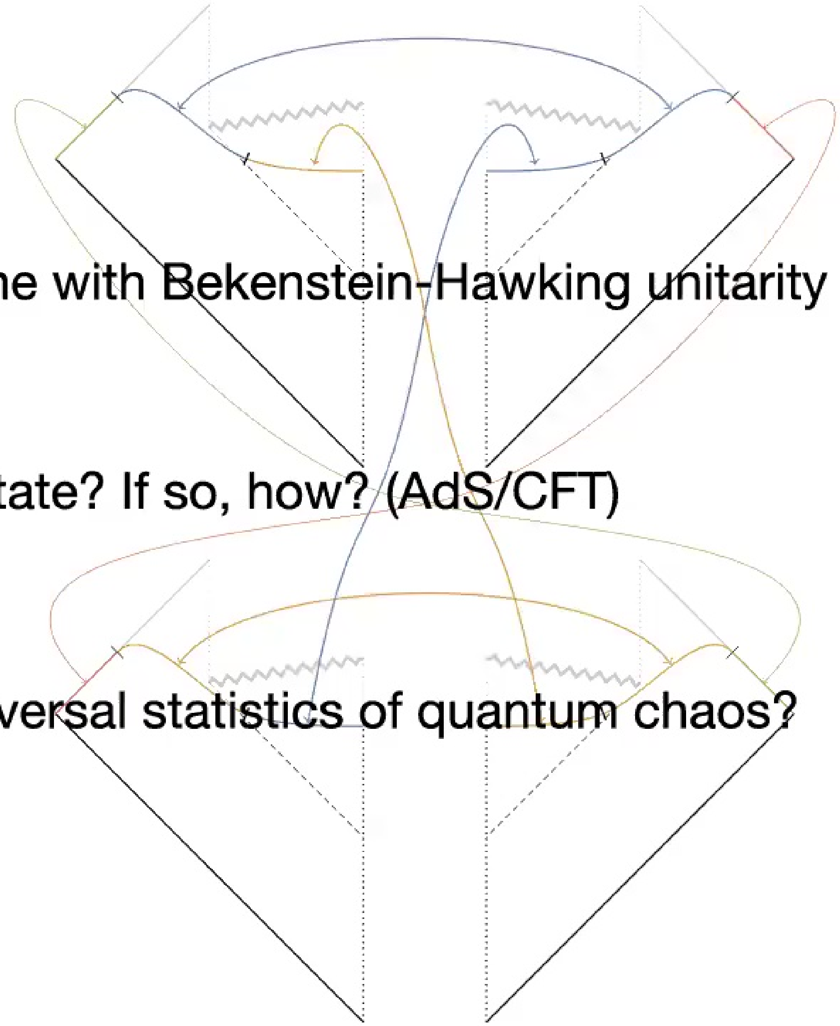
Does a UV completion select a unique $|\alpha\rangle$ state? If so, how? (AdS/CFT)

Quantum error correction: "code" $\leftrightarrow |\alpha\rangle$?

More statistical predictions from gravity? Universal statistics of quantum chaos?

Is unitarity upheld by other observables?

Predictions for an infalling observer?



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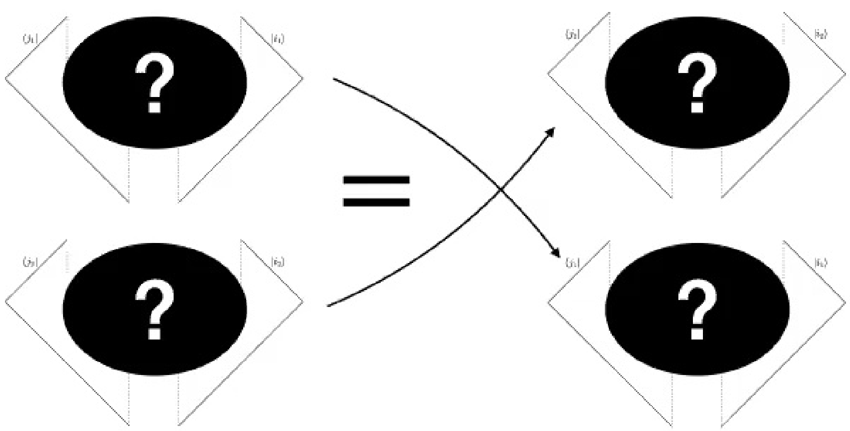

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