Title: Mathematical structure of quantum gravity

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Abstract: A quantum theory of gravity is expected to be described by a Hilbert space endowed with additional mathematical structure appropriate for describing gravitational physics. I discuss aspects of this structure that can be inferred perturbatively, along with connections to arguments for holography and nonperturbative questions.

# Mathematical structure of quantum gravity

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***Quantum Gravity 2020***
***Perimeter Institute***

**July 15, 2020**

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The problem of Quantum Gravity (QG) is one of the most difficult problems in physics.

Traditional approach: General Relativity (GR) $\xrightarrow{\text{“quantize”}}$ Quantum Gravity

Problem: - nonrenormalizability (short distance)

... traditional focus

Deeper problem: - nonunitarity

find w/ HE scattering, BHs

*long distance*

*More central* problem? More profound

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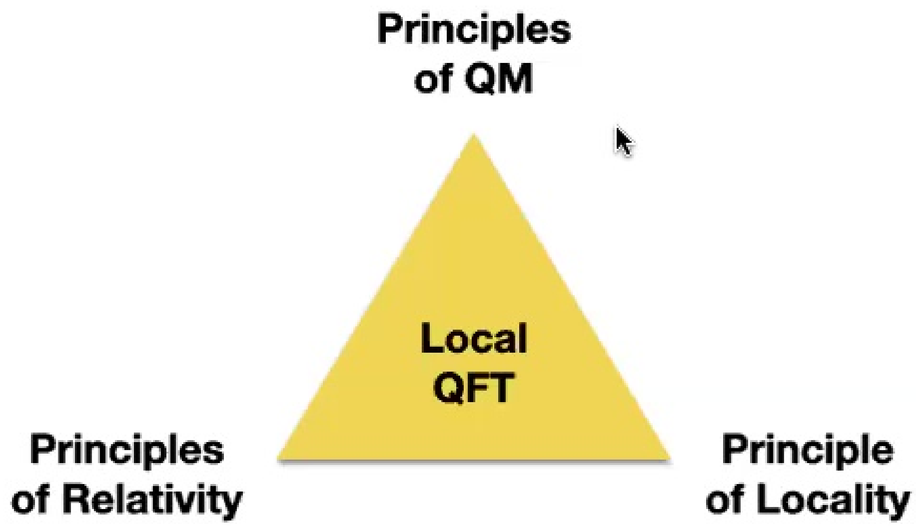
These problems appear to reveal a basic inconsistency:



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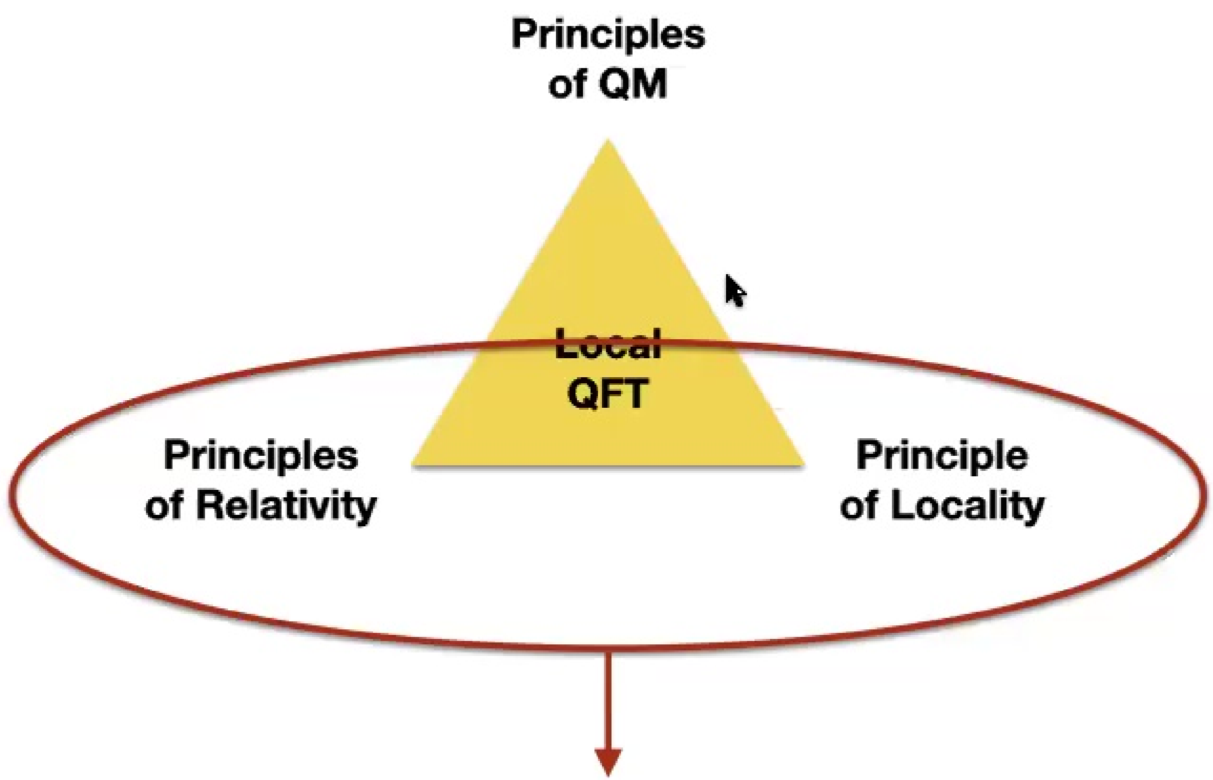


**Classical geometry and topology**

**“spacetime is a manifold”**

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## Various approaches — Strings, Loop QG, etc.:

1) longstanding focus on nonrenormalizability problem

2) don't tell us how to resolve unitarity problem (at least, yet)

3) are still based on *quantizing* classical theory ~ "spacetime"

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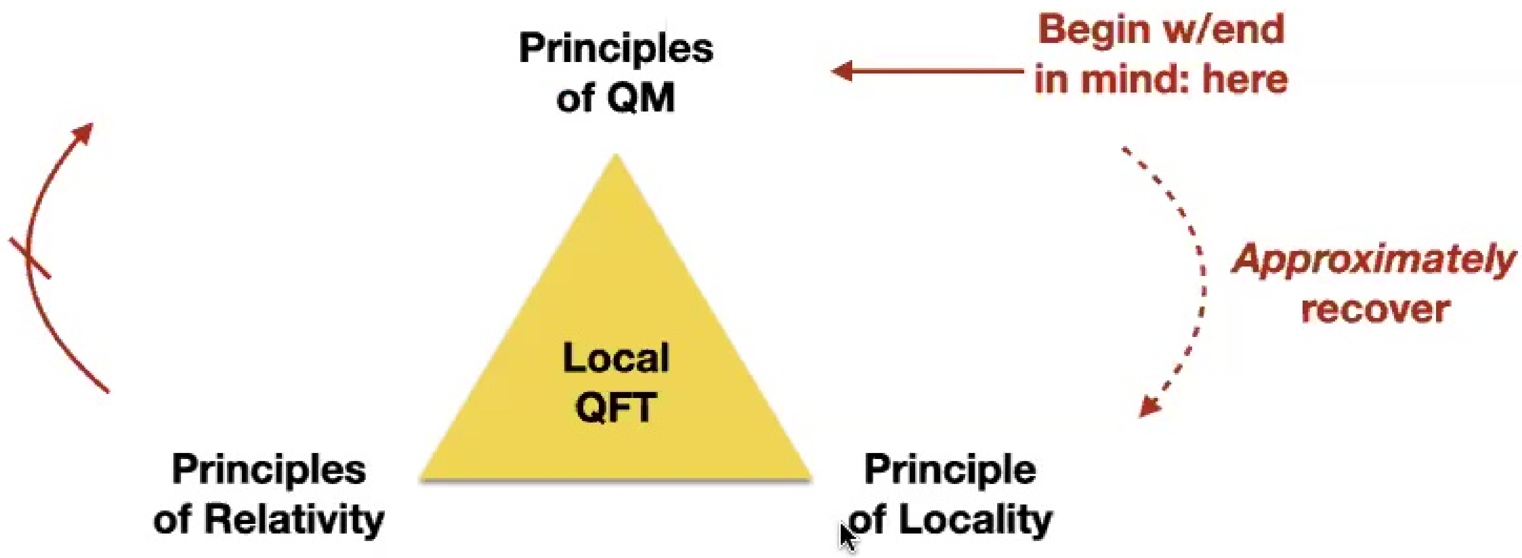
## An alternate approach:




**Use:** ***QM is a tightly constraining framework***

**“Geometrize quantum mechanics, rather than quantizing geometry”**

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## Might call this a "Quantum-first" approach to gravity

[1503.08207;
1803.04973,1805.06900
Carroll+collabs]

**Key postulates:**

1) Quantum-mechanical theory

2) Correspondence (weak gravity) with local QFT (LQFT) + GR

**Central question:**

*What mathematical structure, within QM, gives a consistent theory providing correspondence w/LQFT + GR?*

(Mathematical consistency is an important, and implicit, postulate)

*Plan: infer aspects of this mathematical structure*

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## First, how to think about postulates for QM?

- want to incorporate gravity; don't assume "time," space, etc.

E.g. Hartle, "Generalized QM"
[e.g. gr-qc/0602013]

Not general enough?
e.g. assumes "histories," etc.

"Universal QM" [0711.0757]

**Postulates are remarkably sparse:**

Linear space of states, w/ inner product $\mathscr{H}$

Algebra of q-observables (linear ops.) $\mathscr{A}$ (to distinguish from usual observables)

Unitarity (e.g. of S-matrix, for states with appropriate asymptotics)

**These are plausibly the universal elements of a QM theory.**

***But, clearly more structure is needed to describe physics!***

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## An increasingly important theme for quantum systems:

# Quantum information!

**Key questions:**

How is it localized? (~ basis for kinematics)

How does it transfer? (~ dynamics)

**Concrete examples: is a black hole a quantum subsystem?**

(At least approximately)

how does information transfer out of one ?

(possible observational consequences?! 1905.08807)

**Contrast w/ some current themes:**

There: entanglement —> quantum spacetime

Here: quantum spacetime —> entanglement

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## Localization: what is a *quantum subsystem*?

**In local QFT, answering this question leads to an *example* for the kind of mathematical structure I'm referring to.**

### 1. Quantum subsystems for finite systems (or, locally finite):

$$\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$$

**(sometimes basic postulate — e.g. Zurek, 1412.5206)**

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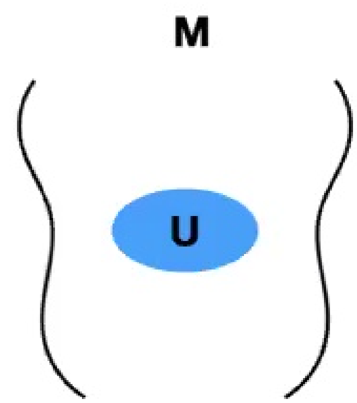
## 2. Quantum subsystems for local QFT:




$$\mathscr{H} \neq \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$$

**(vN type III: "infinite enganglement")**

**Instead, *commuting subalgebras*, associated with open regions**

$U \leftrightarrow \mathscr{A}_U$ **e.g.** $\phi_f = \int d^4x\, f(x)\phi(x)$

**If U and U' are spacelike separated:**

$$[A_U, A_{U'}] = 0$$

**... locality**

***Subalgebras* define "subsystems"**

**Mathematical structure:** **(see, e.g. *Haag, Local Quantum Physics*)**

**inclusions, intersections, etc.:**

**"Net" of subalgebras $\longleftrightarrow$ *topological structure of spacetime* (+ causal)**

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## How do we think about subsystems in quantum gravity?

**Note that it's hard to imagine doing physics without this!**

### Einstein separability:

"... it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space.' Without such an assumption of the mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation."

**-A. Einstein**

**... localization of information, definition of subsystems**

**But we can't do it with classical geometry!**

## So, in gravity:

**If there is a Hilbert space in gravitational physics, is there an analogous structure on it, corresponding to "quantum spacetime," which reduces to the QFT structure in the weak gravity correspondence limit?**

**and *what is this "gravitational substrate"?***

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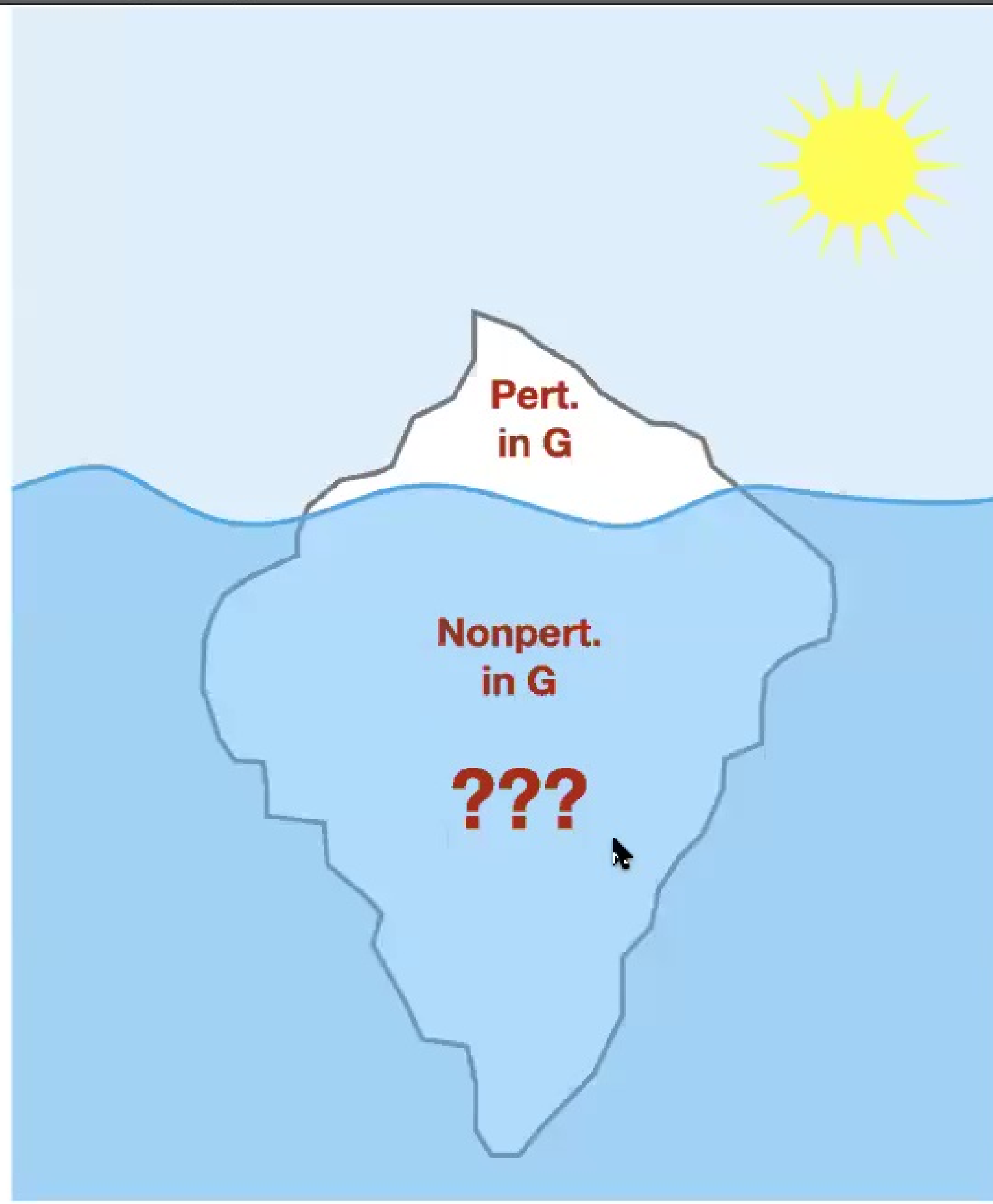
Some modest steps towards inferring this mathematical structure of quantum gravity:




(The tip is a reliable place to begin learning about the structure of icebergs before diving underwater!)

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E.g. consider scalar $\phi$ , coupled to gravity

$G = 0$, **LQFT:** $\phi_f = \int d^4x\, f(x)\phi(x)$, **etc.: local subalgebras of observables**

$G \neq 0$ : $\phi(x)$ **is not a gauge (diff) invariant observable** $\delta_\xi \phi(x) = -\kappa \xi^\mu \partial_\mu \phi(x)$

$\kappa = \sqrt{32\pi G}$

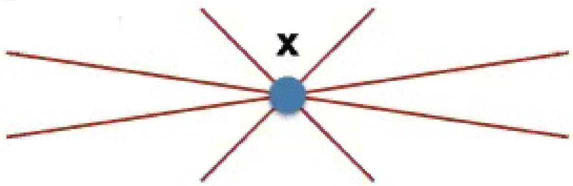
***It isn't physical.***

**Problem:** $\phi(x)$ **creates a particle.**

**A particle is inseparable from its gravitational field.**

**Solution (one):** ***dressed*** **observables: include grav. field:**

$\phi(x) \longrightarrow \Phi(x)$ :



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## Some dressed observable basics:

[1503.08207; 1507.07921, 1607.01025 w/Donnelly]

**Perturbatively:** $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$ $\quad \mathcal{O}(\kappa):$ $\quad \delta h_{\mu\nu} = -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu$

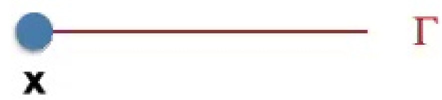
**Want** $\delta_\xi \Phi(x) = 0$ **, or** $[C_\mu(y), \Phi(x)] = 0$ **, with** $C_\mu = G_{0\mu} - 8\pi G T_{0\mu}$

**Solution:** **Construct "dressing:"** $V^\mu[h, x]$ **so that** $\delta V^\mu(x) = \kappa \xi^\mu(x)$

"key condition"

**Then** $\Phi(x) = \phi(x + V^\mu(x))$ **is diff invariant**

**Example:** **let $\Gamma$ be a curve from x to $\infty$**



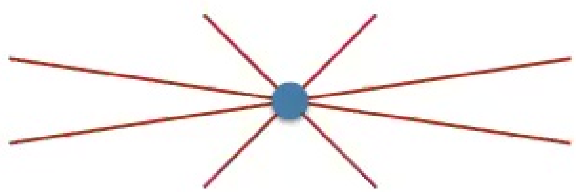

$$V_\mu^\Gamma(x) = \frac{\kappa}{2} \int_x^\infty dx'^\nu \left\{ h_{\mu\nu}(x') + \int_{x'}^\infty dx''^\lambda \left[ \partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'') \right] \right\}$$

"gravitational line"

[1805.06900]

**There are *many* solutions** $V^\mu[h, x]$ **~allowed grav fields of particle**

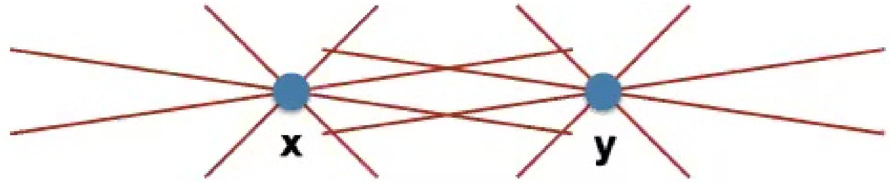
**Another, more physical:** $V_\mu^C(x)$ **"Coulomb" dressing**



**(e.g. spherical average of line)**

## Issues with operator-based def. of subsystems/separability:

$$[\Phi(x), \Phi(y)] \neq 0 \qquad (x-y)^2 > 0$$

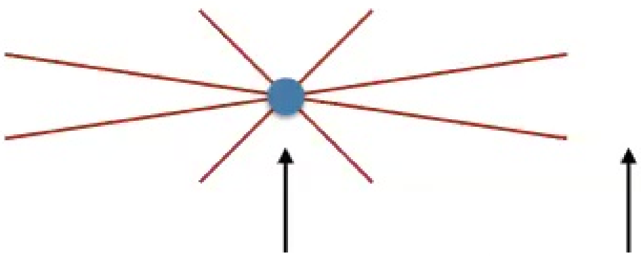



**Gravitational noncommutativity! (but not noncommutative geometry)**

**In NR limit, mass m:** $$[\partial_t \Phi_C(x), \Phi_C(y)] \simeq \frac{Gm}{|x-y|} \partial_t \phi(x) \phi(y)$$

**[1507.07921, w/Donnelly; locality bound]**

**Underlying physical problem:**



**Creating a particle here** **Has a physical effect here**

**(Note also connects to "soft charge" discussion of Strominger, Perry, Hawking …: e.g. measure soft charges at infinity)**

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**So,** ***how do we localize information in gravity?***

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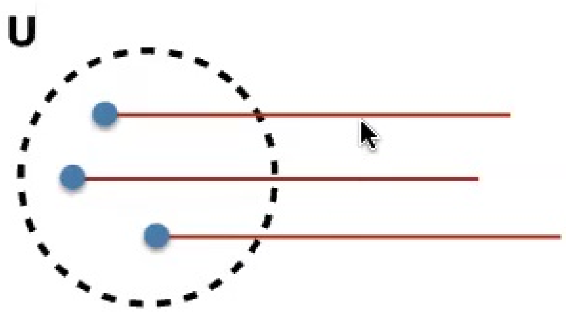
## *Simpler example:* QED w/ charge q scalar

$$\Phi(x) = \phi(x)e^{i\Lambda(x)}$$

gauge invariant

dressing

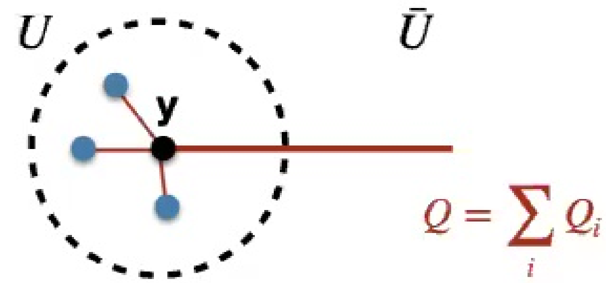
E.g. Faraday line $\Lambda(x) = q\int_x^\infty A$


Can detect charge distribution?

**Not necessarily:** ***many possible choices of dressing***

**Another choice:**




**So, there exists choices of dressing such that EM observables in $\bar{U}$ are insensitive to the charge distribution in $U$**

**I.e. we have independent information associated with neighborhood $U$: "subsystem"**

**But, *not* based on commuting operators!**

**More like** $\mathscr{H} \sim \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$

**Said this was problematic even in LQFT … can we fix?**

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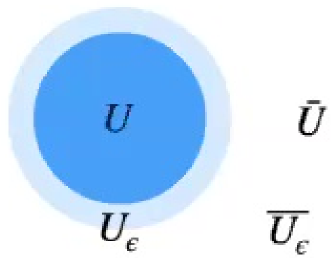
in LQFT (no QED or gravity):




**Said:** $\mathscr{H} \neq \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$

**But, *split vacuum*:** $|U_\epsilon\rangle$ **[Haag, and refs. therein]**

**For** $A \in \mathscr{A}_U$ , $A' \in \mathscr{A}_{\overline{U_\epsilon}}$ **:** $\langle U_\epsilon | AA' | U_\epsilon \rangle = \langle 0 | A | 0 \rangle \langle 0 | A' | 0 \rangle$

**"disentangles" degrees of freedom**

$A_I | U_\epsilon \rangle$ , $A_J | U_\epsilon \rangle$ **indistinguishable via measurements in** $\overline{U_\epsilon}$

$A_I, A_J \in \mathscr{A}_U$

**~ "localized qubit"**

**So, gives localized quantum information.**

**Mathematical structure:** $\mathscr{H}_U \otimes \mathscr{H}_{\overline{U_\epsilon}} \hookrightarrow \mathscr{H}$

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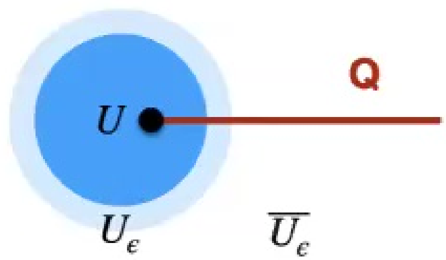
## Now, include EM field:




$$\bigoplus_{Q} \mathscr{H}_{U,Q} \otimes \mathscr{H}_{\overline{U}_\epsilon,Q} \hookrightarrow \mathscr{H}$$

**"Electromagnetic splitting"**

**(build on $|U_\epsilon\rangle$ )**

**Net of EM splittings** $\longleftrightarrow$ **mathematical subsystem structure for QED**

***Notion of localization of quantum information***

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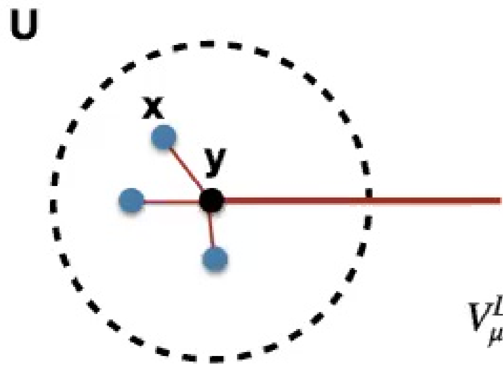
Can we do this for gravity? $\mathcal{O}(\kappa)$ **construction:**

1805.11095 w/ Donnelly, 1903.06160




$$V_\mu^L(x,y) = -\frac{\kappa}{2}\int_y^x dx'^\nu \left\{ h_{\mu\nu}(x') - \int_y^{x'} dx''^\lambda \left[\partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'')\right]\right\}$$

$$V^L(y) = V_\mu^L(y,\infty)$$

**Then:**

$$V^\mu(x) = V_L^\mu(x,y) + V_L^\mu(y) + \frac{1}{2}(x-y)_\nu[\partial^\nu V_L^\mu(y) - \partial^\mu V_L^\nu(y)]$$ **satisfies** $$\delta V^\mu(x) = \kappa\xi^\mu(x)$$

**Creates "standard" grav. field outside U** $\tilde{h}^\mu_{\lambda\sigma}(x)$

**Generalizes to any "standard"** $V_S^\mu(y)$ **satisfying** $\delta V_S^\mu(y) = \kappa\xi^\mu(y)$

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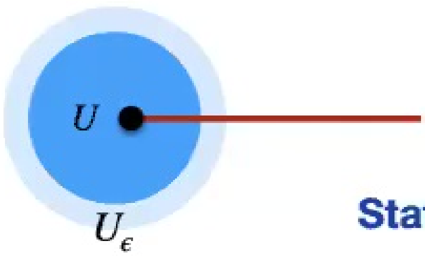
**Subsystems: generalize dressing of field operator:** $\Phi(x) = \phi(x + V^\mu(x))$




$$A \longrightarrow \hat{A} = A + i\int d^3x\, V^\mu(x)\,[T_{0\mu}(x), A] \; + \mathcal{O}(\kappa^2)$$

**State:**

$$|\Psi_I\rangle = A_I|U_\epsilon\rangle \longrightarrow |\widehat{\Psi}_I\rangle = e^{i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)}|\Psi_I\rangle \; + \mathcal{O}(\kappa^2)$$

**Then can show:**

$$[h_{\mu\nu}(\bar{x}), \hat{A}] = -\, i\tilde{h}^\lambda_{\mu\nu}(\bar{x})\,[P_\mu, A] - \frac{i}{2}\partial^\lambda\tilde{h}^\sigma_{\mu\nu}(\bar{x})\,[M_{\lambda\sigma}, A] \qquad \bar{x} \in \bar{U}_\epsilon$$

**Measurements of $h_{\mu\nu}$ outside $U_\epsilon$ only detect total Poincare charges of $A$**

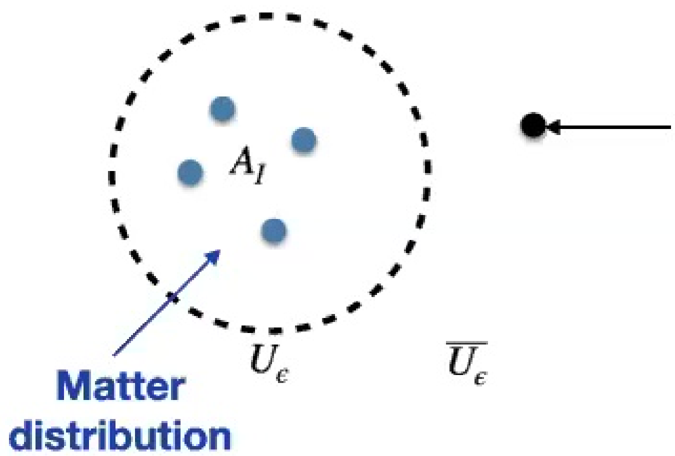
**Likewise** $\langle\hat{\Psi}_I|h_{\mu\nu}(\bar{x}_1)\cdots h_{\mu\nu}(\bar{x}_n)|\hat{\Psi}_J\rangle$

**Depends on:** **1) Matrix elements of products of $P_\mu$, $M_{\mu\nu}$**

**2) The chosen "standard" dressing $\tilde{h}_{\mu\nu}$**

**(to leading order in $\kappa$)**

## I.e. localization of information/definition of subsystem in quantum gravity:




Measurements outside $U_\epsilon$ only depend on total Poincare charges and choice of standard dressing

Don't register other features of matter distribution

(Note includes measurement of soft charges, so no extra information there)

$$|\widehat{\Psi}_I\rangle = \widehat{A}_I|U_\epsilon\rangle$$

**Corresponding mathematical structure?** [1903.06160]

$$\sim \bigoplus_{P_\mu,s,s_z} \mathscr{H}_{U,P_\mu,s,s_z} \otimes \mathscr{H}_{\overline{U}_\epsilon,P_\mu,s,s_z} \hookrightarrow \mathscr{H}$$

**"Gravitational splitting"**

**More generally: suggests *network* of Hilbert space embeddings, *replacing manifold structure of QFT***

**"Quantum spacetime"**

***Basic underlying object is the Hilbert space***

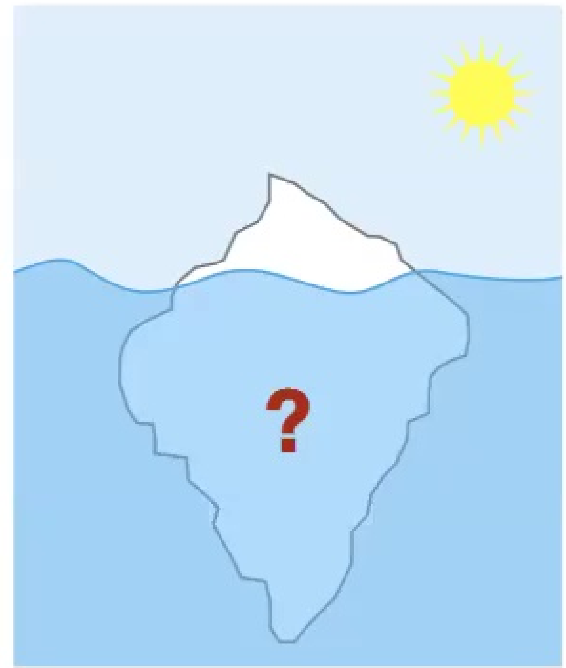
**(At minimum, QG needs to reproduce this structure in the weak limit)**

## Questions

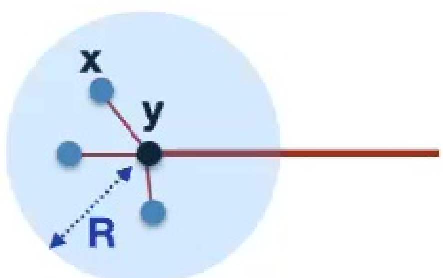
**Is there a simpler way of describing this structure?**

**Higher-order extension?**

**What does nonperturbative extension look like?**




**strong field behavior?**




$U_\epsilon$

**E.g.** $R_s(\langle E \rangle) > R$ **: strong field extends outside;**

**Not all naive QFT states allowed** **(area vs. energy, not information)**

**Related restriction on inclusion maps**

**Smallest Hilbert space?** $R \gtrsim \kappa$ **(some more discussion: 1803.04973)**

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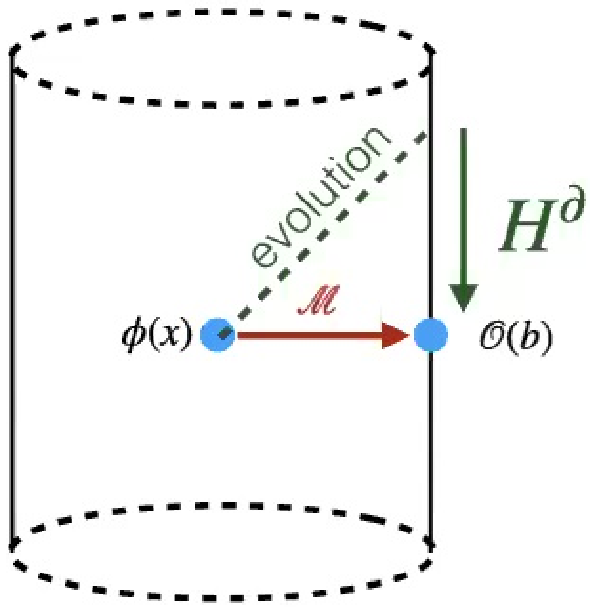
## Connection to holography

[1802.01602 w/ Kinsella, 2004.07843]

**AdS:**




**Key question for AdS/CFT:**

**What is this map $\mathscr{M}$?**

**Best candidate explanation:**

**Marolf [0808.2842, 1308.1977] (+ Jacobson 1212.6944, ...):**

***From gravitational constraints.***

**In gravity, H is a surface term:** $H^\partial$

***if the constraints are solved***

$$C_\mu(x) = G_{0\mu}(x) - 8\pi G\, T_{0\mu}(x) = 0$$

**Recall dressing** $\longleftrightarrow$ $[C_\mu(x), \Phi(y)] = 0$

**Large translation: need all orders in $\kappa$**

$\longleftrightarrow$ **nonperturbative bulk evolution**

**If have, possibly can construct $\mathscr{M}$ ?**

**Though, questions remain ...**

## *Conclusion/summary:*

**Trying to quantize geometry has lead to multiple difficulties**

**Nonrenormalizability; Nonunitarity**

**Instead, begin with QM; ask what mathematical structure needed to approximately recover LQFT+GR**

**"Quantum-first" approach**

**"Einstein separability:" need math. implementation of subsystem structure**

**~ quantum replacement for spacetime** **"Gravitational substrate" on $\mathscr{H}$**

**There are important constraints on this already seen in perturbative gravity!**

***Correspondence***

**Not tensor factorization**

**Not local subalgebras**

**Not "noncommutative geometry"**

**Structure ~ network of Hilbert space inclusions?**

**These are only first steps; more needed to formulate physical theory**

**More complete structure, evolution law, ... (unitarity —> observational signatures?)**

[1905.08807]

So, *how do we localize information in gravity?*

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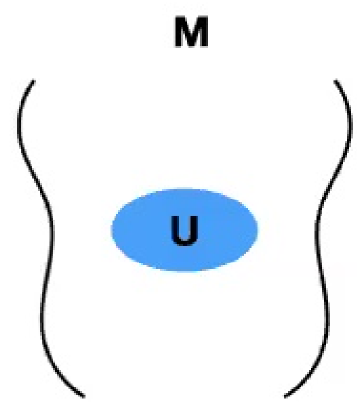
## 2. Quantum subsystems for local QFT:

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