Title: The Quantum Dynamics of Causal Sets: Directions and Challenges

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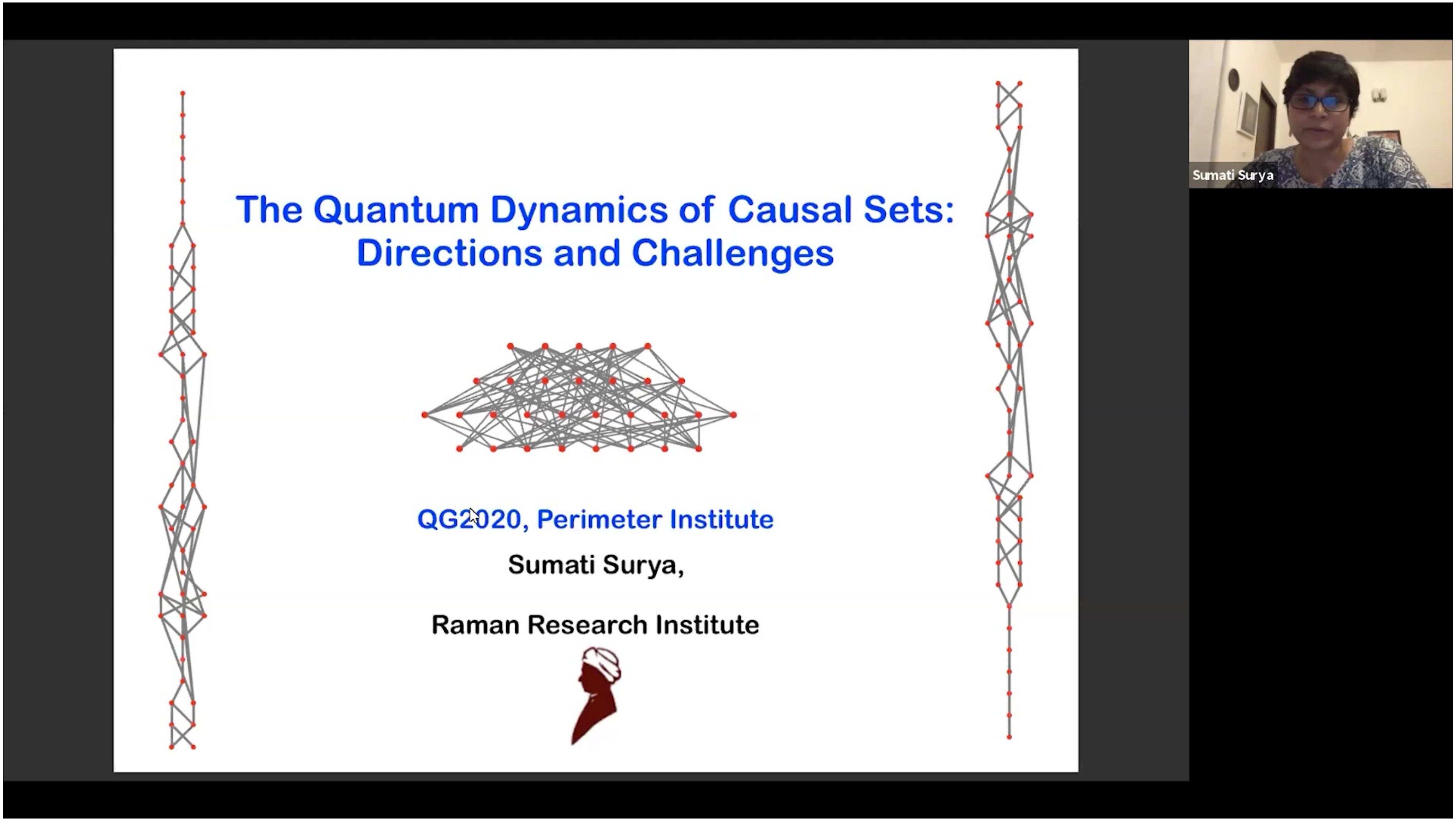
Abstract: I will begin with a short review of causal set theory (CST) focusing on the features that distinguish it from other approaches to quantum gravity. Most striking is a characteristic non-locality due to the Lorentz non-violating Poisson-discreteness in the continuum approximation. The discrete causal structure is rich enough, however, to extract local continuum geometric information, with geometric and topological observables corresponding to order-invariants in the causal set.

These observables provide the necessary equipment in our search for a suitable quantum dynamics of causal sets. I will focus the rest of the talk on recent progress on this journey and the choices and challenges that lie ahead.


The Quantum Dynamics of Causal Sets:
Directions and Challenges
QG2020, Perimeter Institute
Sumati Surya,
Raman Research Institute
Sumati Surya

Outline

- Quantum Gravity: The Choices we make…
- A Brief Tour of Causal Set Theory: Key Features
- Continuum Approximation and Covariant Observables
- Towards a Quantum Dynamics of Causal Sets
  - Lorentzian Statistical Geometry
  - Evaluating the Lorentzian Path Integral
  - Quantum Growth Models

Cunningham, Dowker, Eichhorn, Glaser, Jubb, Mathur, Rideout, AA Singh, Sorkin, Versteegen, Nomann X, Yazdı, Zalel

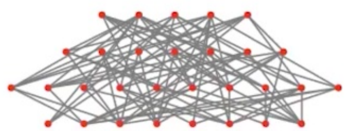
Quantum Gravity: The Choices we make…

Kinematics

g_{ab}, e_a^A ,Triangulations, NC geometry, strings, causal structure ,Tensor Networks….

Dynamics

- *Lorentzian* or *Euclidean*
- *Hamiltonian* or *Path Integral*
- S_{EH} *is fundamental* or *emergent*
- *Quantum Interpretation: Many Worlds, Bohmian, Quantum Measure Theory*



The CST Paradigm

— Bombelli, Lee, Meyer and Sorkin, 1987

- **Spacetime Continuum → locally finite poset**
  - **Acyclic:** $x \prec y, y \prec x \Rightarrow x = y$
  - **Transitive:** $x \prec y, y \prec z \Rightarrow x \prec z$
  - **Local Finiteness:** $|\mathrm{Fut}(x) \cap \mathrm{Past}(y)| < \infty$

- **Continuum Approximation** $C \sim (M, g)$
  - **Order ↔ Causal Structure**
  - **Number ↔ Volume**

- **HKMM theorem:** $(M, \prec) + \mathrm{vol\ element} = (M, g)$

  — Hawking, King, McCarthy, 1976

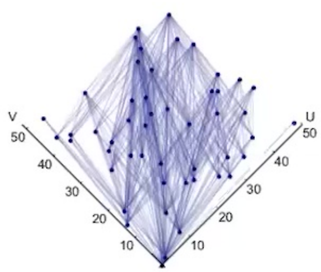
  — Malament, 1977

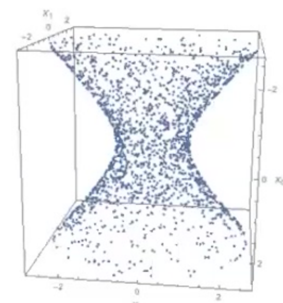
Order + Number ~ Spacetime

- **Poisson Sprinkling**

$$P_V(n) = \frac{(\rho V)^n}{n!} e^{-\rho V}$$

$$\langle N \rangle = \rho V, \qquad \Delta N = \sqrt{\rho V}$$

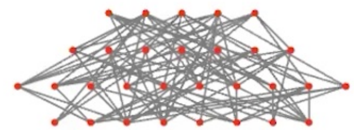




Terminology: Discrete or Continuous ?

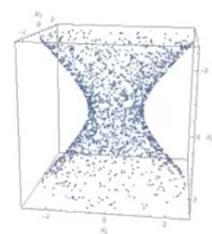
- Mathematics: Discrete ~ Discrete topology

  - Causal sets are not discrete



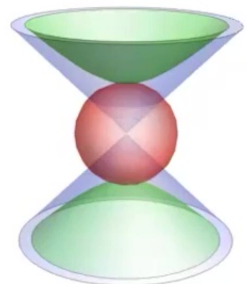
- Physics: Discrete ~ Locally Finite or Countable

  - Causal sets are discrete



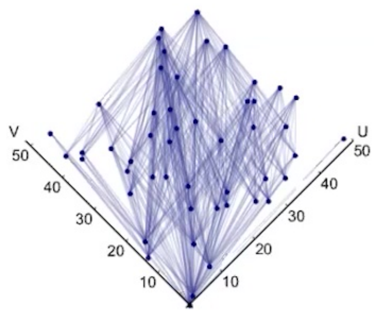
Discreteness ⇒ Lorentz invariance ⇒ Non-locality

- **Poisson preserves Lorentz invariance**
  — *Bombelli, Henson and Sorkin, 2009*



- **Non-locality:**

Graph with non-fixed valency

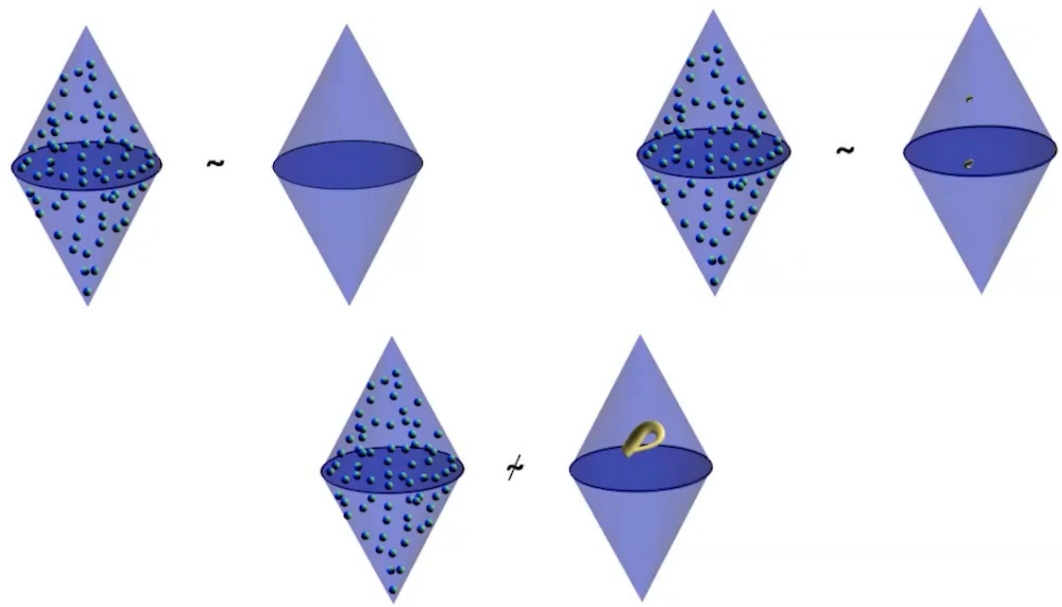


Integral operators

"Cauchy sieves" and not hypersurfaces

The Causal Set Hauptvermutung

$$C \approx (M_1, g_1),\ C \approx (M_2, g_2) \ \Rightarrow \ (M_1, g_1) \sim (M_2, g_2)$$



Lorentzian Gromov Hausdorff Distance

— Bombelli & Noldus 2004

Geometric Reconstruction

Order + Number ~ Spacetime

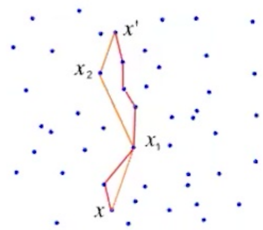
- **Flat Spacetime Dimension**
  - — *Myrrheim, 1978*
  - — *Meyer, 1988*
  - — *Glaser & Surya, 2013*
- **Timelike, Spatial and Spacelike Distance**
  - — *Brightwell & Gregory, 1991*
  - — *Rideout & Wallden, 2009*
  - — *Eichhorn, Surya & Versteegen, 2018*
- **Spatial Homology**
  - — *Major, Rideout & Surya, 2006,2008*
- **Benincasa Dowker Action**
  - — *Benincasa & Dowker, 2010*
  - — *Dowker & Glaser, 2013*
- **Boundary Terms**
  - — *Benincasa, Dowker & Schnitzer, 2011*
  - — *Buck, Dowker, Jubb & Surya, 2015*
  - — *Cunningham, 2018*
- **Scalar Fields : D'Alembertian, Green Functions, SJ Vacuum**
  - — *Benincasa & Dowker, 2010*
  - — *Johnston, 2008,2009*
  - — *Sorkin, 2011*

Order Invariants ~ Covariant Observables

Examples

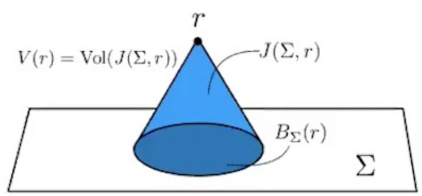
- **Time-like distance**

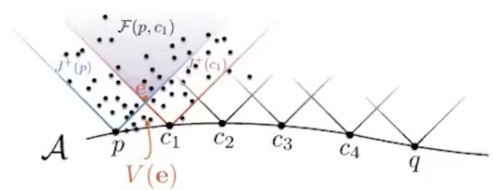
$\tau(p,q) \propto$ **length of longest chain**



— Brightwell & Gregory, 1991
— Myrrheim. 1978
— Meyer, 1988
— Roy, Sinha & Surya, 2013
— Nomaan & Kambor, 2020

- **Spatial distance**





$$d(p,q) = 2\left(\frac{V(e)}{\zeta_{\mathrm{dim}}}\right)^{\frac{1}{\mathrm{dim}}}$$

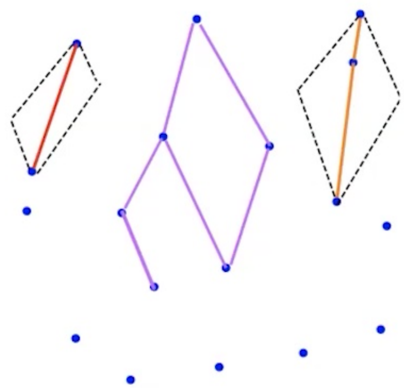
$$D(p,q) = \inf_\gamma d_\gamma(p,q)$$

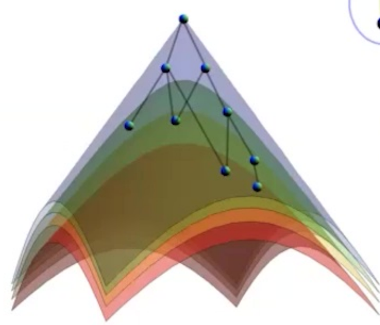
—Eichhorn, Mizera & Surya, 2016
—Eichhorn, Surya & Versteegen, 2018, 2019

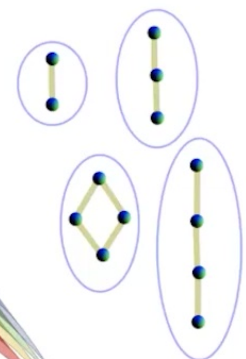
The Benincasa-Dowker Action

— *Benincasa & Dowker, 2010*
— *Dowker & Glaser, 2013*

$$\frac{1}{\hbar}S_{BD} = \frac{4}{\sqrt{6}}\left(N - N_0 + 9N_1 - 16N_2 + 8N_3\right)$$







$$\lim_{\rho_c \to \infty} \hbar \frac{l_c^2}{l_p^2}\langle S_{BD}\rangle = S_{EH} + \text{bdry terms}$$

The 'Real World'

- Sorkin's Prediction for $\Delta\Lambda = \frac{1}{\sqrt{V}} \simeq H^2 \simeq 10^{-120}\, l_p^{-2}$

  — Sorkin, 1987,1991
  — Ahmed, Dodelson, Greene & Sorkin, 2004

- Is the cosmological constant $\Lambda(z)$ a constant?

  —Afshordi, Sorkin & Zwane, 2018

- Cumulative/Secular effects of Discreteness: Swerves

  —Dowker, Henson & Sorkin, 2004

- Non-local Field Theory

  —Belenchia, Benincasa, Liberati, 2015,2016

- Studies on the SJ vacuum

  —Afshordi, Aslanbeigi & Sorkin, 2012
  —Aslanbeigi & Buck, 2013,
  —Afshordi, Buck, Dowker, Rideout, Saravani, Sorkin & Yazdi, 2012
  —Fewster & Verch, 2012,
  —Brum & Fredanhagen, 2013,
  — …..

Rich Phenomenology

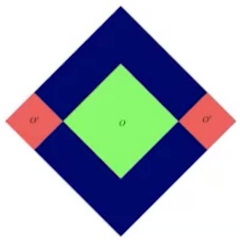
Sorkin's Spacetime Entanglement Entropy

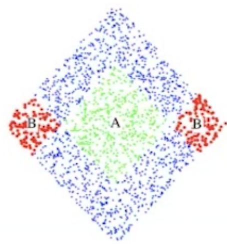
$$S = \sum_{\mu} \mu \ln|\mu|, \quad W_O(x,x')v = i\mu\Delta_O(x,x')v, \quad \Delta_O v \neq 0$$

— Sorkin 2014

Causal Diamonds in d=2

—Saravani Sorkin & Yazdı, 2014



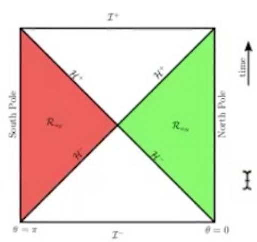


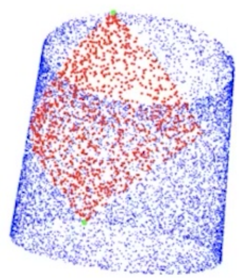
Area Law: $S \approx \frac{1}{3} \ln \frac{\sqrt{N_l}}{4\pi}$

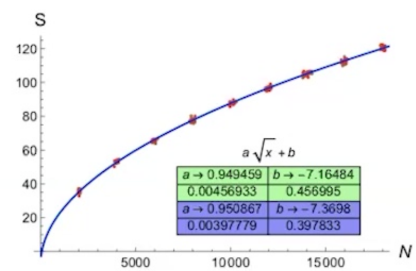
— Sorkin & Yazdı, 2018

Area Law: $S \approx aA$

dS in d=4







— Surya,Yazdı, Nomaan X, in preparation

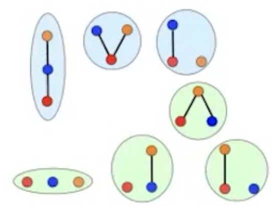
Quantum Dynamics For Causal Sets

Continuum Inspired

Partition Function

$$Z = \sum_{c \in \Omega} e^{i\frac{S(c)}{\hbar}}$$

Sequential Growth



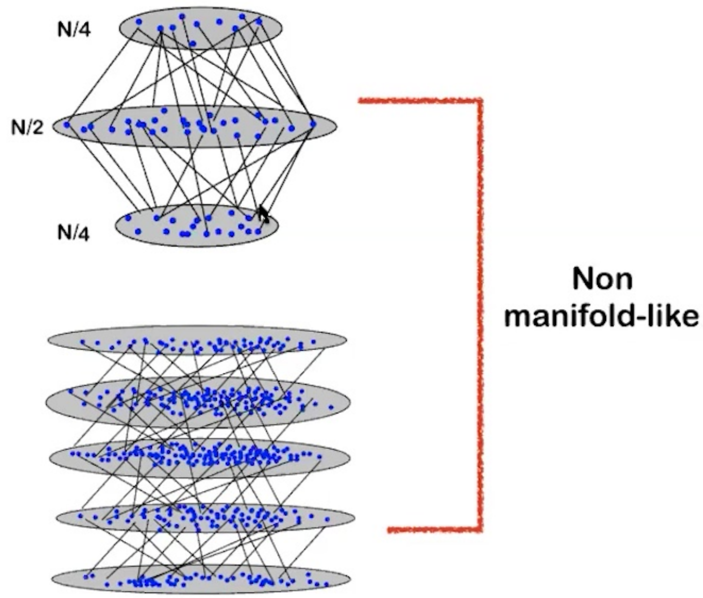
Quantum Measure

$$|\alpha\rangle \in \mathscr{H}, \quad \langle \alpha | \alpha \rangle$$

The Sample Space Ω_N

$|\Omega_N| \sim 2^{\frac{N^2}{4}+o(N^2)}$ **is dominated by KR posets**

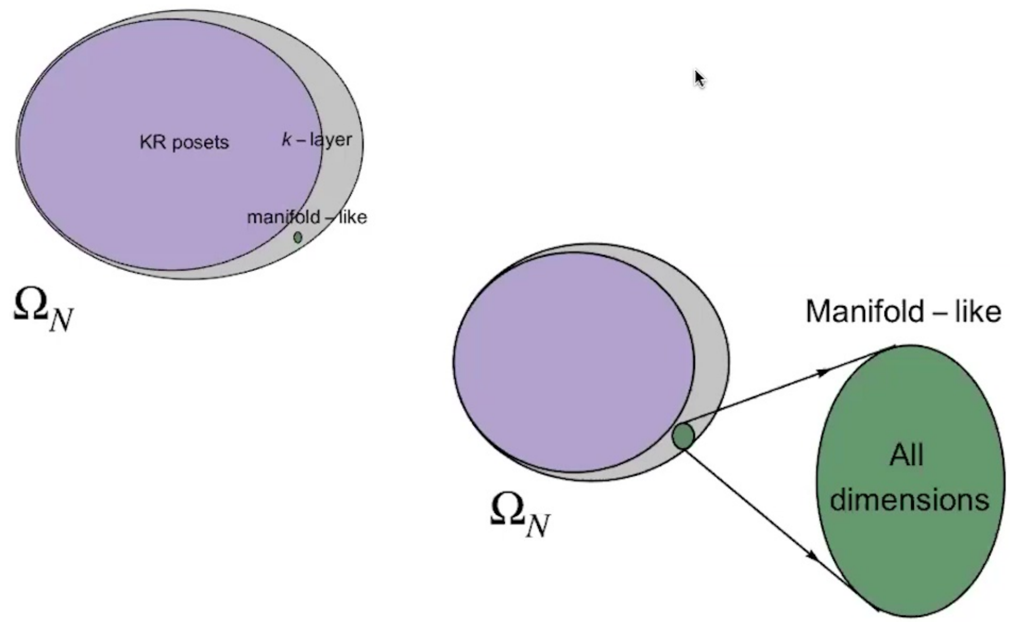
— Kleitmann and Rothschild, 1975



Layered posets are subdominant

— Dhar, 1978, 1980

The Sample Space Ω_N



Lorentzian Statistical Geometry

$$Z \equiv \sum_{c\in\Omega} e^{\frac{i\beta}{\hbar}S(c)} \rightarrow Z_\beta \equiv \sum_{c\in\Omega} e^{-\frac{\beta}{\hbar}S(c)}$$ → **MCMC methods**

Ω_N :

- Using Link and Relational move at $\beta = 0$

  — Henson, Rideout, Surya & Sorkin, 2017

- Directed Acyclic Graphs (DAGS) at $\beta = 0$

  — Cunningham & Surya (ongoing)

$$\Omega_N \rightarrow \Omega_N^{DAG} \supset \Omega_N$$

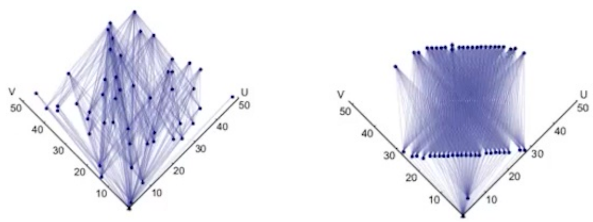
Non-locality is a challenge for thermalisation

Considerable Speed up with DAGs

Dimensional Restriction of Ω

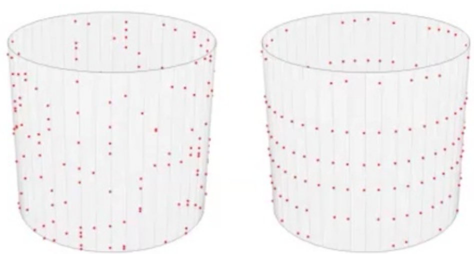
$$Z_\beta \equiv \sum_{c \in \Omega_d} e^{-\frac{\beta}{\hbar} S_d(c)}$$

Causal Diamond
$I^+(x) \cap I^-(y)$

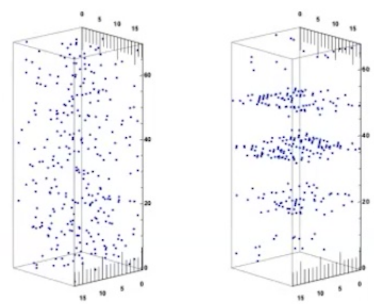


— *Surya, 2012*
— *Glaser & Surya, 2016*
— *Glaser, O'Connor and Surya, 2017*
— *Glaser, 2018*

$S^1 \times \mathbb{R}$



$T^2 \times \mathbb{R}$



— *Cunningham & Surya, 2019*

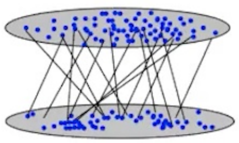
Suppressions in the Lorentzian Path Integral

—Carlip & Loomis,2017

$$Z = \sum_{c\in\Omega} e^{i\frac{S(c)}{\hbar}} = \sum_{c\in A} e^{i\frac{S(c)}{\hbar}} + \sum_{c\in B} e^{i\frac{S(c)}{\hbar}}$$

Is this suppressed?

Bilayer Posets



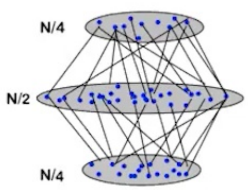
$$\frac{1}{\hbar}S_{BD}(c) = \mu\Big(N + \sum_{j=0}^{j_{max}} \lambda_j N_j\Big) = \mu(N + \lambda_0 N_0)$$

$$Z_{bilayer}[\mu, \lambda_0] \sim \int_0^{1/2} \mathrm{d}p\, |\mathscr{C}_{p,N}| \exp(iS_l(p))$$

Suppression for: $\tan(-\frac{\mu\lambda_0}{2}) > \sqrt{3} \quad \Rightarrow \quad d = 4 : l \geq 1.452 l_p$

$d = 2 :$ **No suppression**

KR Posets



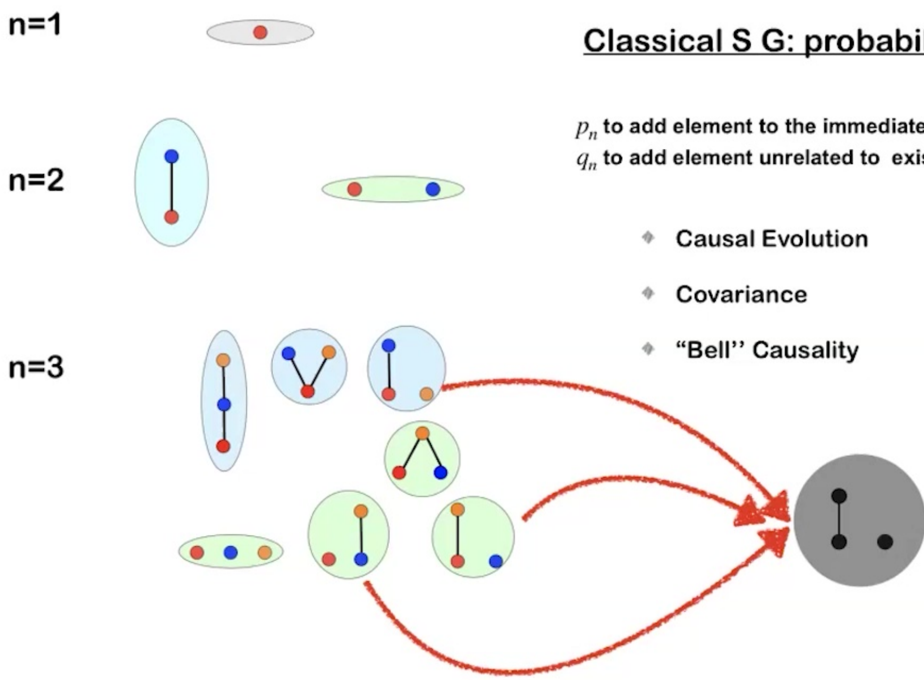
$$\frac{1}{\hbar}S_L(c) = \mu(N + \lambda_0 N_0)$$

KR posets are also suppressed!

—Mathur, Singh & Surya, in prep

Sequential Growth

—Rideout & Sorkin

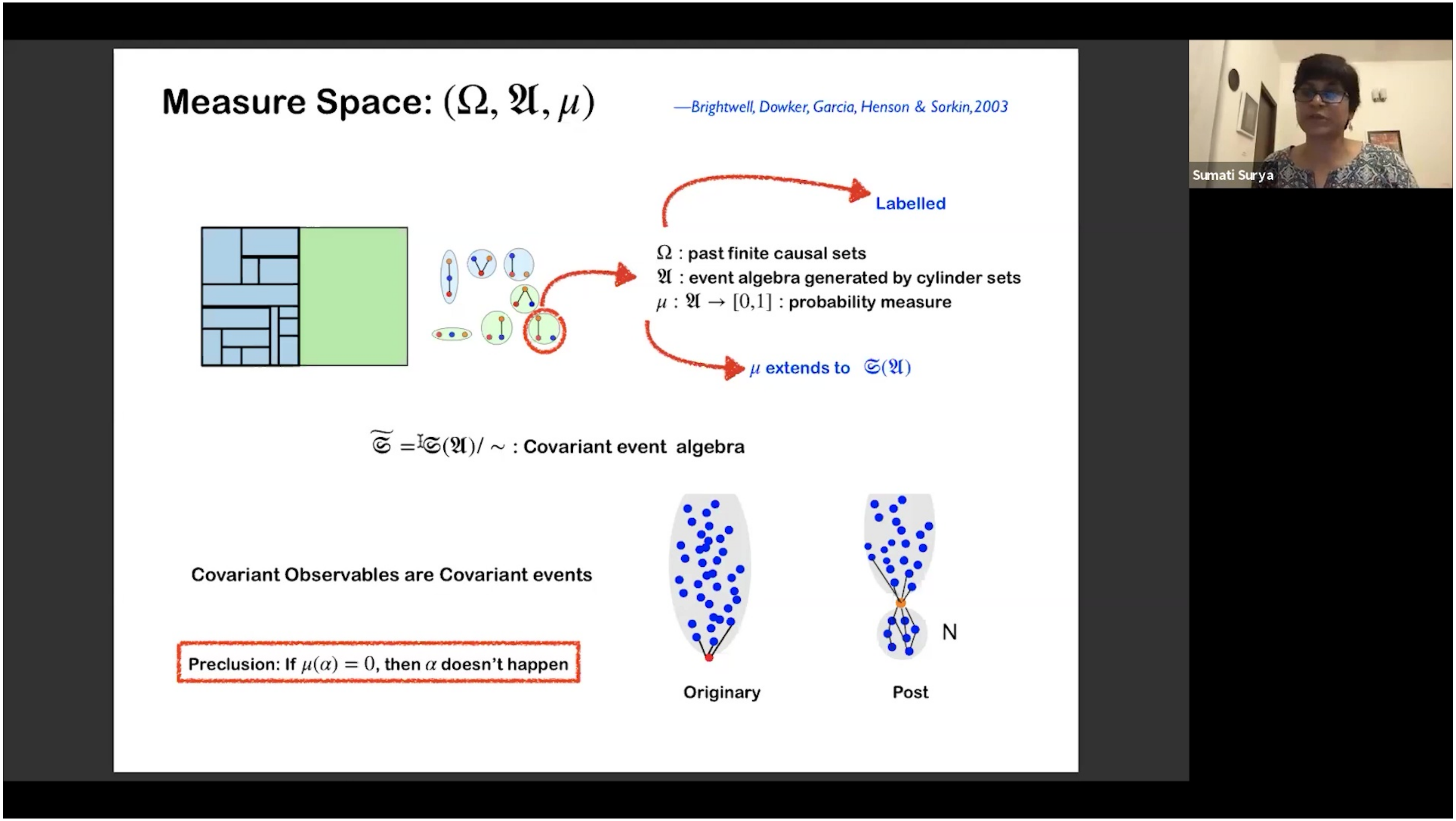


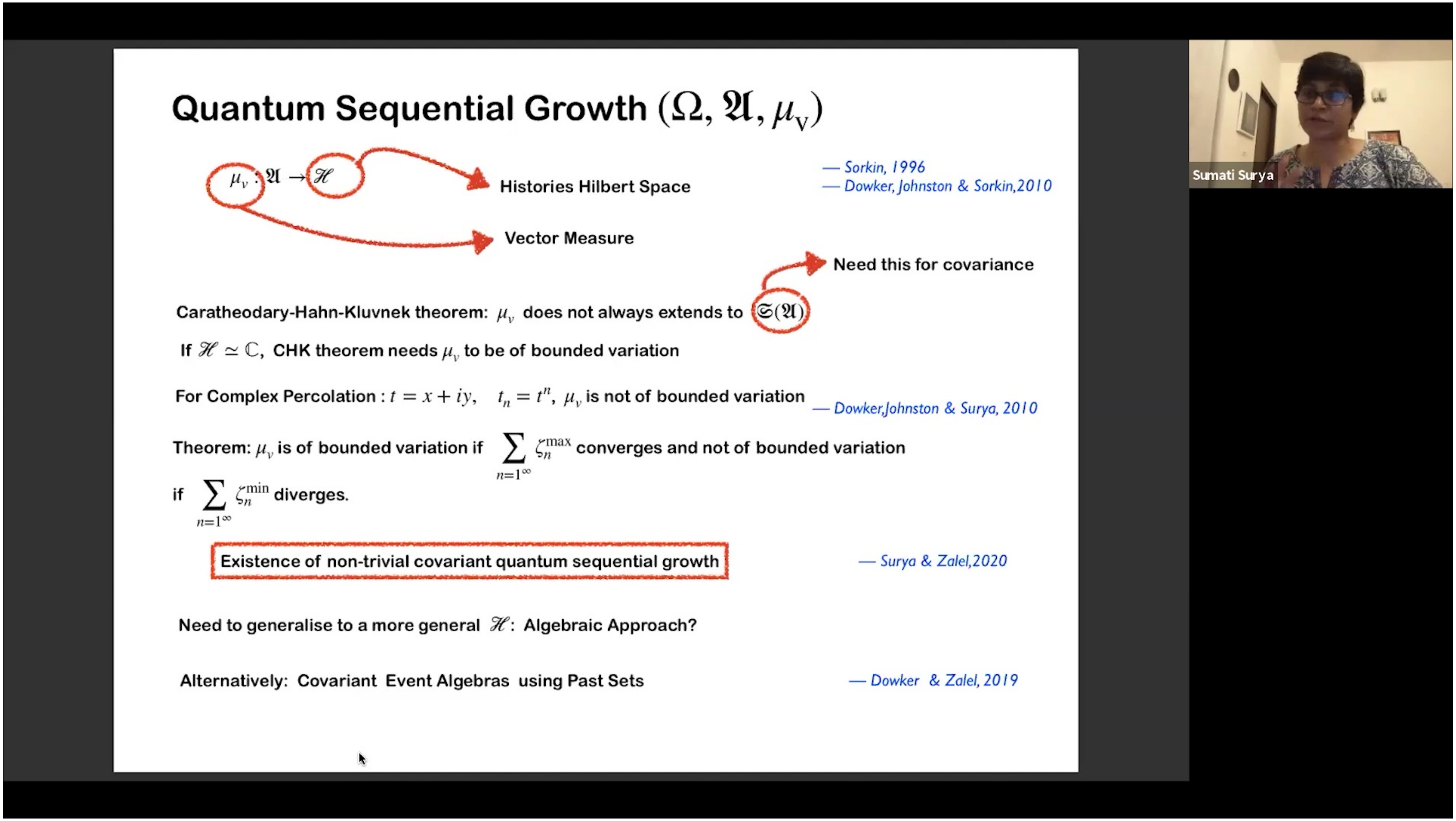
Classical S G: probabilities

p_n to add element to the immediate future of an element
q_n to add element unrelated to existing element

- Causal Evolution
- Covariance $\Rightarrow \{t_1, t_2, \ldots\}$
- "Bell'' Causality

Transitive Percolation $p \in [0,1]$ beats KR entropy!


Measure Space: (Ω, 𝔄, μ)
—Brightwell, Dowker, Garcia, Henson & Sorkin, 2003
Labelled
Ω : past finite causal sets
𝔄 : event algebra generated by cylinder sets
μ : 𝔄 → [0,1] : probability measure
μ extends to 𝔖(𝔄)
𝔖̃ = 𝔖(𝔄)/ ∼ : Covariant event algebra
Covariant Observables are Covariant events
Preclusion: If μ(α) = 0, then α doesn't happen
N
Originary
Post
Sumati Surya


Quantum Sequential Growth $(\Omega, \mathfrak{A}, \mu_v)$
$\mu_v : \mathfrak{A} \to \mathscr{H}$
Histories Hilbert Space
Vector Measure
— Sorkin, 1996
— Dowker, Johnston & Sorkin,2010
Need this for covariance
Caratheodary-Hahn-Kluvnek theorem: μ_v does not always extends to $\mathfrak{S}(\mathfrak{A})$
If $\mathscr{H} \simeq \mathbb{C}$, CHK theorem needs μ_v to be of bounded variation
For Complex Percolation : $t = x + iy$, $t_n = t^n$, μ_v is not of bounded variation
— Dowker,Johnston & Surya, 2010
Theorem: μ_v is of bounded variation if $\sum_{n=1^\infty} \zeta_n^{\max}$ converges and not of bounded variation
if $\sum_{n=1^\infty} \zeta_n^{\min}$ diverges.
Existence of non-trivial covariant quantum sequential growth
— Surya & Zalel,2020
Need to generalise to a more general $\mathscr{H}$: Algebraic Approach?
Alternatively: Covariant Event Algebras using Past Sets
— Dowker & Zalel, 2019
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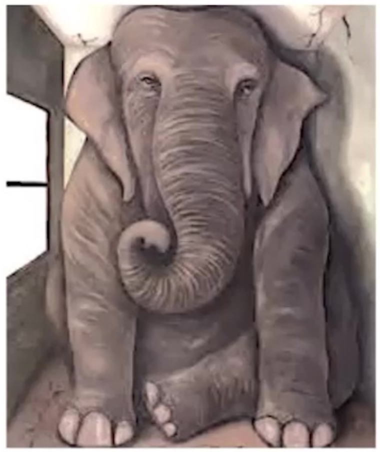
Quantum Interpretation??



Quantum Measure Theory

Principle of Preclusion

Directions & Challenges..

- Library of Order Invariants which have a geometric interpretation
- QFT on causal sets with a Lorentz invariant UV cut-off
- Different approaches to dynamics..
- RG for causal sets and effective actions