Title: Asymptotically Safe Amplitudes from the Quantum Effective Action

Speakers: Frank Saueressig

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Abstract: This talk will feature a brief introduction to the gravitational asymptotic safety program before reviewing the current status of the field. Motivated by recent developments, I will introduce the form factor formulation of the quantum effective action and explain how various quantum gravity programs can be embedded into this framework. Finally, I will discuss a novel gravity-matter model whose scattering amplitudes exhibit all features expected from Asymptotic Safety.

Asymptotically Safe Amplitudes from the Quantum Effective Action

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IN·DEI·NOMINE·FELICITER

NWO
Netherlands Organisation for Scientific Research

T. Draper, B. Knorr, C. Ripken and F.S., arXiv:2007.00733

T. Draper, B. Knorr, C. Ripken and F.S., arXiv:2007.04396

Quantum Gravity 2020, Perimeter Institute, July 14$^{\mathrm{th}}$, 2020

Acknowledgments

J.F. Donoghue, A Critique of the Asymptotic Safety Program, arXiv:1911.02967

- Is the running of G, Λ in conflict with EFT scattering amplitudes?
- Is there a Lorentzian version of Asymptotic Safety?

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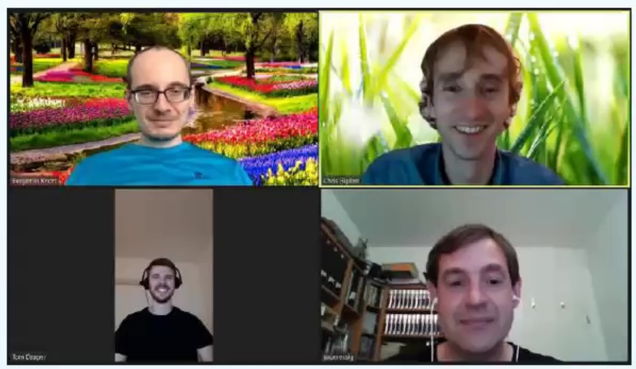
- Is there a Lorentzian version of Asymptotic Safety?

Bonanno et. al., Critical reflections on asymptotically safe gravity, arXiv:2004.06810

- surveys open questions of both technical and conceptual nature

The "Tuesday Night Club" collaboration



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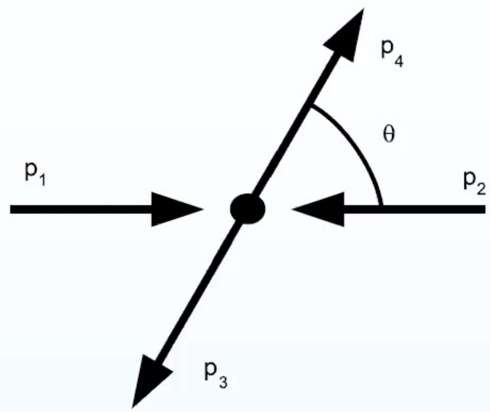
Outline

- Setting the scene: scattering amplitudes in General Relativity
- Asymptotic Safety: introduction and recent developments
- Form Factors in Quantum Gravity
  - Parameterizing the quantum effective action
  - Form Factors for Lorentzian Asymptotic Safety
- Conclusions

scattering of particles



quantum mechanics: amplitude $\mathcal{A}$ relating in- and out-states:

- parameterized by Mandelstam variables

$$\mathfrak{s} = (p_1 + p_2)^2 \qquad \mathfrak{t} = (p_1 + p_3)^2 \qquad \mathfrak{u} = (p_1 + p_4)^2$$

  - $\mathfrak{s}$ = center-of-mass energy
  - massless scalar fields: $\mathfrak{t} = -\mathfrak{s}(1 + \cos\theta)/2 \qquad \mathfrak{u} = -\mathfrak{s}(1 - \cos\theta)/2$

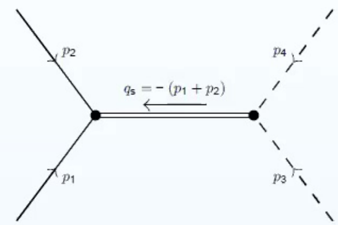
- $\mathcal{A}$ computed via Feynman diagrams

Gravity-mediated particle scattering in general relativity

Starting point: Einstein-Hilbert action with minimally coupled scalar fields:

$$\Gamma = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R + \frac{1}{2} \int d^4x \sqrt{-g} \left[\phi \Delta \phi + \chi \Delta \chi \right]$$

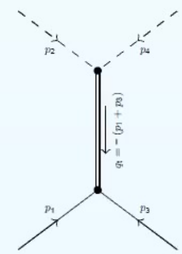
1. amplitude for $\phi\phi \to \chi\chi$-scattering



$$\mathcal{A}_{\mathfrak{s}} = 8\pi G_N \frac{\mathfrak{t}\,\mathfrak{u}}{\mathfrak{s}} \sim G_N\, \mathfrak{s}$$

- grows with the center-of-mass energy $\mathfrak{s}$!

2. amplitude for $\phi\chi \to \chi\phi$-scattering



$$\mathcal{A}_{\mathfrak{t}} = 8\pi G_N \frac{\mathfrak{s}\,\mathfrak{u}}{\mathfrak{t}} \sim G_N \frac{\mathfrak{s}^2}{\mathfrak{t}}$$

- quadratic divergence in the forward-scattering limit

perturbative quantization of general relativity: conclusions

a) treat gravity as effective field theory [J. Donoghue, gr-qc/9405057]

- compute corrections in $s\, G_N \ll 1$ (independent of UV-completion)
- predictivity breaks down at $sG_N \gtrsim 1$

b) quantizing gravity may require new physics concepts:

- String Theory (Veneziano and Virasoro-Shapiro-amplitudes)
- Loop Quantum Gravity

c) gravity makes sense as quantum field theory:

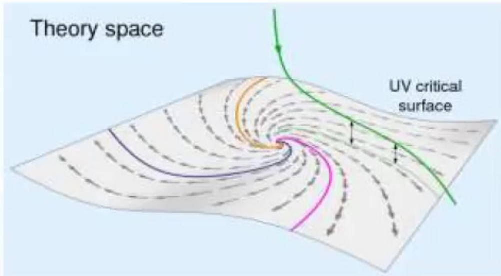
- Asymptotic Safety
- Causal Dynamical Triangulations
- Infinite Derivative Gravity
- . . .

Asymptotic Safety

introduction and recent developments

Asymptotic Safety: key idea

high-energy completion of gravity by Reuter fixed point



- 2 classes of renormalization group trajectories:
  - relevant = end at fixed point in UV
  - irrelevant = go somewhere else...
- predictive power:
  - hitting the fixed point at high energy $\Longleftrightarrow$ relations between couplings

The Asymptotic Safety package

Reuter fixed point

- ensures the absence of UV-divergences

fixed point has finite-dimensional UV-critical surface (predictivity)

- select a RG trajectory with a finite number of measurements

RG flow ends with a effective action compatible with observations (falsifiability)

- tests of general relativity
  - solar system tests, cosmological signatures, gravitational waves, . . .
- compatibility with standard model of particle physics at 1 TeV

structural demands

- resolution of spacetime singularities
- unitarity

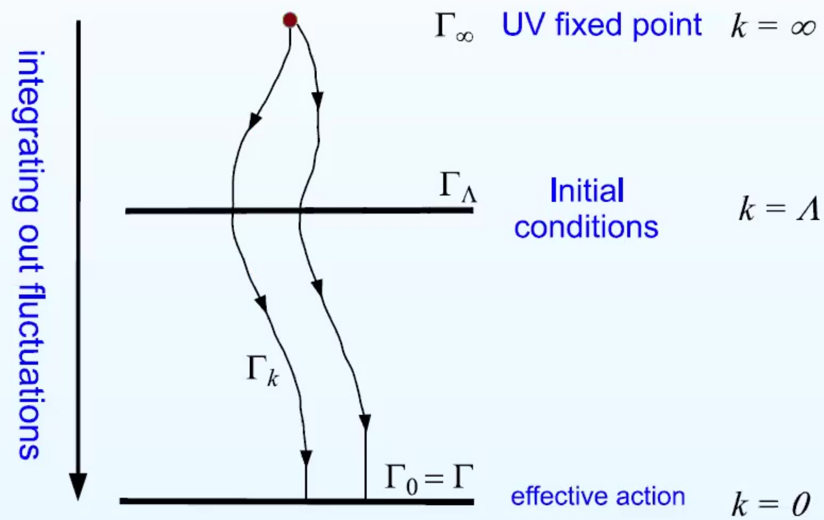
Exploring Asymptotic Safety via Functional Renormalization

M. Reuter, Phys. Rev. D **57** (1998) 971

idea: integrate out quantum fluctuations shell-by-shell in momentum-space

implementation: Wetterich equation for effective average action Γ_k:

$$k\partial_k \Gamma_k[h_{\mu\nu}, \bar{g}_{\mu\nu}] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$



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conceptual remarks

1. fixed points are a property of the solutions of the Wetterich equation
2. physics is in the effective action $\Gamma \equiv \Gamma_{k=0}$
3. the fixed point relates couplings in the effective action

Solving the Wetterich equation: derivative expansion of $\Gamma_k^{\mathrm{grav}}[g]$

Einstein-Hilbert truncation

$\vdots$	$\vdots$		
R^8	$C_{\mu\nu\rho\sigma}\Delta^6 C^{\mu\nu\rho\sigma}$		
R^7	$C_{\mu\nu\rho\sigma}\Delta^5 C^{\mu\nu\rho\sigma}$		
R^6	$C_{\mu\nu\rho\sigma}\Delta^4 C^{\mu\nu\rho\sigma}$		
R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$		
R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$		
R^3	$C_{\mu\nu\rho\sigma}\Delta C^{\mu\nu\rho\sigma}$	$R\,\Delta R$	+ 5 more
R^2	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$	
R			
$\mathbb{1}$			

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Asymptotic Safety: selected results

pure gravity:

- non-Gaussian fixed point persists once 2-loop counterterm is included

  [H. Gies, B. Knorr, S. Lipholdt, and F. Saueressig, arXiv:1601.01800]

- low number of free parameters ($\simeq 3$)

  [R. Percacci and A. Codello, arXiv:0705.1769]
  [D. Benedetti, P.F. Machado and F. Saueressig, arXiv:0901.2984]
  [K. Falls, D. F. Litim, . . ., arXiv:1801.00162; arXiv:1810.08550]

- non-Gaussian fixed point in $d = 2$ is a unitary CFT

  [A. Nink and M. Reuter, arXiv:1512.06805]

- cosmology (Starobinsky inflation) may fix all free parameters

  [G. Gubitosi, R. Ooijer, C. Ripken, F. Saueressig, arXiv:1806.10147]

gravity coupled to matter:

- non-Gaussian fixed point compatible with standard-model matter

  [P. Dona, A. Eichhorn and R. Percacci, arXiv:1311.2898]
  [J. Meibohm, J. M. Pawlowski and M. Reichert, arXiv:1510.07018]

- asymptotically safety could predict parameters of the standard model

  [M. Shaposhnikov and C. Wetterich, arXiv:0912.0208]
  [A. Eichhorn, A. Held, arXiv:1803.04027]
  [A. Eichhorn, arXiv:1810.07615]

Open Questions

- number of free parameters?
- low-energy physics compatible with observations?
- exact matter content supporting asymptotic safety?
- characterization of the quantum geometry?

[talk by R. Loll]

- degrees of freedom associated with the Reuter fixed point?
- unitarity?

investigation method

- derivative expansion of Γ_k
- derivative expansion fails by construction

Derivative expansion $\Longrightarrow$ curvature expansion of $\Gamma_k^{\mathrm{grav}}[g]$

$\vdots$

R^8 — Einstein-Hilbert truncation

R^7 — C^2-form factor

R^6

R^5

R^4

R^3 — $R\,\Delta R$ — + 5 more

R^2 — $C_{\mu\nu\rho\sigma} W[\Delta] C^{\mu\nu\rho\sigma}$ — $R_{\mu\nu} R^{\mu\nu}$

R

$\mathbb{1}$

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Open Questions

- number of free parameters?
- low-energy physics compatible with observations?
- exact matter content supporting asymptotic safety?
- characterization of the quantum geometry associated with NGFP?
- degrees of freedom associated with the NGFP?
- unitarity?

investigation method

- derivative expansion of Γ_k
- derivative expansion fails by construction

derivative expansion $\Longrightarrow$ curvature expansion keeping derivatives

Parameterizing the Quantum Effective action

building blocks for gravity-scalar theory:

$$\Gamma = \Gamma_{\text{grav}} + \Gamma_{\text{gf}} + \Gamma_{\text{matter}}$$

curvature expansion in the gravitational sector

$$\Gamma_{\text{grav}} = \frac{1}{16\pi G_N} \int \sqrt{-g} \left[-R - \frac{1}{6} R\, f_R(\Delta)\, R + \frac{1}{2} C_{\mu\nu\rho\sigma}\, f_C(\Delta)\, C^{\mu\nu\rho\sigma} + \cdots \right]$$

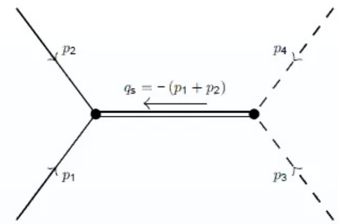
- form factors $f_R(\Delta)$ and $f_C(\Delta)$ fix flat space graviton propagators G_2, G_0

supplemented by two-parameter family of gauge-fixing conditions:

$$\Gamma_{\text{gf}} = \frac{1}{32\pi\alpha} \int \sqrt{-\eta} \left(\partial^\mu h_{\mu\nu} - \frac{1+\beta}{4} \partial_\nu h \right) \left(\partial_\rho h^{\rho\nu} - \frac{1+\beta}{4} \partial^\nu h \right) .$$

fluctuations around flat background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Gravity mediated scattering $\phi\phi \to \chi\chi$



on-shell amplitude

$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi G_N}{3}\Big[-\left(1 + \mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1 + \mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right)$$

$$\times G_2(\mathfrak{s}) \times \left(\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right)$$

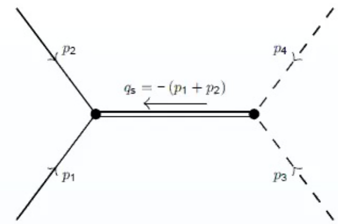
$$+ \left((\mathfrak{s} + 2m_\phi^2)(1 + \mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$

$$\times \left((\mathfrak{s} + 2m_\chi^2)(1 + \mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_0(\mathfrak{s})\Big]$$

properties:

- gauge invariant
- invariant under a momentum-dependent rescaling of the graviton

Gravity mediated scattering $\phi\phi \to \chi\chi$



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$$\times \left((\mathfrak{s} + 2m_\chi^2)(1 + \mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_0(\mathfrak{s}) \Big]$$

properties:

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Gravity mediated scattering $\phi\phi \to \chi\chi$: Stelle gravity

define partial wave amplitudes to separate angular dependence of $\mathcal{A}_{\mathfrak{s}}^{\phi\chi}$:

$$a_j^{\phi\chi}(\mathfrak{s}) \equiv \frac{1}{32\pi} \int_{-1}^{1} \mathrm{d}(\cos\theta)\, P_j(\cos\theta)\, \mathcal{A}_{\mathfrak{s}}^{\phi\chi}(\mathfrak{s}, \cos\theta)\,,$$

- $P_j(x)$: Legendre polynomial of order j

Example: classical Stelle gravity with minimally coupled scalars

$$\Gamma_{\mathrm{grav}}^{\mathrm{Stelle}} = \frac{1}{16\pi G_N} \int \sqrt{-g} \left[-R - \frac{1}{6} R\, f_R(\Delta)\, R + \frac{1}{2} C_{\mu\nu\rho\sigma}\, f_C(\Delta)\, C^{\mu\nu\rho\sigma} + \cdots \right]$$

- non-zero, constant form factors

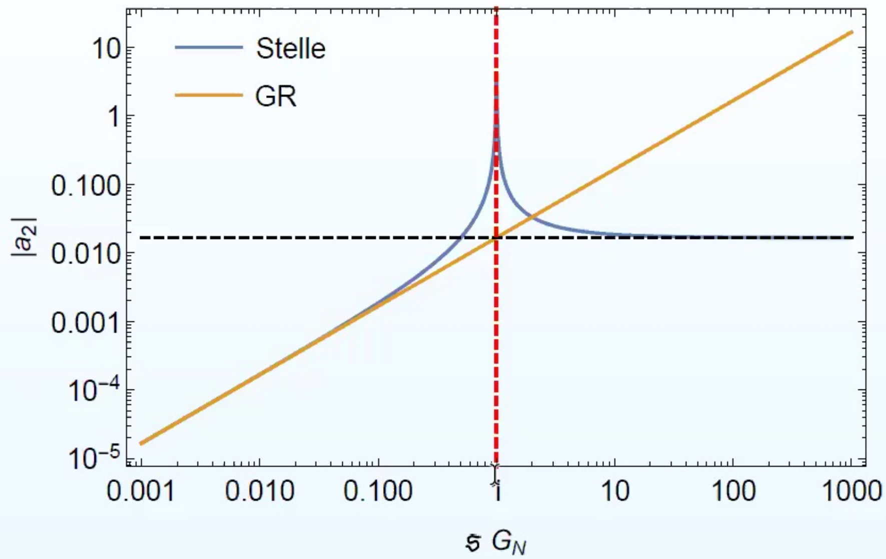
$$f_R = -c_R G_N\,, \qquad f_C = -c_C\, G_N$$

- partial wave amplitudes

$$a_0^{\text{Stelle}} = \frac{G_N}{12} \mathfrak{s}^2 \left[\frac{1}{\mathfrak{s}} - \frac{1}{\mathfrak{s} - (c_R G_N)^{-1}} \right]\,, \qquad a_2^{\text{Stelle}} = -\frac{G_N}{60} \mathfrak{s}^2 \left[\frac{1}{\mathfrak{s}} - \frac{1}{\mathfrak{s} - (c_C G_N)^{-1}} \right]$$

Partial-wave amplitude for classical Stelle gravity

$$a_2^{\text{Stelle}} = -\frac{G_N}{60}\mathfrak{s}^2\left[\frac{1}{\mathfrak{s}} - \frac{1}{\mathfrak{s}-(c_C G_N)^{-1}}\right]$$



growth of amplitude tamed by massive spin-two poltergeist

Form factors: a unifying perspective on quantum gravity

theory	gravity form factors	pole structure	UV behavior
effective field theory	$\log$	pole at cutoff scale	n/a
Stelle gravity	const	massive spin-two ghost d.o.f.	const scale-free
infinite derivative gravity	$\exp$	essential singularity at ∞	$\exp$-falloff $(\mathfrak{s})$, $\exp$-divergent $(\mathfrak{t}/\mathfrak{u})$

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powerful framework to study quantum gravity programs in a unifying way

Blueprint for Asymptotically Safe Amplitudes

requirements

- amplitudes become scale-free at high energy
  $\Longrightarrow$ unitarity, Froissart bound
- no new massive degrees of freedom (ghosts)
  $\Longrightarrow$ unitarity, causality
- locality at high energy

form factors for Lorentzian Asymptotic Safety

- graviton propagators

$$f_R(\mathfrak{s}) = c_R G_N \tanh(c_R G_N \mathfrak{s}) \qquad f_C(\mathfrak{s}) = c_R G_N \tanh(c_C G_N \mathfrak{s})$$

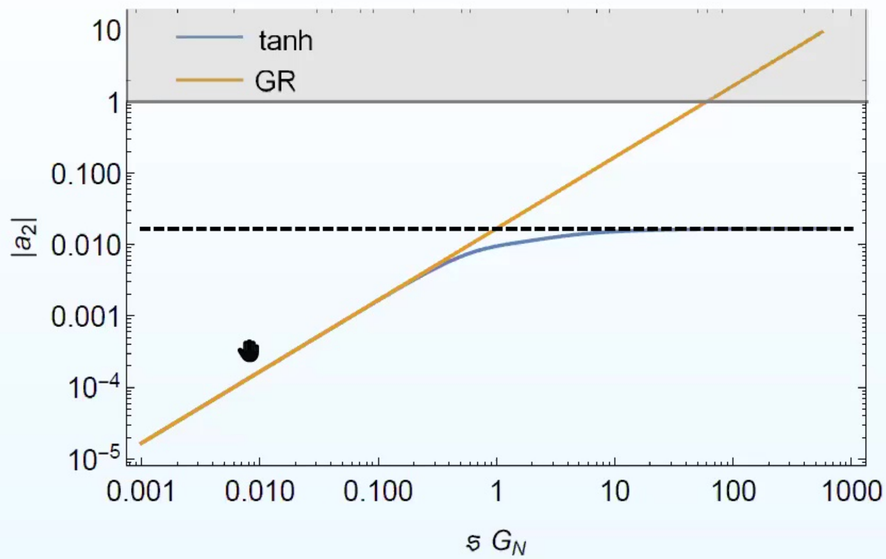
- scalar self-interactions $f_{\phi^2\chi^2}$ contributes

$$\mathcal{A}_4^{\phi\chi}(\mathfrak{s}|\mathfrak{t},\mathfrak{u}) = 4\pi G_N G_2(\mathfrak{s})(\mathfrak{t}^2+\mathfrak{u}^2) f^{\mathrm{int}}(\mathfrak{s}^2+\mathfrak{t}^2+\mathfrak{u}^2)$$

  interpolation function $f^{\mathrm{int}}(x)$ suppresses self-interaction at low energy

Partial-wave amplitude for Lorentzian Asymptotic Safety

$$a_2 = -\frac{G_N}{60} \frac{\mathfrak{s}}{1 + c_C\, G_N \mathfrak{s}\, \tanh(c_C G_N \mathfrak{s})}$$



amplitude becomes scale-free at large center-of-mass energy $G_N \mathfrak{s} \gg 1$

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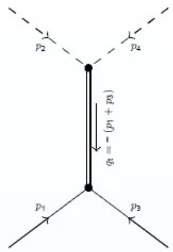
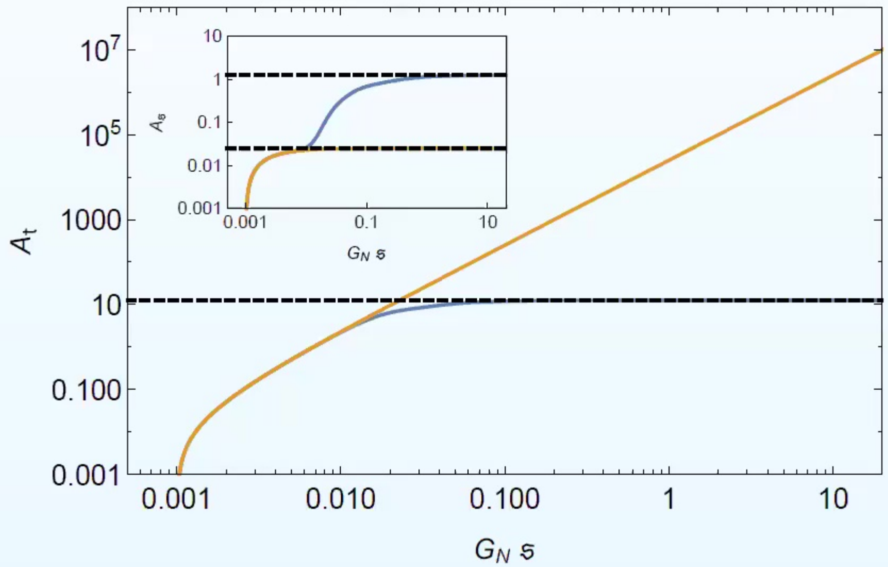
Gravity-mediated scattering $\phi\chi \to \phi\chi$



related to $\phi\phi \to \chi\chi$ by crossing symmetry $\mathfrak{s} \leftrightarrow \mathfrak{t}$

- graviton-contribution to $\mathfrak{t}$-channel amplitude $\mathcal{A}_\mathfrak{t} \sim \mathfrak{s}^2 G_C(\mathfrak{t})$
- divergence canceled by scalar self-interaction



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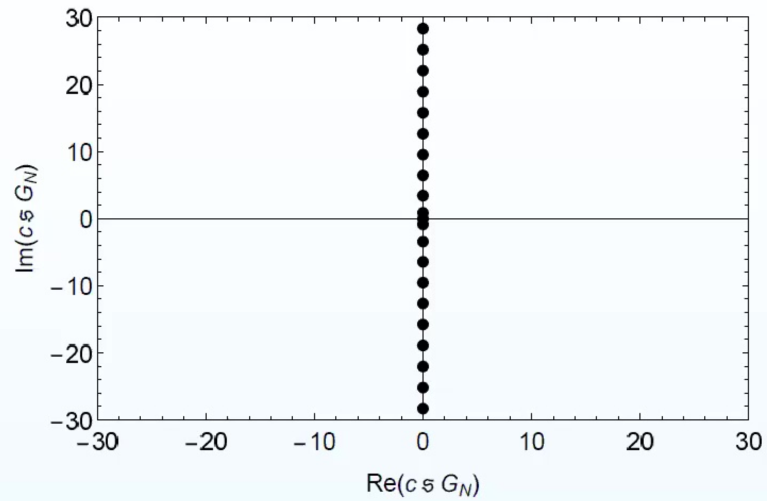
Mechanism underlying Lorentzian Asymptotic Safety



- propagators exhibit an infinite tower of poles at imaginary squared momentum

$$G_2(\mathfrak{s}) \propto \frac{1}{\mathfrak{s}[1 + c_C G_N \mathfrak{s}\, \tanh(c_C G_N \mathfrak{s})]}$$

- poles exhibit Regge-type scaling asymptotically $\hat{\Gamma}_n \propto in$
- avoids causality violation typical for Lee-Wick models

Concluding comments

scale-free amplitudes require relations among propagators and vertices

provided by Reuter fixed point

- evidence from first-principle computations (Wetterich equation)
  - form factor computations

    [L. Bosma, B. Knorr, F. Saueressig, arXiv:1904.04845]
    [B. Knorr, C. Ripken, F. Saueressig, arXiv:1907.02903]

  - works on "effective universality" in Asymptotic Safety

    [J. Meibohm, J.M. Pawlowski, M. Reichert, arXiv:1510.07018]
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    [N. Christiansen, D.F. Litim, J.M. Pawlowski, M. Reichert, arXiv:1710.04669]
    [A. Eichhorn, P. Labus, J.M. Pawlowski, M. Reichert, arXiv:1804.00012]

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getting quantum-corrected amplitudes via “RG-improvement” fails

- RG-information insufficient for reproducing all amplitudes:

$$G(k^2) \longrightarrow f_C(p^2), f_R(p^2), f_{R\phi\phi}(p_1^2, p_2^2, p_3^2), \ldots$$

Conclusions . . .

Structure of the Asymptotic Safety Program

Reuter Fixed Point

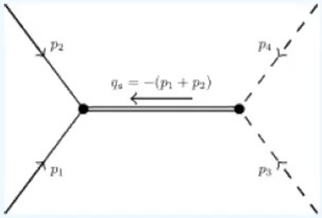
↓ Wetterich equation

quantum effective action

↓ ↓

observable phenomena | fundamental requirements

[Event Horizon Telescope]



Conclusions and Outlook

[discussion session: Daniele Oriti, Thursday]

Asymptotic Safety, Causal Dynamical Triangulations, Causal Set Theory, Loop Quantum Gravity, Group Field Theory, Infinite Derivative Gravity, weakly non-local gravity, ...

↓

quantum effective action

↓ ↓

observable phenomena | fundamental requirements

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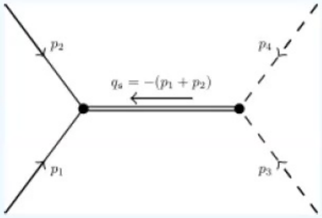

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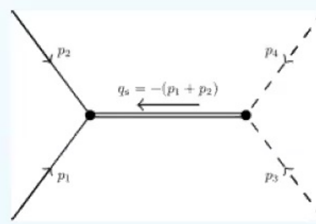

Thank You!

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