

Title: Random tensors, melonic theories and quantum gravity

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Abstract: I will present a brief review of large- N tensor models and their applications in quantum gravity. On the one hand, they provide a general platform to investigate random geometry in an arbitrary number of dimensions, in analogy with the matrix models approach to two-dimensional quantum gravity. Previously known universality classes of random geometries have been identified in this context, with continuous random trees acting as strong attractors. On the other hand, the same combinatorial structure supports a generic family of large- N quantum theories, collectively known as melonic theories. Being largely solvable, they have opened a new window into strongly-coupled quantum theory, and via holography, into quantum gravity. Prime examples are provided by the SYK model and generalizations, which capture essential features of Jackiw-Teitelboim gravity.



Random tensors, melonic theories and quantum gravity

Sylvain Carrozza



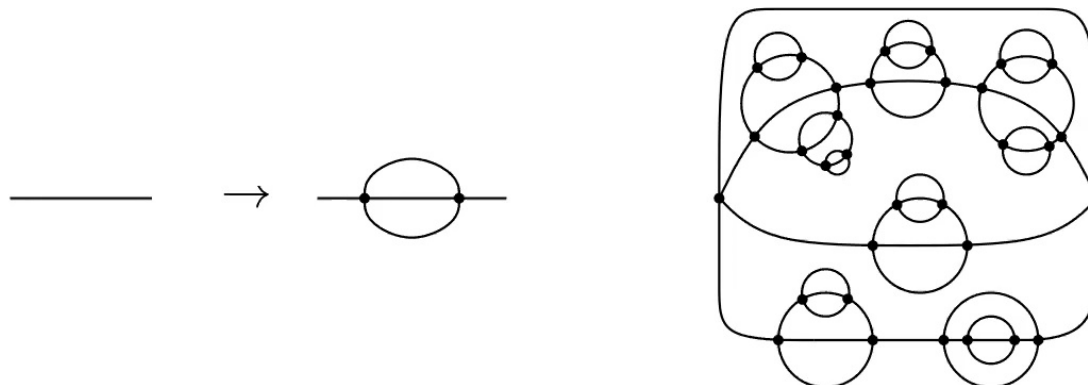
Quantum Gravity 2020
Perimeter Institute for Theoretical Physics
July 14, 2020



LARGE-N THEORIES

- ▶ platform to investigate aspects of quantum gravity.
- ▶ tractable, or even analytically solvable toy-models.

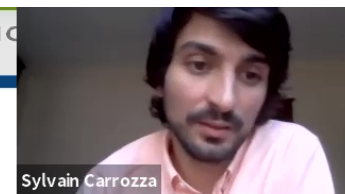
Melonic theories → Feynman expansion dominated by *melon diagrams*



Sachdev-Ye-Kitaev model

Tensor models

Structural glass / machine learning



OUTLINE

Introduction: vector, matrix and tensor models

$(d = 0)$ Random geometry

$(d \geq 1)$ Melonic quantum (field) theories

Questions



TENSORS AND INVARIANTS

Statistical/quantum theories of p -index tensors of size N :

$$T_{a_1 a_2 \dots a_p} = \begin{array}{c} \bullet \\ \diagdown \quad \diagup \\ a_1 \quad a_2 \quad \dots \quad a_p \end{array}$$

$$\sum_{c=1}^N T_{abc} T_{cde} = \begin{array}{c} \textcolor{blue}{c} \\ \bullet \quad \bullet \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ a \quad b \quad d \quad e \end{array}$$

Connected invariants:

$$p = 2 \quad \begin{array}{c} \circlearrowleft \\ \circlearrowright \end{array} \quad \begin{array}{c} \bullet \quad \bullet \\ \circlearrowleft \quad \circlearrowright \end{array} \quad \begin{array}{c} \bullet \quad \bullet \quad \bullet \quad \bullet \\ \circlearrowleft \quad \circlearrowright \end{array} \quad \dots \quad (\text{tr}(M^n))$$



TENSORS AND INVARIANTS

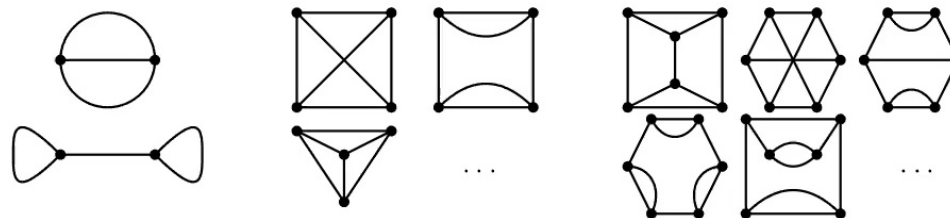
Statistical/quantum theories of p -index tensors of size N :

$$T_{a_1 a_2 \dots a_p} = \text{Diagram of a tensor with } p \text{ legs labeled } a_1, a_2, \dots, a_p$$

$$\sum_{c=1}^N T_{abc} T_{cde} = \text{Diagram of two tensors connected by a leg labeled } c$$

Connected invariants:

$p = 3$



$$\#\{\text{invariants of order } 2n\} \sim \left(\frac{3}{2}\right)^n n!$$

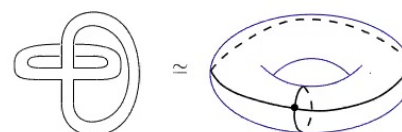
\Rightarrow Rapid growth of theory space for $p \geq 3$. Universal features at large N ?



TOPOLOGICAL EXPANSION OF MATRIX MODELS ['T Hooft '77

$$\mathcal{Z}_N(\lambda) = \int dM e^{-S(M)}, \quad S(M) = \frac{1}{2} \text{tr}(M^2) - \frac{\lambda}{N} \text{tr}(M^4) + \dots$$

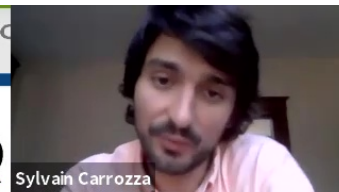
$$\begin{array}{c} i \xrightarrow{\delta_{ii'}} i' \\ j \xrightarrow{\delta_{jj'}} j' \end{array}$$



Universal large- N expansion

$$\ln \mathcal{Z}_N(\lambda) = \sum_{g \in \mathbb{N}} N^{2-2g} \mathcal{F}_g(\lambda) \quad \text{with} \quad \mathcal{F}_g(\lambda) = \sum_{\mathcal{G}, g(\mathcal{G})=g} \lambda^{n_{\mathcal{G}}} A(\mathcal{G})$$

$$N^2 \text{ (sphere) } + N^0 \text{ (torus) } + N^{-2} \text{ (genus 2) } + N^{-3} \text{ (genus 3) } + \dots$$

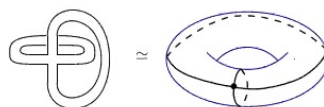


THE LARGE- N LIMIT: A NONPERTURBATIVE TOOL FOR Q

► Random surfaces and QG in $D = 2$

Matrix integral at large $N \rightarrow$ statistical sum of
Feynman graphs \simeq Euclidean space-time geometries

↗



► Strongly-coupled QFT

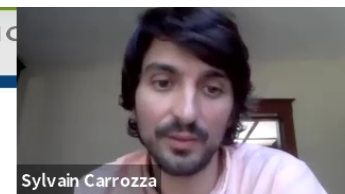
Large number of fields/symmetries e.g. $SU(3) \rightarrow SU(N)$

- perturbation theory in $1/N$
- non-perturbative effects in coupling constants λ

Key probe of **holographic dualities**:

- gauge theory \leftrightarrow Einstein gravity
- vector models \leftrightarrow higher-spin gravity

What are tensor models good for in these two lines of thoughts?



OUTLINE

Introduction: vector, matrix and tensor models

$(d = 0)$ Random geometry

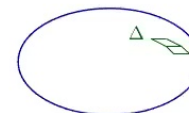
$(d \geq 1)$ Melonic quantum (field) theories

Questions



QG IN $D = 2$ AS A MATRIX INTEGRAL

$$\ln \int dM e^{-N(\frac{1}{2}\text{tr} M^2 - \frac{\lambda}{p}\text{tr} M^p)} \xrightarrow{N \rightarrow \infty} \mathcal{F}_0(\lambda) = \sum_{\Delta} \lambda^{n_{\Delta}}$$



- ▶ Large- N limit \Rightarrow generating function of planar p -angulations Δ , weighted by $n_{\Delta} \sim \text{area}$.
- ▶ Critical regime: $\lambda \rightarrow \lambda_c \Rightarrow$ continuum limit.
- ▶ Double-scaling \Rightarrow non-trivial sum over topologies ("third quantization").

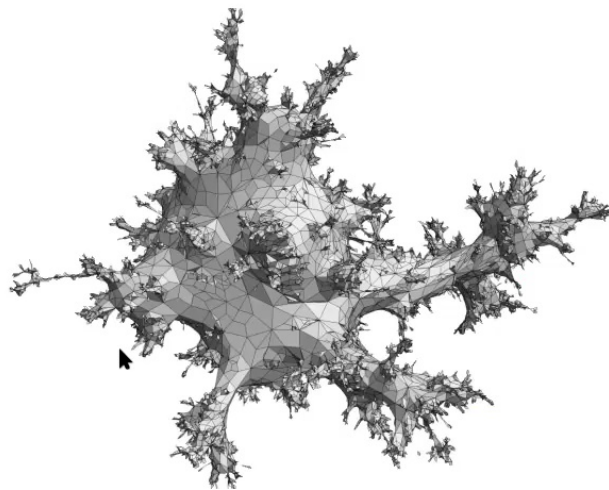
Universality: the distribution over 2d metrics converges to the **Brownian sphere** in the continuum limit, independently of the details of the potential (e.g. value of p).

\rightarrow basic random geometry behind **Liouville QG**.



BROWNIAN SPHERE

[LE GALL, MIERMONT '13]



Credit: T. Budd (<https://hef.ru.nl/~tbudd/gallery/>)

$$\#\{\text{rooted planar } \Delta\} \sim K \lambda_c^{-n_\Delta} n_\Delta^{-5/2}$$

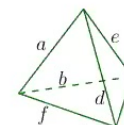
$$d_{\text{spectral}} = 2 \quad ; \quad \text{distance scale} \sim n_\Delta^{1/4} \quad \text{and} \quad d_{\text{Hausdorff}} = 4$$

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QG IN $D \geq 3$ AS A TENSOR INTEGRAL?

$$\mathcal{F}(\lambda) = \ln \int dT \exp \left(-T_{abc}T_{abc} + \frac{\lambda}{N^\alpha} T_{aeb}T_{bfc}T_{ced}T_{dfa} \right)$$



[Ambjørn, Durhuus, Jónsson '91; Gross '91; Sasakura '91;...]

► Challenges:

- matrix techniques not available (spectral representation, orthogonal polynomials...)
- interplay between combinatorics and topology: nice global properties from local Feynman rules?

⇒ no known large- N expansion.

► Path to progress:

[Gurau '09; Gurau, Rivasseau, Bonzom,... '10s]

- more symmetry: $U(N)^D$ -tensors.
- tractable combinatorics, mapping to sufficiently regular topological spaces.

⇒ universal large- N expansion, in any $D \geq 3$.

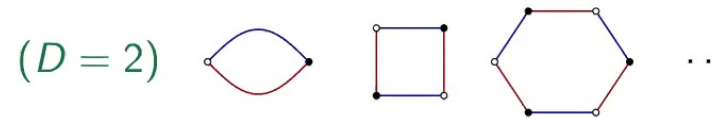


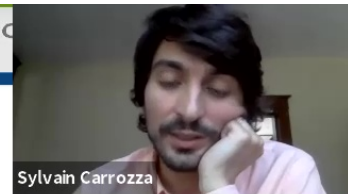
COLORED TENSOR MODELS

[GURAU '09]

$$T_{a_1 a_2 \dots a_D} = \begin{array}{c} \bullet \\ \swarrow \quad \downarrow \quad \searrow \\ a_1 \quad a_2 \quad \dots \quad a_D \end{array} \quad \begin{array}{c} \circ \\ \swarrow \quad \downarrow \quad \searrow \\ a_D \quad \dots \quad a_2 \quad a_1 \end{array} = \bar{T}_{a_1 a_2 \dots a_D}$$

$U(N)^D$ invariants indexed by bubble diagrams \mathcal{B} :



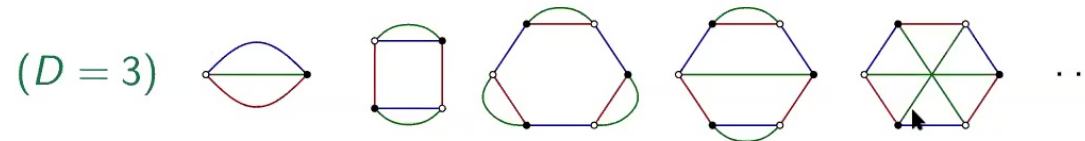


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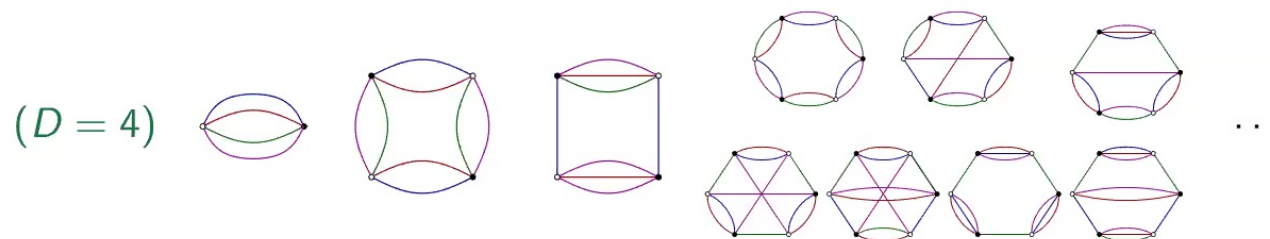


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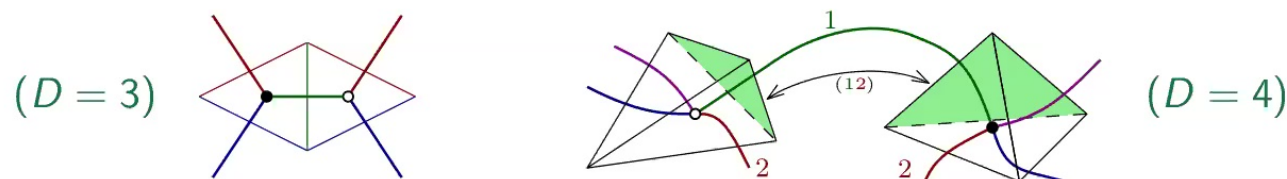


Partition function:

$$\mathcal{F}(\{\lambda_{\mathcal{B}}\}) = \ln \int dT \exp \left(-\bar{T} \cdot T + \sum_{\mathcal{B}} \frac{\lambda_{\mathcal{B}}}{N^{\alpha(\mathcal{B})}} \text{Tr}_{\mathcal{B}}(\bar{T}, T) \right)$$



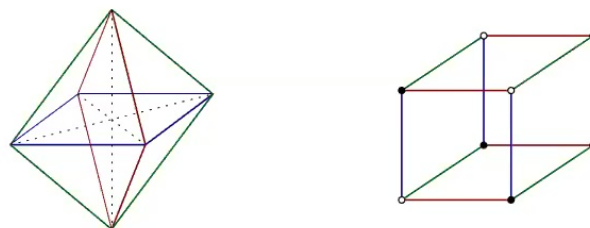
COLORED TRIANGULATIONS



Theorem: [Pezzana '74]
 D -colored graph \Leftrightarrow triangulation Δ of pseudo-manifold of dim. $D - 1$.

Colors \rightarrow unambiguous identification of sub-simplices and their gluings.

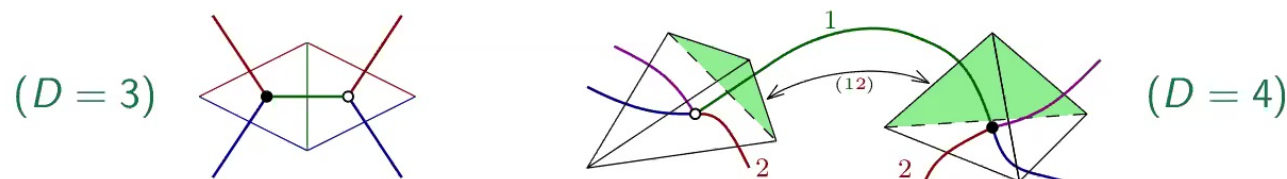
► Bubble $\simeq D$ -colored graph \simeq boundary of D -cell.



► Feynman graph $\simeq (D + 1)$ -colored graph $\simeq \Delta$ of dimension D .



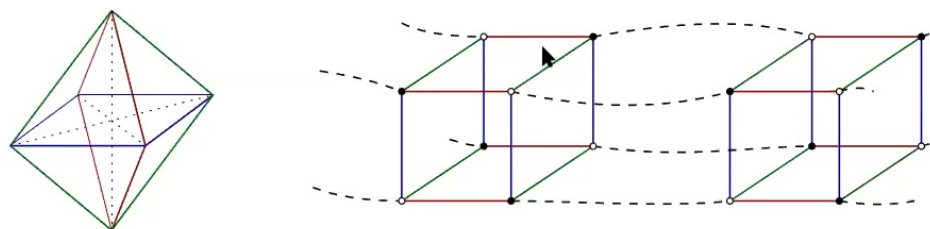
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► Feynman graph $\simeq (D + 1)$ -colored graph $\simeq \Delta$ of dimension D .



LARGE-N EXPANSION

[GURAU '11; BONZOM, GURAU, RIVASSEAU '12]

Scaling of bubbles and Feynman expansion governed by **Gurau degree** ω :

$$\begin{aligned}\mathcal{F}(\{\lambda_{\mathcal{B}}\}) &= \ln \int dT \exp \left(-\bar{T} \cdot T + \sum_{\mathcal{B}} \lambda_{\mathcal{B}} N^{-\frac{2}{(D-2)!}} \omega(\mathcal{B}) \text{Tr}_{\mathcal{B}}(\bar{T}, T) \right) \\ &= \sum_{\omega \in \mathbb{N}} N^{D - \frac{2}{(D-1)!} \omega} \mathcal{F}_{\omega}(\{\lambda_{\mathcal{B}}\})\end{aligned}$$

where

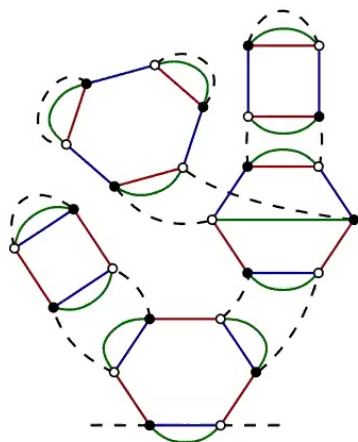
$$\omega(\Delta) = D - n_{D-2}(\Delta) + \frac{D(D-1)}{4} n_D(\Delta)$$

- ▶ $\omega \in \mathbb{N}$
- ▶ generalization of the genus: $D = 2 \Rightarrow \omega = 2g$
- ▶ *not* a topological invariant of Δ when $D \geq 3$
- ▶ however: $\omega = 0 \Rightarrow \Delta$ is a D -sphere



LEADING ORDER

[BONZOM, GURAU, RIELLO, RIVASSEAU '11;...]



$$\omega(\Delta) = 0 \Leftrightarrow \Delta \text{ is melonic}$$

→ special triangulations of the D -sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$G(\lambda) = 1 + \lambda G(\lambda)^{D+1} \quad (\text{Fuss-Catalan})$$

Critical behaviour:

$$G(\lambda_c) - G(\lambda) \underset{\lambda \rightarrow \lambda_c}{\sim} K (\lambda_c - \lambda)^{1/2}$$

$$\Leftrightarrow \#\{\text{rooted melonic } \Delta\} \sim K \lambda_c^{-n_\Delta} n_\Delta^{-3/2}$$

Universal critical exponent $3/2$ associated to combinatorial trees.

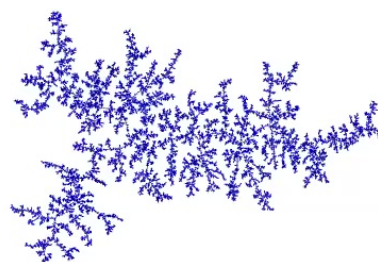


CONTINUUM LIMIT

[GURAU, RYAN '13]

Melons are **branched polymers**

i.e. they converge to the **continuous random tree** [Aldous '91].



Credit: I. Kortchemski (<https://igor-kortchemski.perso.math.cnrs.fr/images.html>)

$$\#\{\text{rooted melonic } \Delta\} \sim K \lambda_c^{-n_\Delta} n_\Delta^{-3/2}$$

$$\boxed{d_{\text{spectral}} = 4/3} \quad ; \quad \text{distance scale} \sim n_\Delta^{1/2} \quad \text{and} \quad d_{\text{Hausdorff}} = 2$$

\Rightarrow **strong universality**: limit independent of D !



FURTHER RESULTS

- ▶ Combinatorial classification of graphs at order $\omega > 0$:
"it's melons all the way down". [Gurau, Schaeffer '13]
- ▶ Double-scaling. [Bonzom, Gurau, Kaminski, Dartois, Oriti, Ryan, Tanasa '13 '14]
- ▶ Schwinger-Dyson eq. \rightarrow analogue of loop equations. [Gurau '11]
- ▶ Non-perturbative treatment. [Gurau '14]
- ▶ Renormalization group approach to large- N . [Eichhorn, Koslowski, Lumma, Pereira...]
- ▶ ...

- ▶ Applications in Group Field Theory:

[Boulatov, Ooguri, '92... Freidel, Gurau, Oriti '00s '10s...]

Melonic behaviour \Rightarrow rigorous renormalization theorems

[Ben Geloun, Rivasseau '11; SC, Oriti, Rivasseau '13;...]

[Review SC '16]

Most recent developments focused on cosmological applications.

[Gielen, Oriti, Sindoni, Sakellariadou, Pithis...]

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BEYOND BRANCHED POLYMERS?

No-go:

- ▶ Non-melonic large- N limits have been explored.
[Bonzom, Delpouve, Rivasseau '15; Bonzom, Lionni '16; Lionni, Thüringen '17]
- ▶ **Universality theorem:** $D = 3 \Rightarrow$ branched polymers for arbitrary spherical bubbles. [Bonzom '18]

Yes go?

- ▶ D even \Rightarrow Brownian sphere, branched polymers and mixtures.
[Bonzom, Delpouve, Rivasseau '15]
- ▶ Simple combinatorial restrictions may change the universality class:
branched polymers $\xrightarrow{2PI}$ Ising on a random surface
[Benedetti, SC, Toriumi, Valette '20]

Major open question:

genuinely new random geometric phase suitable for QG in $D \geq 3$?

[Lionni, Marckert '19]

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SUMMARY

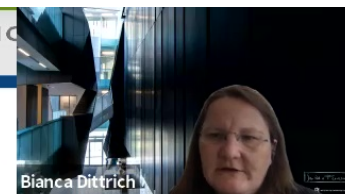
Tensor models for **random geometry**:

- ▶ well-defined generalization of the matrix models approach;
- ▶ reproduce previously known universality classes: continuous random tree, Brownian sphere, and mixtures;
- ▶ tend to be dominated by tree-like combinatorial species \Rightarrow no genuinely new universality class discovered so far...

...but a vast parameter space remains to be explored.

Entry points into the literature:

- ▶ "Random tensors", Gurau, 2016;
- ▶ "The Tensor Track" I-IV, Rivasseau, 2011-2016;
- ▶ "Colored Discrete Spaces", Lionni, 2018.



OUTLINE

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$(d = 0)$ Random geometry

$(d \geq 1)$ Melonic quantum (field) theories

Questions



SACHDEV-YE-KITAEV MODEL

[Sachdev, Ye, Georges, Parcollet '90s...; Kitaev '15, Maldacena, Stanford, Polchinski, Rosenhaus...]

- ▶ Disordered system of N Majorana fermions ψ_a in $d = 0 + 1$

$$H \sim J_{abcd} \psi_a \psi_b \psi_c \psi_d, \quad \langle J_{abcd} \rangle = 0, \quad \langle J_{abcd}^2 \rangle \sim \frac{\lambda^2}{N^3}$$

- ▶ Many interesting properties:

- ▶ solvable at large N
- ▶ emergent conformal symmetry at strong coupling
- ▶ same symmetry breaking as in Jackiw-Teitelboim quantum gravity
→ toy-models of quantum black holes
- ▶ maximal quantum chaos

[Maldacena, Shenker, Stanford]

- ▶ Same melonic large N limit as tensor models

[Witten '16]

→ **SYK-like quantum-mechanical models:**

- ▶ same qualitative properties at large N and strong coupling;
- ▶ **no disorder.**

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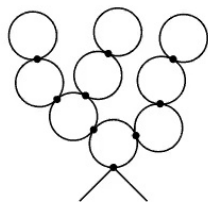


Vector ϕ_a

$$\frac{\lambda}{N} (\phi_a \phi_a)^2$$

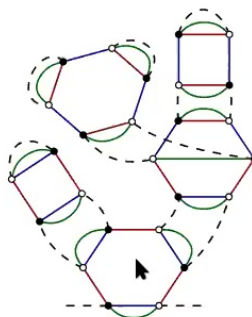


Bubble diagrams

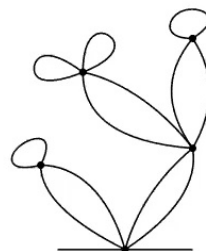


Easy

Tensor T_{abc}

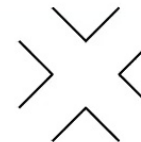


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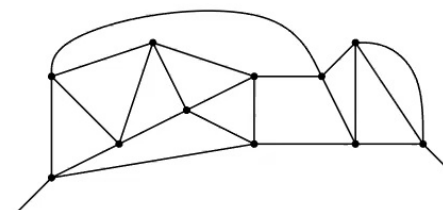


Matrix M_{ab}

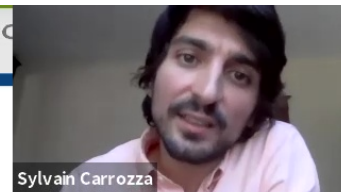
$$\frac{\lambda}{N} M_{ab} M_{bc} M_{cd} M_{da}$$



Planar diagrams



Hard



Vector ϕ_a

$$\frac{\lambda}{N} (\phi_a \phi_a)^2$$



Tensor T_{abc}

$$\frac{\lambda}{N^{3/2}} T_{aeb} T_{bfc} T_{ced} T_{dfa}$$

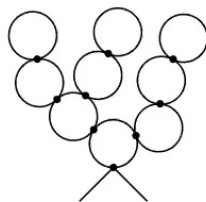


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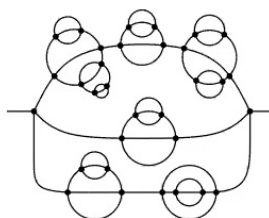


Bubble diagrams



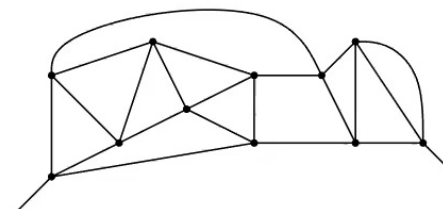
Easy

Melon diagrams



Tractable

Planar diagrams



Hard



Vector ϕ_a

$$\frac{\lambda}{N} (\phi_a \phi_a)^2$$



Tensor T_{abc}

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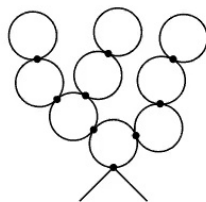


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$$\frac{\lambda}{N} M_{ab} M_{bc} M_{cd} M_{da}$$

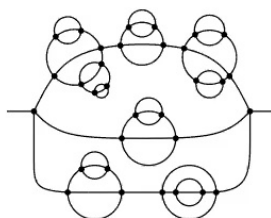


Bubble diagrams



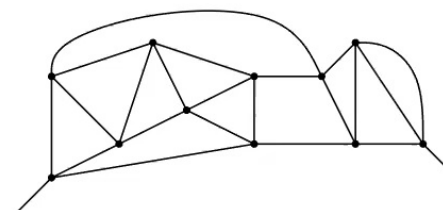
Easy

Melon diagrams



Tractable

Planar diagrams



Hard

Melonic regime \Rightarrow closed and often solvable systems of Schwinger-Dyson equations, capturing bilocal effects.



GENERAL FRAMEWORK FOR 3-INDEX TENSORS

Theorem: A melonic large- N limit exists whenever the tensor representation is **irreducible**.



[Benedetti, SC, Gurau, Kolanovski '17 - Commun. Math. Phys.]

[SC '18; SC, Pozsgay '18]

Motivated by earlier work / conjecture: [Klebanov, Tarnopolsky '17]

- ▶ Applicable to $O(N)$ and $Sp(N)$ tensors, which are **symmetric**, **antisymmetric** or **mixed** under index permutation.
- ▶ Irreducibility: removes **vector modes** and maintains



Key condition missing in models from the 90s.

- ▶ Generalizable to Hermitian or symmetric **multi-matrix models**.

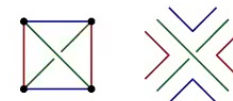
[SC, Ferrari, Tanasa, Valette, '20]



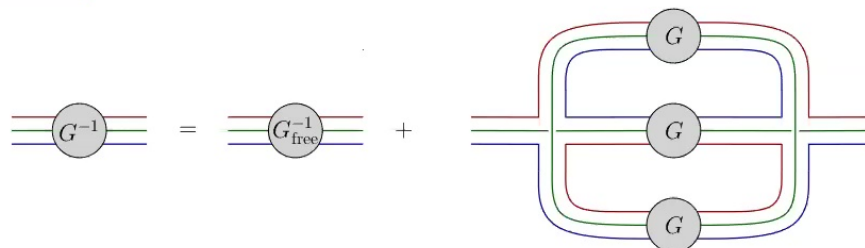
KLEBANOV-TARNOPOLSKY MODEL [KLEBANOV, TARNOPOLSKY '16]

Tensor quantum mechanics of N^3 Majorana fermions:

$$S = \int dt \left(\frac{i}{2} \psi_{i_1 i_2 i_3} \partial_t \psi_{i_1 i_2 i_3} + \frac{\lambda}{4N^{3/2}} \psi_{i_1 i_2 i_3} \psi_{i_4 i_5 i_3} \psi_{i_4 i_2 i_6} \psi_{i_1 i_5 i_6} \right)$$

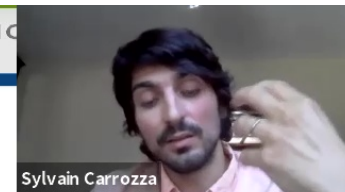


- **Melonic dominance** at large $N \Rightarrow$ closed Schwinger-Dyson equation:
[SC, Tanasa '15]



- SYK **melonic equation**: $\langle T(\psi_{a_1 a_2 a_3}(t_1) \psi_{b_1 b_2 b_3}(t_2)) \rangle \equiv G(t_1, t_2) \prod_{i=1}^3 \delta_{a_i, b_i}$

$$G(t_1, t_2) = G_{\text{free}}(t_1, t_2) + \lambda^2 \int dt dt' G_{\text{free}}(t_1, t) [G(t, t')]^3 G(t', t_2)$$



STRONG-COUPPLING REGIME

$$G(t_1, t_2) = G_{\text{free}}(t_1, t_2) + \lambda^2 \int dt dt' G_{\text{free}}(t_1, t) [G(t, t')]^3 G(t', t_2)$$

- At strong coupling:

$$\lambda^2 \int dt G(t_1, t) [G(t, t_2)]^3 = -\delta(t_1 - t_2)$$

- One special solution (conformal solution):

$$G(t_1, t_2) = - \left(\frac{1}{4\pi\lambda^2} \right)^{1/4} \frac{\text{sgn}(t_1 - t_2)}{|t_1 - t_2|^{2\Delta}}, \quad \Delta = \frac{1}{4}$$

- Emergent conformal invariance: reparametrization $t \mapsto f(t)$

$$G(t_1, t_2) \mapsto |f'(t_1)f'(t_2)|^{1/4} G(f(t_1), f(t_2))$$

⇒ infinite number of solutions?



SYMMETRY BREAKING

$$G(t_1, t_2) = G_{\text{free}}(t_1, t_2) + \lambda^2 \int dt dt' G_{\text{free}}(t_1, t) [G(t, t')]^3 G(t', t_2)$$

- ▶ Spontaneous breaking to $SL(2, \mathbb{R}) \Rightarrow$ Goldstone modes
 $f \in \text{Diff}(\mathbb{R})/SL(2, \mathbb{R})?$
- ▶ G_{free} term \Rightarrow explicit breaking \Rightarrow non-zero Schwarzian effective action

$$S_{\text{eff}}[f] \sim \frac{1}{\lambda} \int dt \{f, t\}$$

[Maldacena, Stanford '16]

$$(\{f, t\} = \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2 \text{ is the Schwarzian derivative})$$

Effective action for boundary gravitons in JT gravity
 \Rightarrow near AdS_2 / near CFT_1 correspondence.



SYK VS TENSOR QUANTUM MECHANICS

- General correspondence between:

disordered SYK-like systems \leftrightarrow **unitary** tensor quantum mechanics

leading to identical large- N Schwinger-Dyson eq.

[Klebanov, Tarnopolsky et al.; Ferrari et al.;...]

- Main specificity of tensor models over disordered models:

Growth of Hilbert space: $\#\{\text{states}\} \sim \exp(\text{cte} \times N^3)$

\Rightarrow **numerically challenging** at finite- N

N	Number of states
1	2
2	36
3	595 354 780

Real $O(2N)^3$
(Klebanov-Tarnopolsky)

N	Number of states
1	3
2	39
3	170 640

Complex $USp(2N)$ symmetric
(SC-Pozsgay)



TENSOR FIELD THEORY

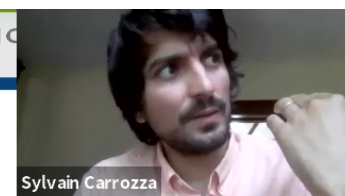
Unlike SYK, tensor models naturally fit in the framework of **local quantum field theory**.

QFT generalization?

Rely on tensor models to construct **melon theories** in $d > 1$.

Why it is interesting:

- ▶ only diagrams that proliferate are **melons** and **ladder diagrams**
⇒ explicit **non-perturbative resummation** sometimes possible
 - ▶ melons are **bi-local**
⇒ **anomalous dimensions** ⇒ **non-trivial CFTs** and **RG flows**
 - ▶ 4-point functions = sums of **ladder diagrams**
⇒ non-perturbative access to the **spectrum**
- ⇒ theoretical insights into **strongly-coupled QFT**.



FIXED POINTS OF WILSON-FISHER TYPE

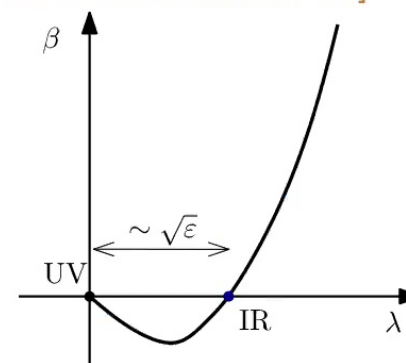
► Four-fermion field theory:

[Prakash, Sinha '17 ; Benedetti, SC, Gurau, Sfondrini '17]

$$\beta^{(\varepsilon)} = -\varepsilon\lambda + \frac{3}{\pi^2}\lambda^3$$

Weakly-interacting IR fixed point in $d = 2 - \varepsilon$.

Presumably flows to strongly-interacting SYK-like phase as $\varepsilon \mapsto 1$.



► Bosonic field theory:

Quartic models in $d = 4 - \varepsilon$

[Giombi, Klebanov, Tarnopolsky '17]

Sextic models in $d = 3 - \varepsilon$

[Giombi, Klebanov, Popov, Prakash, Tarnopolsky '18; Benedetti, Delporte, Harribey, Sinha '19]

∃ IR melonic fixed points, but they are generically unstable.



LONG-RANGE BOSONIC MODELS

[BENEDETTI, GURAU, HARRIBEY '19]

Bosonic tensor field theory in $d < 4$:

$$\mathcal{L} = \frac{1}{2} \varphi_{abc} (-\Delta)^{\zeta} \varphi_{abc} + \frac{m^{2\zeta}}{2} \varphi_{abc} \varphi_{abc} \\ + \frac{i\lambda}{4N^{3/2}} \text{[tetrahedron]} + \frac{\lambda_P}{4N^2} \text{[square]} + \frac{\lambda_D}{4N^3} \text{[double lines]}$$

- ▶ Long-range kinetic term with $\zeta = \frac{d}{4}$. [Gross, Rosenhaus '16] in $d = 1$
- ▶ Imaginary tetrahedral coupling.
- ▶ Large- N melonic limit \Rightarrow flow to **unitary CFT** in the IR:
[Benedetti, Gurau, Harribey, Suzuki '19; Benedetti, Gurau, Suzuki '20]
 - ▶ spectrum and OPE coefficients explicitly computable;
 - ▶ no local stress-tensor.

Glimpse of a new class of large N CFTs? Any relevance for holography?



SUMMARY

Tensor models for **strongly-coupled quantum theory**:

- ▶ third generic family of large- N theories, both rich and tractable;
- ▶ can reproduce SYK-like physics without disorder;
- ▶ generalize to QFT \rightarrow new family of CFTs.

Entry points into the literature:

- ▶ "TASI Lectures on Large N Tensor Models", Klebanov, Popov, Tarnopolsky, 2018;
- ▶ "The Tensor Track" V-VI, Rivasseau, Delporte, 2018-2020;
- ▶ "Notes on Tensor Models and Tensor Field Theories", Gurau, 2019;
- ▶ "Melonic CFTs", Benedetti, 2020.

OUTLINE

Introduction: vector, matrix and tensor models

$(d = 0)$ Random geometry

$(d \geq 1)$ Melonic quantum (field) theories

Questions