Title: Understanding of QG from string theory

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Understanding QG from String Theory

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ER=EPR and Replica Wormholes in Quantum Mechanics and AdS Gravity

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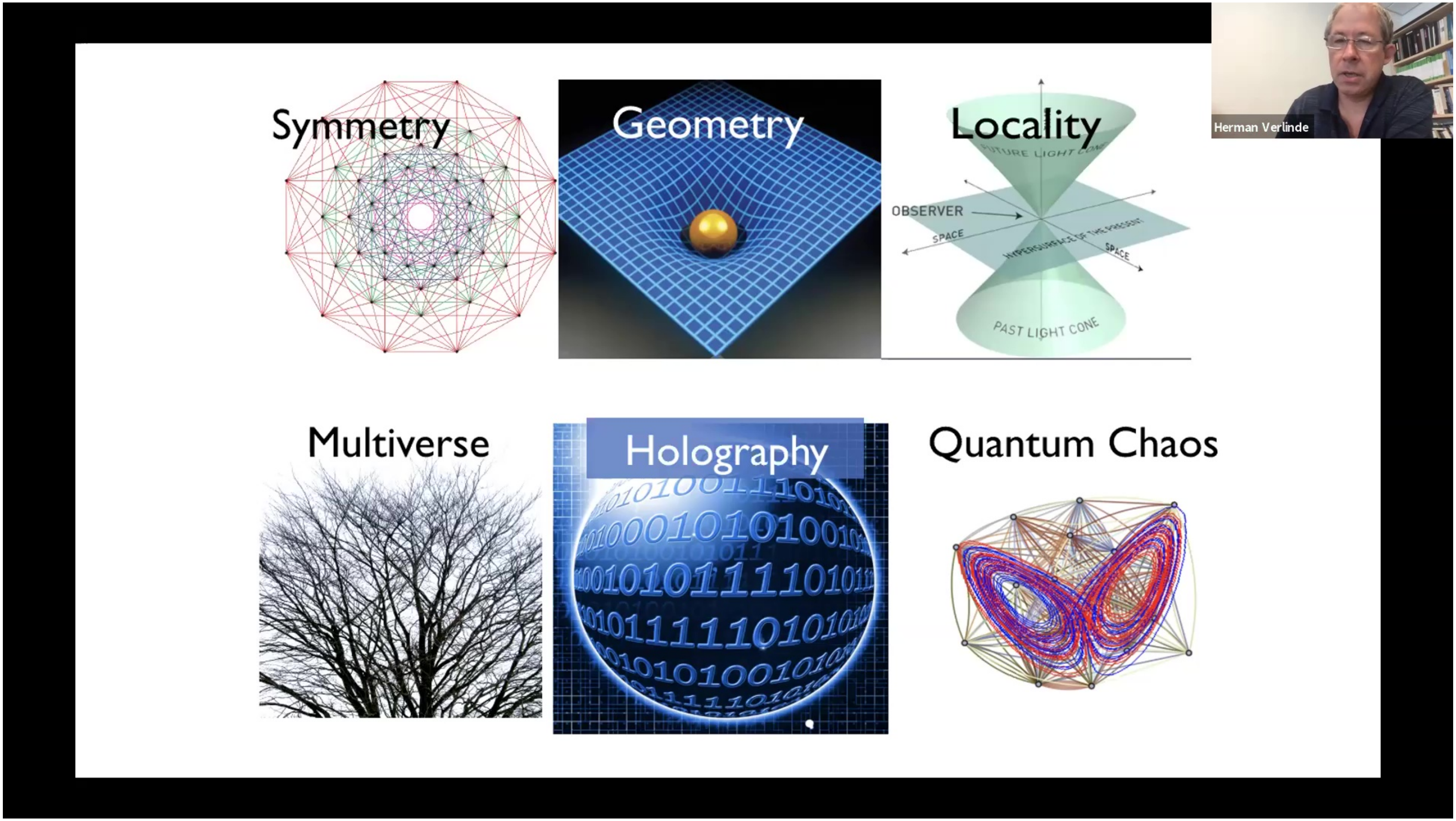
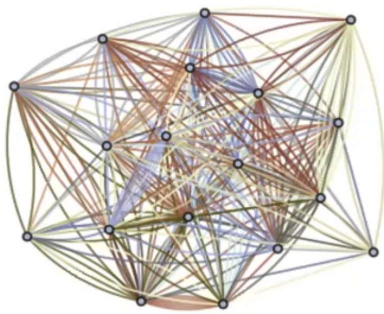
Based on arXiv: 2003.13117 and work in progress with Akash Goel

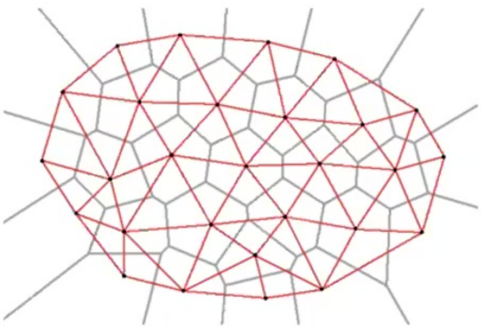

Herman Verlinde
Symmetry
Geometry
Locality
FUTURE LIGHT CONE
OBSERVER
SPACE
HYPERSURFACE OF THE PRESENT
SPACE
PAST LIGHT CONE
Multiverse
Holography
Quantum Chaos

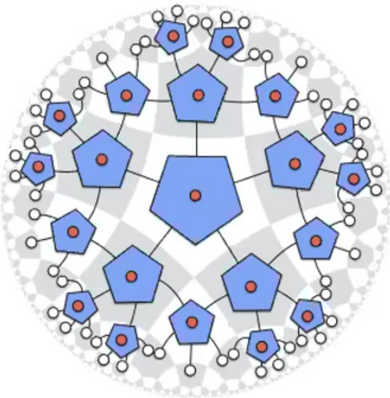
The search for the theory on quantum gravity is guided by basic princi

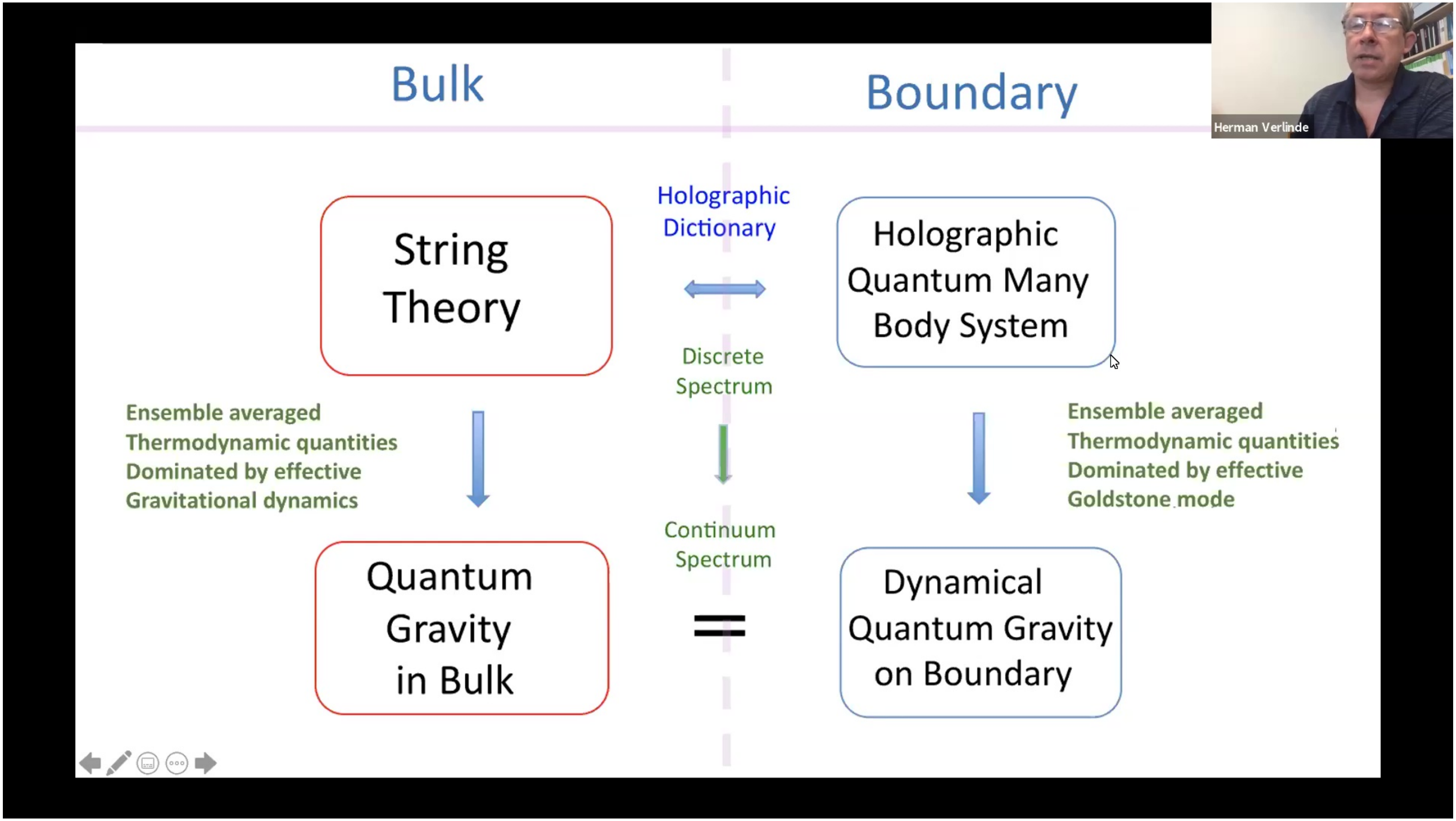
- holography: S = ¼ Area
- quantum information theory
- thermodynamics
- locality and causality
- space-time dynamics
- ....

Some helpful tools: many body QM, CFT bootstrap, large N, tensor networks, ...








Bulk
Boundary
Herman Verlinde
Holographic
Dictionary
String
Theory
Holographic
Quantum Many
Body System
Discrete
Spectrum
Ensemble averaged
Thermodynamic quantities
Dominated by effective
Gravitational dynamics
Ensemble averaged
Thermodynamic quantities
Dominated by effective
Goldstone mode
Continuum
Spectrum
Quantum
Gravity
in Bulk
=
Dynamical
Quantum Gravity
on Boundary

The Bekenstein-Hawking relation

$$S = \frac{A}{4G_N} \quad (1)$$

i) counts the number of microstates of a one-sided black hole with given macroscopic properties

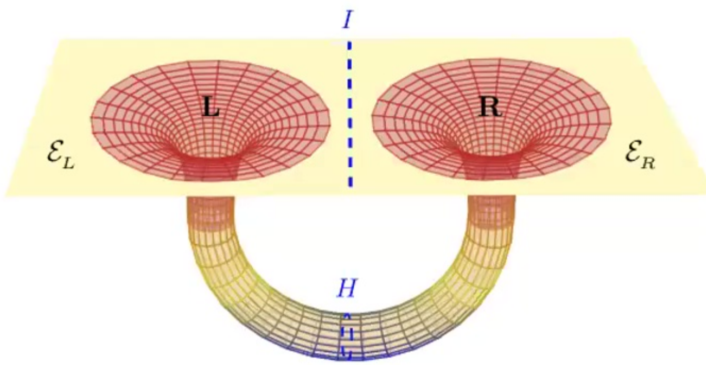
ii) counts the microscopic entanglement across the event horizon connecting the two sides of an eternal black hole.

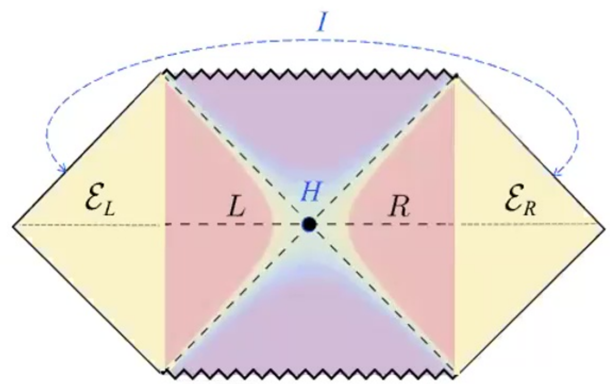
Both proposals are well tested: i) finds support in AdS/CFT and string theory

ii) finds support via the RT formula and the interpretation of the thermofield double

$$|\mathrm{TFD}\rangle = \sum_n \sqrt{p_n}\, |n\rangle_L |n\rangle_R \qquad\qquad p_n = e^{-\beta E_n}/Z$$

as the quantum state of a two-sided black hole with an ER bridge





The TFD is an idealized pure state with zero vN entropy, while a two-sided black hole is a macroscopic object with a large entropy.

So let's ask the question:

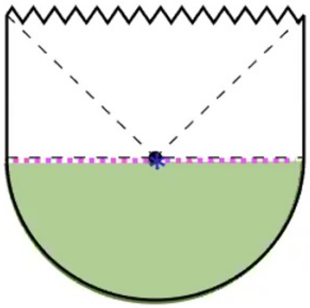
How much entropy can a two sided black hole contain?

Clearly it's bounded by: $0 < S < 2S_{BH}$ with $S_{BH} = A/4G_N$

Three thermal states

1) Thermofield double

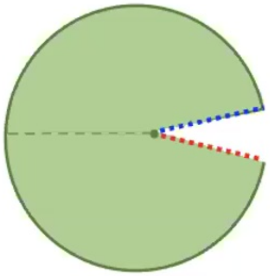
$$|\mathrm{TFD}\rangle = \sum_n \sqrt{p_n}\, |n\rangle_L \, |n\rangle_R$$



Entanglement => Connected!

2) Thermal density matrix

$$\rho_R = \sum_n p_n \, |n\rangle_R \langle n|$$

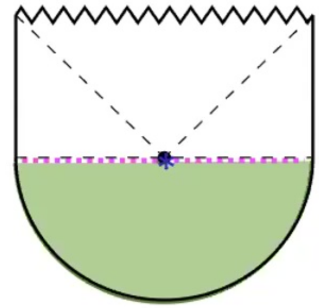


$$\rho_R = \mathrm{tr}_L\big(\rho_{\mathrm{TFD}}\big) \qquad \rho_{\mathrm{TFD}} = |\mathrm{TFD}\rangle\langle\mathrm{TFD}|$$

Three thermal states

1) Thermofield double

$$|\mathrm{TFD}\rangle = \sum_n \sqrt{p_n}\,|n\rangle_L\,|n\rangle_R$$



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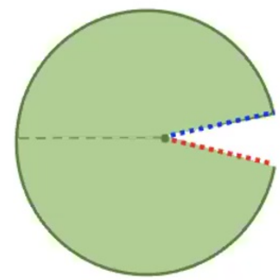
2) Thermal density matrix

$$\rho_R = \sum_n p_n\,|n\rangle_R\langle n|$$



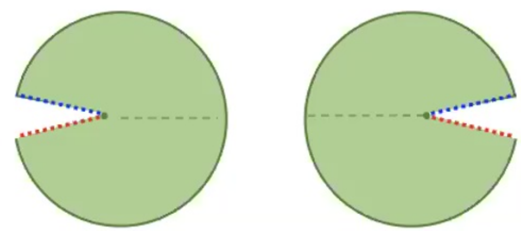
$$S_R = -\mathrm{tr}(\rho_R \log \rho_R) = -\sum p_n \log p_n = S_{BH}.$$

Three thermal states

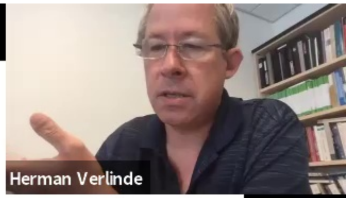
2') Factorized thermal state:

$$\rho_L \otimes \rho_R \; = \; \sum_n p_n \, |n\rangle_L \langle n| \; \otimes \; \sum_{\tilde{n}} p_{\tilde{n}} \, |\tilde{n}\rangle_R \langle \tilde{n}|$$

$$S(\rho_L \otimes \rho_R) \; = \; 2 S_{BH}$$



Uncorrelated => Disconnected!

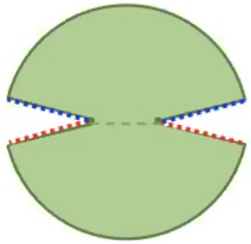


Three thermal states

3) Thermal mixed double state:

$$\rho_{\mathrm{TMD}} = \sum_n p_n \, |n\rangle_L \langle n| \otimes |n\rangle_R \langle n|$$

$$S(\rho_{\mathrm{TMD}}) = S_{BH}$$

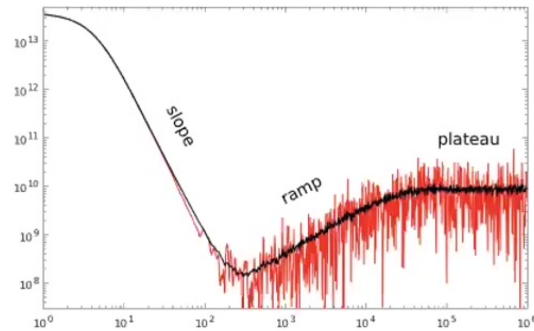


Only classical correlations => Connected via Island!

$$|Z(\beta + it_\alpha)|^2 = |\langle \mathrm{TFD}|\mathrm{TFD}\rangle_\alpha|^2 = \sum_{n,m} p_n p_m \, e^{i(\alpha_n - \alpha_m)}$$

$$e^{i\alpha_n} = e^{-iE_n t_\alpha}$$

Spectral form factor



Saad, Shenker, Stanford

Plateau = late time average

$$\overline{|Z(\beta + it)|^2} = \overline{|\langle \mathrm{TFD}|\mathrm{TFD}\rangle_\alpha|^2} = \mathrm{tr}(\rho_{\mathrm{TMD}}^2)$$

$$\rho_{\mathrm{TMD}} = \overline{|\mathrm{TFD}\rangle_{\alpha\;\alpha}\langle \mathrm{TFD}|} = \sum_n p_n \, |n\rangle_L \langle n| \otimes |n\rangle_R \langle n|$$

$$\overline{e^{i(\alpha_n - \alpha_m)}} = \delta_{nm}.$$

Three thermal states

All three satisfy: $\rho_R = \mathrm{tr}_L(\rho_{\mathrm{TMD}}) = \mathrm{tr}_L(\rho_{\mathrm{TFD}}) = \mathrm{tr}_L(\rho_L \otimes \rho_R)$

Distinguished by amount of mutual information

$$I_{LR} = S_L + S_R - S_{LR}$$

$$I_{LR} = \begin{cases} 2S_{BH} & \text{for TFD} \\ S_{BH} & \text{for TMD} \\ 0 & \text{for } \rho_L \otimes \rho_R \end{cases}$$

$$S(\rho_{\mathrm{TFD}}) = 0$$

$$S(\rho_{\mathrm{TMD}}) = S_{BH}$$

$$S(\rho_L \otimes \rho_R) = 2S_{BH}$$

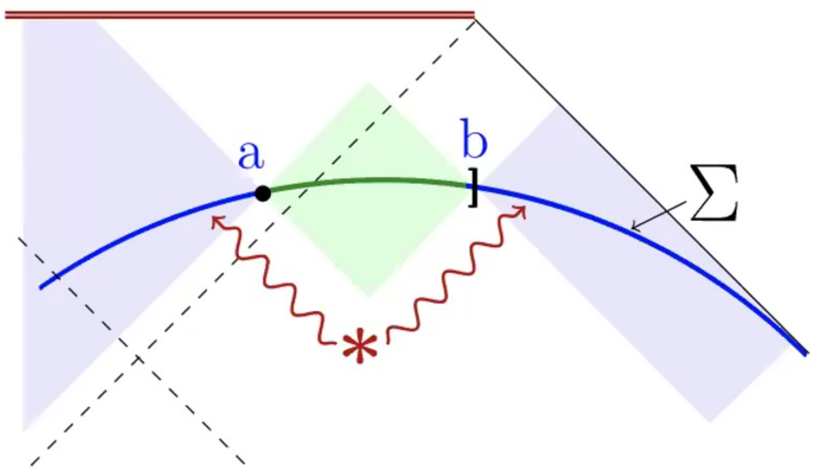
TFD has true entanglement, while the TMD has only classical correlation.

The factorized state has zero correlation between the two sides.

Recent work has uncovered the existence of a new quantum extremal surfa

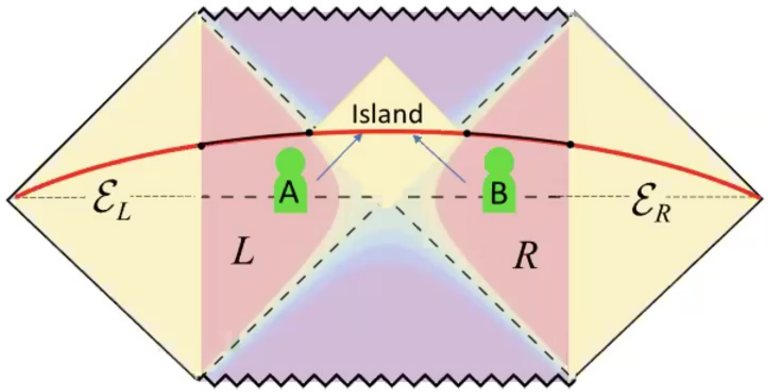
bounding an `Island' inside the black hole, that appears after the Page time.

The quantum information inside the `Island' is contained in the Hawking radiation:



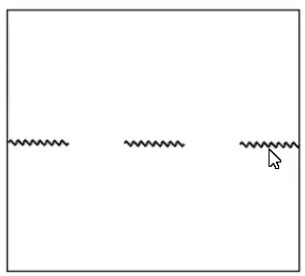
$\Delta t > M \log M$

Two sided black hole with an Island:

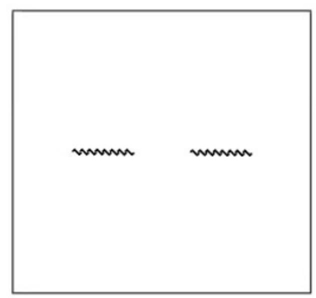


The density matrices of the environment and the CFT are obtained by computing the below two geometric path integrals. The extra cut in the middle is the Island:

$\rho_{\mathcal{E}}$ =



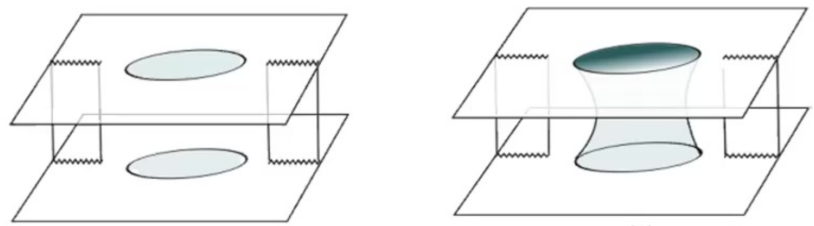
ρ_{CFT} =



The presence of the Island gives rise to new `replica wormhole' saddle po
in the gravitational replica method for computing the Renyi entropy of ρ_{CFT}

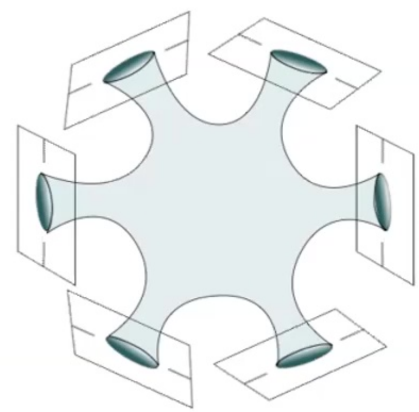
Penington, Shenker, Stanford, Yang

Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini



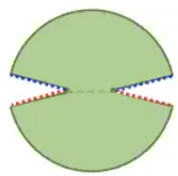
(a) (b)

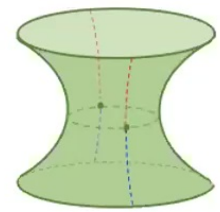
$$\mathrm{tr}(\rho^n_{\mathrm{TMD}}) = \frac{Z(\Sigma_n)}{Z(\beta)^n}$$

$\Sigma_n =$ 

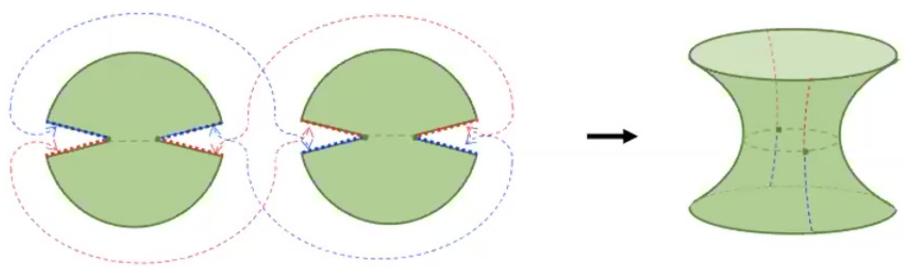
We would like to find the explicit form of this special mixed state!

Let's first consider the second Renyi entropy or purity:

$\rho_{\rm TMD} =$ 

$\mathrm{tr}(\rho^2_{\rm TMD}) =$ 

This can be verified with some minor amount of mental gymnastics:



Replica wormholes in Quantum Mechanics

Commutator Algebra $\quad [X^a, X^c] \;=\; \omega^{ac}$

Casimirs $\quad [J_I, X^a] \;=\; 0$

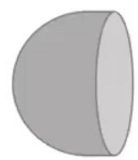
$$\Omega = \frac{1}{2}\,\omega_{ab}\,dX^a \wedge dX^b + dJ \wedge d\tau$$

Symplectic Form

$$Z(\Sigma) \;=\; \int [dX\,d\tau]\; e^{\frac{i}{\hbar} S_\Sigma[X,\tau]}$$

$$S_\Sigma[X,\tau] \;=\; \int_\Sigma \Omega \;-\; \oint_{\partial\Sigma} H\,dt \;=\; \int_\Sigma \omega \;+\; \oint_{\partial\Sigma} (J\,d\tau - H\,dt)$$

Thermal Partition Function $\quad Z(\beta) \;=\; Z(D)$

$D \;=$ 

Disk Partition Function

Replica wormholes in Quantum Mechanics

Commutator Algebra $$[X^a, X^c] \,=\, \omega^{ac}$$

$$\Omega = \frac{1}{2}\omega_{ab}\, dX^a \wedge dX^b + dJ \wedge d\tau$$

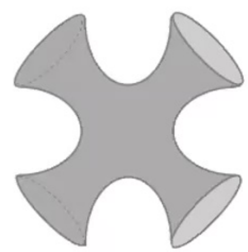
Casimirs $$[J_I, X^a] \,=\, 0$$

Symplectic Form

$$Z(\Sigma) \;=\; \int [dX\, d\tau]\, e^{\frac{i}{\hbar} S_\Sigma[X,\tau]}$$

$$S_\Sigma[X,\tau] \,=\, \int_\Sigma \Omega \,-\, \oint_{\partial\Sigma} H\,dt \,=\, \int_\Sigma \omega \,+\, \oint_{\partial\Sigma} (J\,d\tau - H\,dt)$$

Renyi Entropy $$\mathrm{tr}(\rho^n_{\mathrm{TMD}}) \,=\, \frac{Z(\Sigma_n)}{Z(\beta)^n}$$

$\Sigma_n \,=$  .

Replica Wormhole Partition Function

Replica wormholes in Quantum Mechanics

Commutator Algebra $[X^a, X^c] \,=\, \omega^{ac}$

$$\Omega = \frac{1}{2}\,\omega_{ab}\, dX^a \wedge dX^b + dJ\wedge d\tau$$

Casimirs $[J_I, X^a] \,=\, 0$

Symplectic Form

Poisson sigma model

$$Z(\Sigma) = \int [dX d\eta]\; e^{\frac{i}{\hbar} S_\Sigma[X,\eta]}$$

$$S_\Sigma[X,\eta] \,=\, \int_\Sigma (\eta_a \wedge dX^a + \omega^{ab}\eta_a \wedge \eta_b) \,-\, \oint_{\partial\Sigma} H dt$$

Renyi Entropy $\mathrm{tr}(\rho^n_{\mathrm{TMD}}) \,=\, \dfrac{Z(\Sigma_n)}{Z(\beta)^n}$

$\Sigma_n \,=$  .

Replica Wormhole Partition Function

Replica wormholes in Quantum Mechanics

Commutator Algebra

$$[X^a, X^c] = \omega^{ac}$$

Casimirs

$$[J_I, X^a] = 0$$

$$\Omega = \frac{1}{2}\omega_{ab}\, dX^a \wedge dX^b + dJ \wedge d\tau$$

Symplectic Form

Poisson sigma model

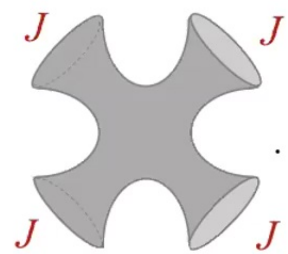
$$Z(\Sigma) = \int [dX d\eta]\, e^{\frac{i}{\hbar} S_\Sigma[X,\eta]}$$

$$S_\Sigma[X,\eta] = \int_\Sigma (\eta_a \wedge dX^a + \omega^{ab}\eta_a \wedge \eta_b) - \oint_{\partial\Sigma} H dt$$

Renyi Entropy

$$\mathrm{tr}(\rho^n_{\mathrm{TMD}}) = \frac{Z(\Sigma_n)}{Z(\beta)^n}$$

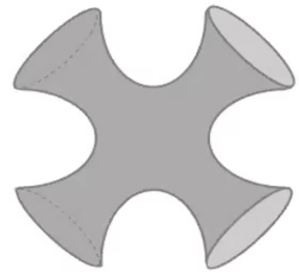
$$\Sigma_n =$$



Replica Wormhole Partition Function

Replica wormholes in 2D CFT

$$\mathrm{tr}(\rho^n_{\mathrm{TMD}}) \;=\; \frac{Z(\Sigma_n)}{Z(\beta)^n} \qquad\qquad \Sigma_n \;=$$



.

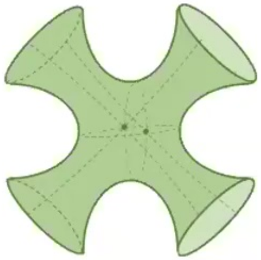
$$Z_{\mathrm{CFT}}(\Sigma_n) \;=\; \sum_{\Delta,\bar{\Delta}} e^{-n\beta(\Delta+\bar{\Delta})} Z_{\Delta}(\Sigma_n)\bar{Z}_{\bar{\Delta}}(\Sigma_n)$$

$$Z_{\Delta}(\Sigma_n) \;=\; \int_{\substack{F(A)=0 \\ C(A)=C_{\Delta}}} [dA]\; e^{-\int_{\Sigma_n}\Omega_{\mathrm{WP}}}$$

From 2D CFT Quantum Mechanics to 3D AdS Gravity

An `exact' definition of the replica wormhole partition function in 2D CFT

$$Z_{\mathrm{CFT}}(\Sigma_n) = \sum_{\Delta,\bar{\Delta}} \int_{\substack{C(\mathbf{A}) = C_\Delta \\ C(\bar{\mathbf{A}}) = C_{\bar{\Delta}}}} [d\mathbf{A} d\bar{\mathbf{A}}] \, e^{-CS(\mathbf{A})+CS(\bar{\mathbf{A}})}$$

$\mathcal{M}_3 =$  $\times \; S^1$

$$C(\mathbf{A}) = \mathrm{tr}\Big(\mathrm{P}\exp \oint_{S^1} \mathbf{A}\Big)$$

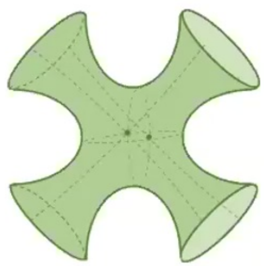
Casimir

From 2D CFT Quantum Mechanics to 3D AdS Gravity

An `exact' definition of the replica wormhole partition function in 2D CFT

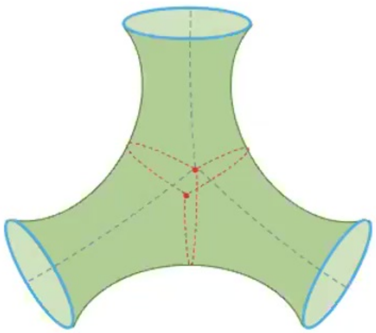
$$Z_{\mathrm{CFT}}(\Sigma_n) \overset{?}{\simeq} \int [d\mathbf{A}d\bar{\mathbf{A}}]\, e^{-CS(\mathbf{A})+CS(\bar{\mathbf{A}})} \overset{?}{\simeq} \int [dg]\, e^{-S_E[g]}$$

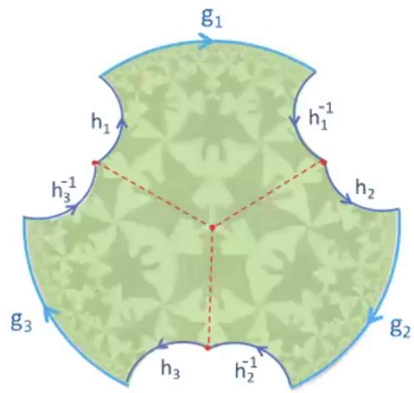
Under which conditions does it reproduce the gravitational prescription?

$\mathcal{M}_3 =$  $\times\ S^1$

$$C(\mathbf{A}) = \mathrm{tr}\Big(\mathrm{P}\exp\oint_{S^1}\mathbf{A}\Big)$$

Casimir





We need to compute the volume
moduli space of cc metrics

$$Z(g_1, g_2, .., g_n) = \mathrm{Vol}\Big(\mathcal{M}(g_1, g_2, .., g_n)\Big)$$

= space of all transition functions subject to the holonomy constraint

$$h_1 g_1 h_1^{-1} \cdot h_2 g_2 h_2^{-1} \cdots h_n g_n h_n^{-1} = 1$$

The computation is straightforward, and in fact standard:

$$\mathrm{Vol}\Big(\mathcal{M}(g_1, g_2, .., g_n)\Big) = \int dh_1\, dh_2 ... dh_n\; \delta(h_1 g_1 h_1^{-1} \cdot h_2 g_2 h_2^{-1} \cdots h_n g_n h_n^{-1} - 1) =$$

$$\int ds\, \rho(s) \int dh_1 ... dh_n\, \chi_s(h_1 g_1 h_1^{-1} \cdots h_n g_n h_n^{-1}) = \int ds\, \rho(s)\, \chi_s(g_1) \chi_s(g_2) ... \chi_s(g_n)$$

Conclusion:

- We have formulated a new holographic relation: an ER bridge can carry microscopic entropy bounded by $S<A/4G_N$.

- The typical state of an ER bridge is given by a thermo-mixed double.

- The Renyi entropy of the TMD coincides with the answer obtained via the replica wormhole calculation.