Title: Quantum gravity from the loop perspective

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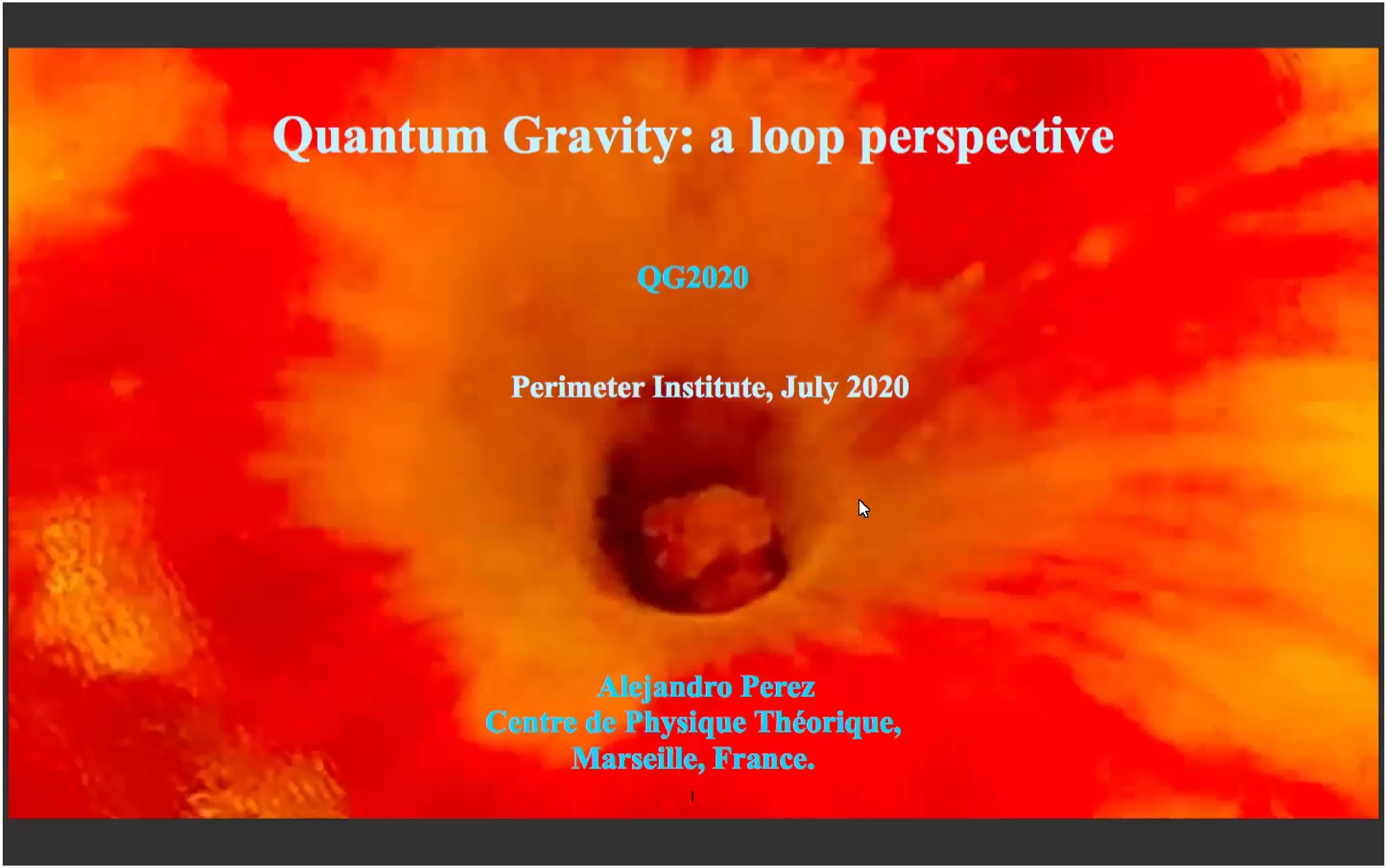
Abstract: I will summarise the main achievements of loop quantum gravity and provide my view on the issues that I consider of central importance for present and future efforts.



Quantum Gravity: a loop perspective
QG2020
Perimeter Institute, July 2020
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Background Independence

- General relativity is diffeomorphism invariant which implies that there is no natural general splitting of the spacetime geometry in terms of a background plus a perturbation.
- Perturbation theory $g_{ab} = \eta_{ab} + h_{ab}$ defines the quantum theory with a background rigid causality. Causal structure is dictated by the background to all orders in perturbation theory.
- In metric variables the theory is naively non-renormalizable.

$$S[g_{ab}] = \frac{1}{2\kappa} \int \sqrt{|g|} \left(R + \Lambda + \alpha_1 R^2 + \alpha_2 R^3 + \cdots + \beta_1 R_{\mu\nu\alpha\sigma} R^{\mu\nu\alpha\sigma} \cdots \right) dx^4,$$

One has to go background independent, BUT...

- Fock space methods are lost (no more nice harmonic oscillators to quantise)
- Euclidean methods (Wick rotations) become unavailable

Can a particle physics scattering approach help?

- Asymptotic background structures (flat, ADS, DS) and a scattering perspective is not viable for stating and trying to solve some key questions in quantum gravity: fate of singularities in BHs and in cosmology, gravity-matter Planckian dynamics and fate of UV QFT singularities.

How to approach the problem? 'CANONICAL' QUANTIZATION

‘Canonical quantization’

what action to start with?

- General relativity in first order variables is naively renormalizable (in the absence of matter and in 4d).

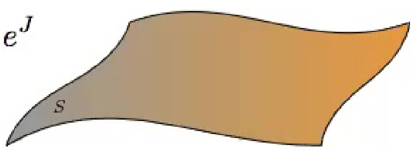
$$
\begin{aligned}
S[e^A_a, \omega^{AB}_a] \;=\; & \frac{1}{2\kappa}\int \overbrace{\epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}(\omega)}^{\text{Einstein}} + \overbrace{\Lambda\, \epsilon_{IJKL} e^I \wedge e^J \wedge e^K \wedge e^L}^{\text{Cosmological Constant}} + \overbrace{\alpha_1\; e_I \wedge e_J \wedge F^{IJ}(\omega)}^{\text{Holst}} \\
+ \; & \underbrace{\alpha_2\; (d_\omega e^I \wedge d_\omega e_I \;-\; e_I \wedge e_J \wedge F^{IJ}(\omega))}_{\text{Nieh-Yan}} + \underbrace{\alpha_3\; F(\omega)_{IJ} \wedge F^{IJ}(\omega)}_{\text{Pontrjagin}} + \underbrace{\alpha_4\; \epsilon_{IJKL} F(\omega)^{IJ} \wedge F^{KL}(\omega)}_{\text{Euler}},
\end{aligned}
$$

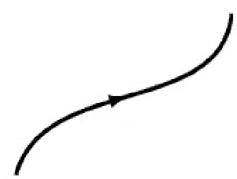
- First order variables are the ones that allow for the coupling of gravity with the matter as we understand it in the standard model (Fermions).

- First order variables carry a natural and non-local geometric meaning (very appealing from the perspective of diffeomorphism invariance)

$$E(\alpha, S) \equiv \int_S \alpha_{IJ} e^I \wedge e^J$$

The fluxes





$$\Lambda(\ell, \omega) \equiv \mathrm{P}\exp - \int_\ell \omega$$

The holonomy

3

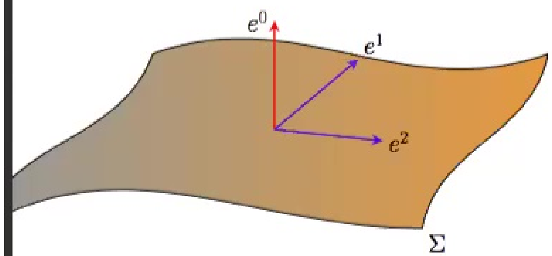
'Canonical quantization'

phase space and discreteness

$$S = \frac{1}{2\kappa}\int \underbrace{(\epsilon_{IJKL} + \frac{1}{\gamma}\eta_{IK}\eta_{JL})}_{p_{IJKL}} (e^I \wedge e^J \wedge F^{KL}(\omega))$$

12 18

$$\delta S = \frac{1}{2\kappa}\int_M \underbrace{2p_{IJKL}\delta e^I \wedge e^J \wedge F^{KL}(\omega) - p_{IJKL}d_\omega(e^I \wedge e^J) \wedge \delta\omega^{KL}}_{\text{e.o.m.}} + \int_{\partial M} \underbrace{\frac{1}{2\kappa}[p_{IJKL}e^I \wedge e^J] \wedge \delta\omega^{KL}}_{p\delta q}$$



Time gauge $e^0_a = n_a$

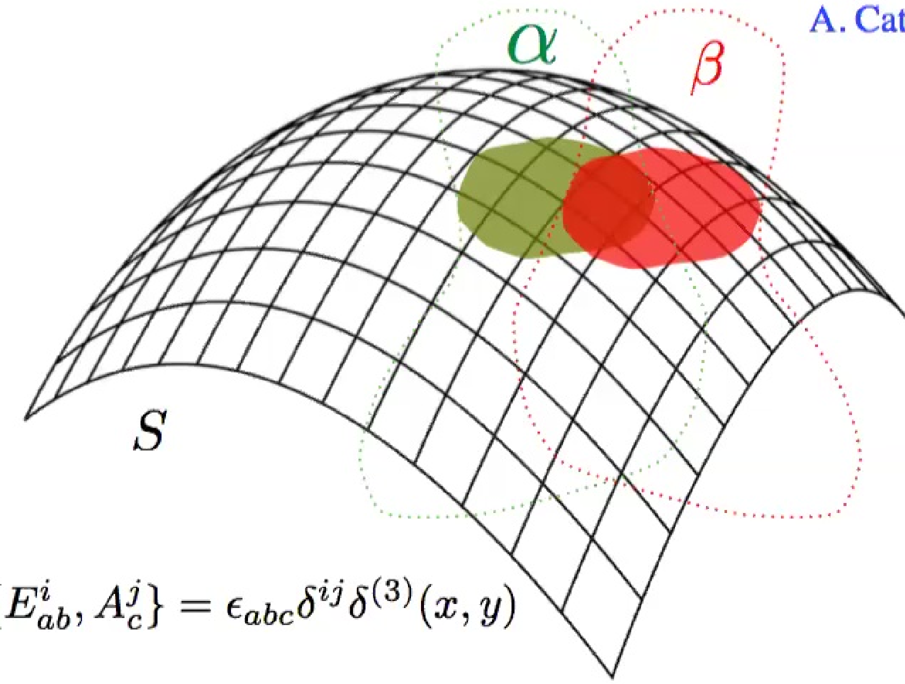
$$\begin{aligned}
\Theta(\delta) &= \frac{1}{\kappa}\int_\Sigma \left(\epsilon_{0jkl}e^0 \wedge e^j \wedge \delta\omega^{kl} + \frac{1}{\gamma}e^0 \wedge e^i \wedge \delta\omega_{0i}\right) - \frac{1}{\kappa}\int_\Sigma \left(\epsilon_{0jkl}e^j \wedge e^k \wedge \delta\omega^{l0} + \frac{1}{\gamma}e^i \wedge e^j \wedge \delta\omega_{ij}\right) \\
&= -\frac{1}{\gamma\kappa}\int_\Sigma [\epsilon_{jkl}e^j \wedge e^k] \wedge \delta\underbrace{(\gamma\omega^{l0} + \epsilon^{lmn}\omega_{mn})}_{\text{Ashtekar-Barbero connection}} \\
&= -\frac{1}{\gamma\kappa}\int_\Sigma [\epsilon_{jkl}e^j \wedge e^k] \wedge \delta A^l
\end{aligned}$$

9

4

Quantization of area in a nut-shell

A. Cattaneo, AP, 2016



$$\{E^i_{ab}, A^j_c\} = \epsilon_{abc}\delta^{ij}\delta^{(3)}(x,y)$$

$$E(\alpha, S) = \int_S \alpha^i e^j \wedge e^k \epsilon_{ijk}$$

$$E^i = \epsilon^i_{jk} e^j \wedge e^k$$

$$\begin{aligned} E(\alpha,S) &\equiv \int_S \mathrm{Tr}[\alpha E] \\ &= \int_{\mathrm{Int}(S)} d(\mathrm{Tr}[\alpha E]) \\ &= \int_{\mathrm{Int}(S)} (\mathrm{Tr}[d_A(\alpha)E] + \mathrm{Tr}[\alpha\, d_A(E)]) \\ &= \int_{\mathrm{Int}(S)} \mathrm{Tr}[(d\alpha + [A,\alpha])E] \end{aligned}$$

$$\{E(\alpha,S), E(\beta,S)\} = \int_{\mathrm{Int}(S)}\int_{\mathrm{Int}(S)} \{\mathrm{Tr}[(d\alpha + [A,\alpha])E], \mathrm{Tr}[(d\beta + [A,\beta])E]\}$$

$$\boxed{\{E(\alpha,S), E(\beta,S)\} = E([\alpha,\beta],S)}$$

Simplest example of edge modes.
[L. Freidel, M. Geiller, E. Livine, D. Pranzetti, AP]

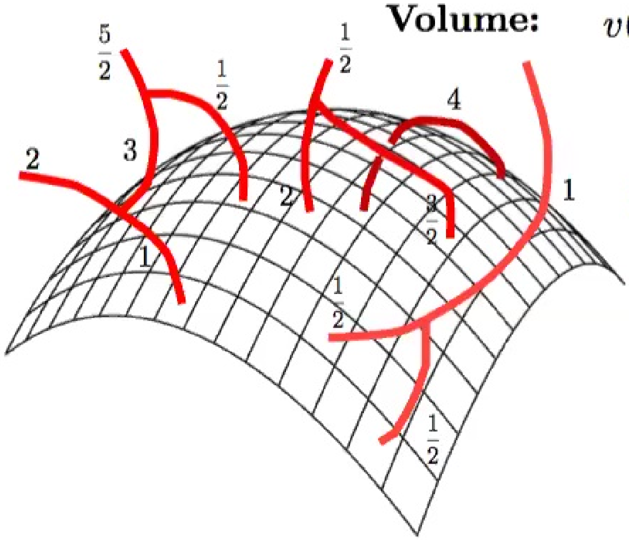
5

Diff-invariant representation of the holonomy-flux algebra

Area: $a(S) = \int_S \sqrt{E_{xy} \cdot E_{xy}}\, dxdy$

Volume: $v(R) = \int_R \sqrt{E_{xy} \cdot (E_{yz} \times E_{zx})}\, dxdydz$

Quantization [Rovelli, Smolin (1994) Ashtekar-Lewandowski (1997)]



Angular Momentum Commutations → **Angular Momentum Discreteness**

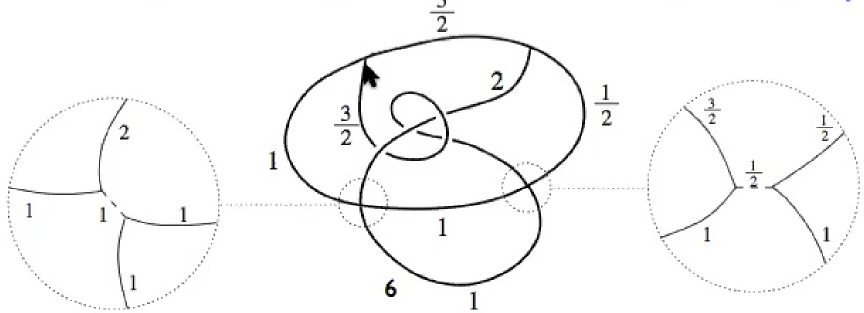
$\{E(S,\alpha), E(S,\beta)\} \approx \kappa\gamma E[[\alpha,\beta],S]$

area quantum $= \gamma \ell_p^2 \sqrt{j(j+1)}$

The Hilbert space of LQG

[Rovelli-Smolin, Ashtekar-Lewandowski, Fleischhack-Lewandowski-Okolow-Sahlmann-Thiemann 1991-2006]

Spin-network basis states admit a geometric interpretation as twisted quantum geometries [Freidel-Speziale (2010)]



There is a (unitarity inequivalent) dual representation where the 'vacuum' is peaked on flat connections. [B. Bahr, B. Dittrich, M. Geiller]

Defining the dynamics:
Quantum constraint algebra

$G(\alpha) \equiv$ generator of $SU(2)$ gauge transformations along α^i

$V(N) \equiv$ generator of 3diffeos along N^a

$S(N) \equiv$ scalar constraint with lapse N

$$\{S(N), S(M)\} = V(S^a)$$

$$S^a \equiv q^{ab}(N\partial_a M - M\partial_a N)$$

Anomaly freeness was too weak → quantisation too ambiguous Thiemann, 96

$0 = \lim_{\epsilon \to 0}[\widehat{S}_\epsilon[N], \widehat{S}_\epsilon[M]] = \lim_{\epsilon \to 0} \widehat{V}_\epsilon(S^a) = 0$ Gambini-Lewandowski-Marolf-Pullin, 97

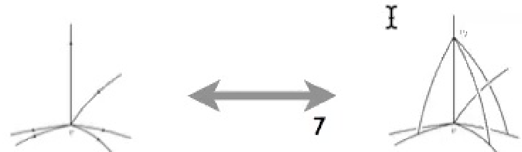
Two issues with Thiemann's seminal quantisation of S[N] that now they seem close to resolved.

ultra local action ≡ no propagating d.o.f. Smolin, 1996

$$[\widehat{S}_\epsilon[N], \widehat{S}_\epsilon[M]] = \widehat{V}_\epsilon(S^a)$$

Laddha-Varadarajan 2011, Tomlin-Varadarajan 2012, Varadarajan 2018 Ashtekar-Varadarajan (unpublished)

- In the Smolin weak coupling model $G = U(1)^3$ anomaly freeness can be checked
- Non trivial propagation is induced by $S[N]$
- Unpublished work in progress indicates that all this works with gravity $G = SU(2)$



7

Dynamics: spin foams

Δ is a cellular decomposition of the spacetime manifold that provides a truncation in the possible spin-network histories being summed over.

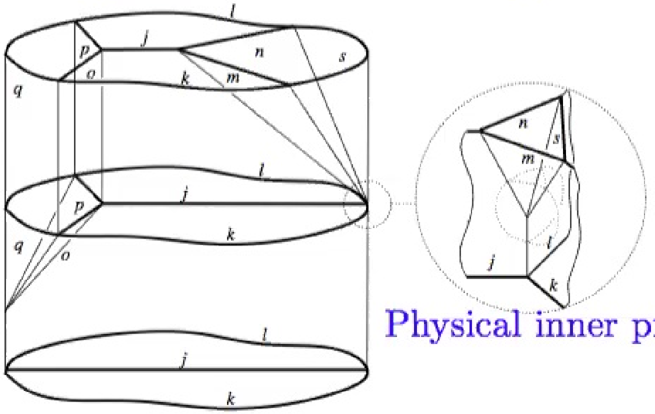
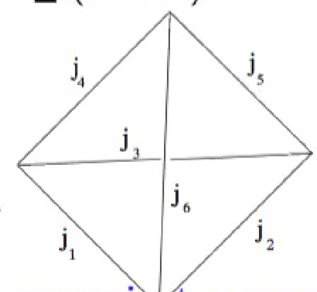


Illustration in 3-dimensions (gravity = BF theory)

no regulator dependence

$$\langle s, s' \rangle_{\text{phys}} = \langle s, Ps' \rangle_{\text{phys}} = A_{\Delta}(s \to s') = A_{\Delta'}(s \to s')$$

$$= \sum_{\{j\}} \prod_{f \subset F_{s \to s'}} (2j_f + 1)^{\frac{\nu_f}{2}} \prod_{v \subset F_{s \to s'}}$$



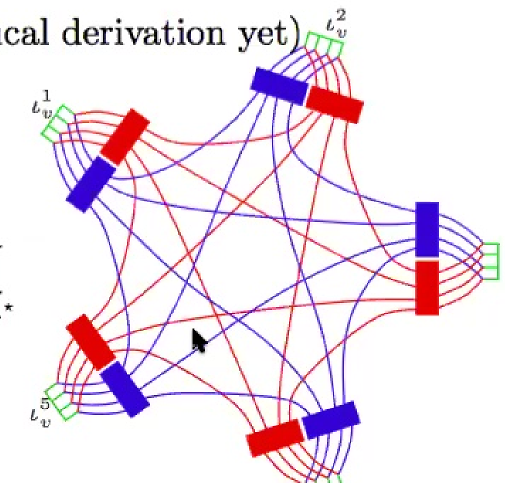
Physical inner product can be derived from the canonical as well as the covariant perspective

Noui-AP 2004 Freidel-Louapre 2002

In 4-dimensions spin foams are derived imposing constraints to BF amplitudes (no canonical derivation yet)

$$Z = \int \mathcal{D}B\mathcal{D}\omega \exp i \left(\int \langle B \wedge F(\omega) \rangle + \text{constraints } B^{\star} \to e \wedge e \right)$$

$$\langle s, s' \rangle^{\Delta}_{\text{phys}} = A_{\Delta}(s \to s') = \sum_{j_f} \sum_{\iota_e} \prod_{f \in \Delta^{\star}} \mathrm{d}_{|1-\gamma|\frac{j}{2}} \mathrm{d}_{(1+\gamma)\frac{j}{2}} \prod_{v \in \Delta^{\star}}$$



Regulator Δ dependence!

(non trivial renormalization needed)

Dynamics: spin foams

$$Z = \int \mathcal{D}B\mathcal{D}\omega \exp i \left(\int \langle B \wedge F(\omega) \rangle + \text{constraints } B^{\star} \to e \wedge e \right)$$

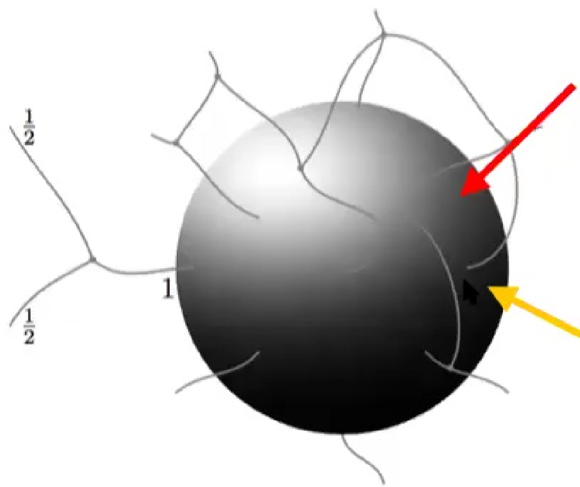
- 4-dimensional spin-foam amplitudes naturally follow from the imposition of the $B \to e \wedge e$ constraints on the topological BF theory amplitudes [Livine-Speziale LS, Freidel-Krasnov FK, Engle-Pereyra-Rovelli-Livine EPRL].
- The vertex amplitude matches the expected semiclassical Einstein amplitudes in the large spin limit (Regge amplitudes for the 4-simplex) [Barrett-Dowdall-Fairbairn-Hellmann, Bahr-Steinhaus, Dona-Speziale]. There is a lot of activity in trying to precisely understand semiclassical limit of amplitudes of a full cellular decomposition Δ [Conrady-Freidel, Bonzom, Hellmann-Kaminski, Han, Oliveira-Engle-Kaminski, Dona-Gozzini-Sarno, Speziale-Dona (work in progress), Asante-Dittrich-Haggard, Han-Huang-Liu-Qu].

  Key question: how to get rid of regulator dependence Δ (continuum limit): The main idea is that renormalisation tools are necessary to answer the question. Continuum limit to be found on its fixed point where Δ independence is achieved (background-free asymptotic safety). All this is work in progress

- Renormalisation is studied in models to build up experience with RG flow in a background independent context (no background geometry to define scales is available). Tensor network techniques, embedding maps... [Dittrich et al., Bahr-Steinhaus et al., Thiemann et al.]
- Analytic simplifications of the LS-FK-EPRL amplitudes open the door for efficient numerical evaluations and first attempts of coarse graining [Dona-Sarno-Speziale, Speziale, Bahr, Steinhaus, Dittrich]

New interesting developments

- There is work on 3d which relate to the ADS/CFT approaches [Dittrich-Goeller-Livine-Riello]
- Spinorial boundary formulation, discreteness of quantum geometry observables in terms of continuous boundary field parametrizations. W. Wieland
- Edge modes quasi-local holography. New boundary charges on embedded surfaces. Promising reacher structure to extend the standard description of LQG and include new degrees of freedom. Freidel-Geiller-Livine-Pranzetti



New edge modes:
Induced 2-metric $\rightarrow SL(2,\mathbb{R})$ charges
Boost Charges (relaxing time-gauge)
and 3-Diffeomorphism charges

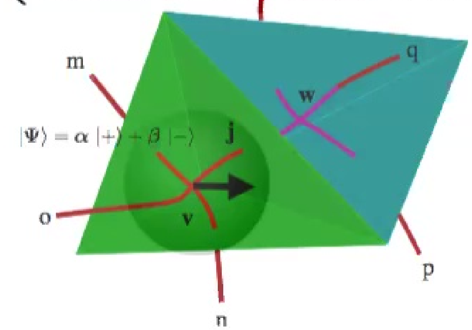
E-Fluxes $\rightarrow SU(2)$ charges

Some important aspects of what we have so far

(personal account)

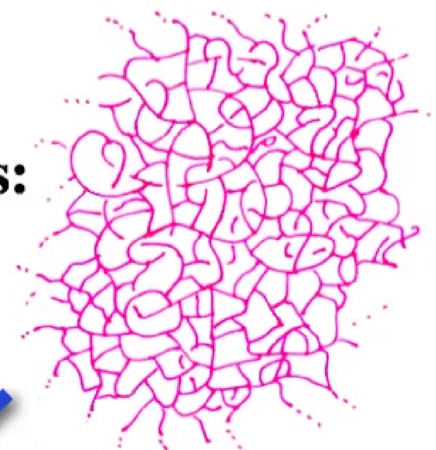
Recovering continuum geometries

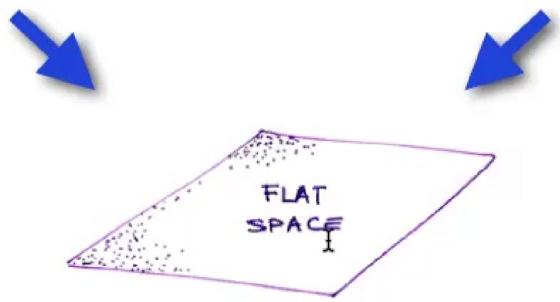
Smooth geometry (with smooth matter fields) is emergent



Weave states
[Ashtekar, Rovelli, Smolin 1992]

Spin-network states:
atoms of geometry



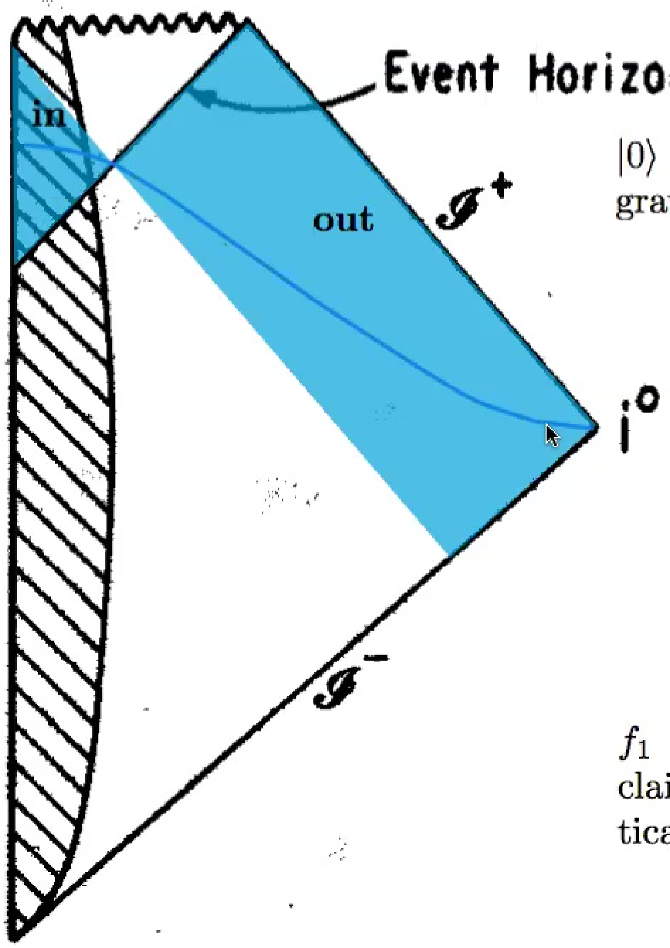


Spin-network states can be thought of as many-body quantum system: a spin system of quantum polyhedra. Semiclassical states require a specific structure in the space correlations (entanglement).
[Bianchi, Guglielmon, Hackl, Yokomizo, Dona, Vilensky]

12

Black hole entropy
non-holographic approach

Sorkin et al. *Phys.Rev.D* 34 (1986) 373-383



$|0\rangle$ is the so-called in-vacuum that idealises the boundary conditions of gravitational collapse.

$$\rho_{\text{out}} \equiv \text{Tr}_{\text{in}}\left[|0\rangle\langle 0|\right]$$

$$S_{ent} \equiv -\text{Tr}_{\text{out}}[\rho_{\text{out}} \log(\rho_{\text{out}})]$$

$$S_{\text{ent}}(\Sigma_2) = \underbrace{f_1 \frac{A}{\epsilon^2} + f_2 \log(\epsilon^{-2} A)}_{\text{Divergent and ambiguous in QFT}} + S_0$$

f_1 is completely ambiguous (regularisation dependent) while f_2 is often claimed to be well defined (is it truly? e.g. ensemble dependence in statistical mechanics).

Answer in loop quantum gravity

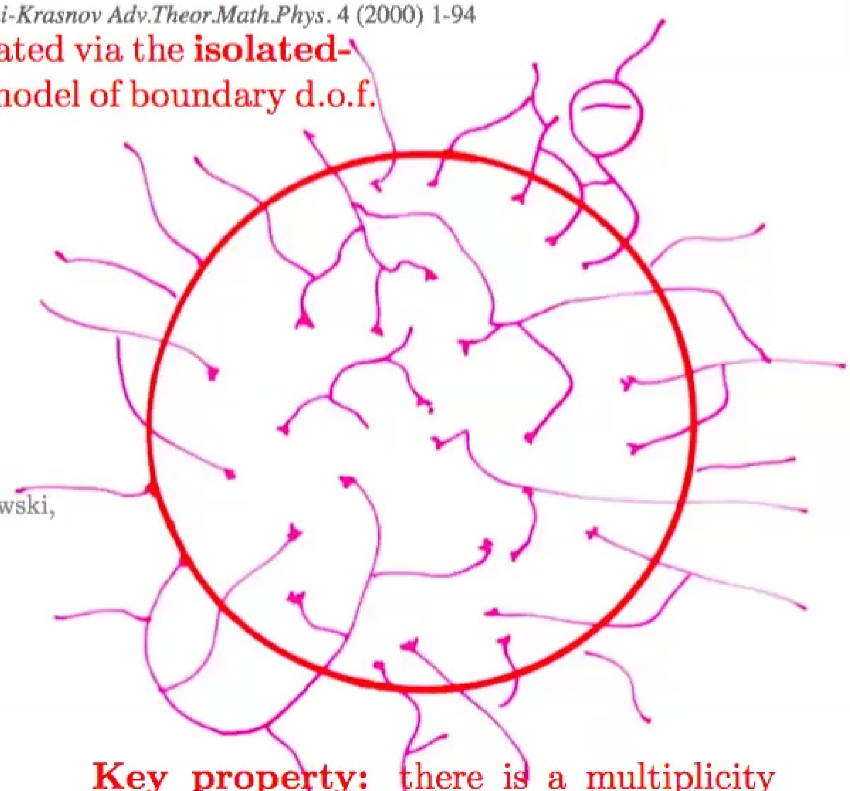
AP. Rept. Prog. Phys. **80** (2017)

$$S_{\rm ent}(\Sigma_2) = \underbrace{f_1 \frac{A}{\epsilon^2} + f_2 \log(\epsilon^{-2} A)}_{} + S_0$$

Dimension of the **stretched** boundary physical Hilbert space

Ashtekar-Baez-Corichi-Krasnov Adv.Theor.Math.Phys. 4 (2000) 1-94

Calculated via the **isolated-horizon** model of boundary d.o.f.



Matter d.o.f. do not contribute in the IH formalism

$$S_{bh} = \frac{\gamma_0}{\gamma} \frac{A}{4\ell_p^2}$$

Rovelli (1996), Ashtekar-Baez-Krasnov-Corichi (1999), Alesci, Agullo, Ashtekar, Baez, Barbero, Bianchi, Bodendorfer, Borja, Corichi, Diaz-Polo, Domagala, Engle, Frodden, Ghosh, Krasnov, Kabul, Lewandowski, Livine, Majumdar, Meissner, Mitra, AP, Pranzetti, Rovelli, Sahlmann, Terno, Thiemann, Villasenor, ...

$$\gamma = \gamma_0$$

Barbero-Villasenor (2008)

There are arguments suggesting that the inclusion of matter can remove the γ dependence and give the Bekenstein-Hawking result. But one would need to generalize the IH calculation where matter does not contribute to counting.

Bianchi (2012), Ghosh-AP (2013), AP (2014)

Key property: there is a multiplicity of micro states (quantum geometries) for each macroscopic classical black hole geometry.

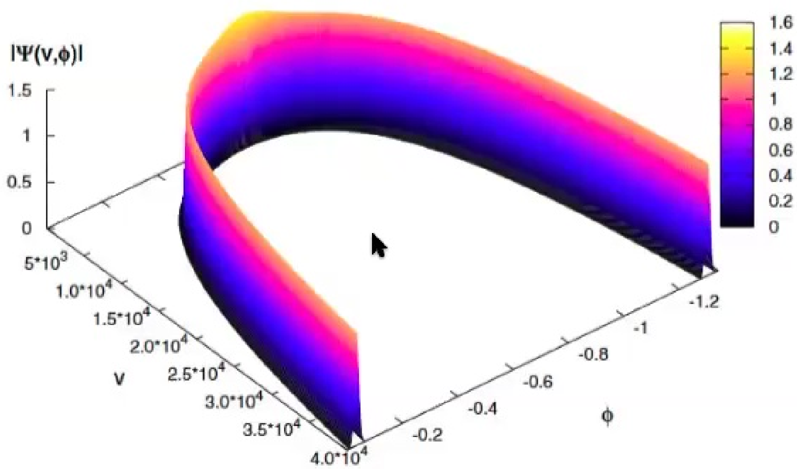
14

Models of Quantum Cosmology

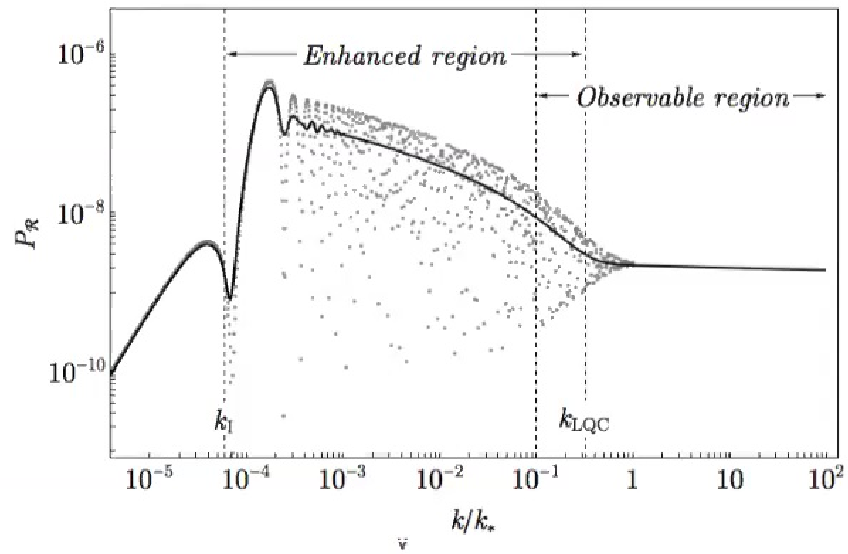
singularity resolution, generality of the bounce

Planckian discreteness generically resolves **big-bang** singularity in the minisuperspace LQG models.

Bojowald (2001), Ashtekar, Singh, etc.



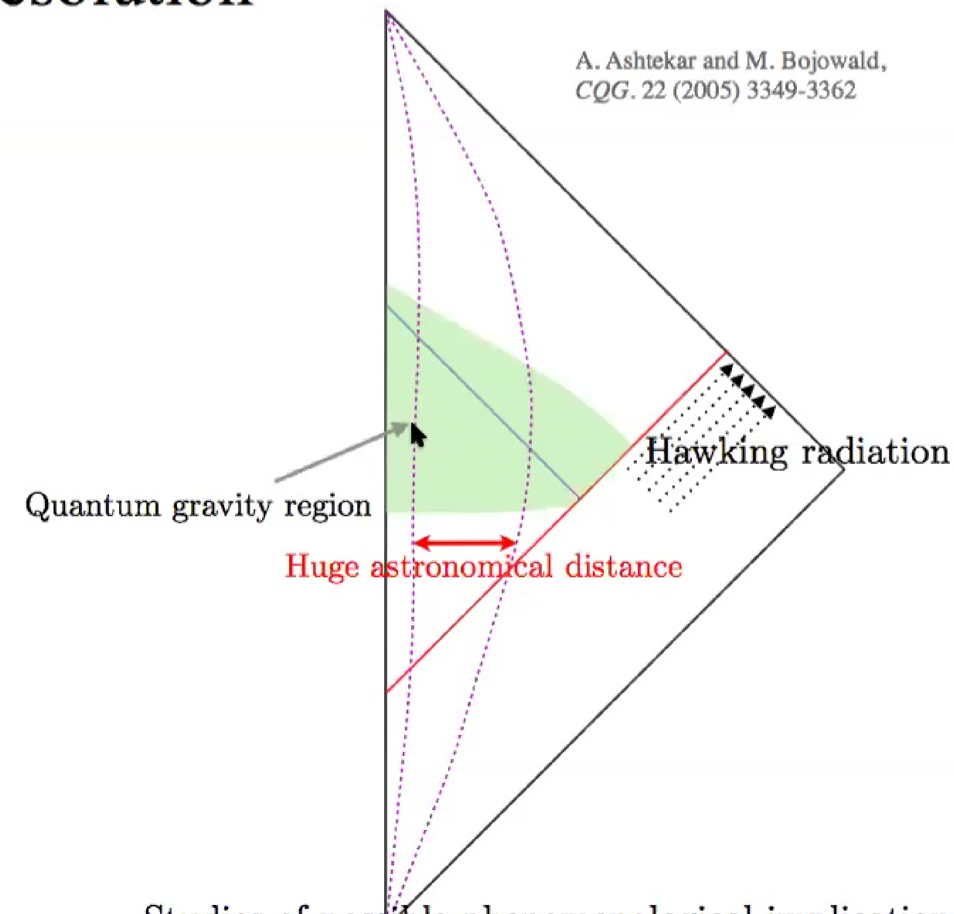
From Ashtekar, Pawlowski, Singh PRD (2006)



From Agullo, Singh PRD (2016)

Gravitational collapse models
singularity resolution

Bojowald, Modesto, Ashtekar, Gambini-Pullin-Olmedo, Haggard-Rovelli-Speziale-Vidotto, Corichi-Singh, Ashtekar-Olmedo-Singh, Bahrami-Alesci-Pranzetti, Bodendorfer-Mele-Munch, Ben-Achur-Liu-Noui, ...



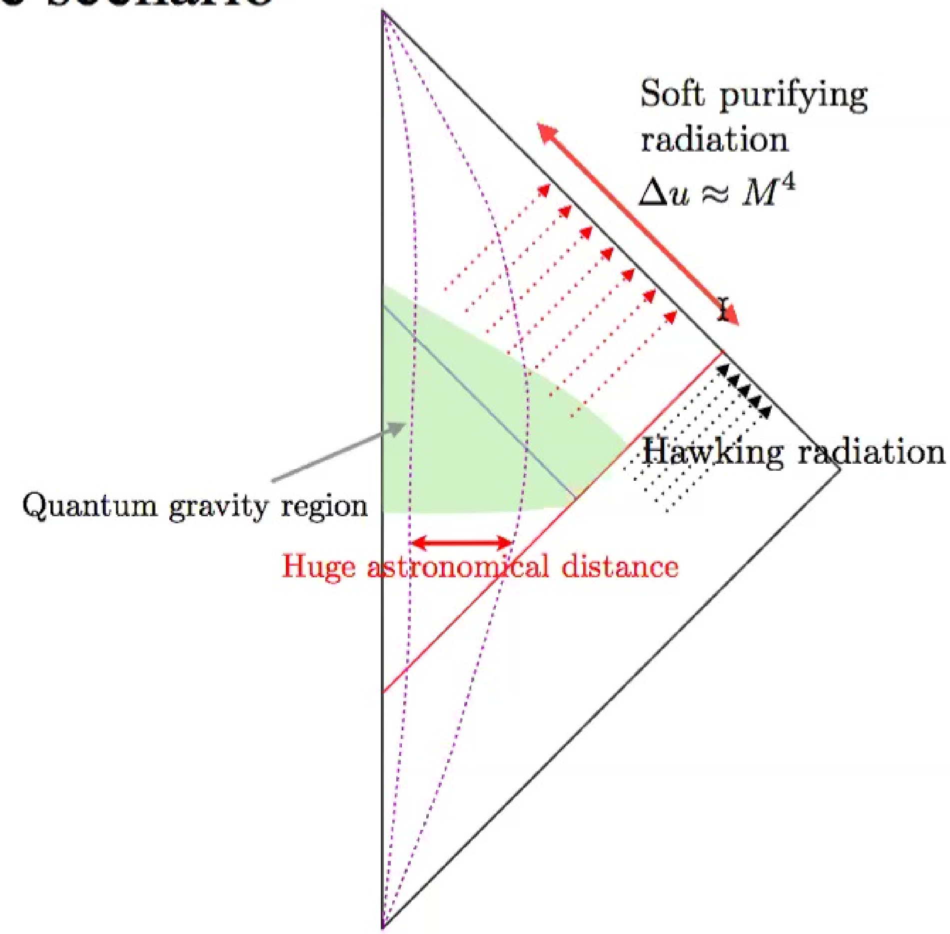
Studies of possible phenomenological implications [Pullin-Gambini, Rovelli-Vidotto-Barrau]

Information in black hole evaporation

a remnant like scenario

Ashtekar-Varadarajan CGHS analysis, Bianchi-Smerlak, Bianchi-Smerlak-DeLorenzo, Haggard.

Information is recovered in QFT modes correlated to the large number of Hawking quanta (with total energy M (initial BH mass)). The cumulative energy of purifying radiation must be at most of the order of m_p from energy conservation. A back of the envelop calculation implies a purification time of the order of M^4. This is confirmed by 2d BH models where Page curve can be exactly described if one assumes QFT propagation through the QG region.

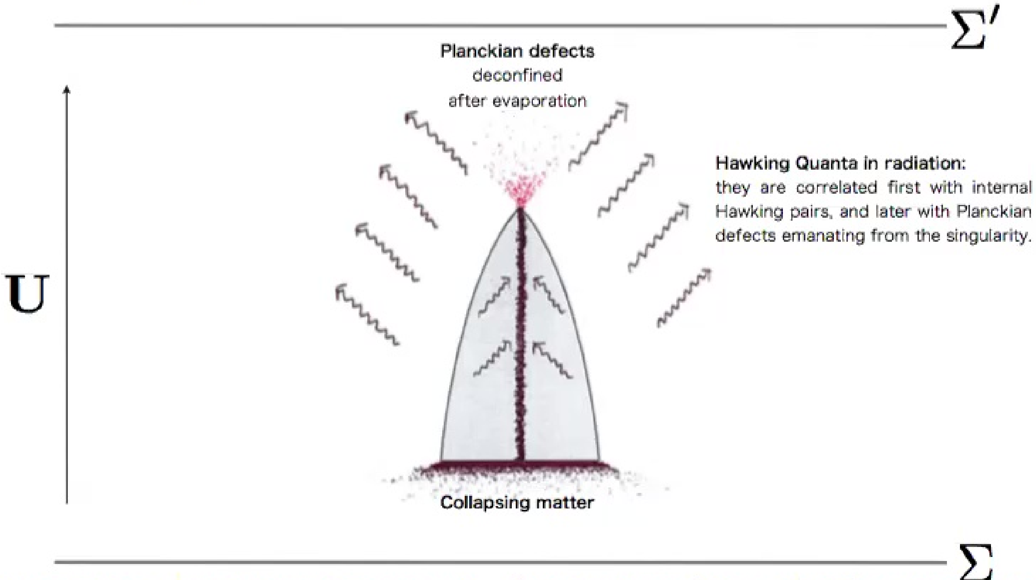


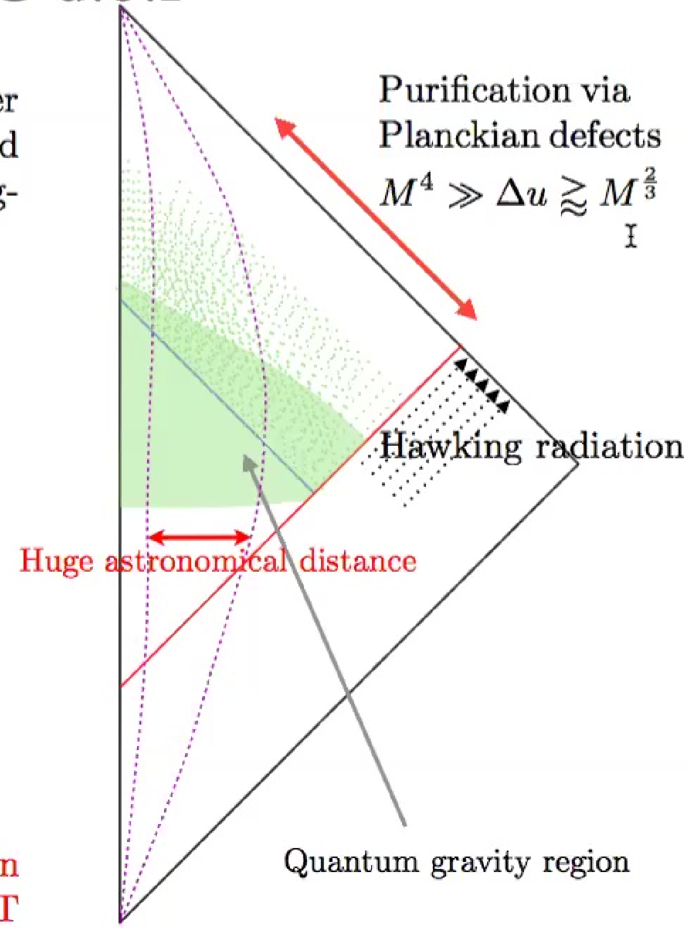
Information in black hole evaporation

purification via Planckian QG d.o.f

AP 2015, Amadei-Liu-AP 2019

Information is recovered in Planckian defects correlated to the large number of Hawking quanta (with total energy M (initial BH mass)). The expected degeneracy of Minkowski vacuum in LQG allows for purification with negligible energy cost. Purification time can be as short as $M^{2/3}$.

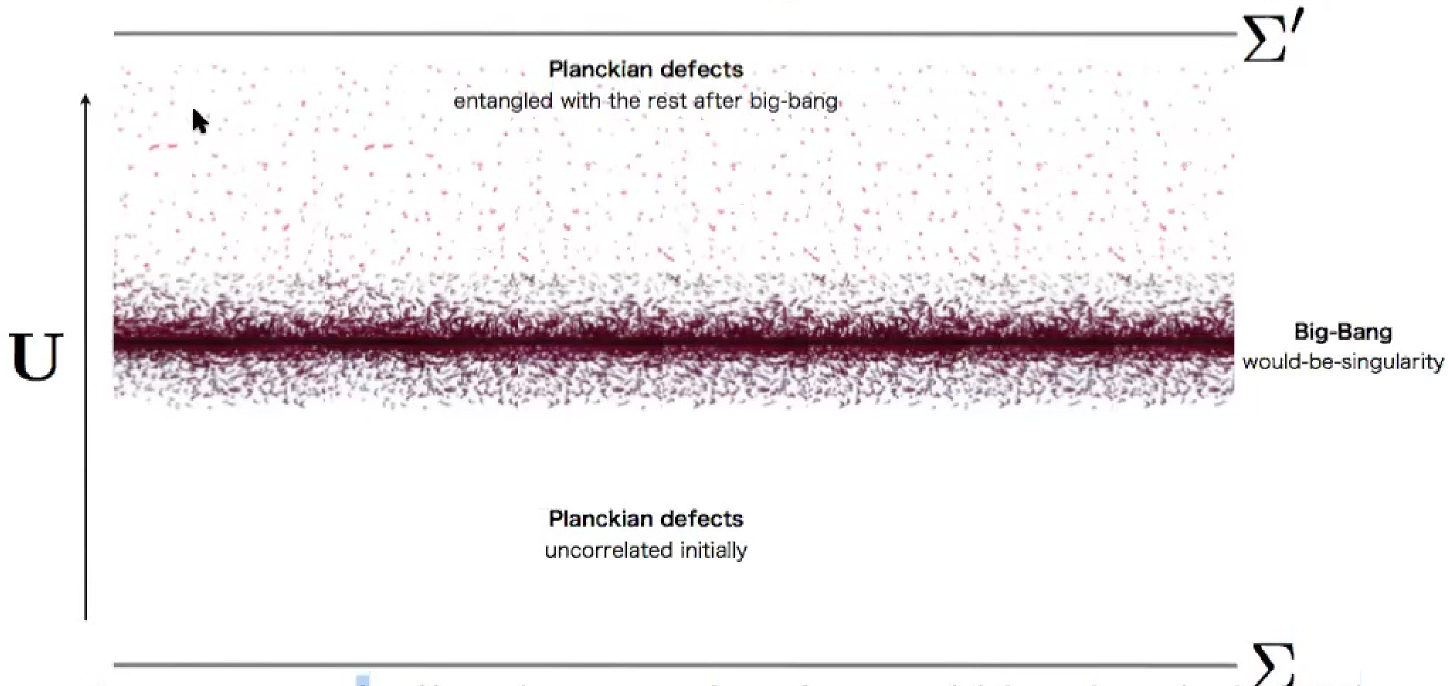




There is no long-lived remnant. As the purifying d.o.f. cannot be capture in an effective QFT description information is degraded from its initial QFT form to correlations with Planckian defects inaccessible to QFT probes.

Information in black hole evaporation

the scenario is realized in quantum cosmology



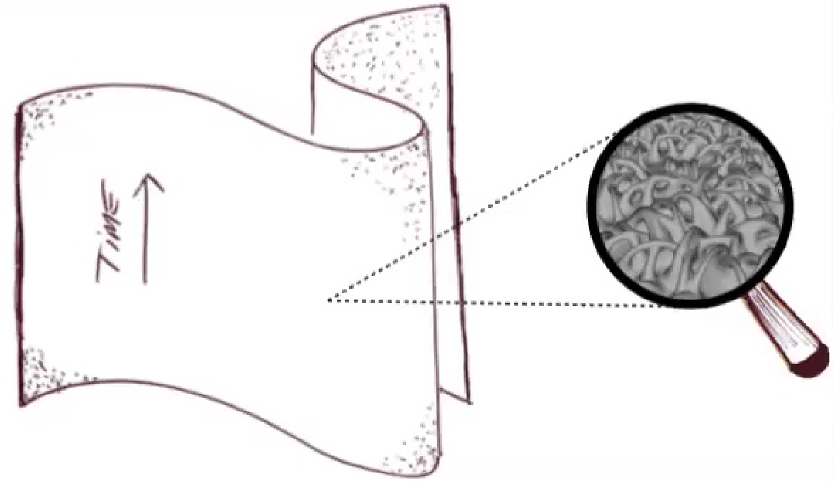
Entropy jumps generically when crossing the would-be-singularity. When asymptotic curvature R is small the entropy jump is proportional to R!

$\rho_{\rm red}(\Sigma)$ is **pure** $\xrightarrow{\text{dynamics}}$ $\rho_{\rm red}(\Sigma')$ is **mixed**

L. Amadei, H. Liu, AP: gr-qc/1912.09750, gr-qc/1911.06059.

Emergence of the cosmological constant
phenomenology of fundamental discreteness

T. Josset, AP and D. Sudarsky, *Phys.Rev.Lett*. 118 (2017) 2, 021102.
AP. D. Sudarsky and J.D. Bjorken *Int.J.Mod.Phys.D* 27 (2018) 14, 1846002
AP and D. Sudarsky, *Phys.Rev.Lett*. 122 (2019) 22, 221302



Trace free Einstein's equations

$$\mathbf{R}_{ab} - \tfrac{1}{4}\mathbf{R}g_{ab} = 8\pi\left(\mathbf{T}_{ab} - \tfrac{1}{4}\mathbf{T}g_{ab}\right)$$

$$\underbrace{\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab}}_{\mathbf{G}_{ab}} + \frac{1}{4}(\mathbf{R}+\mathbf{T})g_{ab} = 8\pi\mathbf{T}_{ab}$$

$$\tfrac{1}{4}\nabla_b\left(\mathbf{R} + 8\pi\mathbf{T}\right) = 8\pi\nabla^a\mathbf{T}_{ab}$$

Need to satisfy the integrability condition $d\mathbf{J} = 0$

$$\mathbf{J}_b \equiv 8\pi\nabla^a\mathbf{T}_{ab}$$

$$\Lambda = \int \mathbf{J}_b dx^b = \frac{2\pi\alpha\hbar}{m_p^2}\int_{t_0}^{t} T\mathbf{R}^2 dt$$

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} = 8\pi\mathbf{T}_{ab} - \underbrace{\left[\Lambda_0 + \int_\ell \mathbf{J}\right]}_{\text{Dark Energy }\Lambda} g_{ab}$$

$$\Lambda \approx \frac{\overline{m}_t^4 T_{ew}^3}{m_p^7} m_p^2 \approx \underbrace{\left(\frac{T_{ew}}{m_p}\right)^7}_{10^{-120}} m_p^2$$

20

Main insights produced

- The isolated-horizon framework that gives a natural description of the origin of BH entropy (not the ultimate yet; matter needs to play a role).
- Singularity resolution in quantum cosmology models.
- Singularity resolution in models of spherically symmetric collapse.
- Potentially rich phenomenology in quantum cosmology and BHs.
- It provides a novel mechanism for the resolution of the information puzzle in BH evaporation.
- Diffusion into Planckian degrees of freedom (fundamental discreteness) provides a simple tentative solution of the cosmological constant problem.

Open problems

- Control of dynamics (regulator independence/4diffeo invariance). Ongoing efforts in the spin foam as well as the scalar constraint quantisation fronts [Dittrich et al., Bahr et al., Thiemann et al.].

Renormalisation:
'Desacralisation' of the
classical Einstein-action

CANONICAL QUANTUM GRAVITY → ~~CANONICAL~~ QUANTUM GRAVITY

- Semiclassical limit (continuum limit). Intertwined with the previous item; complementary ongoing efforts on the kinematical sector (architecture of spacetime [Bianchi et al.])

- How does low energy Lorentz invariance reconcile with fundamental discreteness? [Collins et al.]

- Matter coupling. Anything goes? or are there non trivial consistency requirements in the formalism that could restrict and guide a more fundamental understanding (hierarchy problems in the standard model, physics beyond the standard model). Not so well explored territory (e.g. fermion coupling consistency [Gambini-Pullin 2015], proposals of matter as emergent from loopy pre geometric structures [Bilson-Thompson-Markopoulou-Smolin (2006)])

The status of LQG

- LQG is an ongoing effort to produce a background independent quantisation of gravity in 4 dimensions. The basic variables are extended and non-local (Relational and diff covariant).
- It provides a concrete formalism where all the notions of the theory are clearly defined and carry a concrete geometric interpretation. This produces a framework where the natural questions one would like to ask to a QG theory can be formulated.
- One of the key features is the fundamental discreteness and the necessity of recovering smoothness as an emergent product.
- Recent developments at the canonical level (anomaly free quantisation of the scalar constraint) as well as numerical and analytic progress in the spin foam approach (background independent renormalisation) picture a promise of relevant progress.

Thank you!