Title: Approaches to Quantum Gravity: Key Achievements and Open Issues

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Abstract: This talk will provide an overview of current approaches to quantum gravity, with their respective merits and open problems (`comparative quantum gravity'). To this aim I will focus on some key issues that must be addressed by all approaches

Approaches to Quantum Gravity: Key Achievements and Open Issues

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The precarious state of Quantum Gravity

- Multitude of promising ideas and ansätze, but despite well developed approaches no convergence in sight on what is the 'right answer' (rather the contrary!) and little communication 'across the fence'.

- Nature remains tight-lipped about what comes after Einstein and the Standard Model of particle physics: in spite of intense searches and numerous new data from astrophysics and particle physics so far (almost) no evidence for 'new physics'.

Main obstacle: smallness of Planck scale

$\Rightarrow$ no definite and unambiguous observational signatures for any of the approaches!

The steady progress of Quantum Gravity?

Selected Topics

- UV Completeness

- Perturbative *vs.* Non-Perturbative

- Conceptual Issues

- Emergence

- The Real World

- Outlook

... necessarily very incomplete overview, but see other overview talks and panel discussions at this conference for complementary and more complete information.

UV Completeness (Finiteness)

Perturbative quantum gravity is **non-renormalizable**

$$\Gamma_{div}^{(2)} = \frac{1}{\varepsilon} \frac{209}{2880} \frac{1}{(16\pi^2)^2} \int dV\, C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\tau} C_{\lambda\tau}{}^{\mu\nu}$$

[Goroff & Sagnotti(1985); van de Ven(1992)]

$\Rightarrow$ infinitely many ambiguities and loss of predictivity

Current 'schism' in QG community is result of drawing two opposite conclusions from this basic fact:

- Consistent quantization of gravity requires the replacement of Einstein's theory by a radically different theory at the very shortest distances; or

- UV divergences are artefacts of perturbative treatment $\Rightarrow$ quantum GR can 'stand on its own feet' upon proper *non-perturbative* quantization.

How can UV Completeness be achieved ?

A tall order: unlike previous theories of physics, *this* theory is supposed to be valid *at all* scales !

- Superstring Theory

- $N=8$ Supergravity

- Asymptotic Safety

- Discrete approaches:
  LQG/Spin Foams/GFT/CDT/...

Superstring Theory

(Super-)String magic: quantum gravity path integral reduces to (sum over) *finite*-dimensional integrals:

$$\text{amplitude} = \sum_{g \geq 0} g_s^{2-2g} \int_{(\mathcal{S})\mathcal{M}_{g,n}} d\mu_{(\mathcal{S})g,n} \left\langle V_1(P_1) \cdots V_n(P_n) \right\rangle_{\Sigma_g}$$

where

- $\langle \cdots \rangle$ = CFT correlator on Riemann surface Σ_g of genus g

- $(\mathcal{S})\mathcal{M}_{g,n}$ = moduli or supermoduli space of n-punctured Riemann surface of genus g with suitable measure $d\mu_{(\mathcal{S})g,n}$.

- $V_j(P_j)$ = vertex operator insertion at $P_j \in \Sigma_g$

Necessary ingredient: *modular invariance* with modular group $\mathcal{G}$ ($\mathcal{G} = SL(2, \mathbb{Z})$ for $g = 1$, and $\mathcal{G} \subset Sp(2g, \mathbb{Z})$).

No UV divergences, but in presence of tachyons there are IR divergences $\equiv$ integral over $\mathcal{M}_{g,n}$ does not converge at cusp(s) $\Leftrightarrow$ unstable vacuum ($m_{tachyon}^2 < 0$)

The State of the Art

- Mainly 'quasi-free' world-sheet theories, but difficult for curved backgrounds (*e.g.* $AdS_5 \times S^5$!).

- Higher genus correlators known for low genus g

- For $g \geq 5$ supermoduli space no longer split: superspace measure no longer factorizes into fermionic and bosonic integrals (no "Fubini theorem")
  [for $g = 2$:D'Hoker,Phong(2002); $g > 2$: Donagi,Witten:1304.7798; Sen:1512.00026]

- Sum over genera g (probably) does not converge.

- Non-supersymmetric backgrounds?

- Very sophisticated mathematics even for $g = 1$ (*e.g.* automorphic representations and modular forms)
  [See e.g. P. Fleig et al.: "Eisenstein series and automorphic representations"]

$\Rightarrow$ although path forward seems clear, extremely difficult technical issues $\rightarrow$ still a long way to go ....

$N = 8$ Supergravity

Unique theory (modulo gauging), *most symmetric* known field theoretic extension of Einstein's theory

$$1 \times [2] \oplus 8 \times \left[\frac{3}{2}\right] \oplus 28 \times [1] \oplus 56 \times \left[\frac{1}{2}\right] \oplus 70 \times [0]$$

Large part of work since 1980s on string unification based on, or inspired by this theory and its hidden symmetries [Cremmer,Julia(1979)]

We now know that $N=8$ supergravity is more finite than expected:

- behaves like $N=4$ super-Yang-Mills up to four loops

- However: recent computation at five loops shows divergence at $D_{cr} = \frac{24}{5} \overset{?}{=} 2 + \frac{14}{L} < D_{cr} = \frac{26}{5} = 4 + \frac{6}{L}$ (for $L = 5$)
  [Bern,Carrasco,Chen,Edison,Johansson,Parra-Martinez,Roiban,PRD98(2018)086021]

No fully supersymmetric and fully $E_{7(7)}$ invariant counterterm known (would require an infinite string of higher order interactions), but finiteness would probably require novel symmetries

$\Rightarrow$ **Question of UV finiteness is still up in the air !**

Asymptotic Safety

Approach is closest in spirit to conventional QFT ideas:
→ if true, *no need* for fancy schemes beyond QFT !

[Weinberg(1979),Reuter(1995), ... → Saueressig's talk]

- UV limit of gravity is determined by a non-Gaussian fixed point of the gravitational renormalization group (RG) flow.

- NGFP belongs to a UV critical hypersurface *of finite dimension* within the ∞-dimensional space of *essential couplings* (a coupling is called 'essential' if it cannot be absorbed into a field redefinition).

- Aim: construct scale dependent effective action Γ_k

$$\lim_{k\to\infty} \Gamma_k = \text{bare action} \quad , \quad \lim_{k\to 0} \Gamma_k = \text{effective low energy action}$$

- M_{Planck} is analogous to Λ_{QCD}: *lower end* of asymptotic scaling regime $\Rightarrow$ testable only if some prediction can be made about IR limit as theory flows down from asymptotically safe NGFP.

- Relevant information in universality classes of RG flows.

Circumstantial Evidence

To analyze FRGE truncate to *finite dimensional subspace* of couplings: $\tilde{\Gamma}_k[\bar{g}+h, \ldots] \approx \sum_{j=1}^{n} u_j(k) P_j[\bar{g}, h, \ldots] \Rightarrow$

NGFP's for various gravitational and matter couplings
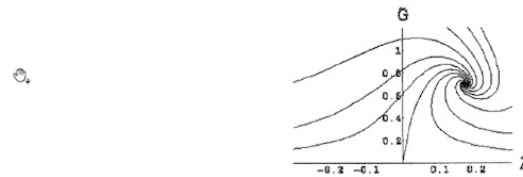
the flow of the dimensionless cosmological constant and Newton constant in the Einstein-Hilbert truncation.

- How to go beyond circumstantial evidence?

- Unitarity and/or ghosts for higher derivative couplings?

- Euclidean *vs.* Lorentzian?

- Physical meaning and universality of running couplings?

- See also: [J.Donoghue, 1911.02967[hep-th]; A.Bonanno et al.,2004.06810[gr-qc]]

Loop Quantum Gravity

Very different quantization scheme $\rightarrow$ issues concerning UV divergences are often obscured.

Kinematical Hilbert space $\mathcal{H}_{kin}$ is *non-separable*:

$$\langle x|x'\rangle = \delta(x - x') \quad \rightarrow \quad {}_{\circ}\langle x|x'\rangle = \begin{cases} 1 & \text{if } x = x' \\ 0 & \text{otherwise} \end{cases}$$

$\rightarrow$ 'pulverize' space (real line) $\Rightarrow$ special features:

- Operators not weakly continuous.

- Cannot 'see' UV divergences (nor anomalies)?

- No negative norm states?

However: if LQG is to recover semi-classical space-time and low energy EFT it *must* face up to the problem of non-renormalizability $\Rightarrow$ if successful, resulting low energy theory via an effective action must reduce to some version of Asymptotic Safety.

Status of Hamiltonian constraint

- **Main success:** definition of regulated Hamiltonian (with $\epsilon > 0$) by means of kinematical operators (volume, etc.) [Thiemann(2000)]

$$\hat{H}[N, \epsilon] = \sum_\alpha N(v_\alpha) \, \epsilon^{mnp} \mathrm{Tr}\Big(\big(h_{\partial P_{mn}(\epsilon)} - h^{-1}_{\partial P_{mn}(\epsilon)} \big) h_p^{-1} \big[h_p, V \big] \Big)$$

$$+ \frac{1}{2}(1 + \gamma^2) \sum_\alpha N(v_\alpha) \, \epsilon^{mnp} \mathrm{Tr}\Big(h_m^{-1} \big[h_m, \bar{K} \big] h_n^{-1} \big[h_n, \bar{K} \big] h_p^{-1} \big[h_p, V \big] \Big)$$

- Diffeomorphism constraint solved formally: $\mathcal{X}_\Gamma = \sum_{\phi \in \mathrm{Diff}} \Psi_{\Gamma \circ \phi}$

- Hamiltonian constraint not defined on $\mathcal{H}_{kin}$, but on distribution space $\mathcal{S}$ ('habitat') = dual of dense subspace $\subset \mathcal{H}_{kin}$.

- Proper definition relies on diffeomeorphism invariance of states $\mathcal{X} \in \mathcal{S} \Rightarrow$ limit $\epsilon \to 0$ exists (at best) as a *weak limit*:

$$\big\langle H^*[N]\mathcal{X} | \Psi \big\rangle = \lim_{\epsilon \to 0} \big\langle \mathcal{X} | \hat{H}[N, \epsilon] \Psi \big\rangle \,, \quad \mathcal{X} \in \mathcal{S}$$

- *Ultralocal* action of unregulated Hamiltonian adds 'spiderwebs' of size $\epsilon \to 0$ to spin network Γ, but cumbersome to evaluate (on $\mathcal{S}$) even for the simplest examples.

Discrete Approaches: Spin Foams,...

'Covariant' version of LQG $\rightarrow$ Aim: emulate (in a rigorous way?) formal gravitational path integral [$\rightarrow$ Perez' talk]

$$\langle \gamma \,|\, \gamma' \rangle = \int_{\gamma}^{\gamma'} \prod_{x \in \mathcal{M}} \mathcal{D} g_{\mu\nu}(x) \mathcal{D}(\text{matter, ghosts}) \exp\left(iS[g; ...]\right)$$

with spatial metrics $\gamma \in \text{Riem}(\Sigma)/\text{Diff}(\Sigma)$ and $\gamma' \in \text{Riem}(\Sigma')/\text{Diff}(\Sigma')$.

Space-time 'slab' $\mathcal{M}$ bounded by 3-manifolds Σ, Σ' $\rightarrow$ analogously, Spin Foam $\mathcal{F}(\Gamma, \Gamma')$ is a 2-complex (consisting of faces, edges, and vertices) bounded above and below by spin networks Γ and Γ'.

By definition, amplitudes must obey composition law

$$\mathcal{A}(\mathcal{F}_1 \circ \mathcal{F}_2) = \mathcal{A}(\mathcal{F}_1) \cdot \mathcal{A}(\mathcal{F}_2)$$

but: (uncountable?) sum over *all* spin foams or refinement limit?

$$\langle \Gamma \,|\, \Gamma' \rangle = \sum_{\text{all foams } \mathcal{F}(\Gamma, \Gamma')} \mathcal{A}\big(\mathcal{F}(\Gamma, \Gamma')\big)$$

Other discrete options: GFT [Oriti], CDT [Loll], Causal Sets [Surya],...

Perturbative *vs.* Non-Perturbative

- Non-string approaches (LQG, spin foams,...) are usually non-perturbative *ab initio* (like the WDW equation!) but have severe difficulties with semi-classical limit and recovering continuous space-time.

- By contrast, string theory as originally formulated is perturbative (expanding in string loops and about a given background space-time), but progress by exploiting duality symmetries and strong/weak coupling dualities as in AdS/CFT (integrability!).

- Defining non-perturbative QG in bulk via CFT on boundary would seem to require *non-perturbative* formulation of $N = 4$ super-Yang-Mills theory?

- Asymptotic Safety: FRG is *a priori* non-perturbative but *de facto* perturbative due to severe truncations.

$g_s \to \infty$ limit of superstring theory?
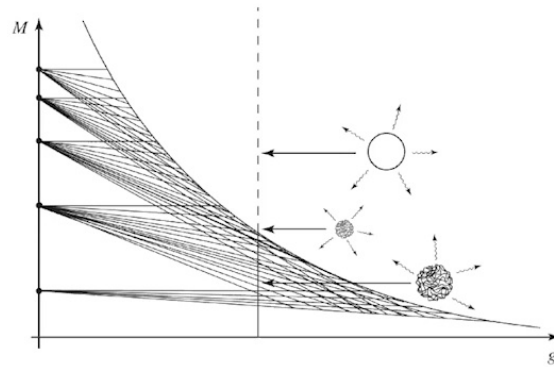
Picture from: T.Damour: 'The entropy of black holes: a primer', hep-th/0401160

As $g_s \to \infty$, extra dimension opens up ($R_{11} \to \infty$ [Witten])

$\to$ discrete string states merge into a continuum

$\Rightarrow$ **Superstring** $\to$ **Supermembrane?** [BST(1985)]

$\to$ SU(∞) matrix model [deWit,Hoppe,HN(1987), BFSS(1996)]

Conceptual Issues

- Making sense of gravitational path integral beyond mere heuristics and 'gedanken mathematics' ?

- Setting up and interpreting the WDW equation

$$\mathcal{H}\Psi \equiv \left(G_{mn\,pq}(\mathbf{g})\Pi^{mn}\Pi^{pq} - \frac{1}{2\kappa}\sqrt{\mathbf{g}}R^{(d)}(\mathbf{g}) + \cdots \right)\Psi = 0$$

- Physical meaning and uniqueness (or not) of Ψ ?

- Interpretation of QM in cosmological context?

- Is QM the right framework, or must it be modified?

- Does matter matter for consistency of QG ?

- BH information paradox? [→ talks by Engelhardt,Maxfield,Wall]

- Background independence?

Background Independence

According to Wikipedia, *Background Independence*, also called Universality, is the concept or assumption, fundamental to all physical sciences, that the nature of reality is consistent throughout space and time. More specifically, no observer can, under any circumstances, perform a measurement that yields a result logically inconsistent with a previous measurement, under a set of rules that are independent of where and when the observations are made. More concretely: a proper formulation of quantum gravity should not depend on a given (space-time) metric or any other given background structure!

Of course, it seems everyone agrees on this *desideratum*, but ....
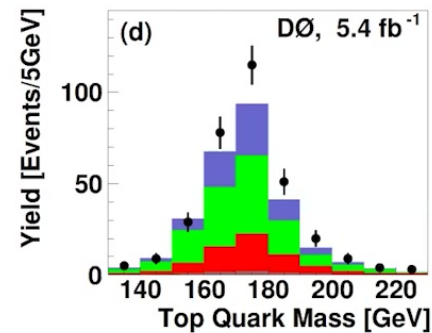
Emergence

Sexy Idea: GR and QFT are effective (low energy) theories arising at large distances from a more fundamental Planck scale theory whose basic degrees of freedom are very different from either GR or QFT, and as yet unknown. QFT, GR, space-time itself as well as general covariance are thus assumed to be 'emergent'.

BUT: Emergence from what? or:

What are the fundamental QG degrees of freedom?

- A zoology of D, NS5, M2, M5,...-branes and BPS solitons?

- AdS/CFT and holography: bulk from boundary?

- Quantum gravity and quantum information? [→ Harlow's talk]

- (Quantum) Gravity from thermodynamics?
  [Damour(1982);Jacobson(1995);Padmanabhan(2009);E.Verlinde(2011)]

- 'Space-time atoms'[→Oriti], cellular automata ['t Hooft],...?

QG and Quantum Information

Mostly in context of AdS/CFT: spatial connectivity from TFD = purification of thermal density matrix

$$\psi = \sum_E e^{-E/2kT} |E\rangle_1 \otimes |E\rangle_2 \ \Rightarrow\ S_{BH} = -\mathrm{Tr}\,\rho \ln\rho \ \text{ for } \rho \equiv \mathrm{Tr}_{\mathcal{H}_1} |\psi\rangle\langle\psi|$$

$$\Rightarrow\quad S_{BH} = \text{entanglement entropy ?}$$

- *Geometric realization* of boundary QFT entanglement entropy via minimal surfaces in AdS [Ryu,Takayanagi(2006)]

- 'Real' black holes and explaining the Page curve: no firewalls after all? [Almheiri et al.:2006.06872]

- SYK model: geometry from a purely quantum mechanical system of Majorana fermions?

Historical note: the idea that physics at the most fundamental level should be formulated in terms of quantum bits is due to C.F.von Weizsäcker (1958)

Emergence from E_{10}?

- Cosmological evolution as one-dimensional motion in the moduli space of 3-geometries [Wheeler,DeWitt,...]

$$\mathcal{M} \equiv \mathcal{G}^{(3)} = \frac{\{\text{spatial metrics } g_{ij}(\mathbf{x})\}}{\{\text{spatial diffeomorphisms}\}}$$

- Formal canonical quantization $\rightarrow$ **WDW equation**.

- BKL-type analysis of Einstein's equations in 'near singularity limit' reveals emergence of ∞–dimensional Kac–Moody symmetry of indefinite type $\Rightarrow$

- Can we understand and 'simplify' $\mathcal{M}$ by means of embedding into a group theoretical coset $G/K(G)$?

- Main conjecture: $G = E_{10}$ and $K(G) = K(E_{10})$ $\Rightarrow$ space-time dependence encoded into ∞-dimensional hyperbolic Kac-Moody Algebra E_{10}? [Damour,Henneaux,HN(2002)]

- $\rightarrow$ a new kind of 'holographic' correspondence?

The Real World

Key question: how to validate/refute a given ansatz?

- New effects: superpartners, large extra dimensions, TeV scale QG, dark matter candidates,...

- Indirect effects (hints in CMB,...?)

- Cumulative effects (deformed dispersion relations,...?)

- Cosmology: origin of inflation? $\Lambda > 0$ *vs.* quintessence?

- The strong gravity regime: hints from gravitational wave observations? [$\rightarrow$ Sakellariadou; also:H.Verlinde,Giddings]

- Standard Model: implications of LHC results ?

Important: must not only be able to produce *some* testable prediction, but also be able to *discriminate* between competing explanations!

(No) News from LHC
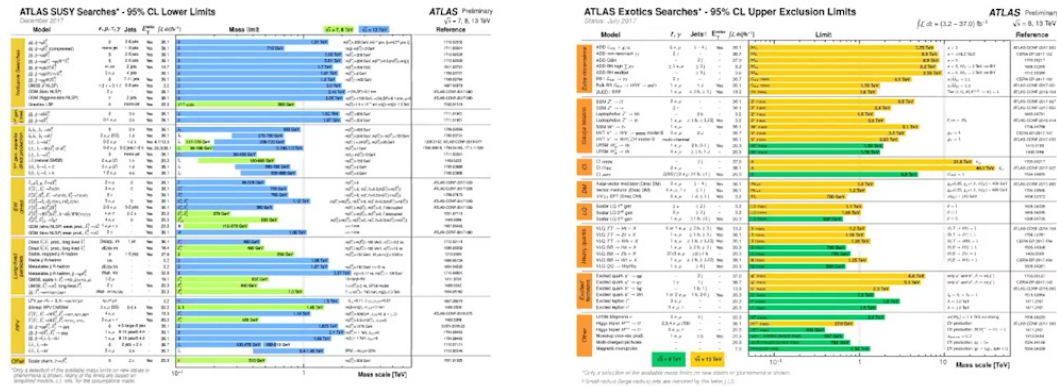
Exclusion limits, nothing but exclusion limits, ...

- No hints whatsoever of new physics

- RG Evolution of (slightly amended) SM couplings: no Landau poles, no instabilities of effective potential up to Planck scale

Conclusion (so far, at least): SM could survive more or less *as is* all the way to Planck scale M_{PL}!

The Multiverse Dilemma

- Very large number of (consistent?) string vacua
  [A.N.Schellekens, "Big numbers in string theory", 1601.02462[hep-th]]

- According to the latest estimate there are (at least!) $\sim 10^{272\,000}$ vacua, typically with gauge groups like
  [W.Taylor,Y.Wang, "Scanning the skeleton of the 4D F theory landscape, 1710.11235]

$$G = (E_8)^{38} \times (F_4)^{94} \times (G_2)^{288} \times SU(2)^{432}$$

- Not even clear whether SM contained in this set !

It is not known which of these vacua are viable: to curtail proliferation of string vacua look for criteria to eliminate UV incomplete theories (weak gravity conjecture, absence of global symmetries,...) [→String-Pheno2020]

Otherwise: must look for very different (and better) ways to link string theory to the real world!

Idem for non-string approaches

... which also suffer from a huge number of ambigui-ties, so far preventing any kind of testable prediction with which the theory will stand or fall:

- LQG: 10^{500} 'consistent' Hamiltonians/spin foam models?

- Discrete Gravity: 10^{500} 'consistent' lattice models?

- Asymptotic Safety: 10^{500} 'consistent' RG flows?

Question *to everyone*: does Nature pick 'right' answer at random from a huge variety of (*soi disant* consis-tent) possibilities, or are there criteria to narrow down the number of possible choices?

$N\!=\!8$ Supergravity: a strange coincidence

$SO(8) \to SU(3) \times U(1)$ breaking and 'family-color locking'

$$(u\,,\,c\,,\,t)_L\ :\qquad \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,,\qquad +\frac{1}{2} = \frac{2}{3} - q$$

$$(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L\ :\qquad \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,,\qquad -\frac{1}{2} = -\frac{2}{3} + q$$

$$(d\,,\,s\,,\,b)_L\ :\qquad \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,,\qquad -\frac{1}{6} = -\frac{1}{3} + q$$

$$(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L\ :\qquad \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,,\qquad +\frac{1}{6} = \frac{1}{3} - q$$

$$(e^-,\mu^-,\tau^-)_L\ :\qquad \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,,\qquad -\frac{5}{6} = -1 + q$$

$$(e^+,\mu^+,\tau^+)_L\ :\qquad \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,,\qquad +\frac{5}{6} = 1 - q$$

$$(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L\ :\qquad \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,,\qquad -\frac{1}{6} = 0 - q$$

$$(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L\ :\qquad \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,,\qquad +\frac{1}{6} = 0 + q$$

M. Gell-Mann (1983): $N = 8$ Supergravity and Standard Model assignments agree if spurion charge is chosen as $q = \frac{1}{6}$!

Mismatch of electric charges can be fixed by enlarging SO(8) to ∞–dimensional R symmetry $K(E_{10}) \subset E_{10}$ [Meissner,Kleinschmidt,HN(2015]

Supermassive Gravitinos?

Beyond $N = 8$ supergravity: gravitinos and spin-$\frac{1}{2}$ fermions belong to one (unfaithful) irreducible spinorial representation of $K(E_{10})$.

Under $\mathbf{SU(3)}_c \times \mathbf{U(1)}_{em}$ gravitinos transform as

$$\left(\mathbf{3}_c, \frac{1}{3}\right) \oplus \left(\bar{\mathbf{3}}_c, -\frac{1}{3}\right) \oplus \left(\mathbf{1}_c, \frac{2}{3}\right) \oplus \left(\mathbf{1}_c, -\frac{2}{3}\right)$$

Unusual features $\rightarrow$ distinct observational signatures?

- Absolutely stable against decay into SM matter

- A novel (and unusual) dark matter candidate ?

- Strong and electromagnetic interactions $\Rightarrow$ would have been seen *unless* mass is very high, and cosmological abundance extremely low $\rightarrow$ can be searched for with *paleo-detectors*.

- could explain UHECRs via gravitino-antigravitino annihilation in the 'skin' of neutron stars (preponderance of ions!)

- Heavy gravitinos = possible seeds for primordial black holes?

Outlook

- Stupendous amount of work and great diversity of results over the past 50 years

- But no convergence of approaches: difficult to see if and how all these ideas could fit together at all!

- → synergistic conferences like this one and debates should be continued and encouraged

Still: *some unmistakable* hint from experiment and observation is badly needed!

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THANK YOU