Title: AdS3 gravity and random CFT

Speakers: Jordan Cotler

Series: Quantum Fields and Strings

Date: June 16, 2020 - 4:00 PM

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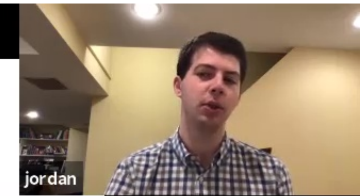
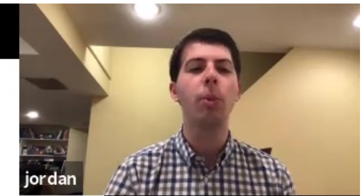
Abstract: We compute the path integral of three-dimensional gravity with negative cosmological constant on spaces which are topologically a torus times an interval. These are Euclidean wormholes, which smoothly interpolate between two asymptotically Euclidean AdS3 regions with torus boundary. From our results we obtain the spectral correlations between BTZ black hole microstates near threshold, as well as extract the spectral form factor at fixed momentum, which has linear growth in time with small fluctuations around it. The low-energy limit of these correlations is precisely that of a double-scaled random matrix ensemble with Virasoro symmetry. Our findings suggest that if pure three-dimensional gravity has a holographic dual, then the dual is an ensemble which generalizes random matrix theory.

To appear tonight [JC, Jensen '20]
1808.03263 [JC, Jensen '18]



AdS$_3$ and Random CFT

JORDAN COTLER



Introduction

[Kitaev '15]
[Maldacena, Stanford '17]
[Jensen '17]
[Maldacena, Stanford, Yang '17]
[Engelsöy, Mertens, Verlinde '17]

- Holographic dualities with disorder

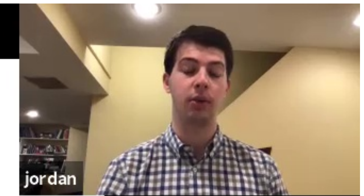
SYK model $\longleftrightarrow$ Nearly-AdS_2 gravity + matter

Matrix model $\longleftrightarrow$ Nearly-AdS_2 JT gravity

[Saad, Shenker, Stanford '18]
[Saad, Shenker, Stanford '19]

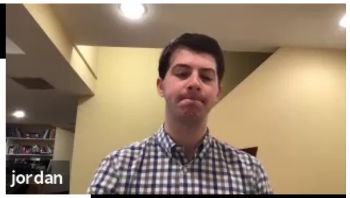
Random CFT $\longleftrightarrow$ Pure AdS_3 gravity

[JC, Jensen '18]
[JC, Jensen '20]



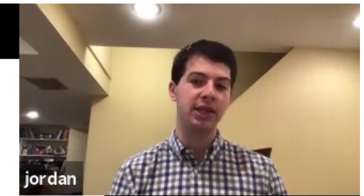
Pure AdS_3 gravity

- Classically soluble
- Has black holes
- Gravitational edge modes (i.e., large diffeomorphisms)



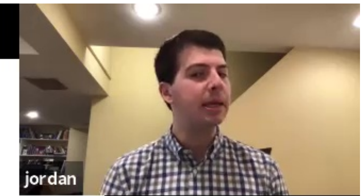
Pure AdS$_3$ gravity

- Classically soluble
- Has black holes
- Gravitational edge modes (i.e., large diffeomorphisms)
  - Marginal interactions $\sim G/L$
- Moduli, sum over topologies



Pure AdS₃ gravity

- Quantum theory has many puzzles
- Not thought to be dual to a (single) CFT – various no-go's
- **Example puzzle:** Euclidean partition function on $D^2 \times \mathbb{S}^1$
  - First studied in [Maloney, Witten '07] (path integral: [JC, Jensen '18])
  - Can be viewed as sum over saddles with torus conformal boundary having complex structure τ, and continuously connected geometries
  - Density of states is not that of a dual unitary compact CFT
  - Spectral gap between vacuum and BTZ threshold, spectrum above is continuous; density of states negative near spectral edge



Pure AdS₃ gravity

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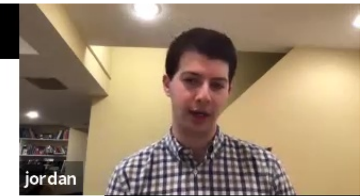
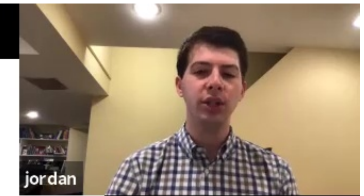
  - Spectral gap between vacuum and BTZ threshold, **spectrum above is continuous**; density of states negative near spectral edge **[Maloney, Witten '07]** **[Keller, Maloney '15]** **[Benjamin, Ooguri, Shao, Wang '19]**



Idea: Pure AdS_3 quantum gravity is dual to an *ensemble*

- What would be compelling evidence of this?

  → compute and examine properties of ***spectral form factor***



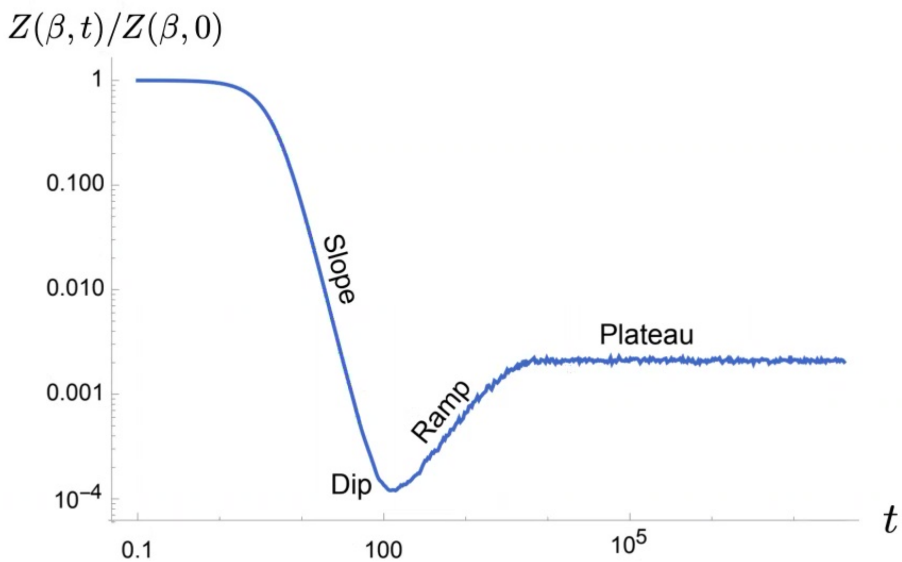
Spectral form factor

- Consider an ensemble of $d \times d$ Hamiltonians with measure dH
- Spectral form factor is given by

$$Z(\beta, t) = \int dH \, \mathrm{tr}\big(e^{-(\beta+it)H}\big)\mathrm{tr}\big(e^{-(\beta-it)H}\big)$$

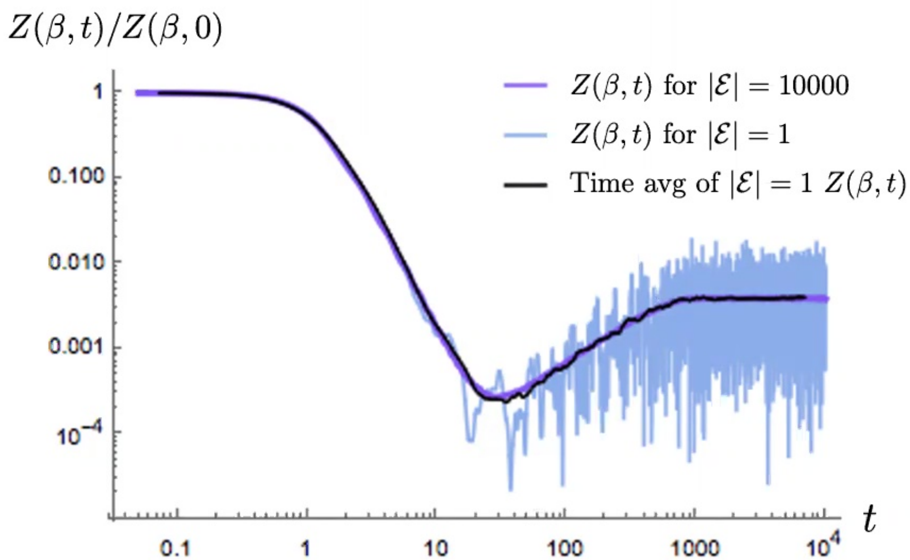
$$= \Big\langle \mathrm{tr}\big(e^{-(\beta+it)H}\big)\mathrm{tr}\big(e^{-(\beta-it)H}\big) \Big\rangle_{\mathrm{ensemble}}$$

SYK $Z(\beta, t)$ for $N = 26$ $(d = 8192)$, $\beta = 5$

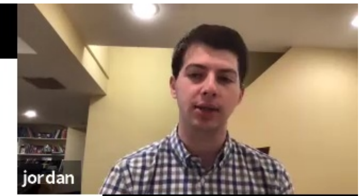


[JC, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka '16]
Figure adapted from [JC, Hunter-Jones, Liu, Yoshida '17]

GUE $Z(\beta,t)$ for $d=500$, $\beta=5$



[Prange '96]



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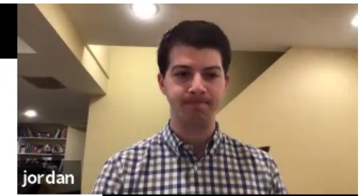


Question: Does AdS₃ gravity produce a spectral form factor ramp with small fluctuations?

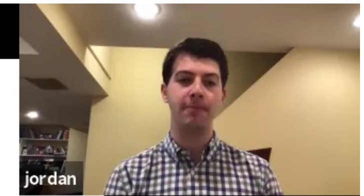
- What do we mean by the spectral form factor in AdS₃ gravity?

$$\left\langle \mathrm{tr}\left(e^{-\mathrm{Im}(\tau_1)H + i\,\mathrm{Re}(\tau_1)\mathsf{P}}\right)\mathrm{tr}\left(e^{-\mathrm{Im}(\tau_2)H + i\,\mathrm{Re}(\tau_2)\mathsf{P}}\right)\right\rangle_{\text{ensemble, conn.}}$$

$$= \left\langle Z^{\mathrm{CFT}}_{\mathbb{T}^2}(\tau_1) Z^{\mathrm{CFT}}_{\mathbb{T}^2}(\tau_2)\right\rangle_{\text{ensemble, conn.}}$$

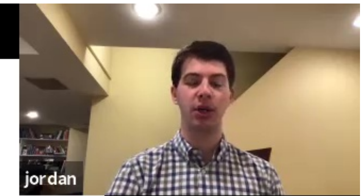
$$\stackrel{?}{=} \boxed{Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2)} + \cdots$$

Euclidean wormhole



Outline

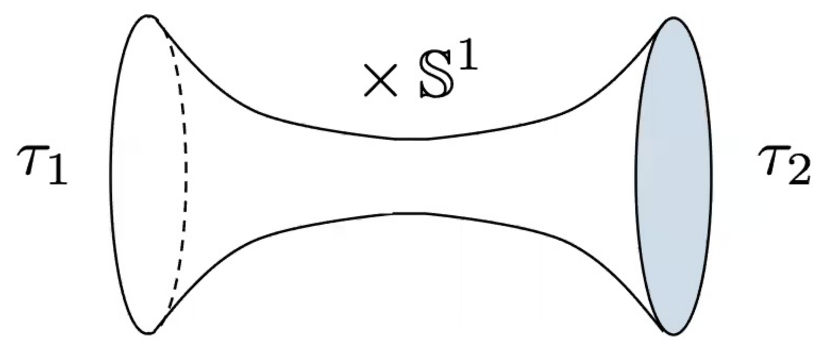
1. Computation of Euclidean wormhole
2. RMT Prediction and Extracting the Ramp
3. Random CFT
4. Discussion

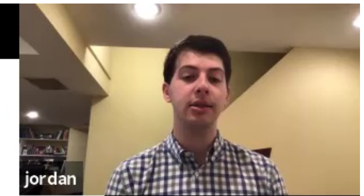


Setup

- Want to compute:

$$Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = \int \frac{[dg]_{\mathbb{T}^2\times I}}{\mathrm{diff}}\, e^{\frac{1}{16\pi G}\int d^3x\,\sqrt{g}(R+2) - S_{\mathrm{bdy}}}$$





No smooth saddle points

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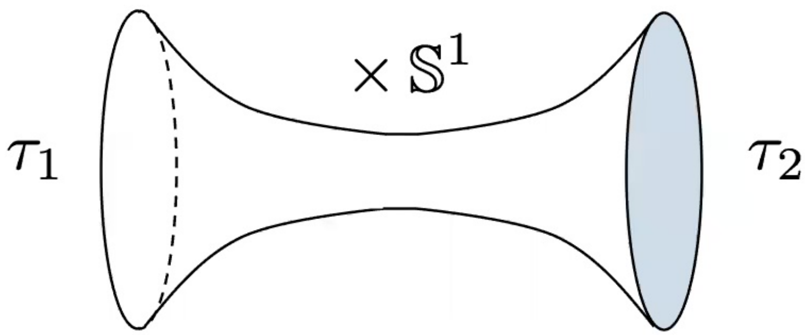
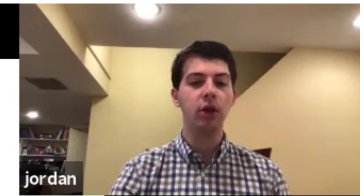




Setup

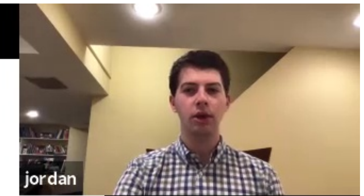
- Let's begin with Lorentzian signature
- Change to first-order variables $(e^A_M, \omega^B{}_{CN})$

$$S = -\frac{1}{16\pi G}\int \varepsilon_{ABC}\left(e^A \wedge (d\omega^{BC} + \omega^B{}_D \wedge \omega^{DC}) + \frac{1}{3}\, e^A \wedge e^B \wedge e^C\right)$$

- Has the form of a phase space path integral

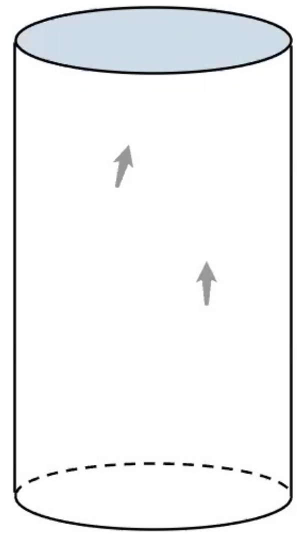
$$S = \int dt\ \left(p_i \dot{q}^i - H(p,q) + \lambda_a\, \mathcal{C}^a(p,q)\right)$$

- Lagrange multipliers in the gravity setting are $e^A_0\ ,\ \omega^A{}_{B0}$



Simpler calculation: $D^2 \times \mathbb{R}$ [JC, Jensen '18]

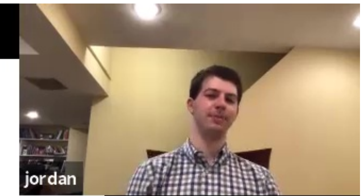
- Integrate out Lagrange multiplier fields, and solve for constraints (compatible with boundary conditions)
- Residual fields $\phi, \bar{\phi}$ are boundary reparameterizations



$$S[\phi, \bar{\phi}] = S_-[\phi] + S_+[\bar{\phi}]$$

$$\boxed{S_{\pm}[\phi] = -\frac{C}{24\pi} \int d^2x \left(\frac{\phi'' \partial_{\pm} \phi'}{\phi'^{\,2}} - \phi' \partial_{\pm} \phi \right)}$$

$$C = \frac{3}{2G}\,,\ {}' = \partial_x\,,\ x^{\pm} = x \pm t\,,\ \phi(x + 2\pi, t) = \phi(x, t) + 2\pi\,,\ \phi' > 0$$



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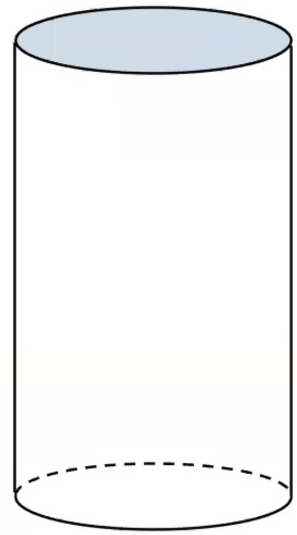
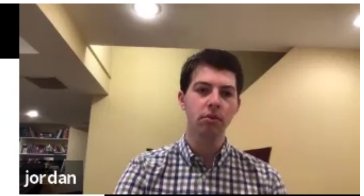


- At fixed time, $\phi, \bar{\phi} \in \mathrm{Diff}(\mathbb{S}^1)/PSL(2;\mathbb{R})$



Simpler calculation: $D^2 \times \mathbb{R}$ [JC, Jensen '18]

- An analog of the Schwarzian action for Lorentzian AdS_3 gravity
- It is also the path integral quantization of the first exceptional coadjoint orbit of Virasoro

[Alekseev, Shatashvili '89]

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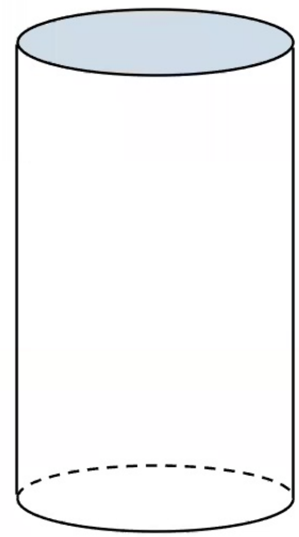
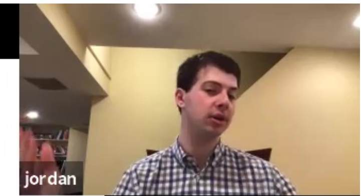


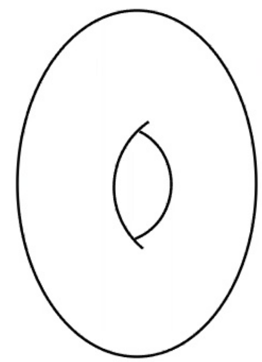


Simpler calculation: $D^2 \times \mathbb{S}^1$ [JC, Jensen '18]

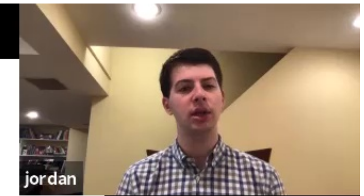
- Go to Euclidean time $t = -iy\,,\ z = x + iy\,,\ z \sim z + 2\pi n + 2\pi m\tau$

$$S_{E,-}[\phi] = \frac{C}{24\pi}\int d^2x \left(\frac{\phi''\bar{\partial}\phi'}{\phi'^{\,2}} - \phi'\bar{\partial}\phi\right)$$

$$\boxed{Z^{\mathrm{AdS}_3}_{D^2\times\mathbb{S}^1}(\tau) = |\chi_{0,c}(\tau)|^2}$$



- 1-loop exact ("quantization" of [Stanford, Witten '17])



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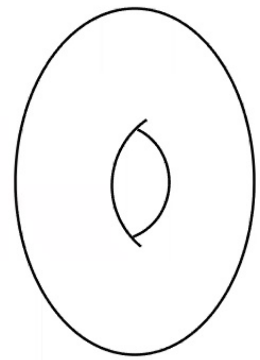
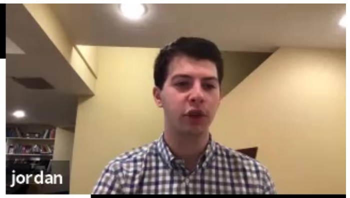


- 1-loop exact ("quantization" of [Stanford, Witten '17])
- Renormalization of central charge $c = C + 13$
- Not modular invariant; sum over choice of contractible cycle (obtain MW)

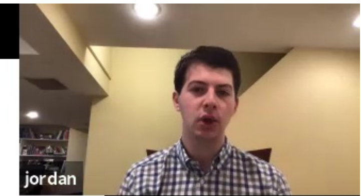


Euclidean Wormhole $\mathbb{T}^2 \times I$ [JC, Jensen '20]

- Integrate out Lagrange multiplier fields, and solve for constraints (compatible with boundary conditions)
- Residual fields $\Phi, \bar{\Phi}$ on each boundary:

$$\Phi_j = b(y)x + \phi_j\,, \quad \bar{\Phi}_j = \bar{b}(y)x + \bar{\phi}_j\,, \quad j = 1,2$$

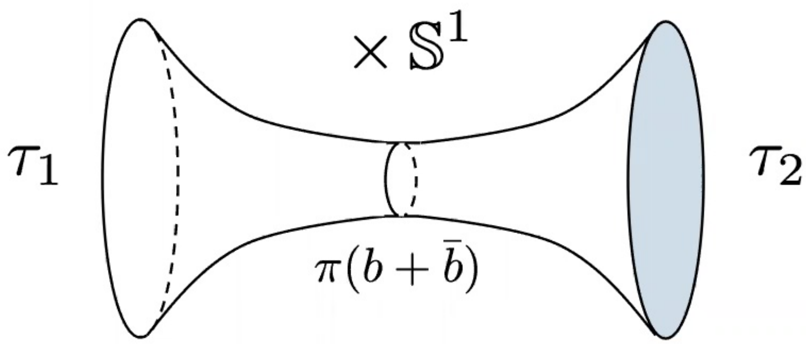
$$\begin{cases} x \sim x + 2\pi\,, \quad y \sim y + 2\pi \\ \phi_j(x,y) \sim \phi_j(x,y) + a(y)\,, \quad \bar{\phi}_j(x,y) \sim \bar{\phi}_j(x,y) + \bar{a}(y) \end{cases}$$

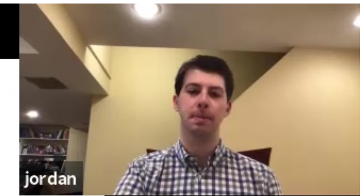


Euclidean Wormhole $\mathbb{T}^2 \times I$ [JC, Jensen '20]

- Sample of a geometry: $b(y) = b\,,\ \ \bar{b}(y) = \bar{b}\,,\ \ \phi = \bar{\phi} = 0$

$$ds^2_{\text{spatial}} = \left(b\bar{b}\sinh^2(\rho) + \frac{1}{4}(b+\bar{b})^2 \right) dx^2 + d\rho^2$$





Euclidean Wormhole $\mathbb{T}^2 \times I$ [JC, Jensen '20]

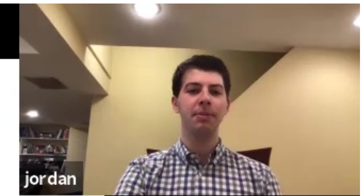
- Full action ($\partial_j = \frac{1}{2}(-i\tau_j\partial_x + i\partial_y)$)

$$S = \frac{C}{24\pi}\int d^2x \bigg(\frac{\Phi_1''\bar{\partial}_1\Phi_1'}{\Phi_1'^2} + \Phi_1'\bar{\partial}_1\Phi_1 + \frac{\bar{\Phi}_1''\partial_1\bar{\Phi}_1'}{\bar{\Phi}_1'^2} + \bar{\Phi}_1'\partial_1\bar{\Phi}_1 + \frac{\Phi_2''\partial_2\Phi_2'}{\Phi_2'^2} + \Phi_2'\partial_2\Phi_2 + \frac{\bar{\Phi}_2''\bar{\partial}_2\bar{\Phi}_2'}{\bar{\Phi}_2'^2} + \bar{\Phi}_2'\bar{\partial}_2\bar{\Phi}_2 \bigg)$$

- Introducing the twists fields

$$\begin{cases} Y(y) = \dfrac{1}{2\pi b(y)}\displaystyle\int_0^{2\pi} dx\,\big(\phi_1(x,y) - \bar{\phi}_1(x,y)\big) \\ \overline{Y}(y) = \dfrac{1}{2\pi \bar{b}(y)}\displaystyle\int_0^{2\pi} dx\,\big(\phi_2(x,y) - \bar{\phi}_2(x,y)\big) \end{cases}$$

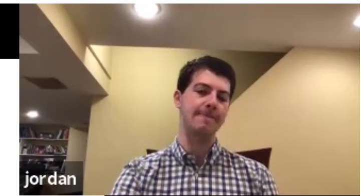
$$S \supset \frac{iC}{24}\int_0^{2\pi} dy\,\big(b\partial_y Y - \bar{b}\partial_y\overline{Y}\big) \Rightarrow \partial_y b = \partial_y \bar{b} = 0$$



Euclidean Wormhole $\mathbb{T}^2 \times I$ [JC, Jensen '20]

- Our partition function can be written as $\left(c = C+1\,,\ \ q = e^{2\pi i \tau}\,,\ \ h = \frac{C(b^2+1)}{24} \right)$

$$Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = \sum_{\text{twist windings}} \int \frac{db^2 dY\, d\bar{b}^2 d\overline{Y}}{(2\pi)^2}\, \omega_{\text{moduli}}\, Z_{\text{AS}}(\tau_1|\bar{b}) Z^*_{\text{AS}}(\bar{\tau}_1|b) Z_{\text{AS}}(\tau_2|\bar{b}) Z^*_{\text{AS}}(\bar{\tau}_2|b)$$



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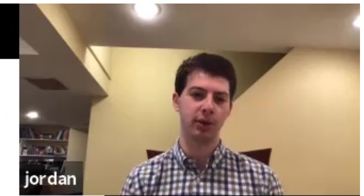
$$Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = \frac{1}{2\pi^2}\, Z_0(\tau_1) Z_0(\tau_2) \sum_{\gamma\in SL(2;\mathbb{Z})} \frac{\mathrm{Im}(\tau_1)\mathrm{Im}(\gamma\tau_2)}{|\tau_1+\gamma\tau_2|^2}$$

$$\left. Z_0(\tau) = \frac{1}{\sqrt{\mathrm{Im}(\tau)}|\eta(\tau)|^2} \right\} \text{partition function of compact boson}$$



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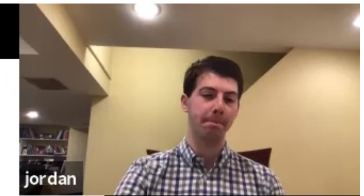
$$Z_0(\tau) = \frac{1}{\sqrt{\mathrm{Im}(\tau)}|\eta(\tau)|^2}$$

partition function of compact boson

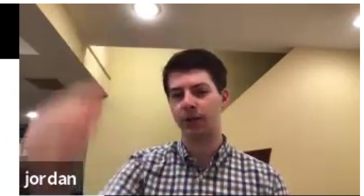
- Final result required a modular sum as per **MW**



Properties of the Result

$$\boxed{Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = \frac{1}{2\pi^2}\, Z_0(\tau_1) Z_0(\tau_2) \sum_{\gamma\in SL(2;\mathbb{Z})} \frac{\mathrm{Im}(\tau_1)\mathrm{Im}(\gamma\tau_2)}{|\tau_1+\gamma\tau_2|^2}}$$

- Sum almost converges; divergence is pure additive constant
- Divergence can be treated using Zeta-function regularization
- Double modular invariance $Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\gamma\tau_1,\gamma'\tau_2) = Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2)$



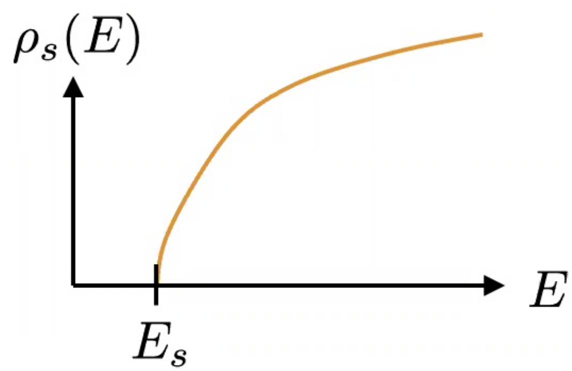
Prediction from Virasoro RMT

- Before extracting the ramp from the wormhole amplitude, let's try to predict what its behavior should be
- Amounts to studying level statistics for RMT with Virasoro symmetry
- For simplicity, let's study the spectral statistics of primary states, each labeled by an energy and momentum

$$[H, \mathsf{P}] = 0 \quad \Longrightarrow \quad \{H_s\}_{s\in\mathbb{Z}}$$

Prediction from Virasoro RMT

- Suppose average density of states for H_s block has the form



$(T \gg \beta)$

[Eynard, Orantin]

$$\boxed{\langle \mathrm{tr}(e^{-(\beta+iT)H_{s_1}})\mathrm{tr}(e^{-(\beta-iT)H_{s_2}})\rangle_{\text{ensemble}} = Z^P_{s_1,s_2}(\beta+iT,\beta-iT) = \frac{T}{4\pi\beta}\, e^{-2\beta E_{s_1}}\delta_{s_1,s_2} + (\text{nonpert.})}$$

Fourier decomposition of $Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2)$

$$Z^{\mathrm{AdS}_3}_{\mathbb{T}^2\times I}(\tau_1,\tau_2)=\frac{1}{2\pi^2}\,Z_0(\tau_1)Z_0(\tau_2)\sum_{s_1,s_2=-\infty}^{\infty}e^{-2\pi i\mathrm{Re}(\tau_1)s_1-2\pi i\mathrm{Re}(\tau_2)s_2}\widetilde{F}_{s_1,s_2}(\mathrm{Im}(\tau_1),\mathrm{Im}(\tau_2))$$

- Fourier coefficients somewhat complicated
- Convenient to consider large $\mathrm{Im}(\tau_1)=\beta_1$, $\mathrm{Im}(\tau_2)=\beta_2$, with β_1/β_2 fixed

$$\widetilde{F}_{s_1,s_2}(\beta_1,\beta_2)=e^{-2\pi|s_1|\beta_1-2\pi|s_2|\beta_2}\left(\frac{\pi\beta_1\beta_2}{\beta_1+\beta_2}\,\delta_{s_1,s_2}+O\left(\frac{1}{\beta}\right)\right)$$

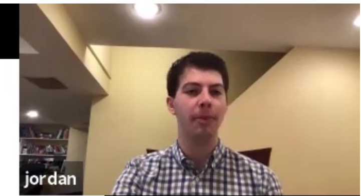
Disentangling the answer

- Wormhole amplitude may be separated into a contribution from primary states alone, and contributions from descendants (determined by symmetry)

$$Z^P_{s_1,s_2}(\beta_1,\beta_2) = \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2}\, e^{-E_{s_1}\beta_1 - E_{s_2}\beta_2}\left(\delta_{s_1,s_2} + O\left(\frac{1}{\beta}\right)\right), \qquad \underbrace{E_s = 2\pi\left(|s| - \frac{1}{12}\right)}_{\text{threshold energy for BTZ black hole at spin s}}$$

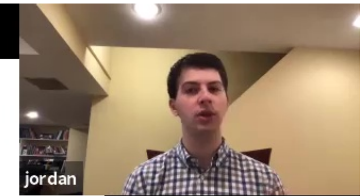
$$\boxed{Z^P_{s_1,s_2}(\beta+iT,\beta-iT) = \frac{T}{4\pi\beta}\, e^{-2\beta E_{s_1}}\delta_{s_1,s_2} + O(T^{-1})}$$

Exactly matches double-scaled Virasoro RMT prediction!



Comments

- More generally, the full low-temperature 2-point fluctuation statistics of BTZ microstates exactly matches the predictions of random matrix theory with Virasoro symmetry (including contribution of descendants)
- Deviations from standard RMT prediction suppressed in time / temperature



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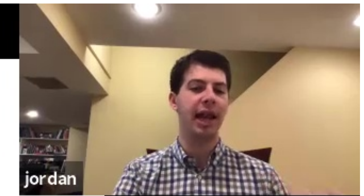
  - AdS_3 analog of JT analysis in [Stanford, Witten '19]



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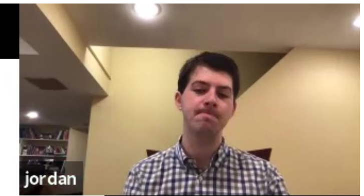
- Deviations from standard RMT prediction suppressed in time / temperature
- Similar story when we gauge time reversal in the bulk, giving us global time reversal symmetry on boundary
  - AdS_3 analog of JT analysis in [Stanford, Witten '19]
- Compatible with nearly-AdS_2 JT limit of AdS_3 [Ghosh, Maxfield, Turiaci '19]



Is AdS_3 dual to a RMT?

- Leading low-temperature limit of wormhole amplitude exactly matches double-scaled RMT with Virasoro symmetry

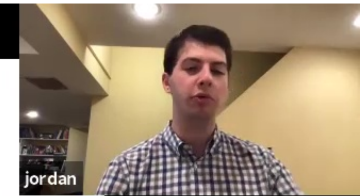
But:

- Euclidean gravity allows arbitrary genus g boundaries
- Power-law suppressed fluctuations of the ramp **(maybe ok)**
- Amplitudes invariant under independent modular transformations

Idea: Pure AdS_3 quantum gravity is dual to an *ensemble of CFTs*

Preface to Conjectures

- Sketch plausible schematic framework, follow logic of statistical physics
- Organizing principle to go beyond $\mathbb{T}^2 \times I$
- For the moment, cast aside pressing technical / conceptual concerns
  - Negativity of **MW** partition function (perhaps resolvable)
  - Lack of knowledge of irrational CFTs at large central charge (probably hard)
- Optimistic hope: maybe many solutions to modular bootstrap that "look" gravitational; can average over this solution space



Ensemble of CFTs

$$\langle Z_{\mathcal{T},\Sigma_{g_1}}(\Omega_1)\cdots Z_{\mathcal{T},\Sigma_{g_n}}(\Omega_n)\rangle_{\text{ensemble}} \equiv \int d[\mathcal{T}]\, Z_{\mathcal{T},\Sigma_{g_1}}(\Omega_1)\cdots Z_{\mathcal{T},\Sigma_{g_n}}(\Omega_n)$$

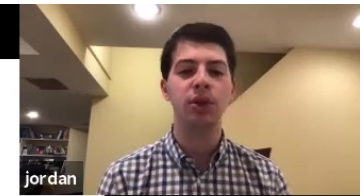
Euclidean AdS₃

$$Z_{\text{AdS}_3}(\Sigma_{g_1},\Omega_1;\ldots;\Sigma_{g_n},\Omega_n) \equiv \sum_{\substack{\text{bulk topologies of } \mathcal{M}_3 \\ \partial\mathcal{M}_3=\Sigma_{g_1}\sqcup\cdots\sqcup\Sigma_{g_n}}} \int_{\Omega_1,\ldots,\Omega_n} d[g]\, e^{-S_{\text{grav}}[g]}$$

Conjecture 1:

$$\langle Z_{\mathcal{T},\Sigma_{g_1}}(\Omega_1)\cdots Z_{\mathcal{T},\Sigma_{g_n}}(\Omega_n)\rangle_{\text{ensemble}} \simeq Z_{\text{AdS}_3}(\Sigma_{g_1},\Omega_1;\ldots;\Sigma_{g_n},\Omega_n)$$

(In a putative $e^{-\#/G}$ expansion)



Special case of torus boundaries

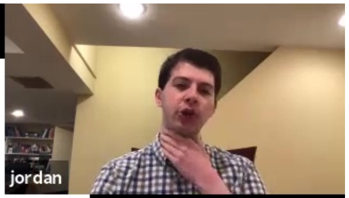
- An ensemble of CFTs induces a Hamiltonian ensemble

$$\langle Z_{\mathcal{T},\mathbb{T}^2}(\tau_1)\cdots Z_{\mathcal{T},\mathbb{T}^2}(\tau_n)\rangle_{\text{ensemble}} = \int dH\,\mathrm{tr}\left(e^{-\mathrm{Im}(\tau_1)H+i\,\mathrm{Re}(\tau_1)\mathsf{P}}\right)\cdots\mathrm{tr}\left(e^{-\mathrm{Im}(\tau_n)H+i\,\mathrm{Re}(\tau_n)\mathsf{P}}\right)$$

- Then **Conjecture 1** implies

Conjecture 2:

$$\int dH\,\mathrm{tr}\left(e^{-\mathrm{Im}(\tau_1)H+i\,\mathrm{Re}(\tau_1)\mathsf{P}}\right)\cdots\mathrm{tr}\left(e^{-\mathrm{Im}(\tau_n)H+i\,\mathrm{Re}(\tau_n)\mathsf{P}}\right) \simeq Z_{\mathrm{AdS}_3}(\mathbb{T}^2,\tau_1;\ldots;\mathbb{T}^2,\tau_n)$$



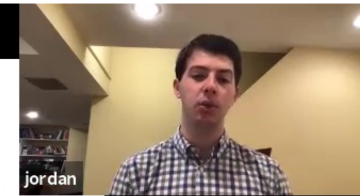
Outline

1. Computation of Euclidean wormhole

2. RMT Prediction and Extracting the Ramp

3. Random CFT

4. $\boxed{\text{Discussion}}$



Discussion

- How to compute partition functions for $\Sigma_{g,n} \times \mathbb{S}^1$, or even 2-manfolds fibered over a circle?
- More general 3-manifolds?
- Generalizations to dS, higher-dimensional AdS wormholes **[JC, Jensen WIP]**
- New non-perturbative tools from phase space path integral
- Random CFT and "mesoscopic quantum gravity"
- Other CFT ensemble dualities? **[Murungan, Stanford, Witten '17]** **[Afkhami-Jeddi, Cohn, Hartman, Tajdini '20]** **[Maloney, Witten '20]**