Title: Soft modes in quantum gravity

Speakers: Monica Pate

Collection: Quantum Gravity 2020

Date: July 16, 2020 - 12:30 PM

URL: http://pirsa.org/20060053

Abstract: I will review advances for gravity in asymptotically flat spacetimes arising from investigations into their structure in the infrared. The recently-discovered infinite-dimensional symmetries of the scattering problem is the central result underlying much of the progress. Key examples include symmetry-based explanations for the previously-observed universal nature of infrared phenomena including soft theorems and memory effects. Moreover, the appearance of a Virasoro symmetry among the symmetries of four-dimensional gravity has led to a proposal for holography in which the scattering amplitudes in quantum gravity are dual to correlation functions of a two-dimensional conformal theory. The other infinite-dimensional symmetry groups place additional non-trivial constraints on the dual theory.

Soft Modes in Quantum Gravity

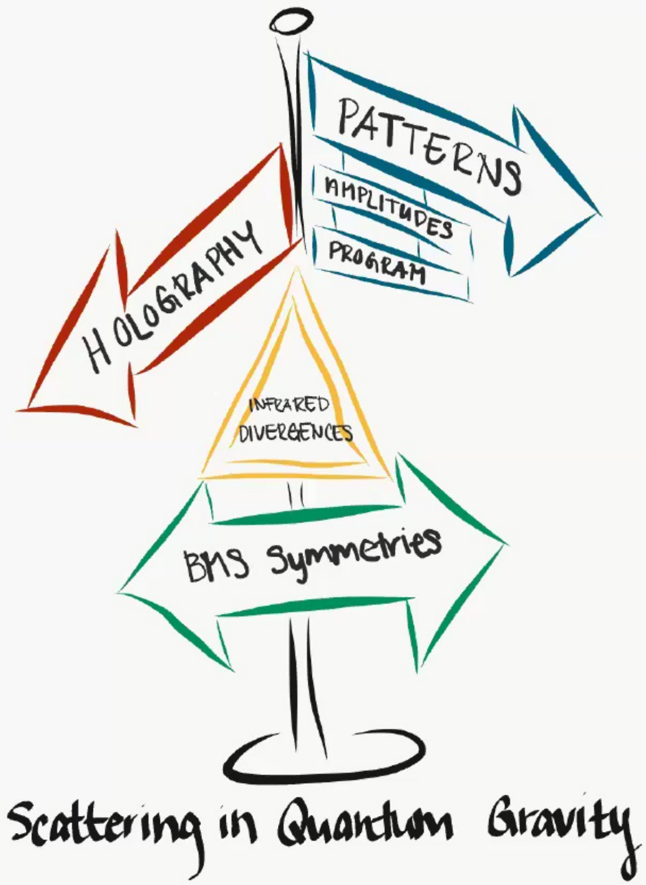
Monica Pate
Harvard University
Quantum Gravity 2020

Focus: soft modes and symmetries in the context of the scattering problem

Probing quantum gravity via scattering

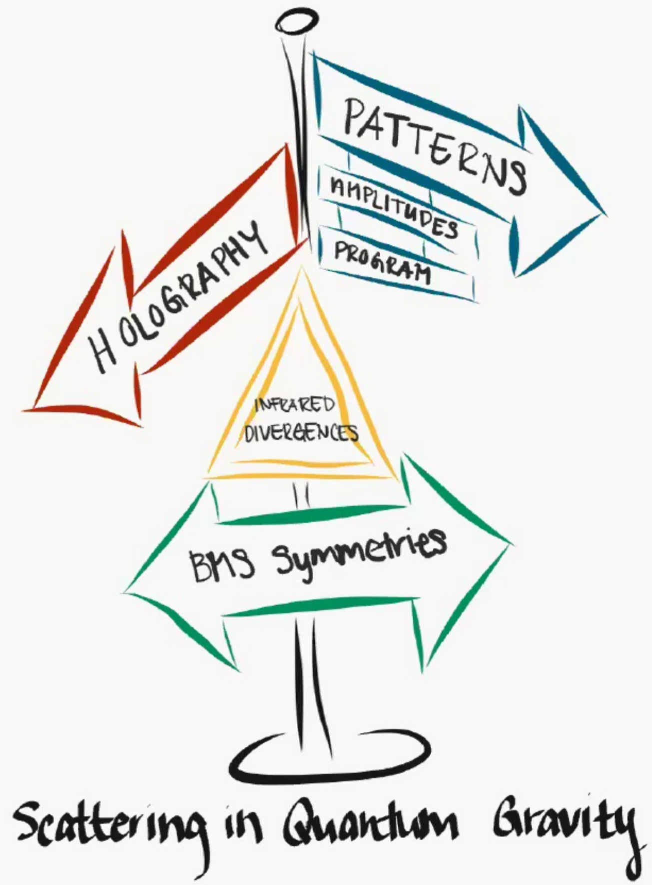
1. **Well-defined question** for quantum gravity in asymptotically flat spacetimes
   - *Amenable* to top-down approach
2. **Answer** with broad implications for quantum gravity
   - *Provides* top-down approach to quantum gravity

Signs, clues, puzzles…



Outline

- Asymptotic symmetries (BMS)
- Symmetries of scattering & soft theorems
- Infrared divergences
- Flatspace holography
  - *Celestial conformal field theory*



BMS symmetries

Method: Asymptotic symmetry analysis

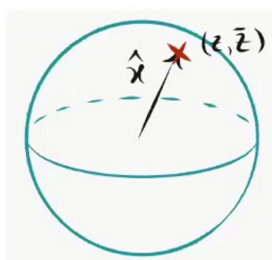
$$\text{asymptotic symmetries} = \frac{\text{allowed diffeomorphisms}}{\text{trivial diffeomorphisms}}$$

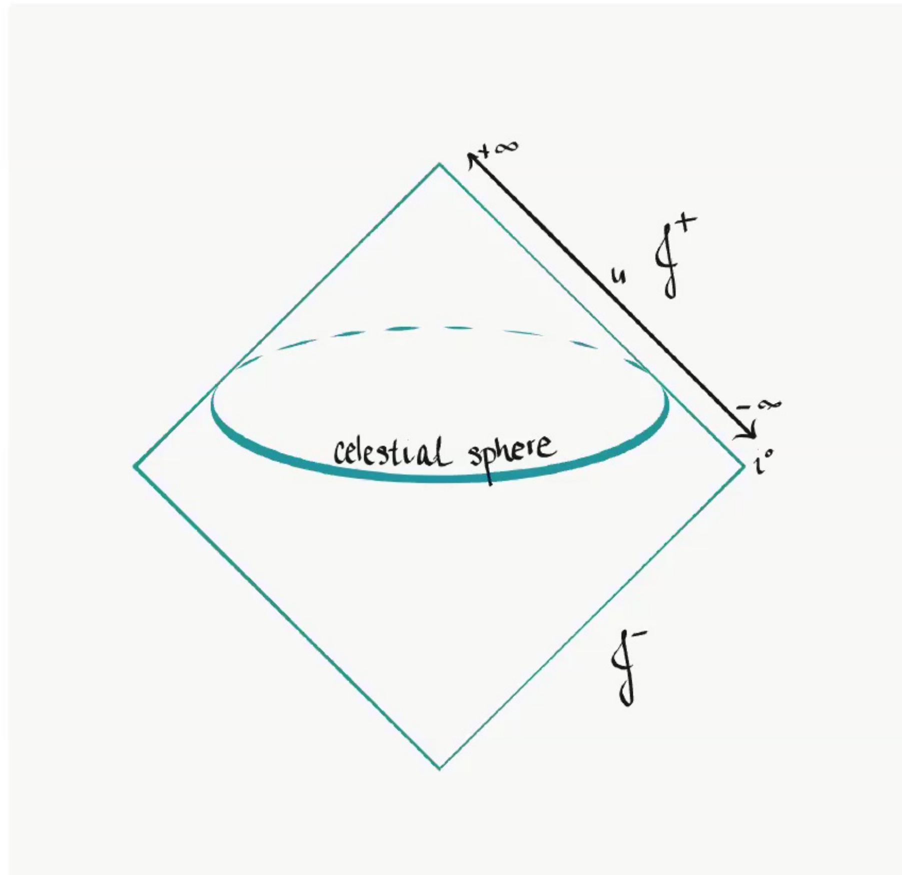
$$\begin{aligned} ds^2 =& - du^2 - 2du dr + 2r^2\gamma_{z\bar{z}} dz d\bar{z} \\ &+ \frac{2m_B}{r} du^2 + rC_{zz} dz^2 + rC_{\bar{z}\bar{z}} d\bar{z}^2 \\ &+ D^z C_{zz} du dz + D^{\bar{z}} C_{\bar{z}\bar{z}} du d\bar{z} + \cdots \end{aligned}$$

Minkowski:

$$u = t - r, \qquad \vec{x} = r\hat{x}(z, \bar{z}),$$

$$\gamma_{z\bar{z}} = \frac{2}{(1 + z\bar{z})^2}$$





[Bondi, van der Burg, Metzner (1962); Sachs (1962)]

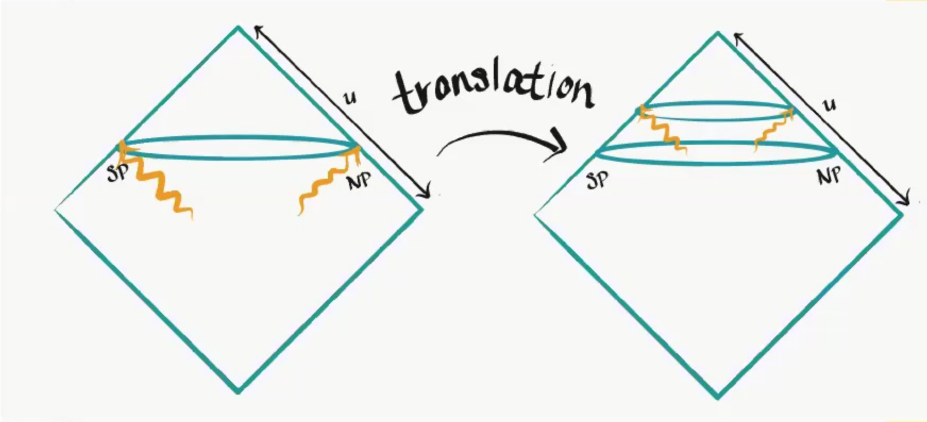
BMS symmetries

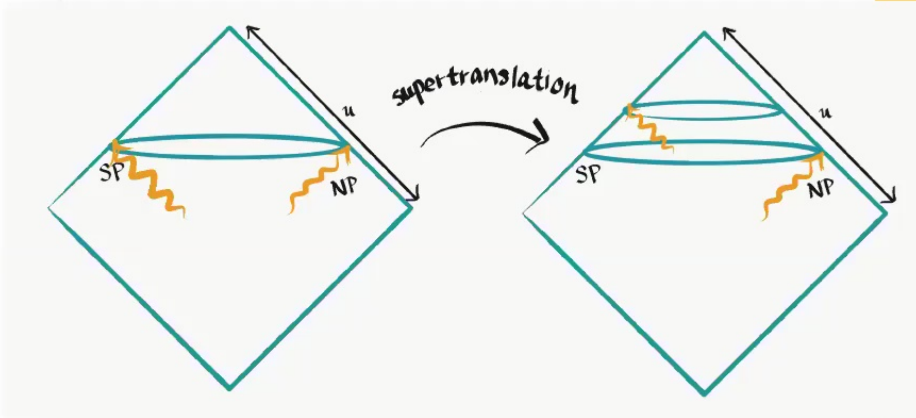
Result: Supertranslations + Lorentz transformations

$$\xi = f(z, \bar{z})\partial_u + \cdots$$

- Independent translation symmetry at every angle on the celestial sphere
- Desynchronization of events at different angles on the celestial sphere
- Infinite-dimensional symmetry group

  ⇒ *Infinitely many constraints?*





[Bondi, van der Burg, Metzner (1962); Sachs (1962)]

Symmetries of the $\mathcal{S}$-matrix?

$\mathcal{S}$-matrix:

$$|\Psi_{\rm out}\rangle = \mathcal{S}|\Psi_{\rm in}\rangle$$

Symmetry of $\mathcal{S}$-matrix:

$$|\Psi_{\rm out/in}\rangle \to |\Psi'_{\rm out/in}\rangle = U^{\pm}|\Psi_{\rm out/in}\rangle$$

$$|\Psi'_{\rm out}\rangle = \mathcal{S}|\Psi'_{\rm in}\rangle$$

Puzzle:

- Analysis at past null infinity:
  - *Additional* infinite-dimensional enhancement of translations
  - Symmetry group $\stackrel{?}{=}$ (Future ST) × (Past ST)
- Scattering problem is ill-defined?
  - Example: $U^- = 1$, $U^+ \neq 1$ (arbitrary)

Resolution:

- *Intuition:* only suitably paired generators on $\mathcal{I}^+$ and $\mathcal{I}^-$ generate symmetries of the $\mathcal{S}$-matrix
- Pairing can be determined from soft theorems
- Soft theorems are statements of invariance under symmetry: $\langle\Psi_{\rm out}|Q^+\mathcal{S} - \mathcal{S}Q^-|\Psi_{\rm in}\rangle = 0$

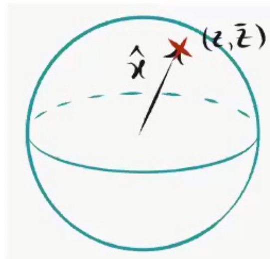
[Strominger, hep-th/1312.2229; He, Lysov, Mitra, Strominger, hep-th/1401.7026]

Soft theorems imply symmetries

- ***Can always interpret soft theorems as statements of invariance of the $\mathcal{S}$-matrix under an infinite-dimensional symmetry***

Weinberg's Soft Graviton Theorem

$$\lim_{\omega\to 0}\omega\langle \text{out}|a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} S_k(\hat{x})\langle \text{out}|\mathcal{S}|\text{in}\rangle, \qquad S_k(z,\bar{z}) = \frac{\kappa}{2}\frac{\varepsilon^{+}_{\mu\nu}p^{\mu}_{k}p^{\nu}_{k}}{\hat{q}\cdot p_k}$$



$q^{\mu} = \omega\hat{q}^{\mu} = \omega(1,\hat{x})$

- Regard soft factor S_k as eigenvalue of single particle state under operator $\mathrm{Q_H}$

$$S_k(z,\bar{z})|p_k\rangle = Q_H(z,\bar{z})|p_k\rangle = \delta_{(z,\bar{z})}|p_k\rangle$$

- RHS gives transformation of single particle states under $\delta_{(z,\bar{z})}$

$$\sum_{k=1}^{n} S_k\langle \text{out}|\mathcal{S}|\text{in}\rangle = \langle \text{out}|[Q_H,\mathcal{S}]|\text{in}\rangle$$

- Soft theorem implies that the $\mathcal{S}$-matrix is invariant under the transformation $\delta_{(z,\bar{z})}$ of single particle states provided that *a soft particle is added.*

[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]

Soft theorems imply symmetries (continued)

Weinberg's Soft Graviton Theorem

$$\lim_{\omega\to 0}\langle \text{out}|\omega a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} S_k \langle \text{out}|\mathcal{S}|\text{in}\rangle = \langle \text{out}|[Q_H,\mathcal{S}]|\text{in}\rangle$$

- Denote operators which adds soft particles Q_S

$$Q_S(z,\bar{z}) \sim -\lim_{\omega\to 0}\omega\left[a_+\left(\omega\hat{x}(z,\bar{z})\right) + a_-^\dagger\left(\omega\hat{x}(z,\bar{z})\right)\right]$$

- Then, the LHS can be written as

$$\lim_{\omega\to 0}\langle \text{out}|\omega a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = -\langle \text{out}|[Q_S,\mathcal{S}]|\text{in}\rangle$$

- Rearranging the soft theorem

$$\langle \text{out}|[Q,\mathcal{S}]|\text{in}\rangle = 0, \qquad Q = Q_H + Q_S$$

⇒ Obtain statement of invariance under symmetry generated by Q

- ***Can always interpret soft theorems as statements of invariance of the S-matrix under an infinite-dimensional symmetry***

[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]

Supertranslations & Weinberg's soft theorem

Construction of local charges

- Parametrize symmetry transformations by functions $f(z,\bar{z})$ rather than points $(z,\bar{z})$
- Determine differential operator that localizes the soft factor for massless $p_k^\mu = \omega_k(1, \hat{x}(z_k, \bar{z}_k))$

$$\mathcal{D}\sum_{k=1}^{n} S_k(z,\bar{z}) = \sum_{k=1}^{n} \omega_k \delta^{(2)}(z - z_k)$$

$$S_k(z,\bar{z}) = \frac{\kappa}{2}\frac{\varepsilon^+_{\mu\nu}p_k^\mu p_k^\nu}{q\cdot p_k}$$

$$\mathcal{D} \sim -\frac{2}{\kappa}D^z D^z$$

- Obtain charges with *local* action (*i.e.* action only depends on f at point $(z_k, \bar{z}_k)$)

$$\delta_f|p_k\rangle = Q_H[f]|p_k\rangle = \int d^2z\gamma_{z\bar{z}} f\mathcal{D}S_k|p_k\rangle$$

[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]

Supertranslations & Weinberg's soft theorem

Equivalence with supertranslations

- When $f = 1$, find total energy

$$\delta_{f=1}|p_k\rangle = \omega_k|p_k\rangle$$

$\Rightarrow$ $Q[f=1]$ generates ordinary time translations

- Generic $f = f(z,\bar{z})$:

$$\delta_f|p_k\rangle = \omega_k f(z_k,\bar{z}_k)|p_k\rangle$$

$\Rightarrow$ f parametrizes amount of translation at every angle

Saddle point approximation

$$e^{ix\cdot p_k} = e^{-i\omega u - i\omega r(1-\hat{x}\cdot\hat{x}_k)} \overset{r\to\infty}{\sim} \frac{1}{i\omega r} e^{-i\omega u}\gamma^{z\bar{z}}\delta^{(2)}(z-z_k)$$

$$\phi(u,z,\bar{z}) \sim \int d\omega\; e^{i\omega u} a^\dagger(\omega,z,\bar{z}) + c.c. \qquad \delta_f\phi \sim f\partial_u\phi \sim \int d\omega\; e^{i\omega u} \underbrace{f\omega a^\dagger(\omega,z,\bar{z})}_{\text{Transformation of asymptotic state}} + c.c.$$

[He, Lysov, Mitra, Strominger, hep-th/1401.7026;
Strominger, hep-th/1703.05448]

Soft modes & infinite-dimensional symmetries

Local soft charges

- By construction, soft charge is of similar form

$$Q_S[f] \sim \int d^2z \gamma_{z\bar{z}} f \mathcal{D} \lim_{\omega \to 0} \omega \left[a(\omega, z, \bar{z}) + a^\dagger(\omega, z, \bar{z})\right]$$

- For some choices $\hat{f}$ of f,

$$\mathcal{D}\hat{f} = 0 \quad \Rightarrow \quad Q_S[\hat{f}] = 0$$

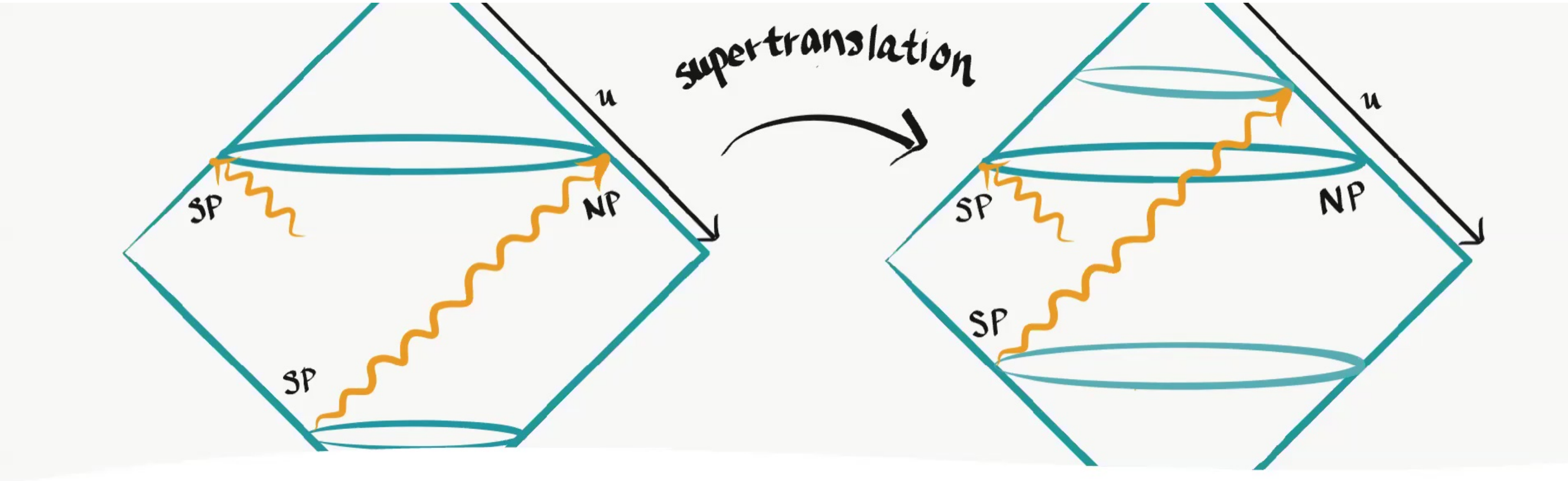
- In previous example, soft charge vanishes for four ordinary translations.

$$\mathcal{D} \sim D^z D^z \quad \Rightarrow \quad \hat{f} = Y_{\ell m}, \quad \ell = 0, 1.$$

- *Finitely many* symmetry transformations preserve particle number, *infinitely many* require addition of soft particles.

  - ***Addition of soft particles was crucial to obtain infinite-dimensional symmetry***
  - ***If restrict to charges with no soft contributions, only find four translational symmetries.***

[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]



Takeaway points:

- Scattering problem is **constrained** by **infinite-dimensional symmetries**
- **Constraints** (*i.e.* Ward identities) are the soft theorems
  - Reveals the essential role played by soft modes
- Can identify **infinite-dimensional symmetries** with asymptotic symmetries obeying an ***antipodal matching condition***
  - Resolves puzzle as to whether the scattering problem is well-posed

Soft modes & degenerate vacua

➢ *Previously found inclusion of soft charge was essential to obtain infinite-dimensional symmetry*

➢ *What is the physical significance?*

$$Q_S(z,\bar{z}) \sim -\lim_{\omega\to 0} \omega \left[a_+ \left(\omega \hat{x}(z,\bar{z})\right) + a_-^\dagger \left(\omega \hat{x}(z,\bar{z})\right) \right]$$

Action on vacuum state:

$$Q_S(z,\bar{z})|\Omega\rangle = |\Omega'\rangle$$

➢ Characterize vacua by eigenvalue under Q_S

$$Q_S(z,\bar{z})|\alpha\rangle = \alpha_{zz}|\alpha\rangle$$

Action on asymptotic states:

$$Q_S(z,\bar{z})|\mathrm{out};\alpha^{\mathrm{out}}\rangle = \alpha^{\mathrm{out}}_{zz}|\mathrm{out};\alpha^{\mathrm{out}}\rangle$$

[Kapec, Perry, Raclariu, Strominger, hep-th/1703.05448;
Choi, Kol, Akhoury, hep-th/1708.05717;
Choi, Akhoury, hep-th/1712.04551]

Soft modes & degenerate vacua

Ward identity:

$$-\langle \text{out}; \alpha^{\text{out}} | [Q_S, \mathcal{S}] | \text{in}; \alpha^{\text{in}} \rangle = \langle \text{out}; \alpha^{\text{out}} | [Q_H, \mathcal{S}] | \text{in}; \alpha^{\text{in}} \rangle$$

$$(\alpha^{\text{in}}_{zz} - \alpha^{\text{out}}_{zz}) \langle \text{out}; \alpha^{\text{out}} | \text{in}; \alpha^{\text{in}} \rangle = \sum_{k=1}^{n} S_k(z, \bar{z}) \langle \text{out}; \alpha^{\text{out}} | \text{in}; \alpha^{\text{in}} \rangle$$

Options:

$$\langle \text{out}; \alpha^{\text{out}} | \text{in}; \alpha^{\text{in}} \rangle = 0 \qquad\qquad (\alpha^{\text{in}}_{zz} - \alpha^{\text{out}}_{zz}) = \sum_{k=1}^{n} S_k(z, \bar{z})$$

- Assuming a unique vacuum state, symmetry constraint implies all S-matrix elements vanish.
- Allowing for vacuum transitions, the shift between `in' and `out' vacua is determined by the soft factor.

[Kapec, Perry, Raclariu, Strominger, hep-th/1703.05448;
Choi, Kol, Akhoury, hep-th/1708.05717;
Choi, Akhoury, hep-th/1712.04551]

BMS vacua and Fadeev-Kulish states

[Kapec, Perry, Raclariu, Strominger, hep-th/1703.05448; Choi, Kol, Akhoury, hep-th/1708.05717; Choi, Akhoury, hep-th/1712.04551]

BMS vacuum states

$$Q_S(z,\bar{z})|\alpha\rangle = \alpha_{zz}|\alpha\rangle$$

- Q_S is zero-frequency component of the radiative gravitational field

$$Q_S(z,\bar{z}) \sim -\lim_{\omega\to 0}\omega\left[a_+(\omega\hat{x}(z,\bar{z})) + a_-^\dagger(\omega\hat{x}(z,\bar{z}))\right] \sim \int du\ \underbrace{\partial_u h_{zz}(u,z,\bar{z})}_{N_{zz}}$$

- Eigenstates of Q_S are coherent states of gravitons with (classical) amplitude α

$$|\alpha\rangle \sim e^{\int \alpha_{zz}(a_- - a_+^\dagger) + c.c.}|0\rangle \sim e^{\int \alpha_{zz}h_{\bar{z}\bar{z}} + c.c.}|0\rangle$$

- Such states were previously studied by Fadeev and Kulish for the purpose of curing infrared divergences which *do* set scattering amplitudes to zero!

Takeaway:

- Symmetry constraints provide a reason for why exclusive scattering amplitudes are IR divergent but states dressed by soft gravitons are not.

Physical intuition:

- Gravitational radiation is emitted in any non-trivial classical scattering process.
- Symmetry constraint in quantum scattering process requires the soft sector to agree with classical result.

Takeaways

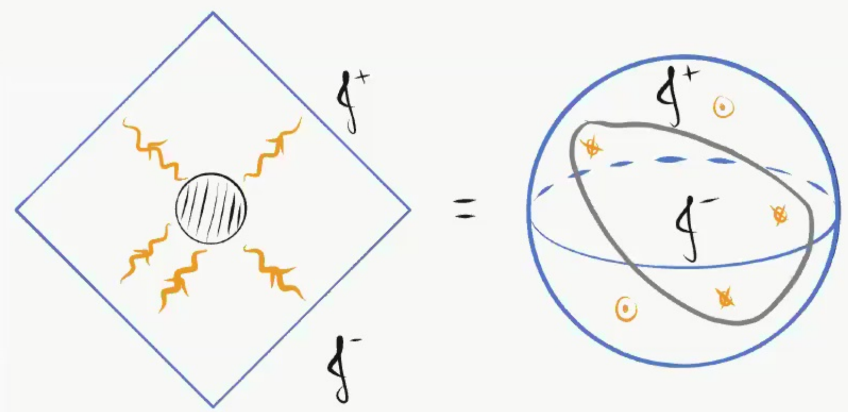
- Symmetry constraints imply the vacuum is infinitely degenerate.
- Generic scattering process induce transitions between inequivalent vacua.
- Infrared divergences arise when failing to account for vacuum transitions.

Outlook thus far...

- Resolved concerns regarding the scattering problem arising from
  - Independent asymptotic symmetries at $\mathscr{I}^+$ and $\mathscr{I}^-$
  - Infrared divergences in scattering amplitudes
- Constraints implied by asymptotic symmetries are soft theorems
  - New soft theorems [Cachazo & Strominger, hep-th/1404.4091]
  - Corrections to soft theorems [Elvang, Jones, Naculich, hep-th/1611.07534; Mitra & Laddha, 1709.03850]

Celestial Amplitudes

- Scattering amplitudes of *boost* (not translation) eigenstates
- Single particle states transform like primary operators in 2D CFT
- Subleading soft graviton theorem is Ward identity for 2D stress tensor
  - Lorentz symmetries enhanced to Virasoro
- Holography
  - Does there exist an intrinsically defined 2D theory whose correlation functions are the 4D scattering amplitudes?
  - How far can the symmetries take us?



[de Boer & Solodukhin, hep-th/0303006; Pasterski, Shao, Strominger, hep-th/1701.00049]

Construction of Celestial Amplitudes (continued)

$$|p, s\rangle = |\omega, z, \bar{z}, s\rangle, \qquad p^\mu(\omega, z, \bar{z}) = \frac{\omega}{1+z\bar{z}}\left(1+z\bar{z}, z+\bar{z}, -i(z-\bar{z}), 1-z\bar{z}\right)$$

- Under Lorentz transformations,

$$p^\mu \to \Lambda^\mu{}_\nu p^\nu, \qquad \frac{\omega}{1+z\bar{z}} \to \frac{\omega}{1+z\bar{z}}|cz+d|^2$$

Seek to trade ω for $SL(2,\mathbb{C})$-invariant label Δ

- Boost along p^μ: $p^\mu \to \lambda p^\mu$
- Generated by: $K = \omega\partial_\omega$
- K is diagonalized by the *Mellin transform*

$$\mathcal{O}_{\Delta,s}(z,\bar{z}) = \int_0^\infty d\omega\ \omega^{\Delta-1}|\omega, z, \bar{z}, s\rangle$$

Celestial Amplitudes

$$\langle\mathcal{O}_{\Delta_1,s_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_n,s_n}(z_n,\bar{z}_n)\rangle = \left(\prod_{k=1}^n \int_0^\infty d\omega_k\ \omega_k^{\Delta_k-1}\right)\mathcal{A}_{s_1\cdots s_n}(p_1,\cdots,p_n)$$

[Kapec, Mitra, Raclariu, & Strominger, hep-th/1609.00282; Cheung, de la Fuente & Sundrum, hep-th/1609.00732]

Symmetry Constraints on Celestial Amplitudes

➢ *What is the fate of the asymptotic symmetry constraints (i.e. soft theorems) in this basis?*

1. **Soft charges**
   - *Conformally soft theorems* characterize poles in conformal dimension

2. **Hard charges**
   - Subleading soft gravitons generate conformal transformations

     ⇒ act *within* a conformal family (labelled by Δ and s)
   - Leading/subsubleading theorems shift conformal dimensions

     ⇒ relate *different* conformal families

Conformally soft theorems

Conventional soft theorems

- Statements about the universality of coefficients $S_{(i)}$ in an expansion in energy of an external graviton

$$\mathcal{A}_{n+1}(\omega, z, \bar{z}) = \left(\frac{1}{\omega} S_{(0)}(z, \bar{z}) + S_{(1)}(z, \bar{z}) + \omega S_{(2)}(z, \bar{z})\right) \mathcal{A}_n + \cdots$$

Property of Mellin transforms

- Coefficients of an asymptotic expansion map to residues of poles in Mellin space

$$f(\omega) = \omega^{-1} a_0 + a_1 + \omega a_2 + \cdots , \qquad a_n = \lim_{\Delta \to -n+1} (\Delta + n - 1) \int_0^\infty d\omega\, \omega^{\Delta - 1} f(\omega).$$

Soft operators

Leading: $\lim_{\omega \to 0} \omega a^\dagger(\omega, z, \bar{z})|0\rangle = \lim_{\Delta \to 1} (\Delta - 1) \mathcal{O}_\Delta(z, \bar{z})$

Subleading: $\lim_{\Delta \to 0} \Delta \mathcal{O}_\Delta(z, \bar{z})$ Subsubleading: $\lim_{\Delta \to -1} (\Delta + 1) \mathcal{O}_\Delta(z, \bar{z})$

- ***Asymptotic symmetries constrain residues of poles in conformal weights***

[Cheung, de la Fuente & Sundrum, 1609.00732; Fan, Fotopoulos & Taylor, 1903.01676; MP, Raclariu & Strominger, 1904.10831; Nandan, Schreiber, Volovich & Zlotnikov, 1904.10940; Adamo, Mason, & Sharma, 1905.09224; Puhm, 1905.09799]

Translations on the celestial sphere

Translations in momentum space

$$\delta|\omega, z, \bar{z}\rangle = \omega|\omega, z, \bar{z}\rangle$$

Translations in boost eigenstate space

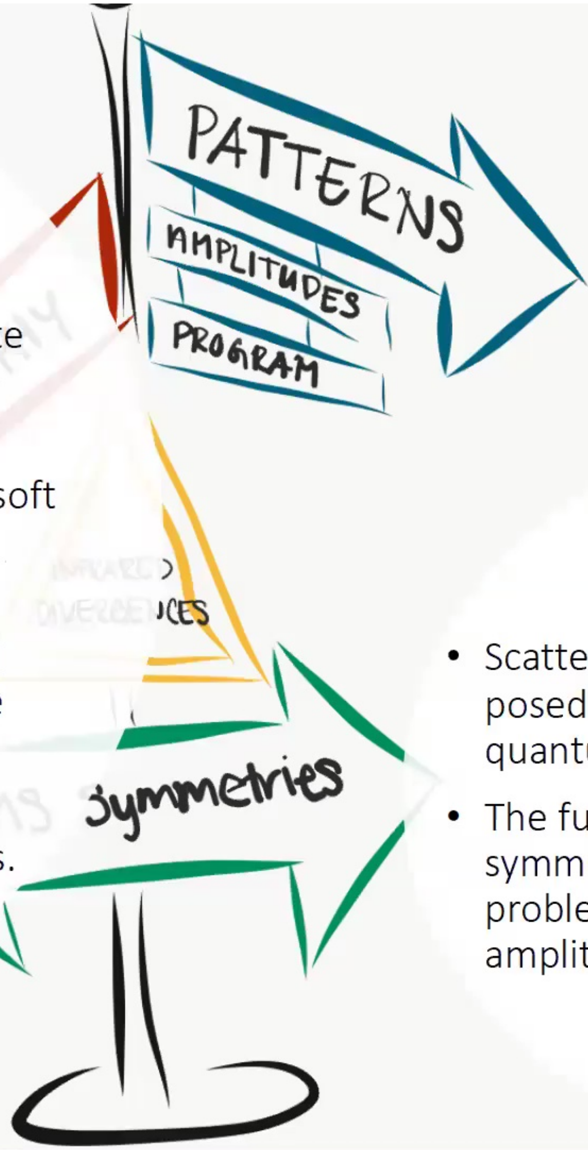
$$\delta\mathcal{O}_\Delta(z, \bar{z}) = \int_0^\infty d\omega\ \omega^{\Delta-1}\delta|\omega, z, \bar{z}\rangle = \int_0^\infty d\omega\ \omega^{\Delta-1}\omega|\omega, z, \bar{z}\rangle = \mathcal{O}_{\Delta+1}(z, \bar{z})$$

➢ *Momentum conservation of celestial amplitudes is captured by shifts in dimension*

[Donnay, Puhm, & Strominger, hep-th/1810.05219; Stieberger & Taylor, hep-th/1812.01080]

Summary

- Asymptotic symmetries imply an infinite number of constraints on scattering problem.
- The constraints can be identified with soft theorems in the standard momentum basis.
- Infrared divergences arise in scattering amplitudes between states that violate the symmetry constraints.
- Asymptotic symmetries constrain OPE (collinear limits) of celestial amplitudes.



Outlook

- Scattering problem remains a well-posed and informative question in quantum gravity.
- The full extent to which asymptotic symmetries constrain the scattering problem (recast in terms of celestial amplitudes) is yet to be determined.

OPE coefficients from symmetry

Constraints from translations

$$\delta\mathcal{O}_\Delta = \mathcal{O}_{\Delta+1} \qquad \mathcal{O}_{\Delta_1}(z_1)\mathcal{O}_{\Delta_2}(z_2) \sim C_{\Delta_1,\Delta_2}(z_{12})\mathcal{O}_{\Delta_1+\Delta_2+n}(z_2)$$

⇒ Recursion relation:

$$C_{\Delta_1+1,\Delta_2} + C_{\Delta_1,\Delta_2+1} = C_{\Delta_1,\Delta_2}$$

+ additional recursion relation from subsubleading soft graviton symmetry

⇒ Graviton-graviton OPE coefficient

$$G^+_{\Delta_1}(z_1,\bar{z}_1)G^+_{\Delta_2}(z_1,\bar{z}_2) \sim B(\Delta_1-1,\Delta_2-1)\frac{\bar{z}_{12}}{z_{12}}G^+_{\Delta_1+\Delta_2}(z_2,\bar{z}_2)$$

➢ *Relations among celestial amplitudes without any soft insertions!*

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Poles in dimension!

[MP, Raclariu, Strominger & Yuan, hep-th 1910.07424]